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Massive fermion model in 3d and higher spin currents

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Abstract: We analyze the 3d free massive fermion theory coupled to external sources. The presence of a mass explicitly breaks parity invariance. We calculate two- and three-point functions of a gauge current and the energy momentum tensor and, for instance, obtain the well-known result that in the IR limit (but also in the UV one) we reconstruct the relevant CS action. We then couple the model to higher spin currents and explicitly work out the spin 3 case. In the UV limit we obtain an effective action which was proposed many years ago as a possible generalization of spin 3 CS action. In the IR limit we derive a different higher spin action. This analysis can evidently be generalized to higher spins. We also discuss the conservation and properties of the correlators we obtain in the intermediate steps of our derivation.

Keywords: Higher Spin Gravity, Chern-Simons Theories, Field Theories in Lower Dimensions

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1 Introduction

In the latest years, field theories, and especially conformal field theories, in 3d have become a favorite ground of research. The motivations for this are related both to gravity and to condensed matter, see for instance [1–3] and references therein, based on AdS/CFT correspondence, where 3d can feature on both sides of duality. Also higher spin/CFT correspondence has raised interest on weakly coupled CFT in 3d, [4–6]. In this context many 3d models, disregarded in the past, are being reconsidered [7, 8]. This paper is devoted to the free massive fermion model in 3d coupled to various sources. Unlike the free massless fermion, [9], this model has not been extensively studied, although examples of research in this direction exist, see for instance [10–12] and also [4], and for the massless scalar model [13]. Its prominent property, as opposed to the massless one,[1] is that the fermion mass parameter m breaks parity invariance, and this feature has nontrivial consequences even when $m \to 0$. In this paper we intend to analyze it more in depth. We will couple it to various external sources, not only to a gauge field and a metric, but also to higher tensor fields.

We are interested in the one-loop effective action, in particular in the local part of its UV and IR limits. These contributions are originated by contact terms of the correlators (for related aspects concerning contact terms, see [16–19]). To do so we evaluate the 2-point correlators, and in some cases also the 3-point correlators, of various currents. Our method of calculation is based on Feynman diagrams and dimensional regularization. Eventually we take the limit of high and low energy compared to the mass m of the fermion. In this way we recover some well-known results, [7, 11, 20], and others which are perhaps not so well-known: in the even parity sector the correlators are those (conformal covariant) expected for the a free massless theory; in the odd parity sector the IR limit of the effective action coincides with the gauge and gravity Chern-Simons (CS) action, but also the UV limit lends itself to a similar interpretation provided we use a suitable scaling limit. We also couple the same theory to higher spin symmetric fields. The result we obtain in this case for the spin 3 current in the UV limit is a generalized CS action. We recover in this way theories proposed long ago from a completely different point of view, [21]. In the IR limit we obtain a different higher spin action.

We remark that in general the IR and UV correlators in the even sector are non-local, while the correlators in the odd-parity sector are local, i.e. made of contact terms (for related aspects, see [7]).

[1] The free massless Majorana model is plagued by a sign ambiguity in the definition of the partition function, [14, 15]. This should not be the case for the massive model. This problem is anyhow under investigation.

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Apart from the final results we find other interesting things in our analysis. For instance the odd parity correlators we find as intermediate results are conformal invariant at the fixed point. However, although we obtain them by taking limits of a free field theory, these correlators cannot be obtained from any known free field theory (using the Wick theorem). Another interesting aspect is connected to the breaking of gauge or diffeomorphism symmetry in the process of taking the IR and UV limits in three-point functions. Although we use analytic regularization, when taking these limits we cannot prevent a breaking of symmetry in the correlators. They have to be 'repaired' by adding suitable counterterms to the effective action.

The paper is organized as follows. The next section is preparatory, we introduce the notation, define the higher spin currents and the generating functions for n-point correlators. Section 3 is devoted to two-point functions of gauge currents, of the e.m. tensor and of the spin three currents. In particular the local odd-parity action extracted from these correlators in the UV limit is identified with an action first introduced in ref. [21]. Section 4 is auxiliary: we discuss CS actions and their invariance analyzed with the tool of perturbative cohomology. Section 5 is devoted to the three-point functions of currents, and to the rather complicated issue of conservation. In section 6 we analyze three-point functions of the e.m. tensor and their IR and UV limits. Finally section 7 contain our conclusions. Several appendices are devoted to particular issues, to introduce auxiliary material or to show explicit calculations.

2 The 3d massive fermion model coupled to external sources

The simplest model is that of a Dirac fermion[2] coupled to a gauge field. The action is

$$S[A] = \int d^3x \, \left[i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi \right], \quad D_\mu = \partial_\mu + A_\mu, \tag{2.1}$$

where $A_\mu = A^a_\mu(x) T^a$ and T^a are the generators of a gauge algebra in a given representation determined by ψ. We will use the antihermitean convention, so $[T^a, T^b] = f^{abc}T^c$, and the normalization $\mathrm{tr}(T^a T^b) = \delta^{ab}$.

The current

$$J^a_\mu(x) = \bar{\psi}\gamma_\mu T^a \psi \tag{2.2}$$

is (classically) covariantly conserved on shell as a consequence of the gauge invariance of (2.1)

$$(DJ)^a = (\partial^\mu \delta^{ac} + f^{abc}A^{b\mu})J^c_\mu = 0. \tag{2.3}$$

The next example involves the coupling to gravity

$$S[g] = \int d^3x \, e \left[i\bar{\psi}E^\mu_a \gamma^a \nabla_\mu \psi - m\bar{\psi}\psi \right], \quad \nabla_\mu = \partial_\mu + \frac{1}{2}\omega_{\mu bc}\Sigma^{bc}, \quad \Sigma^{bc} = \frac{1}{4}\left[\gamma^b, \gamma^c\right]. \tag{2.4}$$

[2]The minimal representation of the Lorentz group in 3d is a real Majorana fermion. A Dirac fermion is a complex combination of two Majorana fermions. The action for a Majorana fermion is $\frac{1}{2}$ of (2.1).

The corresponding energy momentum tensor

$$T_{\mu\nu} = \frac{i}{4}\bar{\psi}\left(\gamma_\mu \overset{\leftrightarrow}{\partial}_\nu + \gamma_\nu \overset{\leftrightarrow}{\partial}_\mu\right)\psi \tag{2.5}$$

is covariantly conserved on shell as a consequence of the diffeomorphism invariance of the action,

$$\nabla^\mu T_{\mu\nu}(x) = 0. \tag{2.6}$$

However we can couple the fermions to more general fields. Consider the free action

$$S = \int d^3x \, \left[i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi\right], \tag{2.7}$$

and the spin three conserved current

$$\begin{aligned} J_{\mu_1\mu_2\mu_3} = & \frac{1}{2}\bar{\psi}\gamma_{(\mu_1}\partial_{\mu_2}\partial_{\mu_3)}\psi + \frac{1}{2}\partial_{(\mu_1}\partial_{\mu_2}\bar{\psi}\gamma_{\mu_3)}\psi - \frac{5}{3}\partial_{(\mu_1}\bar{\psi}\gamma_{\mu_2}\partial_{\mu_3)}\psi \\ & + \frac{1}{3}\eta_{(\mu_1\mu_2}\partial^\sigma\bar{\psi}\gamma_{\mu_3)}\partial_\sigma\psi - \frac{m^2}{3}\eta_{(\mu_1\mu_2}\bar{\psi}\gamma_{\mu_3)}\psi. \end{aligned} \tag{2.8}$$

Using the equation of motion one can prove that

$$\partial^\mu J_{\mu\nu\lambda} = 0, \tag{2.9}$$

$$J_\mu{}^\mu{}_\lambda = \frac{4}{9}m\left(-i\partial_\lambda\bar{\psi}\psi + i\bar{\psi}\partial_\lambda\psi + 2\bar{\psi}\gamma_\lambda\psi\right). \tag{2.10}$$

Therefore, the spin three current (2.8) is conserved on shell and its tracelessness is softly broken by the mass term. Similarly to the gauge field and the metric, we can couple the fermion ψ to a new external source $b_{\mu\nu\lambda}$ by adding to (2.7) the term

$$\int d^3x J_{\mu\nu\lambda}b^{\mu\nu\lambda}. \tag{2.11}$$

Notice that this requires b to have canonical dimension -1. Due to the (on shell) current conservation this coupling is invariant under the (infinitesimal) gauge transformations

$$\delta b_{\mu\nu\lambda} = \partial_{(\mu}\Lambda_{\nu\lambda)}, \tag{2.12}$$

where round brackets stand for symmetrization. In the limit $m \to 0$ we have also invariance under the local transformations

$$\delta b_{\mu\nu\lambda} = \Lambda_{(\mu}\eta_{\nu\lambda)}, \tag{2.13}$$

which are usually referred to as (generalized) Weyl transformations and which induce the tracelessness of $J_{\mu\nu\lambda}$ in any couple of indices.

The construction of conserved currents can be generalized as follows, see [8, 9]. There is a generating function for $J^{(n)}$. Introduce the following symbols

$$u_\mu = \overrightarrow{\partial}_\mu, \qquad v_\mu = \overleftarrow{\partial}_\mu, \qquad \langle uv\rangle = u^\mu v_\mu, \qquad \langle uz\rangle = u^\mu z_\mu, \qquad \langle\gamma z\rangle = \gamma^\mu z_\mu, \qquad \text{etc},$$

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where z^μ are external parameters. Now define

$$J(x;z)=\sum_n J^{(n)}_{\mu_1\dots\mu_n} z^{\mu_1}\dots z^{\mu_n}=\bar\psi\langle\gamma z\rangle F(u,v,z)\psi, \tag{2.14}$$

where

$$F(u,v,z)=e^{(\langle uz\rangle-\langle vz\rangle)}\ f(X),\qquad f(X)=\frac{\sinh\sqrt{X}}{\sqrt{X}},\qquad X=2\langle uv\rangle\langle zz\rangle-4\langle uz\rangle\langle vz\rangle. \tag{2.15}$$

Defining next the operator $\mathcal{D}=\langle(u+v)\frac{\partial}{\partial z}\rangle$, it is easy to prove that, using the free equation of motion,

$$\mathcal{D}J(x;z)=0. \tag{2.16}$$

Therefore all the homogeneous terms in z in $J(x;z)$ are conserved if $m=0$. If $m\neq 0$ one has to replace X with $Y=X-2m^2\langle zz\rangle$. Then we define

$$J_m(x;z)=\sum_n J^{(n)}_{\mu_1\dots\mu_n} z^{\mu_1}\dots z^{\mu_n}=\bar\psi\langle\gamma z\rangle e^{(\langle uz\rangle-\langle vz\rangle)}\ f(Y)\psi \tag{2.17}$$

and one can prove that

$$\mathcal{D}J_m(x;z)=0, \tag{2.18}$$

with $m\neq 0$. The case $J^{(3)}$ in (2.17) coincides with the third order current introduced before.

For any conserved current $J^{(n)}_{\mu_1\dots\mu_n}$ we can introduce an associated source field $b^{\mu_1\dots\mu_n}$ similar to the rank three one introduced above, with a transformation law that generalizes (2.12). However, in this regard, a remark is in order. In fact, (2.12) has to be understood as the transformation of the fluctuating field $b_{\mu\nu\lambda}$, which is the lowest order term in the expansion of a field $B_{\mu\nu\lambda}=b_{\mu\nu\lambda}+\dots$ whose background value is 0. $b_{\mu\nu\lambda}$ plays a role similar to $h_{\mu\nu}$ in the expansion of the metric $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}+\dots$ (see also appendix B). In order to implement full invariance we should introduce in the free action the analog of the spin connection for $B_{\mu\nu\lambda}$ and a full covariant conservation law would require introducing in (2.9) the analog of the Christoffel symbols.

2.1 Generating function for effective actions

The generating function of the effective action of (2.1) is

$$W[A]=\sum_{n=1}^{\infty}\frac{i^{n+1}}{n!}\int\prod_{i=1}^{n}d^3x_i A^{a_1\mu_1}(x_1)\dots A^{a_n\mu_n}(x_n)\langle 0|\mathcal{T}J^{a_1}_{\mu_1}(x_1)\dots J^{a_n}_{\mu_n}(x_n)|0\rangle, \tag{2.19}$$

where the time ordered correlators are understood to be those obtained with the Feynman rules. The full one-loop 1-pt correlator for J^a_μ is

$$\begin{aligned}\langle\!\langle J^a_\mu(x)\rangle\!\rangle &= \frac{\delta W[A]}{\delta A^{a\mu}(x)} \\ &= -\sum_{n=1}^{\infty}\frac{i^n}{n!}\int\prod_{i=1}^{n}d^3x_i A^{a_1\mu_1}(x_1)\dots A^{a_n\mu_n}(x_n)\langle 0|\mathcal{T}J^a_\mu(x)J^{a_1}_{\mu_1}(x_1)\dots J^{a_n}_{\mu_n}(x_n)|0\rangle.\end{aligned} \tag{2.20}$$

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Later on we will need also the one-loop conservation

$$(D^\mu \langle\!\langle J_\mu(x) \rangle\!\rangle)^a = \partial^\mu \langle\!\langle J^a_\mu(x) \rangle\!\rangle + f^{abc} A^b_\mu(x) \langle\!\langle J^{\mu c}(x) \rangle\!\rangle = 0. \tag{2.21}$$

We can easily generalize this to the case of higher tensor currents $J^{(p)}$. The generating function is

$$W^{(p)}[a] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} d^3x_i a^{\mu_{11}\dots\mu_{1p}}(x_1) \dots a^{\mu_{n1}\dots\mu_{np}}(x_n) \\ \times \langle 0|\mathcal{T} J^{(p)}_{\mu_{11}\dots\mu_{1p}}(x_1) \dots J^{(p)}_{\mu_{n1}\dots\mu_{np}}(x_n)|0\rangle. \tag{2.22}$$

