Neutron star sensitivities in Hoava gravity after GW170817

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Hořava gravity breaks boost invariance in the gravitational sector by introducing a preferred time foliation. The dynamics of this preferred slicing is governed, in the low-energy limit suitable for most astrophysical applications, by three dimensionless parameters α, β and λ. The first two of these parameters are tightly bound by solar system and gravitational wave propagation experiments, but λ remains relatively unconstrained ($0 \leq \lambda \lesssim 0.01 - 0.1$). We restrict here to the parameter space region defined by $\alpha = \beta = 0$ (with λ kept generic), which in a previous paper we showed to be the only one where black hole solutions are non-pathological at the universal horizon, and we focus on possible violations of the strong equivalence principle in systems involving neutron stars. We compute neutron star "sensitivities", which parametrize violations of the strong equivalence principle at the leading post-Newtonian order, and find that they vanish identically, like in the black hole case, for $\alpha = \beta = 0$ and generic $\lambda \neq 0$. This implies that no violations of the strong equivalence principle (neither in the conservative sector nor in gravitational wave fluxes) can occur at the leading post-Newtonian order in binaries of compact objects, and that data from binary pulsars and gravitational interferometers are unlikely to further constrain λ.

I. INTRODUCTION

Since Lorentz invariance is widely regarded as a fundamental symmetry of Nature, much effort has been devoted to test it experimentally, both in the pure matter [1–4] and pure gravity [5, 6] sectors, and in matter-gravity interactions [7]. However, bounds on this symmetry are still considerably weaker in gravitational experiments than in particle physics ones. This is due both to the intrinsic weakness of the gravitational interaction (with the resulting difficulties of performing suitable experiments, which need to be very accurate and/or rely on astrophysical systems) and to the absence of a generic parametrization of all possible Lorentz-violating effects in the strong gravity, highly relativistic regime. (See instead e.g. Refs. [8–10] for a Standard Model extension suitable for describing parametrized Lorentz violations in matter.)

One possible approach is to consider the simplest gravitational theory that extends general relativity and which breaks Lorentz symmetry at low energies in a dynamical fashion. Focusing in particular on boost symmetry, the simplest way to break it is to introduce a preferred time direction at each spacetime event, by means of a timelike, unit-norm vector field (the "æther"). To make this vector field dynamical, the gravitational action must include a kinetic term, whose form is fixed (apart from dimensionless coupling constants) if we require that it should be covariant and quadratic in the æther derivatives [11]. Moreover, one may require that the æther should define not only a preferred time direction locally, but also a preferred time foliation of the spacetime manifold. To this purpose, the æther vector needs to be hypersurface orthogonal, i.e. proportional to the gradient of a timelike scalar (which we will refer to as the "khronon" [12]), whose level sets define the preferred time foliation. The theory obtained in this way, known as "khronometric theory" [12–14], is the simplest theory that breaks boost invariance at low energies. A related theory, referred to as "Einstein-æther theory" [11], is obtained by waiving the requirement that the æther be hypersurface orthogonal.

In the following we will focus solely on khronometric theory. Indeed, this theory is not only interesting from a phenomenological point of view as a framework to test Lorentz invariance in the gravitational sector, but it also arises naturally as the low-energy limit of Hořava gravity [13]. The latter is a gravitational theory that, unlike general relativity, is power-counting and perturbatively renormalizable [13, 15]. As such, it provides a possible model for quantum gravity, although it remains to be seen whether the percolation of Lorentz violations from the gravitational sector to the matter one can be sufficiently suppressed to satisfy the tights bounds on Lorentz violations in matter. Note that this percolation may be suppressed via a large energy scale [16], or Lorentz invariance may emerge in the infrared as a result of renormalization group flows [17–19] (see however also Ref. [20]) or accidental symmetries [21].

Bounds on khronometric theory can be obtained in a number of ways. Theoretical requirements such as the absence of classical (gradient) and quantum (ghost) instabilities [22–24], combined with solar system bounds on the preferred frame parameters [22, 25, 26] and the absence of vacuum Cherenkov radiation from ultrahigh energy cosmic rays [27], have been used to constrain the dimensionless coupling parameters (α, β and λ) of the theory. Moreover, Refs. [28, 29] considered the evolution of binary pulsars under gravitational wave emission, and showed that timing of these systems, when combined with cosmological observations (and namely measurements of the abundance of primordial elements produced during Big Bang Nucleosynthesis [30]) allowed for bounding the coupling constants to within a few percent.

More recently, the coincident gravitational wave and electromagnetic detection of GW170817 and GRB 170817A [31, 32] has constrained the propagation speeds of gravitational waves and light to match to within one part in 10^{15} [33]. This bounds one of the parameters of the theory, β, to $|\beta| \lesssim 10^{-15}$ [34, 35]. Further bounds follow from combining the GW170817/GRB 170817A observations with solar system constraints on the preferred frame parameters [34, 35]. Indeed, if one imposes $|\beta| \lesssim 10^{-15}$, the latter are satisfied by $|\alpha| \lesssim 10^{-7}$, at least if $|\lambda| \gg 10^{-7}$; or by $|\alpha| \lesssim 0.25 \times 10^{-4}$ *and* $\lambda \approx \alpha/(1-2\alpha)$ [35]. This second possibility implies small values for all three coupling constants ($|\alpha| \sim |\lambda| \lesssim 10^{-5}$ and $|\beta| \lesssim 10^{-15}$), while the first tightly constrains α and β ($|\alpha| \lesssim 10^{-7}$, $|\beta| \lesssim 10^{-15}$) but leaves λ essentially unconstrained. The only bounds on λ are then $\lambda \lesssim 0.01 - 0.1$, from measurements of the abundance of primordial elements produced during Big Bang Nucleosynthesis [28–30] and from Cosmic Microwave Background observations [36]; and the sign requirement $\lambda \geq 0$ needed to ensure the absence of ghosts [12, 34]. In the light of these constraints, it is therefore quite tempting to simply set α and β exactly to zero while keeping λ finite.

Ref. [35] has recently studied the structure of black holes moving slowly relative to the preferred foliation in khronometric theory. Indeed, those systems are key to understanding gravitational wave emission from black hole binaries at the lowest post-Newtonian (PN) order, and to computing the "sensitivities" [28, 29, 35, 37–42] that parametrize dipole gravitational emission and violations of the strong equivalence principle (i.e. violations of the universality of free fall for strongly gravitating objects). A surprising finding of Ref. [35] is that these slowly moving black holes present an unavoidable finite-area curvature singularity at the "universal horizon" [43, 44] (i.e. at the causal boundary for signals propagating with arbitrarily large speeds in the ultraviolet limit, which exist in Hořava gravity as a result of Lorentz violations), unless the parameters α and β are set exactly to zero. Furthermore, if one sets $\alpha = \beta = 0$ while keeping $\lambda \neq 0$, the geometry of these black holes is exactly the same as in general relativity and their sensitivities vanish, even though the khronon profile is nontrivial [35]. This entails in particular that gravitational wave emission from binary black holes matches general relativity at the leading PN order, and no vacuum gravitational dipole radiation exists for $\alpha = \beta = 0$.

