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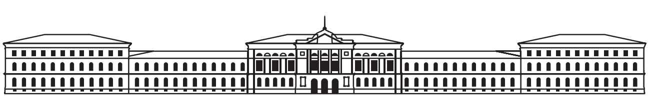

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УПРАВЛЕНИЕ ДИНАМИЧЕСКИМИ СИСТЕМАМИ

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PORTFOLIO OPTIMIZATION IN THE FINANCIAL MARKET WITH SERIALLY DEPENDENT RETURNS UNDER CONSTRAINTS

In this work we consider the optimal portfolio selection problem under hard constraints on trading volume amounts. We assume that the risky asset returns are serially dependent processes with finite conditional moments. The problem of portfolio optimization is stated as a dynamic problem of tracking a financial benchmark. We propose to use the model predictive control (MPC) methodology in order to solve the problem. We also present the numerical modeling results that give evidence of capacity and effectiveness of proposed approach.

Keywords: *investment portfolio*, *serially dependent returns*, *model predictive control.*

The investment portfolio (IP) management is an area of both theoretical interest and practical importance. The basis of the current classical theory of optimal portfolio allocation problem is the single-period "mean variance" approach suggested by Markowitz [1] and the Merton dynamic IP model [2] in continuous time. At present, there exists a variety of models and approaches to the solution of the IP optimization problem, but most of them are the complications and extensions of the Markowitz and Merton approaches to various versions of stochastic models of the prices of risky and risk-free securities and utility functions. The review of the main trends existing in the modern theory of dynamic control of investments is given in [3].

The most of the results presented in these works are limited to the cases without explicit constraints on the trading volume amounts. However it's well-known that realistic investment models must include ones.

In static framework one can take into consideration portfolio constraints that lead to the linear or quadratic programming tasks. Taking into account the portfolio constraints in dynamic models we come to the impractical for actual numerical implementation models, due to the "curse of dimensionality".

We propose to use the model predictive control (also known as receding horizon control) methodology in order to solve the problem. MPC proved to be an appropriate and effective technique to solve the dynamic control problems subject to input and state/output constraints. The main concept of MPC is to solve an open-loop constrained optimization problem with receding horizon at each time instant and implement only the initial optimizing control action of the solution [4].

MPC have begun to be used with success in financial applications such as portfolio optimization and dynamic hedging. Some of the recent works on this subject can be found, for instance, in [5]-[8]. In all these papers authors assume the hypothesis of serially independent returns and consider the explicit form of the model describing the price process of the risky assets (e.g. geometric Brownian motion, e.t.c.). The problem of MPC for discrete-time systems with dependent random parameters is considered in [9]. In that paper the evolution of parameters' vector is described by the linear stochastic difference equation. The results are applied to the IP optimization.

The main novelty of this paper is that the risky asset returns are assumed to be a sequence of stochastic serially dependent variables for which the first and the second conditional distribution moments are known only. The optimal open-loop feedback control strategy is derived subject to hard constraints on the trading volume amounts. Predictive strategies computation includes the decision of the sequence of quadratic programming tasks. This approach thus leads to computationally tractable optimization problems.

We present the numerical modeling results, based on stocks, traded on the Russian Stock Exchanges MICEX, that give evidence of capacity and effectiveness of proposed approach. Numerical examples based on real market have shown that our approach is a theoretically sound and computationally efficient method.

1. Portfolio optimization problem

Consider the investment portfolio consisting on the n risky assets and one risk-free asset (e.g. a bank account or a government bond). Let $u_i(k)$ ($i = 0, 1,2, ..., n$) denote the amount of money invested in the ith asset at time k; $u_0(k)\geq 0$ is the amount invested in a risk-free asset. Then the wealth process $V(k)$ satisfies:

$$V(k) = \sum_{i=1}^{n} u_i(k) + u_0(k). \tag{1}$$

Notice, that if $u_i(k)<0$ ($i = 1, 2, ..., n$), then we use short position with the amount of shorting $|u_i(k)|$.