In particular $a_{\mu\nu} = h_{\mu\nu}$ and $J^{(2)}_{\mu\nu} = T_{\mu\nu}$, and $a_{\mu\nu\lambda} = b_{\mu\nu\lambda}$. The full one-loop 1-pt correlator for J^a_μ is

$$\langle\!\langle J^{(p)}_{\mu_1\dots\mu_p}(x) \rangle\!\rangle = \frac{\delta W[a,p]}{\delta a^{\mu_1\dots\mu_p}(x)} = -\sum_{n=1}^{\infty} \frac{i^n}{n!} \int \prod_{i=1}^{n} d^3x_i a^{\mu_{11}\dots\mu_{1p}}(x_1) \dots a^{\mu_{n1}\dots\mu_{np}}(x_n) \\ \times \langle 0|\mathcal{T} J^{(p)}_{\mu_1\dots\mu_p}(x) J^{(p)}_{\mu_{11}\dots\mu_{1p}}(x_1) \dots J^{(p)}_{\mu_{n1}\dots\mu_{np}}(x_n)|0\rangle. \tag{2.23}$$

The full one-loop conservation law for the energy-momentum tensor is

$$\nabla^\mu \langle\!\langle T_{\mu\nu}(x) \rangle\!\rangle = 0. \tag{2.24}$$

A similar covariant conservation should be written also for the other currents, but in this paper for $p > 2$ we will content ourselves with the lowest nontrivial order in which the conservation law reduces to

$$\partial^{\mu_1} \langle\!\langle J^{(p)}_{\mu_1\dots\mu_p}(x) \rangle\!\rangle = 0. \tag{2.25}$$

Warning. One must be careful when applying the previous formulas for generating functions. If the expression $\langle 0|\mathcal{T} J^{(p)}_{\mu_{11}\dots\mu_{1p}}(x_1) \dots J^{(p)}_{\mu_{n1}\dots\mu_{np}}(x_n)|0\rangle$ in (2.22) is meant to denote the n-th point-function calculated by using Feynman diagrams, a factor i^n is already included in the diagram themselves and so it should be dropped in (2.22). When the current is the energy-momentum tensor an additional precaution is necessary: the factor $\frac{i^{n+1}}{n!}$ must be replaced by $\frac{i}{2^n n!}$. The factor $\frac{1}{2^n}$ is motivated by the fact that when we expand the action

$$S[\eta + h] = S[\eta] + \int d^dx \frac{\delta S}{\delta g^{\mu\nu}}\Big|_{g=\eta} h^{\mu\nu} + \cdots,$$

the factor $\frac{\delta S}{\delta g^{\mu\nu}}\Big|_{g=\eta} = \frac{1}{2} T_{\mu\nu}$. Another consequence of this fact will be that the presence of vertices with one graviton in Feynman diagrams will correspond to insertions of the operator $\frac{1}{2} T_{\mu\nu}$ in correlation functions.

2.2 General structure of 2-point functions of currents

In order to compute the generating function (effective action) W we will proceed in the next section to evaluate 2-point and 3-point correlators using the Feynman diagram approach.

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It is however possible to derive their general structure on the basis of covariance. In this subsection we will analyze the general form of 2-point correlators.

As long as 2-point correlators of currents are involved the conservation law is simply represented by the vanishing of the correlator divergence:

$$\partial^{\mu_1}\langle 0|\mathcal{T}J^{(p)}_{\mu_1\ldots\mu_p}(x)J^{(p)}_{\nu_1\ldots\nu_p}(y)|0\rangle = 0. \tag{2.26}$$

Using Poincaré covariance and this equation we can obtain the general form of the correlators in momentum space in terms of distinct tensorial structures and form factors. Denoting by

$$\tilde{J}_{\mu_1\ldots\mu_p,\nu_1\ldots\nu_p}(k) = \langle \tilde{J}^{(p)}_{\mu_1\ldots\mu_p}(k)\tilde{J}^{(p)}_{\nu_1\ldots\nu_p}(-k)\rangle \tag{2.27}$$

the Fourier transform of the 2-point function, the conservation is simply represented by the contraction of $\tilde{F}_{\mu\ldots}$ with k^μ:

$$k^{\mu_1}\tilde{J}_{\mu_1\ldots\mu_p,\nu_1\ldots\nu_p}(k) = 0. \tag{2.28}$$

The result is as follows. For 1-currents we have

$$\tilde{J}^{ab}_{\mu\nu}(k) = \langle \tilde{J}^a_\mu(k)\tilde{J}^b_\nu(-k)\rangle = \delta^{ab}\left[\tau\left(\frac{k^2}{m^2}\right)\frac{k_\mu k_\nu - k^2\eta_{\mu\nu}}{16|k|} + \kappa\left(\frac{k^2}{m^2}\right)\frac{k^\tau\epsilon_{\tau\mu\nu}}{2\pi}\right]. \tag{2.29}$$

where $|k| = \sqrt{k^2}$ and τ, κ are model dependent form factors.

The most general 2-point function for the energy-momentum tensor has the form

$$\begin{aligned}\left\langle \tilde{T}_{\mu\nu}(k)\,\tilde{T}_{\rho\sigma}(-k)\right\rangle = &\frac{\tau_g\left(k^2/m^2\right)}{|k|}\left(k_\mu k_\nu - \eta_{\mu\nu}k^2\right)\left(k_\rho k_\sigma - \eta_{\rho\sigma}k^2\right)\\ &+\frac{\tau'_g\left(k^2/m^2\right)}{|k|}\left[\left(k_\mu k_\rho - \eta_{\mu\rho}k^2\right)\left(k_\nu k_\sigma - \eta_{\nu\sigma}k^2\right) + \mu\leftrightarrow\nu\right]\\ &+\frac{\kappa_g\left(k^2/m^2\right)}{192\pi}\left[\left(\epsilon_{\mu\rho\tau}k^\tau\left(k_\nu k_\sigma - \eta_{\nu\sigma}k^2\right) + \rho\leftrightarrow\sigma\right) + \mu\leftrightarrow\nu\right].\end{aligned} \tag{2.30}$$

where τ_g, τ'_g and κ_g are model-dependent form-factors. Vanishing of traces over $(\mu\nu)$ or $(\rho\sigma)$ requires $\tau_g + \tau'_g = 0$. Both here and in the previous case, the notation, the signs and the numerical factors are made to match our definition with the ones used in [7].[3]

As for the order 3 tensor currents the most general form of the 2-point function in momentum representation is

$$\begin{aligned}\langle \tilde{J}_{\mu_1\mu_2\mu_3}(k)\tilde{J}_{\nu_1\nu_2\nu_3}(-k)\rangle = &\,\tau_b\left(\frac{k^2}{m^2}\right)|k|^5\pi_{\mu_1\mu_2}\pi_{\mu_3\nu_1}\pi_{\nu_2\nu_3} + \tau'_b\left(\frac{k^2}{m^2}\right)|k|^5\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\\ &+k^4\epsilon_{\mu_1\nu_1\sigma}k^\sigma\left[\kappa_b\left(\frac{k^2}{m^2}\right)\pi_{\mu_2\mu_3}\pi_{\nu_2\nu_3} + \kappa'_b\left(\frac{k^2}{m^2}\right)\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right],\end{aligned} \tag{2.31}$$

[3] Except that we work in spacetime with Lorentzian signature $(+--)$.

where complete symmetrisation of the indices (μ_1, μ_2, μ_3) and (ν_1, ν_2, ν_3) is implicit[4] and

$$\pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \tag{2.32}$$

is the transverse projector. This expression is, by construction, conserved but not traceless. Vanishing of traces requires

$$4\tau_b + 3\tau_b' = 0, \qquad 4\kappa_b + \kappa_b' = 0. \tag{2.33}$$

3 Two-point functions

In this section we compute the the 2-point function of spin 1, 2 and 3 currents using Feynman diagrams with finite mass m. Then we take the limit $m \to 0$ or $m \to \infty$ with respect to the total energy of the process, i.e. the UV and IR limit of the 2-point functions, respectively. These are expected to correspond to 2-point functions of conformal field theories at the relevant fixed points. We will be mostly interested in the odd parity part of the correlators, because in the UV and IR limit they give rise to local effective actions, but occasionally we will also consider the even parity part.

3.1 Two-point function of the current $J^a_\mu(x)$

This case has been treated in [16], therefore we will be brief. The only contribution comes from the bubble diagram with external momentum k and momentum p in the fermion loop. In momentum representation we have

$$\begin{aligned}\tilde{J}^{ab}_{\mu\nu}(k) = &-\int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\gamma_\mu T^a \frac{1}{\not{p} - m}\gamma_\nu T^b \frac{1}{\not{p} - \not{k} - m}\right) = -2\delta^{ab} \\ &\times \int \frac{d^3p}{(2\pi)^3}\frac{p_\nu(p-k)_\mu - p\cdot(p-k)\eta_{\mu\nu} + p_\mu(p-k)_\nu + im\epsilon_{\mu\nu\sigma}k^\sigma + m^2\eta_{\mu\nu}}{(p^2-m^2)((p-k)^2-m^2)}\end{aligned} \tag{3.1}$$

For the even parity part we get

$$\tilde{J}^{ab(even)}_{\mu\nu}(k) = \frac{2i}{\pi}\delta^{ab}\left[\left(1 + \frac{4m^2}{k^2}\right)\mathrm{arctanh}\left(\frac{|k|}{2|m|}\right) - \frac{2|m|}{|k|}\right]\frac{k_\mu k_\nu - k^2\eta_{\mu\nu}}{16|k|}, \tag{3.2}$$

while for the odd parity part we get

$$\tilde{J}^{ab(\mathrm{odd})}_{\mu\nu}(k) = \frac{1}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^\sigma \frac{m}{|k|}\mathrm{arctanh}\left(\frac{|k|}{2|m|}\right) \tag{3.3}$$

where $|k| = \sqrt{k^2}$. The conservation law (2.28) is readily seen to be satisfied. In the following we are going to consider the IR and UV limit of the expressions (3.2) and (3.3) and it is important to remark that we have two possibilities here: we may consider a

[4] When we say that the complete symmetrisation is implicit it means that one should understand, for instance

$$\pi_{\mu_1\mu_2}\pi_{\mu_3\nu_1}\pi_{\nu_2\nu_3} \to \frac{1}{9}\left[\pi_{\mu_1\mu_2}\pi_{\mu_3\nu_1}\pi_{\nu_2\nu_3} + \pi_{\mu_1\mu_3}\pi_{\mu_2\nu_1}\pi_{\nu_2\nu_3} + \dots\right].$$

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timelike momentum ($k^2 > 0$) or a spacelike one ($k^2 < 0$). In the first case, we must notice that the function arctanh $\left(\frac{|k|}{2|m|}\right)$ has branch-cuts on the real axis for $\frac{|k|}{2|m|} > 1$ and it acquires an imaginary part. On the other hand, if we consider spacelike momenta, we will have arctanh $\left(\frac{i|k|}{2|m|}\right) = i \arctan\left(\frac{|k|}{2|m|}\right)$ and arctan $\left(\frac{|k|}{2|m|}\right)$ is real on the real axis. The region of spacelike momenta reproduces the Euclidean correlators. Throughout this paper we will always consider UV and IR limit as being respectively the limits of very large or very small spacelike momentum with respect to the mass scale m. In these two limits we get

$$\tilde{J}^{ab(\text{even})}_{\mu\nu}(k) = \frac{i}{8\pi}\delta^{ab}\frac{k_\mu k_\nu - k^2\eta_{\mu\nu}}{|k|}\begin{cases}\frac{2|k|}{3|m|} & \text{IR}\\ \frac{\pi}{2} & \text{UV}\end{cases}, \tag{3.4}$$

$$\tilde{J}^{ab(\text{odd})}_{\mu\nu}(k) = \frac{1}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^\sigma\begin{cases}\frac{1}{2}\frac{m}{|m|} & \text{IR}\\ \frac{\pi}{2}\frac{m}{|k|} & \text{UV}\end{cases}. \tag{3.5}$$

The UV limit is actually vanishing in the odd case (this is also the case for all the 2-point functions we will meet in the following). However we can consider a model made of N identical copies of free fermions coupled to the same gauge field. Then the result (3.5) would be

$$\tilde{J}^{ab(\text{odd})}_{\mu\nu}(k) = \frac{N}{4}\delta^{ab}\epsilon_{\mu\nu\sigma}k^\sigma\frac{m}{|k|}. \tag{3.6}$$

In this case we can consider the scaling limit $\frac{m}{|k|} \to 0$ and $N \to \infty$ in such a way that $N\frac{m}{|k|}$ is fixed. Then the UV limit (3.6) becomes nonvanishing.

Fourier transforming (3.5) and inserting the result in the generating function (2.19) we get the first (lowest order) term of the CS action

$$\begin{aligned} CS &= \frac{\kappa}{4\pi}\int d^3x \text{Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) \\ &= \frac{\kappa}{4\pi}\int d^3x\epsilon^{\mu\nu\lambda}\left(A^a_\mu\partial_\nu A^a_\lambda + \frac{1}{3}f^{abc}A^a_\mu A^b_\nu A^c_\lambda\right). \end{aligned} \tag{3.7}$$

In particular, from (3.5) we see that in the IR limit $\kappa = \pm\frac{1}{2}$. The CS action (3.7) is invariant not only under the infinitesimal gauge transformations

$$\delta A = d\lambda + [A,\lambda], \qquad \lambda = \lambda^a(x)T^a, \tag{3.8}$$

but also under large gauge transformations when $\kappa \in \mathbb{Z}$. From (3.5) follows that $\kappa_{\text{UV}} = 0$ and $\kappa_{\text{IR}} = \pm 1/2$, which suggests that the gauge symmetry is broken unless there is an even number of fermions. A further discussion of this phenomenon can be found in [7].

3.2 Two-point function of the e.m. tensor

The lowest term of the effective action in an expansion in $h_{\mu\nu}$ come from the two-point function of the e.m. tensor. So we now set out to compute the latter. The correlators of

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the e.m. tensor will be denoted with the letter $\tilde{T}$ instead of $\tilde{J}$. The Feynman propagator and vertices are given in appendix B. For simplicity from now on we assume $m > 0$.

The bubble diagram (one graviton entering and one graviton exiting with momentum k, one fermionic loop) contribution to the e.m. two-point function is given in momentum space by

$$\tilde{T}_{\mu\nu\lambda\rho}(k) = -\frac{1}{64}\int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\frac{1}{\not{p}-m}(2p-k)_\mu\gamma_\nu\frac{1}{\not{p}-\not{k}-m}(2p-k)_\lambda\gamma_\rho\right), \tag{3.9}$$

where symmetrization over (μ,ν) and (λ,ρ) will be always implicit.