Once ascertained that it makes sense to set α and β exactly to zero to satisfy the experimental bounds presented above and to ensure that curvature singularities do not appear in moving black holes, an obvious question is how gravitational wave emission in khronometric theory behaves in systems involving neutron stars, and more in general whether such systems can present violations of the strong equivalence principle. Of course, this is relevant to compare both to pulsar timing observations [28, 29] and to direct gravitational detections of binary neutrons stars, such as GW170817 [31, 32].[1] This problem was studied in detail in Refs. [28, 29], as we have mentioned above, but the parameter space considered in those works does not include the case $\alpha = \beta = 0$ now favored by both the multimessenger detection of GW170817 and GRB 170817A [33–35], and by the results of Ref. [35] on the regularity of moving black holes.

Here, we will fill this gap, and show that for $\alpha = \beta = 0$ stellar sensitivities vanish exactly, which in turn implies that the dynamics of binary neutron stars matches general relativity in both the conservative and dissipative sectors at leading order in the PN expansion. Therefore, in particular, no gravitational dipole radiation is emitted in khronometric gravity in the viable region of parameter space (unlike what happens for generic α and β, c.f. Refs. [28, 29]), and emission at quadrupole order, as well as the conservative Newtonian dynamics, matches the general relativistic prediction. As a result, binary pulsars/GW170817-like observations are unlikely to be useful to constrain the remaining theory parameter λ.

This paper is organized as follows. In Sec. II we will briefly review the action and field equations of khronometric theory; in Sec. III we will review the concept of compact object sensitivities and their effect on the dissipative and conservative dynamics; in Sec. IV we will derive the ansätze for the metric and khronon of stars moving slowly relative to the preferred frame; in Sec. V we will solve the field equations for these stars for $\alpha = \beta = 0$, and show that their geometry matches exactly the predictions of general relativity (implying in particular that the sensitivities vanish exactly). Finally, in Sec. VI we will further discuss our results and put forward our conclusions. Throughout this paper, we will set $c = 1$ and use a metric signature $(+, -, -, -)$.

II. KHRONOMETRIC THEORY

The low energy limit of Hořava gravity, which is suitable for most astrophysical applications involving compact objects [43, 45], can be described covariantly by the khronometric theory action [14]

$$S = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\Big[R + \lambda\,(\nabla_\mu u^\mu)^2 + \beta\nabla_\mu u^\nu \nabla_\nu u^\mu + \alpha\, a_\mu a^\mu\Big] + S_{\rm matter}[g_{\mu\nu}, \Psi]\,, \quad (1)$$

where α, β and λ are dimensionless coupling constants, $a^\mu \equiv u^\nu \nabla_\nu u^\mu$, and the timelike unit-norm "æther" vector field $\boldsymbol{u}$ is given by

$$u_\mu = \frac{\nabla_\mu T}{\sqrt{\nabla^\alpha T \nabla_\alpha T}}\,, \quad (2)$$

[1] Note that the aforementioned bounds on β from GW170817 and GRB 170817A follow from the propagation of the gravitational wave signal alone, but do not exploit its generation.

with T a timelike scalar field (the "khronon") defining the preferred time foliation. The matter fields Ψ are coupled minimally to the four-metric $g_{\mu\nu}$ alone, so as to enforce geodesic motion for matter and the weak equivalence principle at tree level. This minimal coupling implies in particular that the bare gravitational constant G must be related to the locally measured G_N by $G_N = G/(1-\alpha/2)$ [30].

Note that the æther vector can be "eliminated" from this action by choosing coordinates adapted to the khronon, i.e. by choosing the khronon as the time coordinate. This yields $u_\mu = N\delta^T_\mu$ (where N is the lapse), while the action takes the non-covariant form [12, 13, 46]

$$S = \frac{1-\beta}{16\pi G}\int \mathrm{d}T\mathrm{d}^3x\, N\sqrt{\gamma}\Big(K_{ij}\,K^{ij} - \frac{1+\lambda}{1-\beta}K^2 + \frac{1}{1-\beta}{}^{(3)}R + \frac{\alpha}{1-\beta}a_i\,a^i\Big) + S_{\rm matter}[g_{\mu\nu},\Psi]\,, \tag{3}$$

where K^{ij}, ${}^{(3)}R$ and γ_{ij} are the extrinsic curvature, three-dimensional Ricci scalar and three-metric of the $T=$ const foliation; $K = K^{ij}\gamma_{ij}$; and $a_i \equiv \partial_i \ln N$. This action, which is invariant under the foliation-preserving diffeomorphisms

$$T \to \tilde{T}(T)\,, \qquad x^i \to \tilde{x}^i(x,T)\,, \tag{4}$$

but *not* under full four-dimensional diffeomorphisms, is the infrared limit of the original action proposed by Hořava [13]. Nevertheless, in the following we will find it more convenient to use the equivalent covariant action (1).

By varying the action (1) with respect to $g^{\mu\nu}$ while keeping T fixed, one gets the modified Einstein field equations [45, 47]

$$G_{\mu\nu} - T^{\rm kh}_{\mu\nu} = 8\pi G\, T^{\rm matter}_{\mu\nu}\,, \tag{5}$$

where $G_{\mu\nu} = R_{\mu\nu} - R\,g_{\mu\nu}/2$ is the Einstein tensor, $T^{\mu\nu}_{\rm matter} = (-2/\sqrt{-g})(\delta S_{\rm matter}/\delta g_{\mu\nu})$ is the matter stress-energy tensor, and

$$T^{\rm kh}_{\mu\nu} \equiv \nabla_\rho\left[J_{(\mu}{}^{\rho}u_{\nu)} - J^{\rho}{}_{(\mu}u_{\nu)} - J_{(\mu\nu)}u^\rho\right] + \alpha\, a_\mu\, a_\nu + \left(u_\sigma\,\nabla_\rho J^{\rho\sigma} - \alpha\, a_\rho a^\rho\right)u_\mu\, u_\nu + \frac{1}{2}L_{\rm kh}\, g_{\mu\nu} + 2\text{Æ}_{(\mu}u_{\nu)}\,, \tag{6}$$

with

$$J^\rho{}_\mu \equiv \lambda\,(\nabla_\sigma u^\sigma)\,\delta^\rho_\mu + \beta\,\nabla_\mu u^\rho + \alpha\, a_\mu u^\rho\,, \tag{7}$$

$$\text{Æ}_\mu \equiv (g_{\mu\nu} - u_\mu\, u_\nu)\Big(\nabla_\rho J^{\rho\nu} - \alpha\, a_\rho \nabla^\nu u^\rho\Big)\,, \tag{8}$$

$$L_{\rm kh} = \lambda\,(\nabla_\mu u^\mu)^2 + \beta\nabla_\mu u^\nu\nabla_\nu u^\mu + \alpha\, a_\mu a^\mu\,, \tag{9}$$

is the khronon stress-energy tensor.