Let $\eta_i(k+1)$ denote the return of the ith risky asset per period $[k, k+1]$. It is a stochastic unobservable at time k value defined as

$$\eta_i(k+1) = \frac{P_i(k+1) - P_i(k)}{P_i(k)},$$

where $P_i(k)$ denotes the market value of the ith risky asset at time k.

By considering the self-finance strategies (self-financing means that we do not allow wealth to be added to or extracted from the portfolio), the wealth process $V(\cdot)$ at the time $k+1$ is given by (see [9]):

$$V(k+1) = \sum_{i=1}^{n} [1 + \eta_i(k+1)] u_i(k) + [1 + r] u_0(k), \tag{2}$$

where r is a risk-free interest rate of the bond.

Using (1) we can rewrite (2) as follows:

$$V(k+1) = [1 + r] V(k) + \sum_{i=1}^{n} [\eta_i(k+1) - r] u_i(k), \tag{3}$$

here $u_0(k) = V(k) - \sum_{i=1}^{n} u_i(k)$ is the amount invested in the bond.

We impose the following constraints on the control actions:

$$u_i^{\min}(k) \le u_i(k) \le u_i^{\max}(k), (i = \overline{1,n}), \tag{4}$$

$$u_0^{\min}(k) \le V(k) - \sum_{i=1}^{n} u_i(k) \le u_0^{\max}(k). \tag{5}$$

If $u_i^{\min}(k)<0$ ($i=1,2,\dots,n$), so we suppose that the amounts of the short-sale are restricted by $|u_i^{\min}(k)|$; if the short-selling is prohibited then $u_i^{\min}(k)\ge 0$ ($i=1,2,\dots,n$). The amounts of long-sale are restricted by $u_i^{\max}(k)$ ($i=1,2,\dots,n$); $u_0^{\max}(k)\ge 0$ defines the amount we can invest in the risk-free asset; $u_0^{\min}(k) \le 0$ determines the maximum volume of a loan over the risk-free asset. Note, that values $u_i^{\min}(k)$ ($i=0,1,\dots,n$), $u_i^{\max}(k)$ ($i=0,1,\dots,n$) are often depend on common wealth of portfolio in practice. So that we can write $u_i^{\min}(k) = \gamma_i' V(k)$, $u_i^{\max}(k) = \gamma_i'' V(k)$, where γ_i', γ_i'' are constant parameters.

We consider a financial market on a complete filtered probability space $(\Omega, \mathfrak{F}, \{\mathfrak{F}_k\}, \mathbf{P})$, where $\{\mathfrak{F}_k\}$ denotes the complete filtration with σ-field $\mathfrak{F}_k$ generated by the $\{\eta(s)\colon s=0, 1, 2,\dots,k\}$.

Let assume that the risky asset returns $\eta(k)=[\eta_1(k)\ \eta_2(k)\ \dots\ \eta_n(k)]^T$ are serially dependent processes with finite conditional moments:

$$\begin{gathered} E\{\eta(k+i)/\mathfrak{F}_k\} = \overline{\eta}(k+i), \\ E\left\{\eta(k+i)\eta^T(k+j)/\mathfrak{F}_k\right\} = \Theta_{ij}(k), \\ (k = 0,1,2,\dots),(i,j = 1,2,\dots,m). \end{gathered} \tag{6}$$

Our objective is to control the investment portfolio, via dynamics asset allocation among the n stocks and the bond, as closely as possible tracking the deterministic benchmark

$$V^0(k+1) = [1+\mu_0]V^0(k)\,, \tag{7}$$

where μ_0 is a given parameter representing the growth factor, the initial state is $V^0(0)=V(0)$.