3.2.1 The odd parity part

The odd-parity part of (3.9) is

$$\tilde{T}^{(\mathrm{odd})}_{\mu\nu\lambda\rho}(k) = \frac{im}{32}\int_0^1 dx\int\frac{d^3p}{(2\pi)^3}\epsilon_{\sigma\nu\rho}\,k^\sigma\,\frac{(2p+(2x-1)k)_\mu\,(2p+(2x-1)k)_\lambda}{[p^2-m^2+x(1-x)k^2]^2}. \tag{3.10}$$

The evaluation of this integral is described in detail in appendix D. The result is

$$\begin{aligned}\tilde{T}^{(\mathrm{odd})}_{\mu\nu\lambda\rho}(k) = &-\frac{3m}{4|k|}\left[\left(1-\frac{4m^2}{k^2}\right)\mathrm{arctanh}\left(\frac{|k|}{2|m|}\right)+\frac{2|m|}{|k|}\right]\frac{\epsilon_{\mu\lambda\sigma}k^\sigma\left(k_\nu k_\rho-\eta_{\nu\rho}k^2\right)}{192\pi}\\ &-\frac{\mathrm{sign}(m)|m|^2}{64\pi}\epsilon_{\mu\lambda\sigma}k^\sigma\eta_{\nu\rho}.\end{aligned} \tag{3.11}$$

A surprising feature of (3.11) is that if we contract it with k^μ we do not get zero. Let us look closer into this problem.

3.2.2 The divergence of the e.m. tensor: odd-parity part

To see whether the expression of the one-loop effective action is the legitimate one, one must verify that the procedure to obtain it does not break diffeomorphism invariance. The bubble diagram contribution to the divergence of the e.m. tensor is

$$\begin{aligned}k^\mu\tilde{T}_{\mu\nu\lambda\rho}(k) = &-\frac{1}{64}\int\frac{d^3p}{(2\pi)^3}\left[\mathrm{Tr}\left(\frac{1}{\not{p}-m}(2p-k)\cdot k\,\gamma_\nu\frac{1}{\not{p}-\not{k}-m}(2p-k)_\lambda\gamma_\rho\right)\right.\\ &\left.+\,\mathrm{Tr}\left(\frac{1}{\not{p}-m}(2p-k)_\nu\not{k}\frac{1}{\not{p}-\not{k}-m}(2p-k)_\lambda\gamma_\rho\right)\right]+(\lambda\leftrightarrow\rho)\,.\end{aligned} \tag{3.12}$$

Repeating the same calculation as above one finally finds

$$k^\mu\tilde{T}^{(\mathrm{odd})}_{\mu\nu\lambda\rho}(k) = -\frac{\mathrm{sign}(m)|m|^2}{64\pi}\epsilon_{\sigma\nu\rho}\,k^\sigma k_\lambda+(\lambda\leftrightarrow\rho)\,. \tag{3.13}$$

This is a local expression. It corresponds to the anomaly

$$\Delta_\xi = -\frac{\mathrm{sign}(m)|m|^2}{32\pi}\int\epsilon_{\sigma\nu\rho}\xi^\nu\partial^\sigma\partial_\lambda h^{\lambda\rho}. \tag{3.14}$$

The counterterm to cancel it is

$$\mathcal{C} = \frac{\mathrm{sign}(m)|m|^2}{64\pi}\int\epsilon_{\sigma\nu\rho}h^\nu_\lambda\partial^\sigma h^{\lambda\rho}. \tag{3.15}$$

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Once this is done the final result is

$$\langle T_{\mu\nu}(k)\, T_{\lambda\rho}(-k)\rangle_{\text{odd}} = \frac{\kappa_g(k^2/m^2)}{192\pi}\,\epsilon_{\sigma\nu\rho}\,k^\sigma\left(k_\mu k_\lambda - k^2\eta_{\mu\lambda}\right) + \begin{pmatrix}\mu\leftrightarrow\nu\\ \lambda\leftrightarrow\rho\end{pmatrix} \tag{3.16}$$

with

$$\kappa_g(k^2/m^2) = -\frac{3m}{|k|}\left[\left(1-\frac{4m^2}{k^2}\right)\operatorname{arctanh}\left(\frac{|k|}{2|m|}\right)+\frac{2|m|}{|k|}\right]. \tag{3.17}$$

Now (3.16) is conserved and traceless. To obtain (3.17) we have to recall that

$$\tilde{T}_{\mu\nu\lambda\rho}(k) = \frac{1}{4}\langle T_{\mu\nu}(k)\, T_{\lambda\rho}(-k)\rangle, \tag{3.18}$$

which was explained in the warning of section 2.1. To complete the discussion we should also take into account a tadpole graph which might contribute to the two-point function. With the vertex V_{ggff} it is in fact possible to construct such a graph. It yields the contribution

$$\frac{3}{32\pi}\text{sign}(m)|m|^2\, t_{\mu\nu\lambda\rho\sigma}\, k^\sigma. \tag{3.19}$$

This term violates conservation, just as the previous (3.13), but it has a different coefficient. So it must be subtracted in the same way.

3.2.3 The UV and IR limit

Let us set $\lim_{|\frac{m}{k}|\to 0}\kappa_g = \kappa_{\text{UV}}$, and $\lim_{|\frac{k}{m}|\to 0}\kappa_g = \kappa_{\text{IR}}$. We get

$$\kappa_{\text{IR}} = \frac{m}{|m|}, \qquad \kappa_{\text{UV}} = \frac{3}{2}\pi\frac{m}{|k|} = 0 + \mathcal{O}\left(\left|\frac{m}{k}\right|\right). \tag{3.20}$$

As before for the gauge case, in the UV limit we can get a finite result by considering a system of N identical fermions. Then the above 2-point function gets multiplied by N. In the UV limit, $\left|\frac{m}{k}\right| \to 0$, we can consider the scaling limit $N\to\infty, \left|\frac{m}{k}\right|\to 0$ such that

$$\lambda = N\frac{m}{|k|} \tag{3.21}$$

is fixed and finite. In this limit

$$\lim_{N\to\infty,|\frac{m}{k}|\to 0} N\kappa_g(k) = \frac{3\pi}{2}\frac{m}{|m|}\lambda. \tag{3.22}$$

For a unified treatment let us call both UV and IR limits of κ_g simply κ. In such limits contribution to the parity odd part of the effective action can be easily reconstructed by

$$S^{(\text{odd})}_{\text{eff}} = \frac{\kappa}{192\pi}\int d^3x\,\epsilon_{\sigma\nu\rho}\,h^{\mu\nu}\,\partial^\sigma(\partial_\mu\partial_\lambda - \eta_{\mu\lambda}\Box)\,h^{\lambda\rho}. \tag{3.23}$$

This exactly corresponds to a gravitational CS term in 3d, for which at the quadratic order in $h_{\mu\nu}$ we have

$$\begin{aligned} CS &= -\frac{\kappa}{96\pi}\int d^3x\,\epsilon^{\mu\nu\lambda}\left(\partial_\mu\omega_\nu^{ab}\omega_{\lambda ba} + \frac{2}{3}\omega_{\mu a}{}^b\omega_{\nu b}{}^c\omega_{\lambda c}{}^a\right) \\ &= \frac{\kappa}{192\pi}\int d^3x\,\epsilon_{\sigma\nu\rho}\,h^{\lambda\rho}\left(\partial^\sigma\partial_\lambda\partial_b h^{b\nu} - \partial^\sigma\Box h_\lambda^\nu\right) + \dots \end{aligned} \tag{3.24}$$

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Once again we note that the topological arguments combined with path integral quantization force κ to be an integer ($\kappa \in \mathbb{Z}$). From $\kappa_{\rm IR} = \pm 1$ we see that the quantum contribution to the parity-odd part of the effective action in the IR is given by the local gravitational CS term, with the minimal (positive or negative unit) coupling constant. The constant $\frac{3\pi}{2}\lambda$ in the UV has of course to be integer in order for the action to be well defined also for large gauge transformations. Finally we recall that the CS Lagrangian is diffeomorphism and Weyl invariant up to a total derivative. However, note that for the Majorana fermion one would obtain half of the result as for the Dirac fermion, i.e. $\kappa_{\rm IR} = \pm 1/2$.

3.2.4 Two-point function: even parity part

Although in this paper we are mostly interested in the odd-parity amplitudes, for completeness in the following we calculate also the even parity part of the e.m. tensor 2-point correlator.

The even parity part of the two-point function comes from the bubble diagram alone, eq. (3.9). Proceeding in the same way as above one gets

$$\begin{aligned}\tilde{T}^{(\rm even)}_{\mu\nu\lambda\rho}(k) = &\frac{1}{4}\tau_g\left(\frac{k^2}{m^2}\right)\frac{1}{|k|}\left(k_\mu k_\nu - \eta_{\mu\nu}k^2\right)\left(k_\lambda k_\rho - \eta_{\lambda\rho}k^2\right)\\ &+\frac{1}{4}\tau_g'\left(\frac{k^2}{m^2}\right)\frac{1}{|k|}\left[\left(k_\mu k_\lambda - \eta_{\mu\lambda}k^2\right)\left(k_\nu k_\rho - \eta_{\nu\rho}k^2\right) + \mu \leftrightarrow \nu\right]\\ &-\frac{im^3}{48\pi}\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} + 2\eta_{\mu\nu}\eta_{\lambda\rho}\right),\end{aligned} \tag{3.25}$$

where

$$\tau_g\left(\frac{k^2}{m^2}\right) = \frac{i}{64\pi|k|^3}\left[|m|k^2 + 4|m|^3 - \frac{\left(k^2-4m^2\right)^2}{2|k|}\operatorname{arctanh}\left(\frac{|k|}{2|m|}\right)\right], \tag{3.26}$$

$$\tau_g'\left(\frac{k^2}{m^2}\right) = \frac{i}{64\pi|k|^3}\left[4|m|^3 - |m|k^2 + \frac{\left(k^4-16m^4\right)}{2|k|}\operatorname{arctanh}\left(\frac{|k|}{2|m|}\right)\right]. \tag{3.27}$$

Saturating (3.25) with k^μ we find

$$k^\mu\tilde{T}^{(\rm even)}_{\mu\nu\lambda\rho}(k) = -\frac{im^3}{48\pi}\left(k_\lambda\eta_{\nu\rho} + k_\rho\eta_{\nu\lambda} + 2k_\nu\eta_{\lambda\rho}\right). \tag{3.28}$$

The same result can be obtained directly from the even part of (3.12). The term (3.28) is local and corresponds to an anomaly proportional to

$$\mathcal{A}_\xi = \int \xi^\nu\left(\partial_\lambda h^\lambda_\nu + \partial_\nu h\right). \tag{3.29}$$

This can be eliminated by subtracting the counterterm

$$\mathcal{C} = -\frac{1}{2}\int (h^{\lambda\nu}h_{\lambda\nu} + h^2). \tag{3.30}$$

After this we can write

$$\begin{aligned}\langle T_{\mu\nu}(k)\,T_{\lambda\rho}(-k)\rangle_{\rm even} = &\,\tau_g\left(\frac{k^2}{m^2}\right)\frac{1}{|k|}\left(k_\mu k_\nu - \eta_{\mu\nu}k^2\right)\left(k_\lambda k_\rho - \eta_{\lambda\rho}k^2\right)\\ &+\tau_g'\left(\frac{k^2}{m^2}\right)\frac{1}{|k|}\left[\left(k_\mu k_\lambda - \eta_{\mu\lambda}k^2\right)\left(k_\nu k_\rho - \eta_{\nu\rho}k^2\right) + \mu \leftrightarrow \nu\right].\end{aligned} \tag{3.31}$$

The UV limit gives

$$\lim_{|\frac{k}{m}|\to\infty} \tau_g = -\lim_{|\frac{k}{m}|\to\infty} \tau_g' = \frac{1}{256}, \tag{3.32}$$

so that in this limit

$$\begin{aligned}\langle T_{\mu\nu}(k)\, T_{\lambda\rho}(-k)\rangle^{\rm UV}_{\rm even} = -\frac{i}{256}\frac{1}{|k|}\Big(&\left(k_\mu k_\nu - \eta_{\mu\nu}k^2\right)\left(k_\rho k_\lambda - \eta_{\rho\lambda}k^2\right) \\ &- \left[\left(k_\mu k_\rho - \eta_{\mu\rho}k^2\right)\left(k_\nu k_\lambda - \eta_{\nu\lambda}k^2\right) + \mu \leftrightarrow \nu\right]\Big).\end{aligned} \tag{3.33}$$

This represents the two-point function of a CFT in 3d, which is a free theory, the massless limit of the massive fermion theory we are studying.

The IR limit of the form factors (3.26) and (3.27) is

$$\tau_g = \frac{1}{24\pi}\left|\frac{m}{k}\right| + \mathcal{O}\left(\left|\frac{k}{m}\right|\right), \tag{3.34}$$

$$\tau_g' = -\frac{1}{48\pi}\left|\frac{m}{k}\right| + \mathcal{O}\left(\left|\frac{k}{m}\right|\right). \tag{3.35}$$

In this limit we have

$$\begin{aligned}\langle T_{\mu\nu}(k) T_{\lambda\rho}(-k)\rangle^{\rm IR}_{\rm even} = \frac{i|m|}{96\pi}\Big[&\frac{1}{2}\left(\left(k_\mu k_\lambda \eta_{\nu\rho} + \lambda \leftrightarrow \rho\right) + \mu \leftrightarrow \nu\right) - \left(k_\mu k_\nu \eta_{\lambda\rho} + k_\lambda k_\rho \eta_{\mu\nu}\right) \\ &- \frac{k^2}{2}\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda}\right) + k^2 \eta_{\mu\nu}\eta_{\lambda\rho}\Big].\end{aligned} \tag{3.36}$$

The expression (3.36) is transverse but not traceless because $\tau_g + \tau_g' \overset{\rm IR}{\neq} 0$. To have a well-behaved IR limit we may add local counterterms to cancel the whole IR expression. That may be accomplished by simply performing the shifts

$$\tau_g \to \tau_g - \frac{i}{24\pi}\left|\frac{m}{k}\right|, \quad \tau_g' \to \tau_g' + \frac{i}{48\pi}\left|\frac{m}{k}\right|. \tag{3.37}$$

These shifts correspond to the addition of a set of local counterterms in the expression (3.31) and they do not change the UV behavior since they go to zero in that limit.