By varying with respect to T while keeping $g^{\mu\nu}$ fixed, one obtains the khronon equation [45, 47]

$$\nabla_\mu\left(\frac{\text{Æ}^\mu}{\sqrt{\nabla^\alpha T\nabla_\alpha T}}\right) = 0\,. \tag{10}$$

Note however that this equation follows from the modified Einstein equations (5), the Bianchi identity, and the equations of motion for matter (which imply $\nabla_\mu T^{\mu\nu}_{\rm matter} = 0$) [14]. In the following, without any loss of generality, we will therefore solve the modified Einstein equations (5) alone.

The coupling constants α, β and λ must satisfy a number of theoretical requirements (absence of gradient instabilities and ghosts) [22–24] and experimental constraints (absence of vacuum Cherenkov radiation [27], solar system experiments [22, 25, 26], gravitational wave propagation bounds from GW170817 [33–35], and cosmological constraints [28–30, 36]). We refer the reader to Sec. II.A of Ref. [35] for more details, but for our purposes it suffices to recall that β is constrained to be very small ($|\beta| \lesssim 10^{-15}$) by GW170817. Moreover, as already mentioned in Sec. I, by combining the aforementioned theoretical and experimental bounds (and particularly those from solar system tests and GW170817), one finds that either both α and λ need to be small ($|\alpha| \sim |\lambda| \lesssim 10^{-5}$), or α needs to be very small ($|\alpha| \lesssim 10^{-7}$) while λ is relatively unconstrained aside from stability requirements [12, 34] and cosmological bounds [28–30, 36] ($0 \leq \lambda \lesssim 0.01-0.1$).

Taking these bounds into account, it is tempting to set the "small" couplings α and β exactly to zero, and explore the phenomenological consequences of a finite λ. An additional motivation for this choice is that Ref. [35] showed that black holes moving slowly relative to the khronon present curvature singularities at the universal horizon unless $\alpha = \beta = 0$. In the following we will thus set $\alpha = \beta = 0$ and explore the effect of a finite λ on the dynamics of stellar binary systems (and particularly neutron star binaries). Note that the dynamics of neutron star binaries in khronometric theory, as well as the gravitational wave generation from these systems, was studied in Refs. [28, 29], but in a region of parameter space that did *not* include the region $|\alpha| \lesssim 10^{-7}$, $|\beta| \lesssim 10^{-15}$ and $0 \leq \lambda \lesssim 0.01 - 0.1$ currently favored after GW170817.

III. GRAVITATIONAL WAVE GENERATION AND VIOLATIONS OF THE STRONG EQUIVALENCE PRINCIPLE

The main bottleneck for predicting the motion and gravitational wave fluxes of binary systems consists in computing the sensitivities of the binary components [28, 29, 35, 37–42]. Physically, the sensitivities parametrize violations of the strong equivalence principle, i.e. violations of the universality of free fall for self-gravitating bodies. In more detail, while test bodies are bound to follow spacetime geodesics because of the action (1), which presents no direct coupling (at tree level) between the matter fields and the æther/khronon, deviations from geodesic motion may appear beyond tree level, and notably for strongly self-gravitating objects (where the large metric perturbations can mediate effective interactions between the æther/khronon and matter).

This effect, well known in scalar-tensor theories (where it is often referred to as the "Nordtvedt effect" [38, 48, 49]) can be captured by adopting an effective point particle model for the binary components, where the masses are *not* constant, but depend on the velocity relative to the æther [28, 29, 35, 37]. Physically, this parametrization accounts for the fact that the gravitational binding energy of a body (which in turn contributes to the total mass of the object) will in general depend on the æther, since the latter appears in the gravitational action, c.f. Eq. (1). The sensitivities are then defined as [28, 29, 35, 37]

$$\sigma \equiv -\frac{\mathrm{d}\ln\tilde{m}(\gamma)}{\mathrm{d}\ln\gamma}\bigg|_{\gamma=1}, \tag{11}$$

where $\tilde{m}(\gamma)$ is the object's mass, depending on its Lorentz factor in the preferred frame (i.e. $\gamma \equiv \mathbf{U}\cdot\mathbf{u}$, where $\mathbf{U}$ is the body's four velocity). The sensitivities are then expected to be small for objects with weak internal gravity (for which the gravitational binding energy, and thus the æther contribution to the mass, are negligible), and must vanish in the test particle limit (i.e. in the limit in which the self-gravity of the body is neglected).

In general, non-vanishing sensitivities impact the dynamics of a binary system in both the conservative and dissipative sectors. At Newtonian order, the acceleration of a binary component becomes [28, 37, 50]

$$\dot{v}^i_A = -\frac{\mathcal{G} m_B \hat{n}^i_{AB}}{r^2_{AB}}, \tag{12}$$

where A, B range from 1 to 2 and denote the two bodies, $r_{AB} = |\boldsymbol{x}_A - \boldsymbol{x}_B|$, $\hat{n}^i_{AB} = (x^i_A - x^i_B)/r_{AB}$, and we have defined the *active* gravitational masses

$$m_B \equiv (1+\sigma_B)\tilde{m}_B(\gamma_B = 1) \tag{13}$$

and the "strong field" Newton constant

$$\mathcal{G} \equiv \frac{G_N}{(1+\sigma_A)(1+\sigma_B)}. \tag{14}$$

The sensitivities also appear in the binary acceleration at 1PN order [37, 50], together with their derivatives with respect to γ, which one can parametrize with the additional parameters [28, 37]

$$\sigma' \equiv \sigma + \sigma^2 + \frac{d^2\ln\tilde{m}(\gamma)}{d(\ln\gamma)^2}\bigg|_{\gamma=1}. \tag{15}$$

Similarly, the sensitivities appear in the dissipative sector, e.g the energy flux emitted by a quasi-circular binary in gravitational waves is given by [28, 37]

$$\frac{\dot{E}_b}{E_b} = 2\left(\frac{\mathcal{G}Gm_1m_2}{r^3_{12}}\right)\left\{\frac{32}{5}(\mathcal{A}_1 + \mathcal{S}\mathcal{A}_2 + \mathcal{S}^2\mathcal{A}_3)v^2_{12} + (s_1-s_2)^2\left[\mathcal{C} + \left(\frac{21}{5}\mathcal{A}_3 + 18\mathcal{B}\right)V^2_{CM}\right]\right\}, \tag{16}$$

where

$$E_b = -\frac{\mathcal{G}m_1m_2}{2r_{12}}, \tag{17}$$

is the binary's total energy (potential and kinetic); V_{CM} is the velocity of the center of mass relative to the preferred frame; and we have defined the rescaled sensitivities

$$s_A \equiv \frac{\sigma_A}{1+\sigma_A}, \tag{18}$$

as well as the coefficients

$$\mathcal{A}_1 \equiv \frac{1}{c_2} + \frac{3\alpha(\mathcal{Z}-1)^2}{2c_0(2-\alpha)}, \quad \mathcal{A}_2 \equiv \frac{2(\mathcal{Z}-1)}{(\alpha-2)c_0^3}, \tag{19}$$