We use the MPC methodology in order to define the optimal control portfolio strategy. Our objective is:

$$\begin{aligned} J(k+m/k) = E\Bigg\{ & \sum_{i=1}^{m} \left[V(k+i) - V^0(k+i) \right]^2 - \rho(k,i)[V(k+i) - V^0(k+i)] + \\ & + u^T(k+i-1/k)R(k,i-1)u(k+i-1/k)/V(k),V_0(k),\mathfrak{F}_k \Bigg\}, \end{aligned} \tag{8}$$

where m is the prediction horizon, $u(k+i/k)=[u_1(k+i/k), \dots, u_n(k+i/k)]^T$ is the predictive control vector, $R(k,i)>0$ is a positive symmetric matrix of control cost coefficients, $\rho(k,i)>0$ is positive weight coefficient. The performance criterion (8) is composed by a linear combination of a quadratic part, representing the quadratic error between the portfolio value and a benchmark, and a linear part, representing an expected error between the portfolio value and a benchmark which is desired to overcome.

2. Model predictive control strategies design

Theorem. Let the wealth dynamics is given by (3) under constraints (4), (5). Then the MPC policy with receding horizon m, such that it minimizes the objective (8), for each instant k is defined by the equation:

$$u(k)=[I_n \quad 0_n \quad \cdots \quad 0_n]U(k), \tag{9}$$

where I_n is n-dimensional identity matrix; 0_n is n-dimensional zero matrix; $U(k)=[u^T(k/k), \ldots, u^T(k+m-1/k)]^T$ is the set of predictive controls defined from the solving of quadratic programming problem with criterion

$$Y(k+m/k)=\left[2x^T(k)G(k)-F(k)\right]U(k)+U^{\mathrm{T}}(k)H(k)U(k) \tag{10}$$

under constraints

$$U_{\min}(k)\le \overline{S}(k)U(k)\le U_{\max}(k), \tag{11}$$

where
$$x(k)=\left[V(k) \quad V^0(k)\right]^T,$$

$$U_{\min}(k)=[u_{\min}^T(k),0_{n+1\times 1},\ldots,0_{n+1\times 1}]^T, U_{\max}(k)=[u_{\max}^T(k),0_{n+1\times 1},\ldots,0_{n+1\times 1}]^T,$$

$$u_{\min}(k)=\left[u_1^{\min}(k), \quad \ldots \quad u_n^{\min}(k), \quad u_0^{\min}(k)-V(k)\right]^T,$$
$$u_{\max}(k)=\left[u_1^{\max}(k), \quad \ldots \quad u_n^{\max}(k), \quad u_0^{\max}(k)-V(k)\right]^T,$$

$H(k)$, $G(k)$, $F(k)$, $\overline{S}(k)$ are the block matrices of the form

$$H(k)=\left[H_{ij}(k)\right] \quad (i,j=\overline{1,m}),$$
$$G(k)=[G_1(k) \quad G_2(k) \quad \cdots \quad G_m(k)], F(k)=[F_1(k) \quad F_2(k) \quad \ldots \quad F_m(k)],$$
$$\overline{S}(k)=\operatorname{diag}(S(k),0_{n+1\times n},\ldots,0_{n+1\times n})$$

and the blocks satisfy the following recursive equations

$$\begin{aligned}H_{tt}(k)&=R(k,t-1)+\overline{B}^T(k+t)Q(m-t)\overline{B}(k+t)+\\&+E\left\{\tilde{B}^T(k+t)Q(m-t)\tilde{B}(k+t)/\mathfrak{F}_k\right\},\end{aligned} \tag{12}$$

$$\begin{aligned}H_{tf}(k)&=\overline{B}^T(k+t)(A^T)^{f-t}Q(m-f)\overline{B}(k+f)+\\&+E\left\{\tilde{B}^T(k+t)(A^T)^{f-t}Q(m-f)\tilde{B}(k+f)/\mathfrak{F}_k\right\},\ t<f,\end{aligned} \tag{13}$$