3.3 Two-point function of the spin 3 current

Let us recall that we have postulated for the spin 3 current an action term of the form

$$S_{\rm int} \sim \int d^3x\, J_{\mu\nu\lambda} b^{\mu\nu\lambda}, \tag{3.38}$$

where b is a completely symmetric 3rd order tensor (in this subsection we assume $h_{\mu\nu} = 0$ for simplicity). This interaction term gives rise to the following b-field-fermion-fermion vertex V_{bff}

$$\frac{1}{2}\left(\gamma_{(\mu_1} q_{2\mu_2} q_{2\mu_3)} + q_{1(\mu_1} q_{1\mu_2} \gamma_{\mu_3)}\right) - \frac{5}{3} q_{1(\mu_1}\gamma_{\mu_2} q_{2\mu_3)} + \frac{1}{3}\eta_{(\mu_1\mu_2}\gamma_{\mu_3)}\left(q_1 \cdot q_2 + m^2\right), \tag{3.39}$$

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where q_1 and q_2 are the incoming momenta of the two fermions. For a spin n current, the analogous vertex can be obtained from the formula

$$V_{bff} : \langle \gamma z \rangle \, e^{i\langle (q_1-q_2)z\rangle} f\left(-2\langle q_1 q_2\rangle \langle zz\rangle + 4\langle q_1 z\rangle\langle q_2 z\rangle - 2m^2\langle zz\rangle\right) \tag{3.40}$$

by differentiating with respect to z the right number of times (and setting $z=0$).

As usual the contribution from the 2-point function comes from the bubble diagram with incoming and outgoing momentum k_μ. Using the V_{bff} vertex the bubble diagram gives

$$\begin{aligned}\tilde{J}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \int \frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\Bigg(&\frac{1}{\not p - m}\left[\frac{1}{2}\left(\gamma_{(\nu_1}(p-k)_{\nu_2}(p-k)_{\nu_3)} + p_{(\nu_1}p_{\nu_2}\gamma_{\nu_3)}\right)\right.\\ &\left.+\frac{5}{3}p_{(\nu_1}\gamma_{\nu_2}(p-k)_{\nu_3)} - \frac{1}{3}\eta_{(\nu_1\nu_2}\gamma_{\nu_3)}\left(p\cdot(p-k)-m^2\right)\right]\frac{1}{\not p - \not k - m}\\ &\cdot\left[\frac{1}{2}\left(\gamma_{(\mu_1}(p-k)_{\mu_2}(p-k)_{\mu_3)} + p_{(\mu_1}p_{\mu_2}\gamma_{\mu_3)}\right)\right.\\ &\left.+\frac{5}{3}p_{(\mu_1}\gamma_{\mu_2}(p-k)_{\mu_3)} - \frac{1}{3}\eta_{(\mu_1\mu_2}\gamma_{\mu_3)}\left(p\cdot(p-k)-m^2\right)\right]\Bigg).\end{aligned} \tag{3.41}$$

The parity-even part of the final result is given by

$$\begin{aligned}\tilde{J}^{(\mathrm{even})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \tau_b\left(\frac{k^2}{m^2}\right)|k|^5\pi_{\mu_1\mu_2}\pi_{\mu_3\nu_1}\pi_{\nu_2\nu_3} + \tau_b'\left(\frac{k^2}{m^2}\right)|k|^5\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\\ + \mathcal{A}^{(\mathrm{even})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3},\end{aligned} \tag{3.42}$$

where

$$\begin{aligned}\tau_b = \frac{i}{288\pi k^6}\Bigg[&6|k||m|\left(k^4+8k^2m^2-32m^4\right)\\ &-\left(3\left(k^2-4m^2\right)^3+8m^2\left(k^2-6m^2\right)\left(k^2+4m^2\right)\right)\mathrm{arctanh}\left(\frac{|k|}{2|m|}\right)\Bigg],\end{aligned} \tag{3.43}$$

$$\begin{aligned}\tau_b' = \frac{i}{216\pi k^6}\Bigg[&-6|k||m|\left(k^4-\frac{8}{3}k^2m^2+16m^4\right)\\ &+3\left(k^2-4m^2\right)^2\left(k^2+4m^2\right)\mathrm{arctanh}\left(\frac{|k|}{2|m|}\right)\Bigg]\end{aligned} \tag{3.44}$$

and $\mathcal{A}^{(\mathrm{even})}$ corresponds to a set of contact terms that are not transverse but may be subtracted by local counterterms. It is given by

$$\begin{aligned}\mathcal{A}^{(\mathrm{even})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3} = \frac{im^3}{9\pi}\Bigg[&\frac{3}{4}k_{\mu_1}k_{\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} + \frac{7}{8}\left(k_{\mu_1}k_{\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3} + k_{\nu_1}k_{\nu_2}\eta_{\mu_1\mu_2}\eta_{\mu_3\nu_3}\right)\\ &+\frac{32}{15}m^2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3} + \frac{52}{15}m^2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} - \frac{3}{4}k^2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}\Bigg].\end{aligned} \tag{3.45}$$

The parity-odd part is given by

$$\begin{aligned}\tilde{J}^{(\mathrm{odd})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = k^4\epsilon_{\mu_1\nu_1\sigma}k^\sigma\left[\kappa_b\left(\frac{k^2}{m^2}\right)\pi_{\mu_2\mu_3}\pi_{\nu_2\nu_3} + \kappa_b'\left(\frac{k^2}{m^2}\right)\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right]\\ + \mathcal{A}^{(\mathrm{odd})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3},\end{aligned} \tag{3.46}$$

where

$$\kappa_b = \frac{m}{72\pi|k|^5}\left[-20|k|^3|m| + 16|k||m|^3 + \left(k^4 - 32m^4\right)\operatorname{arctanh}\left(\frac{|k|}{2|m|}\right)\right], \quad (3.47)$$

$$\kappa_b' = \frac{m}{18\pi|k|^5}\left[2|k|^3|m| + 8|k||m|^3 - \left(k^2 - 4m^2\right)^2\operatorname{arctanh}\left(\frac{|k|}{2|m|}\right)\right], \quad (3.48)$$

and, as before, $\mathcal{A}^{(\text{odd})}$ corresponds to a set of contact terms that are not transverse but may be subtracted by local counterterms. It is given by

$$\mathcal{A}^{(\text{odd})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3} = -\frac{\text{sign}(m)|m|^2}{16\pi}\epsilon_{\mu_1\nu_1\sigma}k^\sigma\left[(k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3} + k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3}) + \frac{128}{27}m^2\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3} + \frac{32}{27}m^2\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} - k^2\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}\right]. \quad (3.49)$$

3.3.1 Even parity UV and IR limits

In the UV limit, i.e. $\left|\frac{m}{k}\right| \to 0$, we find

$$\lim_{\left|\frac{m}{k}\right|\to 0}\tau_b = -\frac{3}{4}\lim_{\left|\frac{m}{k}\right|\to 0}\tau_b' = \frac{1}{192}. \quad (3.50)$$

In the IR limit, i.e. $\left|\frac{k}{m}\right| \to 0$, we find

$$\tau_b = \frac{8}{135\pi}\left|\frac{m}{k}\right| + \mathcal{O}\left(\left|\frac{k}{m}\right|\right), \quad \tau_b' = -\frac{4}{135\pi}\left|\frac{m}{k}\right| + \mathcal{O}\left(\left|\frac{k}{m}\right|\right). \quad (3.51)$$

As in the case of the IR limit of the 2-point function of the stress-energy tensor, these leading divergent contributions of the form factors give rise to a set of contact terms in the IR that are all proportional to $|m|$. To add counter terms to make the IR well-behaved is equivalent to perform the following shift in the form factors τ_b and τ_b':

$$\tau_b \to \tau_b - \frac{8i}{135\pi}\left|\frac{m}{k}\right|, \quad \tau_b' \to \tau_b' + \frac{4i}{135\pi}\left|\frac{m}{k}\right|. \quad (3.52)$$

3.3.2 Odd parity UV

In the UV limit we find

$$\kappa_b = \frac{1}{144}\frac{m}{|k|} + \mathcal{O}\left(\left|\frac{m}{k}\right|^2\right), \quad \kappa_b' = -\frac{1}{36}\frac{m}{|k|} + \mathcal{O}\left(\left|\frac{m}{k}\right|^2\right). \quad (3.53)$$

As in the previous cases the UV is specified by the leading term in $\frac{m}{|k|}$. We get (after Wick rotation)

$$\tilde{J}^{(\text{odd,UV})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \frac{1}{4}\frac{m}{|k|}\epsilon_{\mu_1\nu_1\sigma}k^\sigma\left[\frac{1}{12}k_{\mu_2}k_{\mu_3}k_{\nu_2}k_{\nu_3} - \frac{2}{9}k^2k_{\mu_3}k_{\nu_3}\eta_{\mu_2\nu_2} + \frac{k^2}{36}\left(k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3} + k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3}\right) + \frac{1}{9}k^4\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3} - \frac{1}{36}k^4\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}\right]. \quad (3.54)$$

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From now on in this section we understand symmetrization among μ_1, μ_2, μ_3 and among ν_1, ν_2, ν_3. The anti-Wick rotation does not yield any change. We can contract (3.54) with any k^{μ_i} and any two indexes μ_i and find zero. Therefore (3.54) is conserved and traceless (it satisfies eq. (2.33)).

We have obtained the same result (3.54) with the method illustrated in appendix E.

3.3.3 Odd parity IR

In the IR limit we find

$$\kappa_b = \frac{8}{27\pi}\frac{m^2}{k^2} + \frac{1}{240\pi} + \mathcal{O}\left(\left|\frac{k}{m}\right|^2\right), \quad \kappa_b' = -\frac{8}{27\pi}\frac{m^2}{k^2} - \frac{2}{135\pi} + \mathcal{O}\left(\left|\frac{m}{k}\right|^2\right). \tag{3.55}$$

Once again the IR limit contain divergent contributions that can be treated by adding local counter terms, which is equivalent to perform the following shifts on the form factors:

$$\kappa_b \to \kappa_b + \frac{8}{27\pi}\frac{m^2}{k^2}, \quad \kappa_b' \to \kappa_b' - \frac{8}{27\pi}\frac{m^2}{k^2}. \tag{3.56}$$

The final result in Lorentzian metric (obtained with the two different methods above) is

$$\begin{aligned}\tilde{J}^{(\text{odd,IR})}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \frac{1}{4\pi}\epsilon_{\mu_1\nu_1\sigma}k^\sigma\Big[&\frac{1}{60}k^4\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} - \frac{8}{135}k^4\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}\\ &-\frac{1}{60}k^2\left(k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3} + k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3}\right) + \frac{16}{135}k^2k_{\mu_2}k_{\nu_2}\eta_{\mu_3\nu_3}\\ &-\frac{23}{540}k_{\mu_2}k_{\mu_3}k_{\nu_2}k_{\nu_3}\Big].\end{aligned} \tag{3.57}$$

The trace of (3.57) does not vanish. However at this point we must avoid a semantic trap. A nonvanishing trace of this kind does not contradict the fact that it represents a fixed point of the renormalization group. An RG fixed point is expected to be conformal, but this means vanishing of the e.m. trace, not necessarily of the trace of the spin three current.

3.3.4 The lowest order effective action for the field B

The odd 2-point correlator in a scaling UV limit similar to (3.22), Fourier anti-transformed and inserted in (2.22), gives rise to the action term

$$\begin{aligned}S^{(UV)} \sim \int d^3x\, \epsilon_{\mu_1\nu_1\sigma}\Big[&3\partial^\sigma B^{\mu_1\mu_2\mu_3}\partial_{\mu_2}\partial_{\mu_3}\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} - 8\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box\partial_{\mu_3}\partial_{\nu_3}B^{\nu_1\nu_3}{}_{\mu_2}\\ &+2\partial^\sigma B^{\mu_1\lambda}{}_\lambda\Box\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} + 4\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box^2 B^{\nu_1}{}_{\mu_2\mu_3}\\ &-\partial^\sigma B^{\mu_1\lambda}{}_\lambda\Box^2 B^{\nu_1\rho}{}_\rho\Big],\end{aligned} \tag{3.58}$$

where $B_{\mu\nu\lambda} = b_{\mu\nu\lambda} + \ldots$. This is the lowest order term of the analog of the CS action for the field B. This theory is extremely constrained. The field B has 10 independent components. The gauge freedom counts 6 independent functions, the conservation equations are 3. The generalized Weyl (g-Weyl) invariance implies two additional degrees of freedom. So altogether the constraints are more than the degrees of freedom. The question is whether such CS actions contain nontrivial (i.e. non pure gauge) solutions.

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In a similar way (3.57) gives rise to the action

$$S^{(IR)} = \frac{1}{32\pi}\frac{1}{540}\int d^3x\,\epsilon_{\mu_1\nu_1\sigma}\Big[-23\partial^\sigma B^{\mu_1\mu_2\mu_3}\partial_{\mu_2}\partial_{\mu_3}\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} \\ +64\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box\partial_{\mu_3}\partial_{\nu_3}B^{\nu_1\nu_3}{}_{\mu_2} - 18\partial^\sigma B^{\mu_1\lambda}{}_\lambda\Box\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} \\ -32\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box^2 B^{\nu_1}{}_{\mu_2\mu_3} + 9\partial^\sigma B^{\mu_1\lambda}{}_\lambda\Box^2 B^{\nu_1\rho}{}_\rho\Big]. \tag{3.59}$$

This action is invariant under (2.12), but not under (2.13).

Remark. The action (3.58) is similar to eq. (30) of [21]. The latter is written in terms of spinor labels, therefore the relation is not immediately evident. After turning to the ordinary notation, eq. (30) of [21] becomes

$$\sim\int d^3x\,\epsilon_{\mu_1\nu_1\sigma}\Big[\frac{3}{2}\partial^\sigma h^{\mu_1\mu_2\mu_3}\partial_{\mu_2}\partial_{\mu_3}\partial_{\nu_2}\partial_{\nu_3}h^{\nu_1\nu_2\nu_3} - 4\partial^\sigma h^{\mu_1\mu_2\mu_3}\Box\partial_{\mu_3}\partial_{\nu_3}h^{\nu_1\nu_3}{}_{\mu_2} \\ +2\partial^\sigma h^{\mu_1\mu_2\mu_3}\Box^2 h^{\nu_1}{}_{\mu_2\mu_3}\Big] \tag{3.60}$$

and one can see that they are equal if we set $B^{\mu\lambda}{}_\lambda = 0$ in (3.58). The reason of the difference is that in [21] the field $h^{\mu\nu\lambda}$ is traceless, while in (3.58) the field $B_{\mu\nu\lambda}$ is not. The presence of the trace part modifies the conservation law and thus the action.