$$\mathcal{A}_3 \equiv \frac{2}{3\alpha(2-\alpha)c_0^5}, \quad \mathcal{B} \equiv \frac{1}{9\alpha\, c_0^5(2-\alpha)}, \tag{20}$$

$$\mathcal{C} \equiv \frac{4}{3c_0^3\,\alpha(2-\alpha)}, \quad \mathcal{S} \equiv s_1\frac{m_2}{m} + s_2\frac{m_1}{m}, \tag{21}$$

$$\mathcal{Z} \equiv \frac{(\alpha_1 - 2\alpha_2)(1-\beta)}{3(2\beta-\alpha)}, \tag{22}$$

which depend in turn on the total mass $m = m_1 + m_2$, on the spin-0 and spin-2 propagation speeds

$$c_2^2 = \frac{1}{1-\beta}, \tag{23a}$$

$$c_0^2 = \frac{(\alpha-2)(\beta+\lambda)}{\alpha(\beta-1)(2+\beta+\lambda)} \tag{23b}$$

and on the preferred-frame parameters [22, 26]

$$\alpha_1 = \frac{4(\alpha-2\beta)}{\beta-1}, \tag{24}$$

$$\alpha_2 = \frac{(\alpha-2\beta)[-\beta^2 + \beta(\alpha-3) + \alpha + \lambda(-1-3\beta+2\alpha)]}{(\beta-1)(\lambda+\beta)(\alpha-2)}. \tag{25}$$

Note that dipole gravitational wave emission is proportional to $\mathcal{C}$ and to $(s_1-s_2)^2$ (as in scalar-tensor theories [38–40]), and that it may dominate over quadupole emission at the low frequencies relevant for binary pulsars, depending on the sensitivities and the theory's coupling parameters. However, in the limit $\alpha, \beta \to 0$ with $\lambda \neq 0$, the spin-0 propagation speed diverges (i.e. the spin-0 mode becomes non-propagating) and one has $\mathcal{A}_1 \to 1$ and $\mathcal{A}_2, \mathcal{A}_3, \mathcal{B}, \mathcal{C} \to 0$. As a result, there exists no dipole gravitational emission in this limit, while quadrupole fluxes match exactly those of general relativity. This means that the damping of the period of binary pulsar systems is not suitable for constraining khronometric theory in this limit.

Nevertheless, timing observations of pulsar systems also constrain modifications to the conservative sector. For instance, in binary systems the acceleration depends on the sensitivities σ_A at Newtonian order, and on σ_A and their "derivatives" σ'_A at 1PN order [37, 50]. As a result, observations of the 1PN dynamics (e.g. periastron precession and preferred frame effects) of binary pulsar systems can in principle constrain khronometric theory [37, 50], provided that one can compute σ_A and σ'_A and relate them to the theory's coupling constants. Note that binary pulsar observations do not constrain the dependence of the Newtonian acceleration on the sensitivities [Eq. (12)], since the Newtonian dynamics of the system is used to infer the masses of the binary components [i.e. the sensitivities are re-absorbed into the mass redefinition given by Eq. (13)], and one has therefore to resort to the 1PN dynamics to test the theory. However, direct bounds on the Newtonian acceleration (and thus on the sensitivities) are provided by the triple system PSR J0337+1715 [51], which consists of a pulsar-white dwarf binary system, and a second white dwarf farther out. Timing of this system allows one to constrain the strong equivalence principle, i.e. to assess whether the two binary components fall with different accelerations toward the third body. The fractional acceleration difference Δ between the binary components is constrained to $|\Delta| < 2.6 \times 10^{-6}$ [51]. This can in turn be translated into a bound on the sensitivities by using Eq. (12) (c.f. also Ref. [50]). In more detail, since the sensitivities are expected to scale with the gravitational binding energy of the star [28, 37], and thus to be much smaller for white dwarfs than for neutron stars, from Eq. (12) one obtains $|\Delta| = |\sigma_{\rm p}| + \mathcal{O}(\sigma_{\rm p})^2$, where $\sigma_{\rm p}$ is the pulsar's sensitivity.

IV. THE ANSÄTZE FOR THE METRIC AND KHRONON OF SLOWLY MOVING STARS

As shown in Refs. [28, 35, 37], the sensitivities can be extracted from solutions to the field equations describing an object moving slowly relative to the preferred foliation (as defined far from the body). In more detail, let us consider a compact object (e.g. a neutron star or a black hole) at rest in the stationary flow of a khronon field moving with speed $-v$ along the z-direction on flat space near spatial infinity. The most generic metric and æther ansätze for such a system are given, through order $\mathcal{O}(v)$ and in Eddington-Finkelstein coordinates, by [35]

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu =& f(r)\mathrm{dv}^2 - 2B(r)\mathrm{d}r\mathrm{dv} - r^2\mathrm{d}\Omega^2 \\ &+ v\Big\{\mathrm{dv}^2 f(r)^2\cos\theta\psi(r) \\ &- 2\mathrm{d}\theta\mathrm{d}r\sin\theta[\Sigma(r) - B(r)\chi(r)] \\ &+ 2\mathrm{d}r\mathrm{dv}f(r)\cos\theta[\delta(r) - B(r)\psi(r)] \\ &+ \mathrm{d}r^2 B(r)\cos\theta[B(r)\psi(r) - 2\delta(r) + 2\Delta(r)] \\ &- 2\mathrm{d}\theta\mathrm{dv}f(r)\sin\theta\chi(r)\Big\} + \mathcal{O}(v^2)\,, \end{aligned} \tag{26}$$

and

$$\begin{aligned} u_\mu \mathrm{d}x^\mu =& \bar{u}_{\rm v}(r)\mathrm{dv} - A(r)B(r)\mathrm{d}r + v\Bigg\{\frac{1}{2}f(r)\cos\theta\times \\ &\left[2\bar{u}^r(r)\left(\frac{B(r)\Delta(r)\bar{u}^r(r)}{\bar{u}_{\rm v}(r)} + \delta(r) - \eta(r)\right)\right. \\ &\left. + \psi(r)\bar{u}_{\rm v}(r)\right]\mathrm{dv} + \frac{1}{2}\cos\theta\times \\ &\left[B(r)\Big(-\frac{2B(r)\Delta(r)\bar{u}^r(r)^2}{\bar{u}_{\rm v}(r)} - 2\delta(r)\bar{u}^r(r)\right. \\ &\left. - \psi(r)\bar{u}_{\rm v}(r)\Big) + 2A(r)f(r)\eta(r)\right]\mathrm{d}r \\ &- \sin\theta\Pi(r)\bar{u}_{\rm v}(r)d\theta\Bigg\} + \mathcal{O}(v^2)\,, \end{aligned} \tag{27}$$

where $\bar{u}_{\rm v} = (1 + fA^2)/(2A)$ and $\bar{u}^r = (-1 + A^2 f)/(2AB)$ are the background [i.e. $\mathcal{O}(v^0)$] æther components. Hypersurface orthogonality of the æther then requires [35]

$$\begin{aligned} \eta(r) =& -\frac{2\bar{u}^r(r)^3 B(r)^3\Delta(r) - 2\bar{u}_{\rm v}(r)^3\Pi'(r)}{2f(r)\bar{u}_{\rm v}(r)} \\ &- \frac{B(r)^2\bar{u}_{\rm v}(r)\bar{u}^r(r)\left[2\delta(r)\bar{u}^r(r) + \psi(r)\bar{u}_{\rm v}(r)\right]}{2f(r)\bar{u}_{\rm v}(r)}\,. \end{aligned} \tag{28}$$