$$H_{tf}(k)=H_{ft}^{\ T}(k), t>f, \tag{14}$$

$$G_t(k)=(A^t)^TQ(m-t)\overline{B}(k+t), \tag{15}$$

$$F_t(k)=\sum_{j=t}^{m}R_2(k,j)A^{j-t}\overline{B}(k+t), \tag{16}$$

$$SS(k)=\begin{bmatrix}1 & 0 & \ldots & 0\\ 0 & 1 & \ldots & 0\\ \ldots & \ldots & \ldots & \ldots\\ 0 & 0 & \ldots & 1\\ -1 & -1 & \ldots & -1\end{bmatrix}, \tag{17}$$

where

$$A=\operatorname{diag}(1+r,1+\mu_0), B[\eta(k),k]=\begin{bmatrix}\eta_1(k)-r & \eta_2(k)-r & \ldots & \eta_n(k)-r\\ 0 & 0 & \ldots & 0\end{bmatrix}, \tag{18}$$

$$\overline{B}(k+t) = E\{B[\eta(k+t), k+t]/\mathfrak{F}_k\}, \widetilde{B}(k+t) = B[\eta(k+t), k+t] - \overline{B}(k+t), \quad (19)$$

$$Q(t) = A^T Q(t-1)A + R_1, (t = \overline{1,m}), Q(0) = R_1, \quad (20)$$

$$R_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, R_2(k,i) = \rho(k,i)[1 \quad -1]. \quad (21)$$

Remark. In virtue of the linear dependence of matrix on its argument, the conditional mathematical expectations in expressions above are easily calculated.

Proof. The system equations (3), (7) can be written in the matrix form

$$x(k+1) = Ax(k) + B[\eta(k+1), k+1]u(k), \quad (22)$$

where x, A, B, u are defined by

$$x(k) = \begin{bmatrix} V(k) & V^0(k) \end{bmatrix}^T,$$

$$A = \mathrm{diag}(1+r, 1+\mu_0),$$

$$B[\eta(k), k] = \begin{bmatrix} \eta_1(k) - r & \eta_2(k) - r & \dots & \eta_n(k) - r \\ 0 & 0 & \dots & 0 \end{bmatrix},$$

$$u(k) = [u_1(k) \quad u_2(k) \quad \dots \quad u_n(k)]^T.$$

Criterion (8) can be transformed into

$$J(k+m/k) = E\left\{\sum_{i=1}^{m} x^T(k+i)R_1 x(k+i) - R_2(k,i)x(k+i)\right.$$
$$\left. + u^T(k+i-1/k)R(k,i-1)u(k+i-1/k)/x(k), \mathfrak{F}_k\right\}, \quad (23)$$

with

$$x(k+i) = A^i x(k) + A^{i-1}B[\eta(k+1), k+1]u(k) + A^{i-2}B[\eta(k+2), k+2]u(k+1) + \dots +$$
$$+ B[\eta(k+i), k+i]u(k+i-1), (i = \overline{1,m})$$

and matrices R_1, $R_2(k,i)$ of the form (21).

We can re-express (23) as follows

$$J(k+m/k) = E\{X^T(k+1)\Delta_1 X(k+1) - \Delta_2(k+1)X(k+1) + U^T(k)\Delta(k)U(k)/x(k), \mathfrak{F}_k\}, \quad (24)$$

with
$$X(k+1) = \Psi x(k) + \Phi[\Xi(k+1), k+1]U(k), \quad (25)$$

where

$$X(k+1) = \begin{bmatrix} x(k+1) \\ x(k+2) \\ \dots \\ x(k+m) \end{bmatrix}, \quad U(k) = \begin{bmatrix} u(k/k) \\ u(k+1/k) \\ \dots \\ u(k+m-1/k) \end{bmatrix}, \quad \Xi(k+1) = \begin{bmatrix} \eta(k+1) \\ \eta(k+2) \\ \dots \\ \eta(k+m) \end{bmatrix}, \quad \Psi = \begin{bmatrix} A \\ A^2 \\ \dots \\ A^m \end{bmatrix},$$