4 Chern-Simons effective actions

In the previous section we have seen that the odd parity 2-point correlators of the massive fermion model, either in the IR or UV limit, are local and give rise to action terms which coincide with the lowest (second) order of the gauge CS action and gravity CS action for the 2-point function of the gauge current and the e.m. tensor, respectively; and to the lowest order of a CS-like action for the rank 3 tensor field B. It is natural to expect that the n-th order terms of such CS actions will originate in a similar way from the n-point functions of the relevant currents. In particular the next to leading (third order) term in the CS actions is expected to be determined by the 3-point functions of the relevant currents. This is indeed so, but in a quite nontrivial way, with complications due both to the regularization and to the way we take the IR and UV limit.

The purpose of this section is to elaborate on the properties of the gauge and gravity CS actions, (3.7) and (3.24), respectively, in order to prepare the ground for the following discussion. The point we want to stress here is that in order to harmonize the formalism with the perturbative expansion in quantum field theory we need perturbative cohomology. The latter is explained in detail in appendix C. It consists of a sequence of coboundary operators which approximate the full cohomology: in the case of a gauge theory the sequence reduces to two elements, in the case of gravity or higher tensor theories the sequence is infinite.

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4.1 CS term for the gauge field

Let us start with the gauge case. The action (3.7) splits into two parts, $CS = CS^{(2)} + CS^{(3)}$, of order two and three, respectively, in the gauge field A. The second term is expected to come from the 3-point function of the gauge current. Gauge invariance splits as follows

$$\delta^{(0)} CS^{(2)} = 0, \qquad \delta^{(1)} CS^{(2)} + \delta^{(0)} CS^{(3)} = 0. \tag{4.1}$$

These equations reflect themselves in the conservation laws, which also split into two equations. The conservation law for the 2-point function is simply the vanishing of the divergence (on any index) of the latter, while for the 3-point function it does not consist in the vanishing of the divergence of the latter, but involves also contributions from 2-point functions. More precisely

$$\begin{aligned} &\partial_x^\mu \langle 0|\mathcal{T} J_\mu^a(x) J_\nu^b(y) J_\lambda^c(z)|0\rangle \\ &\qquad = i f^{abc'} \delta(x-y) \langle 0|\mathcal{T} J_\nu^{c'}(x) J_\lambda^c(z)|0\rangle + f^{acc'} \delta(x-z) \langle 0|\mathcal{T} J_\lambda^{c'}(x) J_\nu^b(y)|0\rangle, \end{aligned} \tag{4.2}$$

which in momentum space becomes

$$-iq^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) + f^{abc'} \tilde{J}^{c'c}_{\nu\lambda}(k_2) + f^{acc'} \tilde{J}^{c'b}_{\lambda\nu}(k_1) = 0, \tag{4.3}$$

where $q = k_1 + k_2$ and $\tilde{J}^{ab}_{\mu\nu}(k)$ and $\tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2)$ are Fourier transform of the 2- and 3-point functions, respectively.

4.2 Gravitational CS term

Let us consider next the gravitational CS case. Much as in the previous case we split the action (3.24) in pieces according to the number of $h_{\mu\nu}$ contained in them. This time however the number of pieces is infinite:

$$CS_g = \kappa \int d^3x\, \epsilon^{\mu\nu\lambda} \left(\partial_\mu \omega_\nu^{ab} \omega_{\lambda ba} + \frac{2}{3} \omega_{\mu a}{}^b \omega_{\nu b}{}^c \omega_{\lambda c}{}^a \right) = CS_g^{(2)} + CS_g^{(3)} + \dots, \tag{4.4}$$

where

$$CS_g^{(2)} = \frac{\kappa}{2} \int d^3x\, \epsilon_{\sigma\nu\rho}\, h^{\lambda\rho} \left(\partial^\sigma \partial_\lambda \partial_b h^{b\nu} - \partial^\sigma \Box h_\lambda^\nu \right) \tag{4.5}$$

and

$$\begin{aligned} CS_g^{(3)} = \frac{\kappa}{4} \int d^3x\, \epsilon^{\mu\nu\lambda} \Big(& 2\partial_a h_{\nu b} \partial_\lambda h_\sigma^b \partial_\mu h^{\sigma a} - 2\partial_a h_\mu^b \partial^c h_{b\nu} \partial^a h_{c\lambda} - \frac{2}{3} \partial_a h_\mu^b \partial_b h_\nu^c \partial_c h_\lambda^a \\ & - 2\partial_\mu \partial^b h_\nu^a (h_a^c \partial_c h_{b\lambda} - h_b^c \partial_c h_{a\lambda}) + \partial_\mu \partial^b h_\nu^a (h_\lambda^c \partial_a h_{bc} - \partial_a h_\lambda^c h_{bc}) \\ & + \partial_\mu \partial^b h_\nu^a \left(\partial_b h_\lambda^c h_{ac} - h_\lambda^c \partial_b h_{ac} \right) - h_\lambda^\rho h_\rho^a \partial_\mu \left(\Box h_{a\nu} - \partial_a \partial_b h_\nu^b \right) \Big). \end{aligned} \tag{4.6}$$

Invariance of CS_g under diffeomorphisms also splits into infinite many relations. The first two, which are relevant to us here, are

$$\delta_\xi^{(1)} CS_g^{(0)} = 0, \qquad \delta_\xi^{(1)} CS_g^{(0)} + \delta_\xi^{(0)} CS_g^{(1)} = 0, \tag{4.7}$$

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where ξ is the parameter of diffeomorphisms. Similar relations hold also for Weyl transformations.

Such splittings correspond to the splittings of the Ward identities for diffeomorphisms and Weyl transformations derived from the generating function (2.22). The lowest order WI is just the vanishing of the divergence of the 2-point e.m. tensor correlators. The next to lowest order involves 2-point as well as 3-point functions of the e.m. tensor:

$$\begin{aligned}
&\langle 0|\mathcal{T}\{\partial^\mu T_{\mu\nu}(x)T_{\lambda\rho}(y)T_{\alpha\beta}(z)\}|0\rangle \\
&= i\Big\{2\frac{\partial}{\partial x^{(\alpha}}\left[\delta(x-z)\langle 0|\mathcal{T}\{T_{\beta)\nu}(x)T_{\lambda\rho}(y)\}|0\rangle\right] + 2\frac{\partial}{\partial x^{(\lambda}}\left[\delta(x-y)\langle 0|\mathcal{T}\{T_{\rho)\nu}(x)T_{\alpha\beta}(z)\}|0\rangle\right] \\
&\quad -\frac{\partial}{\partial x_\tau}\delta(x-z)\eta_{\alpha\beta}\langle 0|\mathcal{T}\{T_{\tau\nu}(x)T_{\lambda\rho}(y)\}|0\rangle - \frac{\partial}{\partial x_\tau}\delta(x-y)\eta_{\lambda\rho}\langle 0|\mathcal{T}\{T_{\tau\nu}(x)T_{\alpha\beta}(z)\}|0\rangle \\
&\quad + \frac{\partial}{\partial x^\nu}\delta(x-z)\langle 0|\mathcal{T}\{T_{\lambda\rho}(y)T_{\alpha\beta}(x)\}|0\rangle + \frac{\partial}{\partial x^\nu}\delta(x-y)\langle 0|\mathcal{T}\{T_{\lambda\rho}(x)T_{\alpha\beta}(z)\}|0\rangle\Big\}.
\end{aligned} \tag{4.8}$$

In momentum space, denoting by $\tilde{T}_{\mu\nu\lambda\rho}(k)$ and by $\tilde{T}_{\mu\nu\lambda\rho\alpha\beta}(k_1,k_2)$ the 2-point and 3-point function, respectively, this formula becomes

$$\begin{aligned}
iq^\mu\tilde{T}_{\mu\nu\lambda\rho\alpha\beta}(k_1,k_2) = {}& 2q_{(\alpha}\tilde{T}_{\beta)\nu\lambda\rho}(k_1) + 2q_{(\lambda}\tilde{T}_{\rho)\nu\alpha\beta}(k_2) - \eta_{\alpha\beta}k_2^\tau\tilde{T}_{\tau\nu\lambda\rho}(k_1) \\
& -\eta_{\lambda\rho}k_1^\tau\tilde{T}_{\tau\nu\alpha\beta}(k_2) + k_{2\nu}\tilde{T}_{\alpha\beta\lambda\rho}(k_1) + k_{1\nu}\tilde{T}_{\lambda\rho\alpha\beta}(k_2),
\end{aligned} \tag{4.9}$$

where round brackets denote symmetrization normalized to 1.

From the action term (4.6), by differentiating three times with respect to $h_{\mu\nu}(x)$,$h_{\lambda\rho}(y)$ and $h_{\alpha\beta}(z)$ and Fourier-transforming the result one gets a sum of local terms in momentum space (see appendix F), to be compared with the IR and UV limit of the 3-point e.m. tensor correlator.

4.3 CS term for the B field

Here we would like to understand the nature of the "CS-like" terms obtained in the IR and UV limits, and especially to understand how it is possible that they are different in the spin-3 case, unlike what we saw in spin-1 and spin-2.[5] For this purpose, we use a higher-spin "geometric" construction originally developed in [36]. In the spin-3 case the linearised "Christoffel connection" is given by the so-called second affinity defined by

$$\begin{aligned}
\Gamma_{\alpha_1\alpha_2;\beta_1\beta_2\beta_3} = \frac{1}{3}\Big\{&\partial_{\alpha_1}\partial_{\alpha_2}B_{\beta_1\beta_2\beta_3} - \frac{1}{2}\left(\partial_{\alpha_1}\partial_{\beta_1}B_{\alpha_2\beta_2\beta_3} + \partial_{\alpha_1}\partial_{\beta_2}B_{\alpha_2\beta_1\beta_3}\right. \\
&+\partial_{\alpha_1}\partial_{\beta_3}B_{\alpha_2\beta_1\beta_2} + \partial_{\alpha_2}\partial_{\beta_1}B_{\alpha_1\beta_2\beta_3} + \partial_{\alpha_2}\partial_{\beta_2}B_{\alpha_1\beta_1\beta_3} + \partial_{\alpha_2}\partial_{\beta_3}B_{\alpha_1\beta_1\beta_2}) \\
&\left.+\partial_{\beta_1}\partial_{\beta_2}B_{\alpha_1\alpha_2\beta_3} + \partial_{\beta_1}\partial_{\beta_3}B_{\alpha_1\alpha_2\beta_2} + \partial_{\beta_2}\partial_{\beta_3}B_{\alpha_1\alpha_2\beta_1}\right\}.
\end{aligned} \tag{4.10}$$

[5] In the literature one can find two kinds of generalizations of the CS action in 3d to higher spins (for a general review on higher spin theories, see [25–32]). One leads to quadratic equations of motion, the other to higher derivative equations of motion. The first kind of theories are nicely summarized in [33]. The second kind of theories, to our best knowledge, was introduced in [21] (following [34]). This splitting was already shadowed in [35].

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Under the gauge transformation (2.12) this "connection" transforms as

$$\delta_\Lambda \Gamma_{\alpha\beta;\mu\nu\rho} = \partial_\mu\partial_\nu\partial_\rho \Lambda_{\alpha\beta}. \tag{4.11}$$

The natural generalisation of (the quadratic part of) the spin-1 and the spin-2 CS action term to the spin-3 case is given by

$$\begin{aligned} I_{\rm CS}[B] &\equiv \int d^3x\, \epsilon^{\mu\sigma\nu}\, \Gamma^{\alpha\beta}{}_{;\mu\rho\lambda}\, \partial_\sigma \Gamma^{\rho\lambda}{}_{;\nu\alpha\beta} \\ &= \frac{1}{3}\int d^3x\, \epsilon_{\mu\sigma\nu}\big(\partial_\alpha\partial_\beta B^{\mu\alpha\beta}\, \partial^\sigma\partial_\rho\partial_\lambda B^{\nu\rho\lambda} + 2\,\partial_\alpha \Box B^{\mu\alpha\beta}\, \partial^\sigma\partial_\rho B^{\nu\rho}{}_\beta \\ &\qquad + \Box B^{\mu\alpha\beta}\, \partial^\sigma \Box B^{\nu}{}_{\alpha\beta}\big) + \text{ (boundary terms)}. \end{aligned} \tag{4.12}$$

From (4.11) directly follows that this CS term is gauge invariant (up to boundary terms). In the spin-3 case one can construct another 5-derivative CS term by using Fronsdal tensor (or spin-3 "Ricci tensor") defined by

$$\begin{aligned} \mathcal{R}_{\mu\nu\rho} &\equiv \Gamma^{\alpha}{}_{\alpha;\mu\nu\rho} \\ &= \frac{1}{3}\big\{\Box B_{\mu\nu\rho} - \partial^\alpha(\partial_\mu B_{\alpha\nu\rho} + \partial_\nu B_{\alpha\rho\mu} + \partial_\rho B_{\alpha\mu\nu}) \\ &\qquad + \partial^\mu\partial^\nu B_{\rho\alpha}{}^{\alpha} + \partial^\rho\partial^\mu B_{\nu\alpha}{}^{\alpha} + \partial^\nu\partial^\rho B_{\mu\alpha}{}^{\alpha}\big\}. \end{aligned} \tag{4.13}$$

Using this tensor one can defined another CS action term

$$\begin{aligned} I'_{\rm CS}[B] &\equiv \int d^3x\, \epsilon^{\mu\sigma\nu}\, \mathcal{R}_{\mu\rho\lambda}\, \partial_\sigma \mathcal{R}_\nu{}^{\rho\lambda} \\ &= \frac{1}{9}\int d^3x\, \epsilon_{\mu\sigma\nu}\big(2\,\partial_\alpha\partial_\beta B^{\mu\alpha\beta}\, \partial^\sigma\partial_\rho\partial_\lambda B^{\nu\rho\lambda} + 2\,\partial_\alpha \Box B^{\mu\alpha\beta}\, \partial^\sigma\partial_\rho B^{\nu\rho}{}_\beta \\ &\qquad - 2\,\partial_\alpha\partial_\beta B^{\mu\alpha\beta}\, \partial^\sigma \Box B^{\nu\rho}{}_\rho + \Box B^{\mu\alpha\beta}\, \partial^\sigma \Box B^{\nu}{}_{\alpha\beta} \\ &\qquad + \Box B^{\mu\alpha}{}_\alpha\, \partial^\sigma \Box B^{\nu\rho}{}_\rho\big) + \text{ (boundary terms)}. \end{aligned} \tag{4.14}$$

The presence of two CS terms in the spin-3 case explains why there is a priori no reason to expect from UV and IR limits to lead to the same CS-like term.