Considering an infinitesimal gauge transformation with generator $\xi^\mu\partial_\mu = v\Omega(r)(-r\cos\theta\partial_r + \sin\theta\partial_\theta) + vK(r)\cos\theta\partial_{\rm v}$, one can set $\Pi = \Delta = 0$ by tweaking Ω and K and re-defining the other free functions appearing in the ansatz [35]. Note also that by requiring the spacetime to be asymptotically flat [i.e. $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$] and the æther to asymptote to $u^\mu\partial_\mu = \partial_t - v\partial_z + \mathcal{O}(v)^2$ in suitably defined "Cartesian'" coordinates $(t, x, y, z = r\cos\theta)$ near spatial infinity, one finds that the potentials in the ansatz above must satisfy the boundary conditions $\psi, \Sigma \to 0$, $\delta \to -1$ and $\chi/r \to -1$ for $r \to \infty$ [35].[2]

In more detail, the perturbative solution to the field equations (5) near spatial infinity that satisfies these boundary conditions is [35]

$$\delta(r) = -1 - \frac{2(\beta + \lambda)(-r_s + 2\chi_0)}{(1 - 3\beta - 2\lambda)r} + \mathcal{O}\left(\frac{1}{r^2}\right)\,, \tag{29a}$$

$$\chi(r) = -r + \chi_0 + \mathcal{O}\left(\frac{1}{r}\right)\,, \tag{29b}$$

$$\psi(r) = \frac{-3\beta(3 - 2a_2)r_s^2 - 2\Sigma_1}{3\,r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)\,, \tag{29c}$$

$$\Sigma(r) = \frac{\Sigma_1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)\,, \tag{29d}$$

[2] This is easy to see in the following manner. These conditions, inserted into the ansätze (26) and (27) lead to $ds^2 \approx dt^2 - dr^2 - r^2 d\Omega^2 - 2vdtdz$ and $u_\mu dx^\mu \approx dt$, where we have changed the time coordinate to $t \approx {\rm v} - r$ and defined $z = r\cos\theta$. Further changing the time coordinate to $t' = t - vz$ yields the flat line element, while the æther transforms to $u^\mu\partial_\mu \approx \partial_{t'} - v\partial_z$ asymptotically, i.e. at spatial infinity the æther moves with velocity $-v$ with respect to the flat asymptotic metric.

$$f(r) =1 - \frac{2r_s}{r} - \frac{\alpha r_s^3}{6r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) \tag{29e}$$

$$B(r) =1 + \frac{\alpha r_s^2}{4r^2} + \frac{2\alpha r_s^3}{3r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) \tag{29f}$$

$$A(r) =1 + \frac{r_s}{r} + \frac{a_2 r_s^2}{r^2} + (24a_2 + \alpha - 6)\,\frac{r_s^3}{12r^3} + \mathcal{O}\left(\frac{1}{r^4}\right), \tag{29g}$$

where $r_s = G_N M$ (with M the object's mass), while χ_0, χ_2, Σ_1 and a_2 are parameters that describe the object, and which in practice are to be extracted from the full (and typically numerical) "strong-field" solution [28, 35]. The sensitivities can then be read off this solution, and in particular from χ_0, as [35]

$$\begin{aligned}\sigma =& \frac{\alpha - \beta - 3\alpha\beta + 5\beta^2 + \lambda - 2\alpha\lambda + 3\beta\lambda}{(2-\alpha)(1-3\beta-2\lambda)} \\ & - \frac{2(1-\beta)(\beta+\lambda)}{(2-\alpha)(1-3\beta-2\lambda)}\frac{\chi_0}{r_s}\,.\end{aligned} \tag{30}$$

For a star, unlike for a black hole, the background is static, i.e. one has $A = 1/\sqrt{f}$ and $\bar{u}^r = 0$ [28, 52], and it is convenient to express the metric and æther in Schwarzschild coordinates, which are related to the Eddington-Finkelstein coordinates used above and in Ref. [35] by the simple transformation $\mathrm{dv} = \mathrm{d}t + B(r)/f(r)\mathrm{d}r$. Let us also use spatial coordinates comoving with the fluid, i.e. let us propagate the spatial coordinates of the initial spatial hypersurface along the fluid's worldlines, as discussed in Ref. [28], so that the fluid 4-velocity is simply $\boldsymbol{U}_{\text{fluid}} \propto \partial_{\mathrm{v}} = \partial_t$. Note that this coordinate choice is compatible with the gauge transformations used above to set $\Pi = \Delta = 0$ (which consist indeed of a time independent transformation of the spatial coordinates – thus equivalent to relabeling the spatial coordinates on the initial foliation – and an infinitesimal redefinition of the time coordinate v). This can also be checked explicitly by noting that $\boldsymbol{U}_{\text{fluid}} \propto \partial_{\mathrm{v}} = \partial_t$ is unaffected by a change of coordinates with generator $\xi^\mu\partial_\mu = v\Omega(r)(-r\cos\theta\partial_r + \sin\theta\partial_\theta) + vK(r)\cos\theta\partial_{\mathrm{v}}$, which one needs to set $\Pi = \Delta = 0$.

At this point, unlike for a black hole, we can require that the ansätze for the metric and æther be left unchanged by the joint transformations $t \to -t$ and $v \to -v$, i.e. by a joint reflection of the (Schwarzschild) time and of the velocity [28]. It can easily be checked that this requires $\Sigma = \psi = 0$, which in turn implies $\eta = 0$ due to the orthogonality condition (28). Note that this requirement cannot be imposed for black holes since the background æther has a radial component, i.e. a time reflection transforms the æther from ingoing to outgoing [35, 43]. Similarly, invariance under a joint reflection of time and velocity implies that perturbations to the density and pressure of the fluid should vanish at $\mathcal{O}(v)$ [28].

