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On the semi-classical origin of the electron anomalous magnetic moment

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For modern theoretical physics there is tendency to review methods of describing quantum-mechanical systems in terms of relationship with classical analogues. We attempted to calculate the anomalous magnetic moment from a semiclassical view (with consideration for Zitterbewegung) on the basis of electromagnetic fields generated by the electron in the immediate vicinity of the charge point. Derived value of anomalous magnetic moment is in full agreement with the Schwinger's calculations.

Keywords: *radiation, anomalous magnetic moment, Zitterbewegung, semi-classical theory.*

1 Introduction

In 1941 W. Pauli [1] showed that it's possible to add a relative invariant an additional to Bohr magneton $\mu_0 = e\hbar/2m_0c$ magnetic moment part μ_a describing interaction with external electromagnetic field. At first it seemed to be purely abstract theoretical construct. In 1947 G. Breit [2], analyzing experimental data for measuring the hyperfine structure of the hydrogen atom spectrum, suggested that the magnetic moment of the electron has the intrinsic anomalous part:

$$\mu = \mu_0 + \mu_a.$$

The first theoretical calculations of the anomalous magnetic moment of the electron were provided by J. Schwinger [3] in 1948. These calculations were excellent application of the developing quantum electrodynamics. The result obtained by Schwinger:

$$\mu_a = \mu_0(1 + \frac{\alpha}{2\pi}),$$

where $\alpha = e^2/\hbar c$ – fine structure constant, which was in good agreement with experiment [4]. Further calculations and measurements, which took into account higher approximations of the fine structure constant [5], depending on the presence of a strong external fields and the number of electron energy level [6] resulted only with a refinement of the obtained expressions for the Schwinger's anomalous magnetic moment (see review works [7, 8] and others).

The discovery of the anomalous magnetic moment was a milestone in the construction of modern quantum field theory. Attempts of better visibility of the semi-classical presentation goes back to the Welton and Koba works [9,10] (see also [8]). Of course, the Welton-Koba method still represents only a qualitative interest for the interpretation of the anomalous magnetic moment. And it can not claim to be serious quantitative results.

In our work to obtain the theoretical value of the anomalous magnetic moment used exact methods of purely relativistic classical electrodynamics [11]. Only at the last stage of calculations the electron's self-field is determined by semi-classically introducing Compton wavelength. This allows to define more precisely the effective electromagnetic mass of an electron, and with it the anomalous magnetic moment. But the classical goal is not important in itself. The quantum mechanical picture when expressed in terms of invariance principles will show the relationship between the classical variables. This is our main motivation for the classical analysis of spinning particles: to finally obtain a thorough quantum scheme [12].

2 Electomagnetic self-action of an electron in classical electrodynamics

In the classical field theory interaction of a charged particle with an external and its own electromagnetic field can be formulated using a differential conservation law of momentum density [13]:

$$D_\nu \mathcal{P}^{\mu\nu}_e + D_\nu \mathcal{P}^{\mu\nu}_{ext} + D_\nu \bar{\mathcal{P}}^{\mu\nu} + D_\nu \tilde{\bar{\mathcal{P}}}^{\mu\nu} + D_\nu \tilde{\mathcal{P}}^{\mu\nu} = 0, \quad (1)$$

here

$$\mathcal{P}^{\mu\nu}_e = M_0 \int v^\mu(\tau) v^\nu(\tau) \delta(R^\rho - r^\rho_e(\tau)) d\tau$$

– momentum density tensor of particle with effective mass M_0,

$$\mathcal{P}^{\mu\nu}_{ext} = -\frac{1}{4\pi c}(H^{\mu\rho} H_\rho{}^\nu + \frac{1}{4} g^{\mu\nu} H_{\alpha\beta} H^{\alpha\beta})$$

– momentum density tensor of external field with stress $H^{\mu\nu}_{ext}$. The last three members in the eq. 1 take into account the field produced by the charged particle. It is composed of the convective field $\tilde{H} \sim \frac{1}{\tilde{r}^2}$ and the radiation field $\bar{H} \sim \frac{1}{\bar{r}}$. Therefore, the momentum density tensor of the self-field of the charge consists of three parts

$$\mathcal{P}^{\mu\nu}_{self} = \bar{\mathcal{P}}^{\mu\nu} + \tilde{\bar{\mathcal{P}}}^{\mu\nu} + \tilde{\mathcal{P}}^{\mu\nu},$$

where

$$\bar{\mathcal{P}}^{\mu\nu} = \frac{1}{4\pi}\frac{e^2c^3}{(\tilde{r}_\rho v^\rho)^4}(\frac{1}{2}g^{\mu\nu} - \frac{\tilde{r}^\mu v^\nu + \tilde{r}^\nu v^\mu}{\tilde{r}_\rho v^\rho} - c^2\frac{\tilde{r}^\mu\tilde{r}^\nu}{(\tilde{r}_\rho v^\rho)^2}) \sim \frac{1}{\tilde{r}^4}$$