Now it is easy to see that the following combination

$$5\, I_{\rm CS}[B] - 3\, I'_{\rm CS}[B] \tag{4.15}$$

exactly gives the effective action term (3.58) which we obtained from the one-loop calculation.

To understand why the combination (4.15) represents a generalization of the spin-2 CS term (gravitational CS term), one has to take into account the symmetry under generalized Weyl (g-Weyl) transformations, which for spin-3 is given by (2.13). It can be shown that the CS terms (4.12) and (4.14) are not g-Weyl-invariant, but that (4.15) is the *unique g-Weyl-invariant* linear combination thereof.

It is then not surprising that the effective current $J_{\mu\nu\rho}$ obtained from (4.15) is proportional to the spin-3 "Cotton tensor" studied in [37]. It is the gauge- and g-Weyl- invariant conserved traceless totally symmetric tensor with the property that if it vanishes then the

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gauge field is g-Weyl-trivial. With this we have completed the demonstration that on the linear level the spin-3 CS term is a natural generalisation of the spin-2 CS term.

For completeness we add that the combination

$$\frac{1}{192\pi}\left(-\frac{41}{3}I_{\rm CS}[B]+9I'_{\rm CS}[B]\right) \tag{4.16}$$

reproduces (3.59), which is not g-Weyl invariant.

5 Three-point gauge current correlator: odd parity part

In this section we explicitly compute the 3-point current correlator of the current $J^a_\mu(x)$. The 3-point correlator for currents is given by the triangle diagram. The three external momenta are q, k_1, k_2. q is incoming, while k_1, k_2 are outgoing and of course momentum conservation implies $q = k_1 + k_2$. The relevant Feynman diagram is

$$\tilde{J}^{1,abc}_{\mu\nu\lambda}(k_1,k_2)=i\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\gamma_\mu T^a\frac{1}{\not p-m}\gamma_\nu T^b\frac{1}{\not p-\not k_1-m}\gamma_\lambda T^c\frac{1}{\not p-\not q-m}\right) \tag{5.1}$$

to which we have to add the cross graph contribution $\tilde{J}^{2,abc}_{\mu\nu\lambda}(k_1,k_2)=\tilde{J}^{1,acb}_{\mu\lambda\nu}(k_2,k_1)$. From this we extract the odd parity part and perform the integrals. The general method is discussed in subsection 6.2, here we limit ourselves to the results. Such results have already been presented in [16], but since they are important for the forthcoming discussion we summarize them below. For simplicity we set $k_1^2 = k_2^2 = 0$, so the total energy of the process is $E=\sqrt{q^2}=\sqrt{2k_1\cdot k_2}$.

Near the IR fixed point we obtain a series expansion of the type

$$\tilde{J}^{abc(\mathrm{odd})}_{\mu\nu\lambda}(k_1,k_2)\approx i\frac{1}{32\pi}\sum_{n=0}^{\infty}\left(\frac{E}{m}\right)^{2n}f^{abc}\tilde{I}^{(2n)}_{\mu\nu\lambda}(k_1,k_2) \tag{5.2}$$

and, in particular, the relevant term in the IR is

$$I^{(0)}_{\mu\nu\lambda}(k_1,k_2)=-12\epsilon_{\mu\nu\lambda}. \tag{5.3}$$

The first thing to check is conservation. The current (2.2) should be conserved also at the quantum level, because no anomaly is expected in this case. It is evident that the contraction with q^μ does not give a vanishing result, as we expect because we must include also the contribution from the 2-point functions, (4.2). But even including such contributions we get

$$-\frac{3}{8\pi}f^{abc}q^\mu\epsilon_{\mu\nu\lambda}+\frac{1}{4\pi}f^{abc}\epsilon_{\nu\lambda\sigma}k_2^\sigma+\frac{1}{4\pi}f^{abc}\epsilon_{\nu\lambda\sigma}k_1^\sigma\neq 0. \tag{5.4}$$

Conservation is violated by a local term. Thus we can recover it by adding to $I^{(0)}_{\mu\nu\lambda}(k_1,k_2)$ a term $4\epsilon_{\mu\nu\lambda}$. This corresponds to correcting the effective action by adding a counterterm

$$-2\int dx\epsilon^{\mu\nu\lambda}f^{abc}A^a_\mu A^b_\nu A^c_\lambda. \tag{5.5}$$

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Adding this to the result from the 2-point correlator we reconstruct the full CS action (3.7).

This breakdown of conservation is surprising, therefore it is important to understand where it comes from. To this end we consider the full theory for finite m. The contraction of the 3-point correlator with q^μ is given by

$$q^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) = -i\int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\not{q}T^a\frac{1}{\not{p}-m}\gamma_\nu T^b\frac{1}{\not{p}-\not{k}_1-m}\gamma_\lambda T^c\frac{1}{\not{p}-\not{q}-m}\right). \quad (5.6)$$

Replacing $\not{q} = (\not{p}-m)-(\not{p}-\not{q}-m)$ considerably simplifies the calculation. The final result for the odd parity part (after adding the cross diagram contribution, $1\leftrightarrow 2, b\to c, \nu\leftrightarrow\lambda$) is

$$\begin{aligned} q^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) = & -\tfrac{i}{4\pi}f^{abc}\epsilon_{\lambda\nu\sigma}k_1^\sigma \tfrac{2m}{k_1}\mathrm{arccoth}\left(\tfrac{2m}{k_1}\right) \\ & -\tfrac{i}{4\pi}f^{abc}\epsilon_{\lambda\nu\sigma}k_2^\sigma \tfrac{2m}{k_2}\mathrm{arccoth}\left(\tfrac{2m}{k_2}\right). \end{aligned} \quad (5.7)$$

Therefore, as far as the odd part is concerned, the 3-point conservation (4.3) reads

$$\begin{aligned} & -iq^\mu \tilde{J}^{(\mathrm{odd})abc}_{\mu\nu\lambda}(k_1,k_2) + f^{abd}\tilde{J}^{(\mathrm{odd})dc}_{\nu\lambda}(k_2) + f^{acd}\tilde{J}^{(\mathrm{odd})db}_{\lambda\nu}(k_1) \\ & \quad = -\frac{1}{4\pi}f^{abc}\epsilon_{\lambda\nu\sigma}\left(k_1^\sigma\frac{2m}{k_1}\mathrm{arccoth}\left(\frac{2m}{k_1}\right)+k_2^\sigma\frac{2m}{k_2}\mathrm{arccoth}\left(\frac{2m}{k_2}\right)\right) \\ & \quad\quad + \frac{1}{4\pi}f^{abc}\epsilon_{\lambda\nu\sigma}\left(k_1^\sigma\frac{2m}{k_1}\mathrm{arccoth}\left(\frac{2m}{k_1}\right)+k_2^\sigma\frac{2m}{k_2}\mathrm{arccoth}\left(\frac{2m}{k_2}\right)\right) = 0. \end{aligned} \quad (5.8)$$

Thus conservation is secured for any value of the parameter m. The fact that in the IR limit we find a violation of the conservation is an artifact of the procedure we have used (in particular of the limiting procedure) and we have to remedy by subtracting suitable counterterms from the effective action. These subtractions are to be understood as (part of) the definition of our regularization procedure.

Something similar can be done also for the UV limit. However in the UV limit the resulting effective action has a vanishing coupling $\sim \frac{m}{E}$, unless we consider an $N\to\infty$ limit theory, as outlined above. In order to guarantee invariance under large gauge transformations we have also to fine tune the limit in such a way that the κ coupling be an integer. But even in the UV we meet the problem of invariance breaking.

We will meet the same problem below for the 3-point function of the e.m. tensor.

6 Three-point e.m. correlator: odd parity part

We go now to the explicit calculation of the 3-point e.m. tensor correlator. The three-point function is given by three contributions, the bubble diagram, the triangle diagram and the cross triangle diagram. We will focus in the sequel only on the odd parity part.

6.1 The bubble diagram: odd parity

The bubble diagram is constructed with one V_{gff} vertex and one V_{ggff} vertex. It has an incoming line with momentum $q = k_1+k_2$ with Lorentz indices μ,ν, and two outgoing lines

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have momenta k_1, k_2 with Lorentz labels λ, ρ and α, β, respectively. The internal running momentum is denoted by p. The corresponding contribution is

$$D_{\lambda\rho\alpha\beta\mu\nu}(k_1,k_2) = \frac{i}{128}\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left[\frac{1}{\not p - m}t_{\lambda\rho\alpha\beta\sigma}(k_2-k_1)^\sigma\frac{1}{\not p - \not q - m}((2p_\mu - q_\mu)\gamma_\nu + \mu\leftrightarrow\nu)\right] \quad (6.1)$$

where

$$t_{\lambda\rho\alpha\beta\sigma} = \eta_{\lambda\alpha}\epsilon_{\rho\beta\sigma} + \eta_{\lambda\beta}\epsilon_{\rho\alpha\sigma} + \eta_{\rho\alpha}\epsilon_{\lambda\beta\sigma} + \eta_{\rho\beta}\epsilon_{\lambda\alpha\sigma}. \quad (6.2)$$

The odd part gives (the metric is Lorentzian)

$$\tilde{D}_{\lambda\rho\alpha\beta\mu\nu}(k_1,k_2) = \frac{m}{256\pi}t_{\lambda\rho\alpha\beta\sigma}(k_2-k_1)^\sigma\left(\eta_{\mu\nu}\left(2m - \frac{q^2-4m^2}{|q|}\mathrm{arctanh}\frac{|q|}{2m}\right)\right.$$
$$\left. + q_\mu q_\nu\left(\frac{2m}{q^2} + \frac{q^2-4m^2}{|q|^3}\mathrm{arctanh}\frac{|q|}{2m}\right)\right). \quad (6.3)$$

Saturating with q^μ we get

$$q^\mu\tilde{D}_{\lambda\rho\alpha\beta\mu\nu}(k_1,k_2) = \frac{m^2}{256\pi}t_{\lambda\rho\alpha\beta\sigma}(k_2-k_1)^\sigma 2q_\nu. \quad (6.4)$$

This corresponds to an anomaly

$$\mathcal{A}_\xi \sim \int d^3x\,\partial_\nu\xi^\nu\epsilon_{\rho\beta\sigma}h^{\lambda\rho}\partial^\sigma h^\beta_\lambda \quad (6.5)$$

which we have to subtract. This gives

$$\tilde{D}_{\lambda\rho\alpha\beta\mu\nu}(k_1,k_2) = \frac{1}{256\pi}t_{\lambda\rho\alpha\beta\sigma}(k_2-k_1)^\sigma\left(q_\mu q_\nu - \eta_{\mu\nu}q^2\right)\left(\frac{2m^2}{q^2} + m\frac{q^2-4m^2}{|q|^3}\mathrm{arctanh}\frac{|q|}{2m}\right). \quad (6.6)$$

Taking the limit of the form factor (last round brackets) for $\frac{m}{|q|}\to 0$ (UV), we find 0 (the linear term in $\frac{m}{|q|}$ vanishes). Taking the limit $\frac{m}{|q|}\to\infty$ (IR) we find

$$\tilde{D}^{(IR)}_{\lambda\rho\alpha\beta\mu\nu}(k_1,k_2) = \frac{2}{3}\frac{1}{256\pi}t_{\lambda\rho\alpha\beta\sigma}(k_2-k_1)^\sigma\left(q_\mu q_\nu - \eta_{\mu\nu}q^2\right). \quad (6.7)$$

This corresponds to the action term

$$\sim\int d^3x\,(\Box h - \partial_\mu\partial_\nu h^{\mu\nu})\,t_{\lambda\rho\alpha\beta\sigma}\left(h^{\lambda\rho}\partial^\sigma h^{\alpha\beta} - \partial^\sigma h^{\lambda\rho}\,h^{\alpha\beta}\right). \quad (6.8)$$

6.2 Triangle diagram: odd parity

It is constructed with three V_{gff} vertices. It has again an incoming line with momentum $q = k_1 + k_2$ with Lorentz indices μ, ν. The two outgoing lines have momenta k_1, k_2 with Lorentz labels λ, ρ and α, β, respectively. The contribution is formally written as

$$\tilde{T}^{(1)}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2) = -\frac{1}{512}\int\frac{d^3p}{(2\pi)^3}\,\mathrm{tr}\left[\left(\frac{1}{\not p - m}\big((2p-k_1)_\lambda\gamma_\rho + (\lambda\leftrightarrow\rho)\big)\,\frac{1}{\not p - \not k_1 - m}\right.\right.$$
$$\left.\left.\times\big((2p-2k_1-k_2)_\alpha\gamma_\beta + (\alpha\leftrightarrow\beta)\big)\,\frac{1}{\not p - \not q - m}\big((2p-q)_\mu\gamma_\nu + (\mu\leftrightarrow\nu)\big)\right)\right], \quad (6.9)$$

to which the cross graph contribution $\tilde{T}^{(2)}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2) = \tilde{T}^{(1)}_{\mu\nu\lambda\rho\alpha\beta}(k_2,k_1)$ has to be added.