In summary, the metric, æther and fluid ansätze for a star therefore become, in Schwarzschild coordinates,

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu =& f(r)\mathrm{d}t^2 - \frac{B(r)^2}{f(r)}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2 \\ & + 2vf(r)\mathrm{d}t\left[\cos\theta\delta(r)\mathrm{d}r - \sin\theta\chi(r)\mathrm{d}\theta\right] + \mathcal{O}(v^2)\,, \end{aligned} \tag{31}$$

$$u_\mu \mathrm{d}x^\mu = \sqrt{f(r)}\mathrm{d}t + \mathcal{O}(v^2)\,, \quad U^\mu_{\text{fluid}}\partial_\mu = \frac{\partial_t}{\sqrt{f(r)}} + \mathcal{O}(v^2)\,, \tag{32}$$

$$\rho(r) = \bar{\rho}(r) + \mathcal{O}(v^2)\,, \quad p(r) = \bar{p}(r) + \mathcal{O}(v^2)\,, \tag{33}$$

where overbars denote background $\mathcal{O}(v^0)$ quantities.

Note that these ansätze, while obtained in a slightly different way, reduce to those successfully used by Ref. [28] to compute neutron star sensitivities for generic α, β and λ, with the additional "dipole" assumption $V(r,\theta) \propto \delta(r)\cos\theta$ and $S(r,\theta) \propto \chi(r)\sin\theta$. This dipolar structure corresponds to taking $n = 1$ in Eqs. (145), (146) and (147) of Ref. [28], and indeed it is from those $n = 1$ equations that Ref. [28] extracts neutron star sensitivities.

V. SOLUTIONS FOR SLOWLY MOVING STARS AND THEIR SENSITIVITIES

For $\alpha = \beta = 0$, the stellar exterior background solution is given by $f(r) = 1 - 2r_s/r$ and $B(r) = 1$ [35, 53]. The field equations for the potentials δ and χ reduce to the homogeneous coupled system

$$\begin{aligned}\chi''(r) =& \frac{1}{r(2r_s - r)}\{\delta'(r)(1+2\lambda)r(2r_s - r) \\ & + 2\delta(r)[(\lambda-2)r_s - 2\lambda r] + 4r_s\chi'(r) + 4\lambda\chi(r)\}\,,\end{aligned} \tag{34}$$

$$\begin{aligned}\delta''(r) =& \frac{1}{\lambda r^2(r-2r_s)^2}\{\delta(r)[5\lambda r_s^2 + 2(1-4\lambda)r_s r \\ & + (2\lambda-1)r^2] + r(2r_s - r)[2\lambda(r_s + r)\delta'(r) \\ & - (2\lambda+1)\chi'(r)] + 2\lambda(5r_s - 2r)\chi(r)\}\,.\end{aligned} \tag{35}$$

Let us define, without loss of generality, $\delta(r) = \chi'(r) + \lambda h(r)$. This redefinition makes the equations above of third order in the derivatives of χ, but the $\chi'''(r)$ terms can be eliminated by using the derivative of Eq. (34), thus yielding

$$\begin{aligned}\chi''(r) =& \frac{1}{2r(2r_s - r)}\{(2\lambda+1)r(r-2r_s)h'(r) \\ & + h(r)[4\lambda r - 2(\lambda-2)r_s] - 2(r_s - 2r)\chi'(r) - 4\chi(r)\}\,,\end{aligned} \tag{36}$$

$$\begin{aligned}h''(r) =& \frac{1}{r^2(r-2r_s)^2}[7r_s r(2r_s - r)h'(r) \\ & + 2h(r)\left(-10r_s^2 + 2r_s r + r^2\right)]\,.\end{aligned} \tag{37}$$

The second equation can be solved analytically to give

$$h(r)=\frac{k_1 r^2\left(3r_s^2-6r_s r+2r^2\right)}{3r_s^3(r-2r_s)^2}+\frac{k_2 r^{5/2}}{\sqrt{r-2r_s}}\,, \tag{38}$$

with k_1 and k_2 integration constants.

From the boundary conditions at spatial infinity, $\delta\to -1$ and $\chi/r\to -1$ for $r\to\infty$, it follows that it must be $k_2=-2k_1/(3r_s^3)$. For this choice of the integration constants, the equation for χ [Eq. (36)] can be solved analytically with Lagrange's method of the variation of constants. The explicit solution is quite cumbersome and hardly enlightening, so we will only present here its perturbative expansion near spatial infinity, which reads

$$\chi(r)=\frac{C_0 r}{r_s}+\frac{k_1-2\lambda k_1-6C_0}{12}+\frac{(7-4\lambda)k_1 r_s}{24r}+\frac{(2+3\lambda)k_1 r_s^2\ln(r/r_s)}{9r^2}+\frac{C_1 r_s^2}{r^2}+\mathcal{O}\left(\frac{\ln r}{r^3}\right)\,, \tag{39}$$

with C_0 and C_1 integration constants. Since the system given by Eqs. (36) and (37) is homogeneous, its solutions are invariant under a global rescaling, i.e. if the pair (h,χ) is a solution, $(\Lambda h,\Lambda\chi)$, with Λ an arbitrary constant, is also a solution. Rescaling therefore Eq. (39) by a factor $-r_s/C_0$ [in order to match Eq. (29b)], one can then compute the sensitivity, once the exterior solution for χ and h is known, via Eq. (30). Setting $\alpha=\beta=0$ and [from Eq. (39)] $\chi_0=-r_s(k_1-2\lambda k_1-6C_0)/(12C_0)$, one obtains

$$\sigma=\frac{\lambda k_1}{12C_0}\,. \tag{40}$$

As a sanity check, note that for the special choice $k_1=0$, one gets $\sigma=0$. This makes perfect sense, because $k_1=0$ implies $h(r)=0$ and thus $\delta(r)=\chi'(r)$. An infinitesimal gauge transformation $\mathrm{v}'=\mathrm{v}+v\,\chi(r)\cos\theta+\mathcal{O}(v)^2$ can then be used to eliminate all $\mathcal{O}(v)$ terms in the metric (31), i.e. that metric becomes simply (in the exterior) the Schwarzschild metric in Eddington-Finkelstein coordinates [since for $\alpha=\beta=0$ one has $f(r)=1-2r_s/r$ and $B(r)=1$]. In general, however, k_1 may *not* be zero. In fact, the exact values of the integration constants k_1 and C_0 must be obtained by matching the (analytically known) exterior solution for h and χ to the stellar interior, at the surface of the star. This was indeed the procedure followed in Refs. [28, 29].