$$\Phi[\Xi(k+1), k+1] = \begin{bmatrix} B[\eta(k+1), k+1] & 0_{n\times 2} & \dots & 0_{n\times 2} \\ AB[\eta(k+1), k+1] & B[\eta(k+2), k+2] & \dots & 0_{n\times 2} \\ \dots & \dots & \dots & \dots \\ A^{m-1}B[\eta(k+1), k+1] & A^{m-2}B[\eta(k+2), k+2] & \dots & B[\eta(k+m), k+m] \end{bmatrix},$$

$$\Delta_1 = \begin{bmatrix} R_1 & 0_{2\times2} & \dots & 0_{2\times2} \\ 0_{2\times2} & R_1 & \dots & 0_{2\times2} \\ \dots & \dots & \dots & \dots \\ 0_{2\times2} & 0_{2\times2} & \dots & R_1 \end{bmatrix}, \Delta(k) = \begin{bmatrix} R(k,0) & 0_{n\times n} & \dots & 0_{n\times n} \\ 0_{n\times n} & R(k,1) & \dots & 0_{n\times n} \\ \dots & \dots & \dots & \dots \\ 0_{n\times n} & 0_{n\times n} & \dots & R(k,m-1) \end{bmatrix},$$

$$\Delta_2(k+1) = [R_2(k,1) \quad R_2(k,2) \quad \dots \quad R_2(k,m)].$$

Using (25) we can write (24) as follows

$$J(k+m/k) = x^T(k)\Psi^T\Delta_1\Psi x(k) +$$
$$+\left[2x^T(k)\Psi^T\Delta_1 - \Delta_2(k+1)\right]E\{\Phi[\Xi(k+1),k+1]/x(k),\mathfrak{F}_k\}U(k) +$$
$$+U^T(k)\left[E\left\{\Phi^T[\Xi(k+1),k+1]\Delta_1\Phi[\Xi(k+1),k+1]/x(k),\mathfrak{F}_k\right\} + \Delta(k)\right]U(k). \quad (26)$$

Denote the following matrices

$$H(k) = E\left\{\Phi^T[\Xi(k+1),k+1]\Delta_1\Phi[\Xi(k+1),k+1]/x(k),\mathfrak{F}_k\right\} + \Delta(k),$$
$$G(k) = \Psi^T\Delta_1 E\{\Phi[\Xi(k+1),k+1]/x(k),\mathfrak{F}_k\},$$
$$F(k) = \Delta_2(k+1)E\{\Phi[\Xi(k+1),k+1]/x(k),\mathfrak{F}_k\}.$$

It can be shown that the blocks of the matrices $H(k)$, $G(k)$, $F(k)$ satisfy the recursive equations (12) – (20).

Thus we have that the problem of minimizing the criterion (26) subject to (4) is equivalent to the quadratic program problem with criterion

$$Y(k+m/k) = \left[2x^T(k)G(k) - F(k)\right]U(k) + U^T(k)H(k)U(k)$$

subject to (11). Therefore we obtain the desired result, completing the proof of the theorem.

3. Numerical examples

This section tests the proposed approach on a simple example where we consider the situation of an investor who has to allocate his wealth among five risky assets and one risk-free asset. The updating of the portfolio based on the MPC is executed once every trading day. We used five largest companies risky assets traded on the Russian Stock Exchanges MICEX: Gazprom, VTB, LUKOIL, Sberbank, NorNickel. We tested the results on daily actual closing prices over a period of time from August 18, 2010 to January 14, 2011. The risk-free asset considered here as bank account with risk-free rate r_1 =4% per annum.

First we need to estimate the required model parameters (6) over the predictive horizon m.