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The odd part of (6.9) is

$$\tilde{T}^{(1,odd)}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2) = -\frac{m}{512}\int \frac{d^3p}{(2\pi)^3}\,\mathrm{tr}\,\big[\not{p}\gamma_\rho(\not{p}-\not{k}_1)\gamma_\beta\gamma_\nu + \gamma_\rho(\not{p}-\not{k}_1)\gamma_\beta(\not{p}-\not{q})\gamma_\nu \\ + \not{p}\gamma_\rho\gamma_\beta(\not{p}-\not{q}\gamma_\nu + m^2\gamma_\rho\gamma_\beta\gamma_\nu\big]\,\frac{(2p-k_1)_\lambda(2p-2k_1-k_2)_\alpha(2p-q)_\mu}{(p^2-m^2)((p-k_1)^2-m^2)((p-q)^2-m^2)}, \quad (6.10)$$

where the symmetrization $\lambda\leftrightarrow\rho, \alpha\leftrightarrow\beta, \mu\leftrightarrow\nu$ is understood. In order to work out (6.10) we introduce two Feynman parameters: u integrated between 0 and 1, and v integrated between 0 and $1-u$. The denominator in (6.10) becomes

$$\big[(p-(1-u)k_1-vk_2)^2 + u(1-u)k_1^2 + v(1-v)k_2^2 + 2uv\,k_1\cdot k_2 - m^2\big]^3.$$

After taking the traces, we get

$$\tilde{T}^{(1,odd)}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2) = \frac{im}{128}\int_0^1 du\int_0^{1-u} dv\int\frac{d^3p}{(2\pi)^3}\,\big[\epsilon_{\rho\sigma\nu}\left(-2p_\beta k_1^\sigma + k_{1\beta}q^\sigma + q_\beta k_1^\sigma\right) \\ + 2\epsilon_{\sigma\beta\nu}p_\rho k_2^\sigma + \epsilon_{\rho\beta\nu}\left(-5p^2 + (2p-q)\cdot k_1 + m^2\right) + \eta_{\rho\nu}\epsilon_{\sigma\beta\tau}k_1^\sigma k_2^\tau\big] \\ \cdot\frac{(2p-k_1)_\lambda(2p-2k_1-k_2)_\alpha(2p-q)_\mu}{[(p-(1-u)k_1-vk_2)^2+\Delta)^3}, \quad (6.11)$$

where $\Delta = u(1-u)\,k_1^2 + v(1-v)\,k_2^2 + 2uv\,k_1\cdot k_2 - m^2$.

So we can shift $p\to p' = p-(1-u)k_1-vk_2$ and integrate over p'. The p-integrals can be easily carried out, see appendix D. Unfortunately we are not able to integrate over u and v in an elementary way. So, one way to proceed is to use Mathematica, which however is not able to integrate over both u and v unless some simplification is assumed. Therefore we choose the condition $k_1^2 = 0 = k_2^2$. In this case the total energy of the process is $E = \sqrt{q^2} = \sqrt{2k_1\cdot k_2}$.

An alternative way is to use Mellin-Barnes representation for the propagators in (6.10) and proceed in an analytic as suggested by Boos and Davydychev, [22–24]. This second approach is discussed in appendix E. In all the cases we were able to compare the two methods they give the same results (up to trivial terms).

6.3 The IR limit

The IR limit corresponds to $m\to\infty$ or, better, $\frac{m}{E}\to\infty$ where $E=\sqrt{2k_1\cdot k_2}$. In this limit we find one divergent term $\mathcal{O}(m^2)$ and a series in the parameter $\frac{m}{E}$ starting from the 0-th order term. The $\mathcal{O}(m^2)$ term is (after adding the cross contribution)

$$\sim\frac{m^2}{16\pi}\Big[16\epsilon_{\sigma\beta\nu}k_2^\sigma\left(\eta_{\rho\lambda}\eta_{\alpha\mu}+\eta_{\rho\alpha}\eta_{\lambda\mu}+\eta_{\rho\mu}\eta_{\lambda\alpha}\right) + 16\epsilon_{\sigma\rho\nu}k_1^\sigma\left(\eta_{\beta\lambda}\eta_{\alpha\mu}+\eta_{\beta\alpha}\eta_{\lambda\mu}+\eta_{\beta\mu}\eta_{\lambda\alpha}\right) \\ -\epsilon_{\rho\beta\nu}\left(-\frac{112}{3}(k_1-k_2)_\mu\eta_{\lambda\alpha}+\frac{16}{3}(11k_1+7k_2)_\alpha\eta_{\lambda\mu}-\frac{16}{3}(7k_1+11k_2)_\lambda\eta_{\alpha\mu}\right)\Big]. \quad (6.12)$$

This term has to be symmetrized under $\mu\leftrightarrow\nu, \lambda\leftrightarrow\rho, \alpha\leftrightarrow\beta$. It is a (non-conserved) local term. It must be subtracted from the action. Once this is done the relevant term for us

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is the 0-th order one. Let us call it $\tilde{T}^{(\mathrm{odd,IR})}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2)$. Its lengthy explicit form is written down in appendix F.2.

If we compare this expression plus the contribution from the bubble diagram with the one obtained from $CS_g^{(3)}$ in F.1, which it is expected to reproduce, we see that they are different. This is not surprising in view of the discussion of the gauge case: the next to leading order of the relevant CS action is not straightaway reproduced by the relevant 3-point correlators, but need corrections. This can be seen also by contracting $\tilde{T}^{(\mathrm{odd,IR})}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2)$ with q^μ and inserting it in the WI (4.9): the latter is violated.

Now let us Fourier antitransform $\tilde{T}^{(\mathrm{odd,IR})}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2)$ and insert the result in the $W[g]$ generating function. We obtain a local action term of 3rd order in $h_{\mu\nu}$, which we may call $\widetilde{CS}_g^{(3)}$. Having in mind (4.1), we find instead

$$\delta_\xi^{(1)} CS_g^{(2)} + \delta_\xi^{(0)} \widetilde{CS}_g^{(3)} = \mathcal{Y}^{(2)}(\xi) \neq 0, \tag{6.13}$$

where $\mathcal{Y}^{(2)}(\xi)$ is an integrated local expression quadratic in $h_{\mu\nu}$ and linear in the diffeomorphism parameter ξ. It is clear that in order to reproduce (4.1) we must add counterterms to the action, as we have done in the analogous case in section 5. The question is whether this is possible. We can proceed as follows, we subtract from (6.13) the second equation in (4.9) and obtain

$$\delta_\xi^{(0)}(\widetilde{CS}_g^{(3)} - CS_g^{(3)}) = \mathcal{Y}^{(2)}(\xi). \tag{6.14}$$

Therefore $\widetilde{CS}_g^{(3)} - CS_g^{(3)}$ is the counterterm we have to subtract from the action in order to satisfy (4.9) and simultaneously reconstruct the gravitational Chern-Simons action up to the third order. The procedure seems to be tautological, but this is simply due to the fact that we already know the covariant answer, i.e. the gravitational CS action, otherwise we would have to work our way through a painful analysis of all the terms in $\mathcal{Y}^{(2)}(\xi)$ and find the corresponding counterterms.[6]

The just outlined procedure is successful but somewhat disappointing. For the purpose of reproducing $CS_g^{(3)}$ the overall three-point calculation seems to be rather ineffective. One can say that the final result is completely determined by the two-point function analysis. Needless to say it would be preferable to find a regularization as well as a way to take the IR and UV limits that do not break covariance. We do not know it this is possible.

On the other hand the three-point function analysis is important for other reasons. For brevity we do not report other explicit formulas about the coefficients of the expansion in $\frac{E}{m}$ and $\frac{m}{E}$. They all look like correlators, which may be local and non-local. The analysis of these expressions opens a new subject of investigation.

7 Conclusion

In this paper we have calculated two- and three-point functions of currents in the free massive fermion model in 3d. We have mostly done our calculations with two different

[6] Of course in the process of defining the regularization and the IR limiting procedure, we are allowed to subtract all the necessary counterterms (with the right properties) except fully covariant action terms (like the CS action itself, for one).

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methods, as explained in appendix D and E, and obtained the same results. When the model is coupled to an external gauge potential and metric, respectively, we have extracted from them, in the UV and the IR limit, CS action terms for gauge and gravity in 3d. We have also coupled the massless fermion model to higher spin potentials and explicitly worked out the spin 3 case, by obtaining a very significant new result in the UV limit: the action reconstructed from the two-point current correlator is a particular case of higher spin action introduced a long time ago by [21]; this is one of the possible generalizations of the CS action to higher spin. It is of course expected that carrying out analogous calculations for higher spin currents we will obtain the analogous generalizations of Chern-Simons to higher spin. Our result for the spin 3 case in the IR is an action with a higher spin gauge symmetry, different from the UV one; we could not recognize it as a well-known higher spin action.

Beside the results concerning effective actions terms in the UV and IR limit, there are other interesting aspects of the correlators we obtain as intermediate steps. For instance, the odd parity current correlators at fixed points are conformal invariant and are limits of a free theory, but they cannot be obtained from any free theory using the Wick theorem. There are also other interesting and not understood aspects. For instance, the two-point e.m. tensor correlators of the massive model can be expanded in series of E/m or m/E, where E is the relevant energy, the coefficients in the expansion being proportional always to the same conformal correlator. For the three-point functions the situation is more complicated, there is the possibility of different limits and the expansion coefficients are also nonlocal. Still, however, we have a stratification similar to the one in the two-point functions with coefficient that look like conserved three-point correlators (but have to be more carefully evaluated). One would like to know what theories these correlators refer to.

Finally it would be interesting to embed the massive fermion model in an AdS_4 geometry. One can naively imagine the AdS_4 space foliated by 3d submanifolds with constant geodesic distance from the boundary and a copy of the theory defined on each submanifold with a mass depending of the distance. This mass could be generated, for instance, by the vev of a pseudoscalar field. This and the previous question certainly deserve further investigation.

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A Gamma matrices in 3d

In $2+1$ dimensions we may take the gamma matrices, [38], as

$$\gamma_0 = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma_1 = -i\sigma_1 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \quad \gamma_2 = i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{A.1}$$

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They satisfy the Clifford algebra relation for the anticommutator of gamma matrices, namely

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu},$$

For the trace of three gamma matrices we have

$$\mathrm{tr}\,(\gamma_\mu\gamma_\nu\gamma_\rho) = -2\mathrm{i}\epsilon_{\mu\nu\rho},$$

Properties of gamma matrices in 3d:

$$\begin{aligned}
\mathrm{tr}\,(\gamma_\mu\gamma_\nu) &= 2\eta_{\mu\nu}, \\
\mathrm{tr}\,(\gamma_\mu\gamma_\nu\gamma_\rho) &= -2i\epsilon_{\mu\nu\rho}, \\
\mathrm{tr}\,(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) &= 2\left(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right), \\
\{\gamma^\mu, \Sigma^{\rho\sigma}\} &= -i\epsilon^{\mu\rho\sigma}. \\
\gamma_\sigma\gamma_\mu\gamma_\nu &= -i\epsilon_{\sigma\mu\nu} + \eta_{\mu\sigma}\gamma_\nu - \eta_{\sigma\nu}\gamma_\mu + \eta_{\mu\nu}\gamma_\sigma \qquad \text{(A.2)} \\
\mathrm{tr}\,(\gamma_\sigma\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho) &= -2i\left(\epsilon_{\mu\nu\lambda}\eta_{\sigma\rho} + \eta_{\mu\nu}\epsilon_{\sigma\lambda\rho} - \eta_{\mu\lambda}\epsilon_{\sigma\nu\rho} + \eta_{\nu\lambda}\epsilon_{\sigma\mu\rho}\right) \qquad \text{(A.3)}
\end{aligned}$$

Identity for ϵ and η tensors:

$$\eta_{\mu\nu}\epsilon_{\lambda\rho\sigma} - \eta_{\mu\lambda}\epsilon_{\nu\rho\sigma} + \eta_{\mu\rho}\epsilon_{\nu\lambda\sigma} - \eta_{\mu\sigma}\epsilon_{\nu\lambda\rho} = 0$$

Finally, to make contact with the spinorial label notation of ref. [21] one may use the symmetric matrices

$$(\tilde{\gamma}_0)_{\alpha\beta} = i(\gamma_0)_\alpha{}^\gamma\epsilon_{\gamma\beta}, \qquad (\tilde{\gamma}_1)_{\alpha\beta} = (\gamma_1)_\alpha{}^\gamma\epsilon_{\gamma\beta}, \qquad (\tilde{\gamma}_2)_{\alpha\beta} = -(\gamma_2)_\alpha{}^\gamma\epsilon_{\gamma\beta}, \qquad \text{(A.4)}$$

where ϵ is the antisymmetric matrix with $\epsilon_{12} = -1$, and write

$$h_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6} = h_{abc}(\tilde{\gamma}^a)_{\alpha_1\alpha_2}(\tilde{\gamma}^b)_{\alpha_3\alpha_4}(\tilde{\gamma}^c)_{\alpha_5\alpha_6}, \qquad \partial_a(\tilde{\gamma}^a\epsilon)_\alpha{}^\beta = \partial_\alpha{}^\beta, \qquad \text{etc.} \qquad \text{(A.5)}$$

Wick rotation. Among the various conventions for the Wick rotation to compute Feynman diagram, we think the simplest one is given by the following formal rule on the metric: $\eta_{\mu\nu} \to -\eta^{(E)}_{\mu\nu}$. This implies

$$k^2 \to -(k^{(E)})^2, \qquad p_\mu p_\nu \to \frac{1}{3}(p^{(E)})^2\,\eta^{(E)}_{\mu\nu}, \ldots$$

We have also to multiply any momentum integral by i. For the sake of simplicity we always understand the superscript $^{(E)}$.

B Invariances of the 3d free massive fermion

In the theory defined by 2.4 there is a problem connected with the presence of $\sqrt{g} = e$ in the action. When defining the Feynman rules we face two possibilities: 1) either we incorporate $\sqrt{e}$ in the spinor field ψ, so that the factor $\sqrt{g}$ in fact disappears from the action, or, 2), we keep the action as it is.

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In the first case we define a new field $\Psi = \sqrt{e}\psi$. The new action becomes

$$S = \int d^3x \, \left[i\bar{\Psi} E^\mu_a \gamma^a \nabla_\mu \Psi - m\bar{\Psi}\psi \right]. \tag{B.1}$$

due essentially to the fact that $\nabla_\lambda g_{\mu\nu} = 0$. The action is still diff-invariant provided Ψ transforms as

$$\delta_\xi \Psi = \xi^\mu \partial_\mu \Psi + \frac{1}{2} \nabla_\mu \xi^\mu \Psi \tag{B.2}$$

In the case $m = 0$ we also have Weyl invariance with

$$\delta_\omega \Psi = \frac{1}{2}\omega\Psi, \qquad \text{instead of} \qquad \delta_\omega \psi = \omega\psi, \tag{B.3}$$

So the simmetries are classically preserved while passing from ψ to Ψ. From a quantum point of view this might seem a Weyl transformation of Ψ, but it is not accompanied by a corresponding Weyl transformation of the metric. So it is simply a field redefinition, not a symmetry operation.