The field equations for h and χ in the presence of a perfect fluid source are only slightly more involved than the vacuum system (36)–(37). To derive them, one can first note that the field equations of khronometric theory for static spherically symmetric stars only differ from their general-relativistic counterparts if $\alpha\neq 0$ [28, 29, 52]. Since we are considering here the case $\alpha=\beta=0$, the background quantities $f(r)$, $B(r)$, $\bar\rho(r)$ and $\bar p(r)$ will therefore be the same as in general relativity, i.e. they can be easily computed by solving the TolmanOppenheimerVolkoff equations [54, 55] while imposing regularity at $r=0$. The field equations for h and χ can then be obtained by the same procedure as in vacuum, i.e. replace the ansatz $\delta(r)=\chi'(r)+\lambda h(r)$ in the $\mathcal{O}(v)$ equations, and use the derivatives of the equations themselves to eliminate the χ''' terms. Making also repeated use of the $\mathcal{O}(v)^0$ equations (i.e. the TolmanOppenheimerVolkoff equations) and their derivatives, after some algebra one obtains

$$\begin{aligned}\chi''(r)=&-\frac{1}{2r^2f(r)}\Big\{B(r)^2\Big[rh(r)\Big(3\lambda+4\pi(4\lambda+3)r^2G\bar p(r)\\&-4\pi(2\lambda+1)r^2G\bar\rho(r)+2\Big)\\&+r\Big(16\pi r^2G\bar p(r)-8\pi r^2G\bar\rho(r)+3\Big)\chi'(r)-4\chi(r)\Big]\\&+rf(r)\Big[(2\lambda+1)rh'(r)+(\lambda-2)h(r)+\chi'(r)\Big]\Big\}\,,\end{aligned} \tag{41}$$

$$\begin{aligned}h''(r)=&\frac{1}{2r^2f(r)^2}\Big\{B(r)^2f(r)\Big[rh'(r)\Big(-32\pi r^2G\bar p(r)\\&+24\pi r^2G\bar\rho(r)-7\Big)\\&+8h(r)\Big(-\pi r^2G\bar p(r)+\pi r^3G\bar\rho'(r)-5\pi r^2G\bar\rho(r)+2\Big)\Big]\\&+2B(r)^4h(r)\Big[8\pi r^2G\bar p(r)+1\Big]\Big[8\pi r^2G\bar\rho(r)-1\Big]\\&+f(r)^2\Big[7rh'(r)-10h(r)\Big]\Big\}\end{aligned} \tag{42}$$

Remarkably, just like in vacuum, the equation for h does not depend on χ.

Like the TolmanOppenheimerVolkoff equations, Eqs. (41)–(42) have a singular point at $r=0$. In principle, we should thus solve these equations perturbatively near the center of the coordinates, imposing regularity by requiring that h and χ (as well as $\bar p$, $\bar\rho$, F and B) are analytic functions near $r=0$. As we will show below, the general regular perturbative solution for h and χ depends on two integration constants, one of which can be fixed by exploiting the homogeneity of the system (41)–(42), i.e. the fact that if (h,χ) is a solution for the interior geometry, $(\Lambda h,\Lambda\chi)$, with Λ an arbitrary constant, is also a solution. One may then use the perturbative solution, which now depends on only one undetermined integration constant, to "move away" from the center, i.e. to provide initial conditions at small $r\neq 0$ for an outward numerical integration. One could then try to tune that undetermined integration constant to match the numerical interior solution to the asymptotically flat exterior at the stellar surface, e.g. by a shooting method (as was done in similar problems in Refs. [28, 29, 35, 43, 56]). However, while counting the degrees of freedom of the perturbative solution would seem to allow for this procedure, the latter is doomed to fail in our particular case, as we will now explain, because of the structure of the system (41)–(42).

Indeed, let us look in more detail at the perturbative solution for h near the origin. To obtain it, one needs first to solve the background TolmanOppenheimerVolkoff equations imposing regularity at $r=0$, i.e. with the ansätze $\bar{p}(r) = \sum_{n=0}^{+\infty} \bar{p}_{2n} r^{2n}$, $f(r) = \sum_{n=0}^{+\infty} f_{2n} r^{2n}$ and $B(r) = \sum_{n=0}^{+\infty} B_{2n} r^{2n}$ (with the odd powers set to zero by the field equations, and with the density expressed in terms of the pressure via a suitable equation of state). The background solution can then be inserted into Eq. (42), and one can look for a solution of the form $h(r) = \sum_{n=0}^{+\infty} h_{2n} r^{2n}$ (note that the field equations forbid again the presence of odd powers). This procedure yields

$$h(r) = h_2 \left[r^2 + \frac{2}{15} \pi r^4 (7G\bar{\rho}_0 - 33G\bar{p}_0) + \mathcal{O}(r^6) \right] , \quad (43)$$

where $\bar{\rho}_0$ and $\bar{p}_0$ are the central density and pressure, h_2 is an undetermined integration constant, while the higher order terms are proportional to h_2 and are otherwise completely determined by the perturbative background solution for f, B, $\bar{p}$ and $\bar{\rho}$ near the center (for which we have chosen a gauge in which $f(0) = 1$). Therefore, the perturbative regular solution for h near the origin has only one integration constant, h_2.

Because of the homogeneity of the system (41)–(42), we can then rescale $h_2 = 1$, in which case the solution for h near the center is completely determined.[3] Therefore, even though solving Eq. (41) for χ perturbatively near $r = 0$ will yield a second integration constant for the system (41)–(42) (i.e. the general solution turns out to be $\chi(r) = \sum_{n=0}^{+\infty} \chi_{2n+1} r^{2n+1}$ with all the coefficients χ_{2n+1} given in terms of h_2 and $\chi'(0) = \chi_1$), there is not enough freedom (at least for generic matter equations of state) to match the interior solution for h to an asymptotically flat exterior [i.e. to Eq. (38) with $k_2 = -2k_1/(3r_s^3)$].

Alternatively, another equivalent way of understanding why the matching is not possible is the following. One is of course free to keep h_2 generic (rather than set it 1 as done above), but that will only provide a global rescaling of the interior solution [as is obvious from the homogeneity of the system (41)–(42)]. Similarly, the exterior asymptotically flat solution given by Eq. (38) with $k_2 = -2k_1/(3r_s^3)$ depends on just one integration constant (k_1), which is also a global amplitude because of homogeneity. One may then try to match the two solutions by imposing continuity of h and h' at the star's surface, which will yield two conditions $h_2 H_1 = k_2 H_2$ and $h_2 H_3 = k_2 H_4$, where H_1, H_2, H_3 and H_4 depend on the interior and exterior solutions and their derivatives evaluated at the matching point. Clearly, this system can only be satisfied if $H_2/H_1 = H_4/H_3$. Since the interior solution will depend on the matter equation of state (e.g. for a neutron star, the equation of state of nuclear matter), such a "miracle" cannot happen for generic equations of state.

More in general, note that the matching turns out to be impossible because the equation for h does not depend on χ, i.e. if the equation for h were coupled to χ, the perturbative solutions for h and χ may both depend on the same two integration constants, which would leave us with enough freedom to match to the exterior solution. We have verified this also by changing the interior equation (42) "by hand", introducing an artificial source term depending on χ on the right-hand side.

In any case, as a result, we obtain that for generic equations of state the only solution for h that is regular at the center and asymptotically flat is the trivial one, $h(r) = 0$. This thus yields vanishing sensitivities by virtue of Eq. (40). Moreover, since $h(r) = 0$ implies that $\delta = \chi'$, one can perform an infinitesimal gauge transformation $\mathrm{v}' = \mathrm{v} + v\,\chi(r)\cos\theta + \mathcal{O}(v)^2$ to eliminate all $\mathcal{O}(v)$ terms in the metric (31), i.e. not only do the sensitivities vanish, but the entire geometry for slowly moving stars matches that of the corresponding general relativistic solution when $\alpha = \beta = 0$ and $\lambda \neq 0$.