The practical difficulty with implementing obtained result is the choice of estimation parameter approach. There are essentially three ways to solve this problem. One approach considered in the statistics literature is to estimate the parameters using simple averaging. Second, the unknown parameters can be estimated using different model specifications describing the return asset evolution [10]. Third, one can use complex nonparametric methods, as implied in [9]. In reality, it is impossible to obtain precision parameter estimates.

In order to simplify our example, we used the following approach. We computed the expected returns using 4-day simple averaging of past historical return data and assume that the expected returns remain constant over the predictive horizon m. To obtain the expected asset returns we use the adjusted procedure, updating the estimates at each decision time *k*. This procedure allows to adaptively track the behavior of risky asset returns. We obtained the second moments up to 6 order using the past 200 trading days prior to the tracking period. These parameters are assumed to be stationary over the investment horizon and equal to the initial empirical estimates, based on backwards data. Our estimates were

$$\Theta = 10^{-4}\begin{bmatrix} 0.4814 & -0.0906 & -0.0351 & 0.0710 & -0.0201 & -0.0562 \\ 0.4569 & 0.0249 & -0.0317 & 0.0618 & -0.0432 & -0.0092 \\ 0.3564 & -0.0107 & -0.0089 & 0.0611 & -0.0179 & -0.0328 \\ 0.5041 & 0.0152 & -0.0276 & 0.0927 & -0.0293 & -0.0212 \\ 0.5817 & -0.0368 & 0.0138 & 0.0345 & -0.0362 & -0.0385 \end{bmatrix}.$$

The rows consist of the second moments of the risky assets returns Gazprom, VTB, LUKOIL, Sberbank, NorNickel.

We set the tracking target to return 0.2 % per day (μ_0=0.002). For our portfolio, we assumed an initial wealth of $V(0)=V^0(0)=1$.The weight coefficients are set as R=diag(10^{-5}, …,10^{-5}), $\rho(k,i)$=0.2. We impose hard constraints on the tracking portfolio problem with parameters $\gamma_i'=0$, $\gamma_i''=3$ (i=1, …,5), $\gamma_0'=3$. In this example no short-selling is allowed. We use Theorem 3.1 in order to define the optimal control portfolio strategy. For the on-line finite horizon problems MPC we used a horizon of m=10, and numerically solved it in MATLAB by using the quadprog.m function.

The results are summarized in three figures. Figure 1 plots portfolio (bold line) and benchmark values (dotted line). In figure 2 we have investments in the risky assets Sberbank (solid line), Gazprom (dotted line). Figure 3 illustrates returns of risky assets Sberbank (solid line), Gazprom (dotted line).

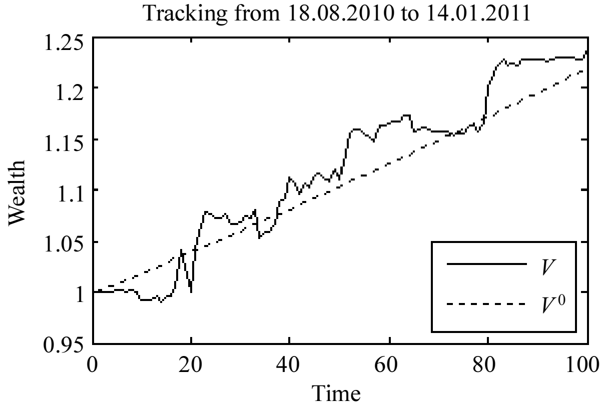

Fig. 1. 100 days performance of benchmark tracking, no short-selling is allowed (V– portfolio values, V^0 – benchmark values).

We find that on actual data the proposed approach is reasonable. The value of the portfolio is effectively tracked the benchmark and respected the constraints. Figure 2 shows that the amounts of short-selling are significantly reduced. It is important to acknowledge that, at least in this unsophisticated example, where we use simple averaging for parameters estimation, the tracking performance appears to be rather efficient. The obvious appeal of our approach is its simplicity and the fact that all of the models and statistical techniques designed for the describing asset return evolution can be applied directly to estimate unknown parameters.