– corresponds to the convective field,

$$\tilde{\mathcal{P}}^{\mu\nu} = -\frac{1}{4\pi c}\frac{e^2}{(\tilde{r}_\rho v^\rho)^2}(c^2\frac{(\tilde{r}_\rho w^\rho)^2}{(\tilde{r}_\rho v^\rho)^4} - \frac{w_\rho w^\rho}{(\tilde{r}_\rho v^\rho)^2})\tilde{r}^\mu\tilde{r}^\nu \sim \frac{1}{\tilde{r}^2}$$

– describes radiation field,

$$\tilde{\bar{\mathcal{P}}}^{\mu\nu} = -\frac{1}{4\pi}\frac{e^2c^2}{(\tilde{r}_\rho v^\rho)^3}(\frac{\tilde{r}^\mu w^\nu - \tilde{r}^\nu w^\mu}{\tilde{r}_\rho v^\rho} - \frac{\tilde{r}^\mu v^\nu + \tilde{r}^\nu v^\mu}{(\tilde{r}_\rho v^\rho)^2}\tilde{r}_\rho w^\rho - \frac{2c^2\tilde{r}^\mu\tilde{r}^\nu\tilde{r}_\rho w^\rho}{(\tilde{r}_\rho c^\rho)^3}) \sim \frac{1}{\tilde{r}^3}$$

– corresponds to part of mutual interference. In $\mathcal{P}^{\mu\nu}_{self}$ field are defined in the retarded time and therefore for them $\tilde{D}^\mu = \tilde{\partial}^\mu + \frac{\tilde{r}^\mu}{\tilde{r}_\rho v^\rho}\frac{d}{d\tau}$. To obtain the equations of motion of a charged particle the eq. 1 should be integrated throughout the four-dimensional space around the world line of the segment of length $cd\tau$. In the first two members integration is carried out using a four-δ-function and a second pair of Maxwell equations in the second term. The result of these procedure is the usual Lorentz force. The remaining three members are integrated with the theorem of Gauss-Ostrogradsky in the volume enclosed between the two light cones and the time-like hyper band (see fig.). It is called Bha-bha tube with elements of the hypersurface:

$$d\sigma = \varepsilon^2((1 + \frac{\varepsilon}{c^2}w_\rho e^\rho)e_\nu + \frac{\varepsilon}{c^3}w_\rho e^\rho v_\nu)d\Omega c d\tau,$$

where $d\Omega$ - element of solid angle in the rest frame of the particle, e_ν – Rorlich's vector and

$$e_\nu e^\nu = 1,\ e^\nu = -c\frac{\tilde{r}^\nu}{\tilde{r}_\rho v^\rho} - \frac{1}{c}v^\nu,\ v_\nu e^\nu = 0.$$

Invariant quantity $\varepsilon = \tilde{r}_\rho e^\rho$ plays the role of a spherical radius.

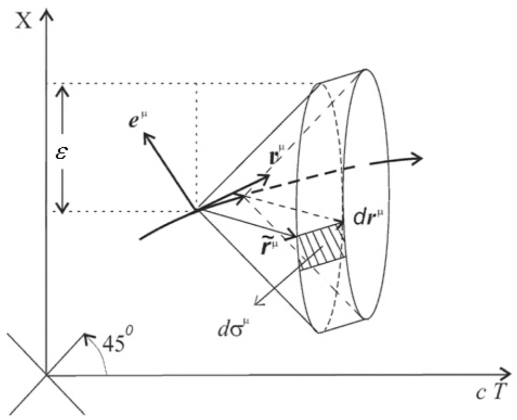

Figure 1: Bha-Bha tube.