We stress that even though the solution for h is trivial, that for $\delta = \chi'$ is not. Indeed, the solution for χ can be obtained explicitly by setting $h(r) = 0$ in Eq. (41), and then by integrating numerically the resulting equation outward from the center of the star (imposing regularity there and using the homogeneity of the equation to set the one undetermined constant appearing in the perturbative solution – χ_1 – to a given value, e.g. $\chi_1 = 1$). The integration can proceed through the stellar surface and then in the exterior [where the solution, as mentioned, can be obtained also analytically by Lagrange's method of the variation of constants, c.f. Eq. (39) for its series expansion]. The fact that $\delta = \chi'$ is not trivial was of course to be expected from the boundary conditions ($\delta = \chi' \to -1$ at spatial infinity), which correspond to an asymptotically flat geometry and *moving* æther. In other words, for $\alpha = \beta = 0$ and $\lambda \neq 0$ the geometry of the star is the same as in general relativity, but the æther is non-trivial (and in particular it corresponds to a flow with velocity $-v$ at spatial infinity). This is the same situation that Ref. [35] found for black holes when $\alpha = \beta = 0$ and $\lambda \neq 0$, although the non-trivial æther configurations differ in the two cases. Like for black holes, however, it is clear that the æther stress energy tensor vanishes at order $\mathcal{O}(v)$, so that the metric matches the general relativistic solution. In this sense the æther behaves as a "stealth" field on the spacetime of stars moving slowly relative to the preferred foliation, when $\alpha = \beta = 0$.

VI. DISCUSSION AND CONCLUSIONS

We have studied the structure of slowly moving stars in khronometric theory, i.e. the low-energy limit of Hořava gravity, in the region of parameter space favored by experimental tests (and notably by solar system tests and

[3] Note that rescaling $h_2 = 1$ is justified as it corresponds to dividing the original system (41)–(42) by $h''(0)/2$. One can then introduce new variables $h_{\rm new} = h/(h''(0)/2)$ and $\delta_{\rm new} = \delta/(h''(0)/2)$. Because of the linearity and homogeneity of the system, the resulting equations will then still be given by Eqs. (41)–(42), but with $h \to h_{\rm new}$ and $\delta \to \delta_{\rm new}$ and with the extra constraint $h''_{\rm new}(0) = 2$. This latter condition then forces h_2 in Eq. (43).

the multimessenger bounds on gravitational wave propagation from GW170817) and by theoretical considerations (and namely by the results of Ref. [35], which found that two of the dimensionless coupling constants of the theory – α and β – need to vanish exactly for black holes in motion relative to the preferred frame to present regular universal horizons).

Solutions for stars moving slowly relative to the preferred frame are necessary to calculate the "sensitivities" – i.e. the parameters characterizing the effective coupling of the star to the Lorentz-violating scalar degrees of freedom of the theory – and therefore assess possible violations of the strong equivalence principle. Non-zero sensitivities may generally impact the dynamics of binary systems both in the conservative and dissipative sectors (e.g. they can trigger dipole gravitational emission, as well as deviations away from the quadrupole formula and the conservative Newtonian and PN dynamics of general relativity).

While neutron star sensitivities had previously been computed in Refs. [28, 29], the parameter space considered in those works did not include the case $\alpha = \beta = 0$ currently favored by GW170817 and by the black hole regularity results of Ref. [35]. In this work, we have indeed found that stellar sensitivities for the $\alpha = \beta = 0$ case can be obtained quite easily, without having to study the stellar interior in detail (unlike what had to be done in the general case by Refs. [28, 29]). We have found that for generic matter equations of state, the sensitivities vanish exactly when $\alpha = \beta = 0$, irrespective of the value of the third coupling constant of the theory, λ. This implies in particular that no deviations from general relativity appear in the generation of gravitational waves from neutron star/pulsar binaries at the lowest PN order, i.e. no dipole fluxes are present and no deviations from the quadrupole formula appear. Similarly, no deviations from general relativity arise in the conservative dynamics at Newtonian order.

We note that this result generalizes the similar result of Ref. [35], which recently found that *black hole* sensitivities vanish exactly when $\alpha = \beta = 0$. In fact, exactly as in the black hole case [35], we find that the geometry of an isolated star moving slowly relative to the preferred frame matches the predictions of general relativity (at least once asymptotically flat boundary conditions are imposed), and thus coincides with the geometry of a star at rest relative to the preferred frame. This fact, obvious in general relativity because of Lorentz symmetry, is non-trivial in khronometric theory, where Lorentz symmetry is violated. We also stress that while the metric shows no deviations from general relativity at first order in velocity, the æther field is non-trivial and corresponds to a flow moving toward the star at spatial infinity. Necessarily, however, the æther stress energy (and thus its backreaction on the metric) vanish everywhere, i.e. the æther behaves as a "stealth" field.

Let us also note that our findings confirm explicitly, in this particular case, the results of Refs. [57, 58], which showed, via a Hamiltonian analysis, that khronometric theory (in vacuum and with asymptotically flat boundary conditions) should reduce to general relativity exactly for $\alpha = \beta = 0$. We note, however, that the results presented here for the exterior geometry, and namely the exact solution for h [Eq. (38)], show that even in vacuum there exist asymptotically flat solutions to khronometric theory with $\alpha = \beta = 0$ that do *not* match those of general relativity, even though they generically present (presumably naked) singularities. These vacuum solutions can indeed be obtained by setting $k_2 = -2k_1/(3r_s^3)$ in Eq. (38).

Finally, let us note that even if one were willing to accept curvature singularities at the universal horizon of moving black holes in khronometric theory, the dimensionless parameters α and β would still have to be very small to pass experimental bounds ($|\alpha| \lesssim 10^{-5}$, $|\beta| \lesssim 10^{-15}$). Therefore, the sensitivities for finite but experimentally viable α and β are expected to be of the order of $|\sigma| \approx |\sigma(\alpha = \beta = 0) + (\partial\sigma/\partial\alpha)\alpha + (\partial\sigma/\partial\beta)\beta| \lesssim 10^{-5}$ (assuming the derivatives in this Taylor expansion are of order ~ 1). These values seem outside the reach of not only existing gravitational wave detectors, but also of third generation ones/LISA [59, 60], because in the limit $\alpha, \beta \to 0$ and $\lambda \neq 0$ the spin-0 speed diverges and the khronon becomes non-dynamical, which further suppresses deviations from general relativity in the gravitational fluxes (c.f. Sec. III). However, sensitivities $\sigma \sim 10^{-5}$ may be testable with observations of the Newtonian dynamics of triple pulsar systems (for instance PSR J0337+1715 [51]). Tests of the 1PN dynamics of binary pulsars may also yield useful constraints, though they require computing not only the sensitivities but also their derivatives σ'. Moreover, further constraints on khronometric gravity might come from cosmological observations, which might have the potential to further constrain λ, whose viable range ($|\lambda| \lesssim 0.01 - 0.1$) is still sizable.

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