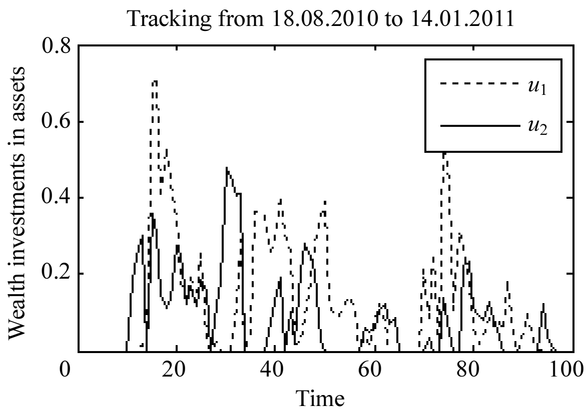

Fig. 2. Asset allocation decision, no short-selling is allowed (u_1 – Sberbank, u_2 – Gazprom).

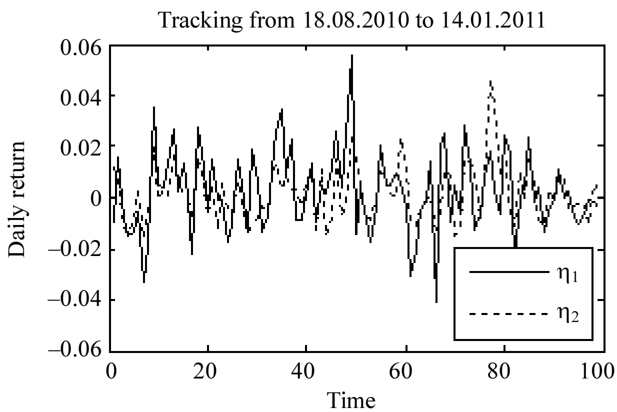

Fig. 3. Risky assets returns (η_1 – Sberbank, η_2 – Gazprom).

Conclusion

In this paper we studied a discrete-time portfolio selection problem subject to constraints on trading volume amounts. We propose to use the MPC methodology in order to solve the problem. The optimal open-loop feedback portfolio control strategy is derived.

We present the numerical modeling results, based on stocks, traded on the Russian Stock Exchanges MICEX, that give evidence of capacity and effectiveness of proposed approach.

The main features of the model are (a) the flexibility of dealing with portfolio constraints, (b) the generality of stochastic return models that can be used with the method, and (c) the efficiency in numerical solution.

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Домбровский В.В., Объедко Т.Ю. (Томский государственный университет). **Оптимизация инвестиционного портфеля на финансовом рынке с сериально зависимыми доходностями при ограничениях.**

Ключевые слова: инвестиционный портфель, сериально зависимые доходности, управление с прогнозирующей моделью.

Рассматривается задача управления инвестиционным портфелем с учетом явных ограничений на объемы торговых операций. При этом предполагается, что доходности рисковых финансовых активов представляют собой последовательность зависимых случайных параметров, для которых известны только первые и вторые моменты распределений. Проблема управления инвестиционным портфелем формулируется как динамическая задача слежения за некоторым эталонным портфелем, имеющим заданную инвестором доходность.

В работе получены уравнения синтеза стратегий управления инвестиционным портфелем с учетом явных ограничений на объемы торговых операций. Для решения задачи используется метод управления с прогнозирующий моделью (управление со скользящим горизонтом). Такой подход позволяет достаточно просто учитывать явные ограничения на управляющие переменные – объемы вложений и займов. Синтез стратегий управления с прогнозированием сводится к последовательности задач квадратичного программирования.

Приведены результаты численного моделирования с использованием реальных данных российского фондового рынка. Численное моделирование подтверждает работоспособность и эффективность предложенного подхода.