As a result of integration over the four-dimensional volume, taking into account the delay of the radiation we get

$$M_0w^\mu(\tau) = \frac{e}{c}H^{\mu\nu}_{ext}v_\nu(\tau) - \frac{2e^2}{3c^5}(w_\alpha w^\alpha v^\mu + \frac{3c^3}{4\varepsilon}w^\mu)_{\tau^*\to\tau}.$$

The last term on the right side in the radiation friction force is a divergent expression, since $\varepsilon = c(\tau^* - \tau) \to 0$ then, to be excluded from the definition of force, let taking into account the delay of the radiation

$$w^\mu(\tau^*) = w^\mu(\tau) - \kappa\dot{w}^\mu(\tau)\Delta\tau,$$

where κ – constant factor, which can be determined from the condition of the space-like radiation friction force. Putting $\kappa = 4/3$, we obtain

$$(M_0 + \frac{e^2}{2\varepsilon c^2})w^\mu(\tau) = \frac{e}{c}H^{\mu\nu}_{ext}v_\nu(\tau) + \frac{2e^2}{3c^5}(w^\mu - \frac{1}{c^2}w_\nu w^\nu v^\mu).$$

Thus on the right side we get the Lorentz force and the standard expression of the radiative friction force, on the left - the definition of four-dimensional force vector with a massive multiplier that allows for self-action of the electromagnetic field. Interesting that the delay parameter of the radiation

$$\frac{4}{3}\Delta\tau = \frac{4\varepsilon}{3c} = \frac{2e^2}{\tilde{m}_0c^3} = \tau_0$$

coincides with the quantum of time that proposed by

P. Caldirola [14], if under $\tilde{m}_0$ is understood electromagnetic mass of an electron:

$$\tilde{m}_0 = \frac{e^2}{2\varepsilon c^2}.$$

To obtain the standard expression for the Lorentz force, taking into account the radiative friction force should take

$$M_0 + \tilde{m}_0 = m_0.$$

3 Semi-classical self-action and the anomalous magnetic moment

Thus, we have a well-known equation

$$m_0 w^\mu = \frac{e}{c} H^{\mu\nu} v_\nu + \tilde{F}^\mu$$

which is satisfied if in the original equation the effective mass equals to

$$M_0 = m_0 (1 - \frac{e^2}{2\varepsilon m_0 c^2}).$$

In fact, that it is mass of "naked" particle in the absence of the self-interaction with the electromagnetic field. It is obvious that the magnetic moment of an electron with an anomalous addition is determined by this mass:

$$\mu = \frac{e\hbar}{2M_0 c} \cong \mu_0 (1 + \frac{e^2}{2\varepsilon m_0 c^2}).$$

The idea, that the anomalous magnetic moment of the electron is directly related to the effective mass of the particle, found its confirmation in quantum electrodynamics [15]. This idea has been actively developed in classical electrodynamics by MacGregor [16], but his calculations have essentially approximate nature.

Until now we have been using only the methods of relativistic electrodynamics. Our further calculations will be associated with the refinement of the concept of radius of the spherical electromagnetic field around the electron. Clearly, this is the minimum distance at which the electromagnetic field, leaving the electron leaves it "naked". Compton wavelength of an electron is known to be an indicator of the distance at which quantum effects become important, considering

$$\varepsilon = \frac{1}{2}\lambda_C = \frac{\pi\hbar}{m_0 c}$$

we get

$$M_0 = m_0 (1 - \frac{\alpha}{2\pi}),$$

then according to the definition

$$\mu = \mu_0 (1 + \frac{\alpha}{2\pi}).$$

It is Schwinger's result for the anomalous magnetic moment. It's very interesting that the value of ε can be associated with the Schrodinger's Zitterbewegung, considering

$$\hbar\omega_{Zb} = 2m_o c^2.$$

Then we get the same value ε, if we take

$$\varepsilon = cT_{Zb} = \frac{2\pi c}{\omega_{Zb}} = \frac{\pi\hbar}{m_0 c}.$$

4 Conclusion

Feature of our work is that the derivating of the anomalous magnetic moment of the electron, we started from a purely classical theory - exact relativistic radiation. Only at the last stage of calculations we attracted semiclassical postulates to describe the interaction of an electron with a quantized electromagnetic field.

Our result confirms the correctness of our assumptions and the validity of the principle of correspondence of classical and quantum field theory. It opens up new possibilities in the study of the effects of phenomena such as electromagnetic mass of an electron, Zitterbewegung and quantization of time and etc.

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О ПОЛУКЛАССИЧЕСКОМ ПРОИСХОЖДЕНИИ АНОМАЛЬНОГО МАГНИТНОГО МОМЕНТА ЭЛЕКТРОНА

В данной работе предпринята попытка рассчитать аномальный магнитный момент электрона с полуклассической точки зрения с учётом явления Zitterbewegung и на основании электромагнитных полей, создаваемых электроном в непосредственной близости от заряда. Полученное значение аномального магнитного момента находится в полном согласии с первоначальными расчетами Швингера.

Ключевые слова: *конвективное электромагнитное поле, аномальный магнитный момент, Zitterbewegung, полуклассическая теория.*

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