Alternative 1) is the procedure of Delbourgo-Salam. The action can be rewritten

$$S = \int d^3x \, \left[\frac{i}{2} \bar{\Psi} E^\mu_a \gamma^a \overset{\leftrightarrow}{\partial} \Psi - m\bar{\Psi}\Psi + \frac{1}{2} E^\mu_a \omega_{\mu bc} \epsilon^{abc} \bar{\Psi}\Psi \right]. \tag{B.4}$$

In this case we have one single graviton-fermion-fermion vertex V_{gff} represented by

$$\frac{i}{8} \left[(p+p')_\mu \gamma_\nu + (p+p')_\nu \gamma_\mu \right] \tag{B.5}$$

and one single 2-gravitons-2-fermions vertex V_{ggff} given by

$$\frac{1}{16} t_{\mu\nu\mu'\nu'\lambda} (k-k')^\lambda \tag{B.6}$$

where

$$t_{\mu\nu\mu'\nu'\lambda} = \eta_{\mu\mu'}\epsilon_{\nu\nu'\lambda} + \eta_{\nu\mu'}\epsilon_{\mu\nu'\lambda} + \eta_{\mu\nu'}\epsilon_{\nu\mu'\lambda} + \eta_{\nu\nu'}\epsilon_{\mu\mu'\lambda}, \tag{B.7}$$

the fermion propagator being

$$\frac{i}{\not{p} - m + i\epsilon}$$

The convention for momenta are the same as in [17, 19].

Alternative 2) introduces new vertices. In this case the Lagrangian can be written

$$\begin{aligned} L = & \frac{i}{2} \bar{\psi} \gamma^a \overset{\leftrightarrow}{\partial}_a \psi - i\, m\, \bar{\psi}\psi \\ & + \frac{i}{4} \bar{\psi} \gamma^a h^\mu_a \overset{\leftrightarrow}{\partial}_\mu \psi + \frac{i}{4} h^\lambda_\lambda \bar{\psi} \gamma^a \overset{\leftrightarrow}{\partial}_a \psi - \frac{i}{2} h^\lambda_\lambda\, m\, \bar{\psi}\psi \\ & + \frac{i}{8} h^\lambda_\lambda \bar{\psi} \gamma^a h^\mu_a \overset{\leftrightarrow}{\partial}_\mu \psi - \frac{1}{16} \bar{\psi}\, h^\lambda_c \partial_a h_{\lambda b} \psi\, \epsilon^{abc} \end{aligned} \tag{B.8}$$

As a consequence we have three new vertices. A vertex V'_{gff} coming from the mass term

$$-\frac{i}{2}\, m\, \eta_{\mu\nu} \mathbf{1}, \tag{B.9}$$

another V''_{gff} coming from the kinetic term

$$\frac{i}{4}\eta_{\mu\nu}(\not{p}+\not{p}') \tag{B.10}$$

and a new V'_{ggff}

$$\frac{i}{8}\eta_{\mu'\nu'}\left[(p+p')_\mu\gamma_\nu+(p+p')_\nu\gamma_\mu\right] \tag{B.11}$$

An obvious conjecture is that the two procedures lead to the same results, up to trivial terms. But this has still to be proved.

In this paper we follow alternative 1 only.

C Perturbative cohomology

In this appendix we define the form of local cohomology which is needed in perturbative field theory. Let us start from the gauge transformations.

$$\delta A = d\lambda + [A,\lambda], \qquad \delta\lambda = -\frac{1}{2}[\lambda,\lambda]_+, \qquad \delta^2 = 0, \qquad \lambda = \lambda^a(x)T^a \tag{C.1}$$

To dovetail the perturbative expansion it is useful to split it. Take A and λ infinitesimal and define the perturbative cohomology

$$\begin{aligned} &\delta^{(0)}A = d\lambda, && \delta^{(0)}\lambda = 0, && (\delta^{(0)})^2 = 0 \\ &\delta^{(1)}A = [A,\lambda], && \delta^{(1)}\lambda = -\frac{1}{2}[\lambda,\lambda]_+ && \\ &\delta^{(0)}\delta^{(1)} + \delta^{(1)}\delta^{(0)} = 0, && (\delta^{(1)})^2 = 0 && \end{aligned} \tag{C.2}$$

The full coboundary operator for diffeomorphisms is given by the transformations

$$\delta_\xi g_{\mu\nu} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\nu, \qquad \delta_\xi\xi^\mu = \xi^\lambda\partial_\lambda\xi^\mu \tag{C.3}$$

with $\xi_\mu = g_{\mu\nu}\xi^\nu$. We can introduce a perturbative cohomology, or graded cohomology, using as grading the order of infinitesimal, as follows

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + h^\mu_\lambda h^{\lambda\nu} + \dots \tag{C.4}$$

The analogous expansions for the vielbein is

$$e^a_\mu = \delta^a_\mu + \chi^a_\mu + \frac{1}{2}\psi^a_\mu + \dots,$$

Since $e^a_\mu\eta_{ab}e^b_\nu = h_{\mu\nu}$, we have

$$\chi_{\mu\nu} = \frac{1}{2}h_{\mu\nu}, \qquad \psi_{\mu\nu} = -\chi^a_\mu\chi_{a\nu} = -\frac{1}{4}h^\lambda_\mu h_{\lambda\nu}, \qquad \dots \tag{C.5}$$

This leads to the following expansion for the spin connection ω_μ^{ab}

$$\begin{aligned}\omega_\mu^{ab} &= \frac{1}{2}e^{\nu a}\left(\partial_\mu e_\nu^b - \partial_\nu e_\mu^b\right) - \frac{1}{2}e^{\nu b}\left(\partial_\mu e_\nu^a - \partial_\nu e_\mu^a\right) - \frac{1}{2}e^{\rho a}e^{\sigma b}\left(\partial_\rho e_{\sigma c} - \partial_\sigma e_{\rho c}\right)e_\mu^c \qquad \text{(C.6)}\\ &= -\frac{1}{2}\left(\partial^a h_\mu^b - \partial^b h_\mu^a\right) - \frac{1}{8}\left(h^{\sigma a}\partial_\mu h_\sigma^b - h^{\sigma b}\partial_\mu h_\sigma^a\right) + \frac{1}{4}\left(h^{\sigma a}\partial_\sigma h_\mu^b - h^{\sigma b}\partial_\sigma h_\mu^a\right)\\ &\quad -\frac{1}{8}\left(h^{\sigma a}\partial_\sigma h_\mu^b - h^{\sigma b}\partial_\sigma h_\sigma^a\right) - \frac{1}{8}h_\mu^c\left(\partial^a h_c^b - \partial^b h_c^a\right)\\ &\quad -\frac{1}{8}\left(\partial^b(h_\mu^\lambda h_\lambda^a) - \partial^a(h_\mu^\lambda h_\lambda^b)\right) + \ldots\end{aligned}$$

Inserting the above expansions in (C.3) we see that we have a grading in the transformations, given by the order of infinitesimals. So we can define a sequence of transformations

$$\delta_\xi = \delta_\xi^{(0)} + \delta_\xi^{(1)} + \delta_\xi^{(2)} + \ldots$$

At the lowest level we find immediately

$$\delta_\xi^{(0)} h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \qquad \delta_\xi^{(0)}\xi_\mu = 0 \qquad \text{(C.7)}$$

and $\xi_\mu = \xi^\mu$. Since $(\delta_\xi^{(0)})^2 = 0$ this defines a cohomology problem.

At the next level we get

$$\delta_\xi^{(1)} h_{\mu\nu} = \xi^\lambda \partial_\lambda h_{\mu\nu} + \partial_\mu \xi^\lambda h_{\lambda\nu} + \partial_\nu \xi^\lambda h_{\mu\lambda}, \qquad \delta_\xi^{(1)}\xi^\mu = \xi^\lambda \partial_\lambda \xi^\mu \qquad \text{(C.8)}$$

One can verify that

$$(\delta_\xi^{(0)})^2 = 0 \qquad \delta_\xi^{(0)}\delta_\xi^{(1)} + \delta_\xi^{(1)}\delta_\xi^{(0)} = 0, \qquad (\delta_\xi^{(1)})^2 = 0 \qquad \text{(C.9)}$$

Proceeding in the same way we can define an analogous sequence of transformations for the Weyl transformations. From $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\delta_\omega h_{\mu\nu} = 2\omega g_{\mu\nu}$ we find

$$\delta_\omega^{(0)} h_{\mu\nu} = 2\omega\eta_{\mu\nu}, \qquad \delta_\omega^{(1)} k_{\mu\nu} = 2\omega h_{\mu\nu}, \qquad \delta_\omega^{(2)} h_{\mu\nu} = 0, \ldots \qquad \text{(C.10)}$$

as well as $\delta_\omega^{(0)}\omega = \delta_\omega^{(1)}\omega = 0, \ldots$.

Notice that we have $\delta_\xi^{(0)}\omega = 0, \delta_\xi^{(1)}\omega = \xi^\lambda\partial_\lambda\omega$. As a consequence we can extend (C.9) to

$$(\delta_\xi^{(0)} + \delta_\omega^{(0)})(\delta_\xi^{(1)} + \delta_\omega^{(1)}) + (\delta_\xi^{(1)} + \delta_\omega^{(1)})(\delta_\xi^{(0)} + \delta_\omega^{(0)}) = 0 \qquad \text{(C.11)}$$

and $\delta_\xi^{(1)}\delta_\omega^{(1)} + \delta_\omega^{(1)}\delta_\xi^{(1)} = 0$, which together with the previous relations make

$$(\delta_\xi^{(0)} + \delta_\omega^{(0)} + \delta_\xi^{(1)} + \delta_\omega^{(1)})^2 = 0 \qquad \text{(C.12)}$$

For what concerns the higher tensor field $B_{\mu\nu\lambda}$ in this paper we use only the lowest order transformations given by (2.12) and (2.13).

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D Useful integrals

The Euclidean integrals over the momentum p we use for the 2-point function are:

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+\Delta)^2} = \frac{1}{8\pi}\frac{1}{\sqrt{\Delta}}, \tag{D.1}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{(p^2+\Delta)^2} = -\frac{3}{8\pi}\sqrt{\Delta}, \tag{D.2}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{p^4}{(p^2+\Delta)^2} = \frac{5}{8\pi}\Delta^{3/2}, \tag{D.3}$$

where $\Delta = m^2 + x\,(1-x)\,k^2$ and for the 3-point functions

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+\Delta)^3} = \frac{1}{32\pi}\frac{1}{\Delta^{\frac{3}{2}}} \tag{D.4}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{(p^2+\Delta)^3} = \frac{3}{32\pi}\frac{1}{\sqrt{\Delta}} \tag{D.5}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{p^4}{(p^2+\Delta)^3} = \frac{15}{32\pi}\sqrt{\Delta} \tag{D.6}$$

where $\Delta = m^2 + u(1-u)k_1^2 + v(1-v)k_2^2 + 2uvk_1\cdot k_2$. In these formulae x, u, v are Feynman parameters.

Sample calculation. As an example of our calculations we explain here some details of the derivation in 3.2. To make sense of the integral in (3.10) we have to go Euclidean, which implies $p^2 \to -p^2, k^2 \to -k^2, \eta_{\mu\nu} \to -\eta_{\mu\nu}$ and $d^3p \to id^3p$. Therefore

$$\tilde{T}^{(\text{odd})}_{\mu\nu\lambda\rho}(k) = \frac{m}{32}\int_0^1 dx \int \frac{d^3p}{(2\pi)^3}\left[\epsilon_{\sigma\nu\rho}\,k^\sigma\,\frac{\frac{4}{3}p^2\eta_{\mu\lambda} + (2x-1)^2k_\mu k_\lambda}{[p^2+m^2+x(1-x)k^2]^2} + \begin{pmatrix}\mu\leftrightarrow\nu\\ \lambda\leftrightarrow\rho\end{pmatrix}\right]. \tag{D.7}$$

Next we use the appropriate Euclidean integrals above to integrate over p and get

$$\begin{aligned}\tilde{T}^{(\text{odd})}_{\mu\nu\lambda\rho}(k) = &-\frac{m}{256\pi}\int_0^1 dx\,\epsilon_{\sigma\nu\rho}\,k^\sigma \\ &\times\left(4\eta_{\mu\lambda}(m^2+x(1-x)k^2)^{\frac{1}{2}} + k_\mu k_\lambda\frac{(2x-1)^2}{(m^2+x(1-x)k^2)^{\frac{1}{2}}}\right) + \begin{pmatrix}\mu\leftrightarrow\nu\\ \lambda\leftrightarrow\rho\end{pmatrix}\end{aligned} \tag{D.8}$$

The x integrals are well defined:

$$\int_0^1 dx\,(m^2+x(1-x)k^2)^{\frac{1}{2}} = \frac{1}{2}m + \frac{1}{4}\frac{k^2+4m^2}{|k|}\arctan\frac{|k|}{2m} \tag{D.9}$$

$$\int_0^1 dx\,\frac{(2x-1)^2}{(m^2+x(1-x)k^2)^{\frac{1}{2}}} = -2\frac{m}{k^2} + \frac{k^2+4m^2}{|k|^3}\arctan\frac{|k|}{2m} \tag{D.10}$$

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Therefore the result is

$$\begin{aligned}\tilde{T}^{(\text{odd})}_{\mu\nu\lambda\rho}(k) &= \frac{m}{256\pi}\,\epsilon_{\sigma\nu\rho}\,k^{\sigma}\left[-\eta_{\mu\lambda}\left(2m+\frac{k^2+4m^2}{|k|}\arctan\frac{|k|}{2m}\right)\right.\\ &\qquad\left.+\frac{k_\mu k\nu}{k^2}\left(-2m+\frac{k^2+4m^2}{|k|}\arctan\frac{|k|}{2m}\right)\right]+\begin{pmatrix}\mu\leftrightarrow\nu\\ \lambda\leftrightarrow\rho\end{pmatrix}\\ &= \frac{m}{256\pi}\,\epsilon_{\sigma\nu\rho}\,k^{\sigma}\left[2m\left(-\eta_{\mu\lambda}-\frac{k_\mu k_\lambda}{k^2}\right)\right.\\ &\qquad\left.+\left(-\eta_{\mu\lambda}+\frac{k_\mu k_\lambda}{k^2}\right)\frac{k^2+4m^2}{|k|}\arctan\frac{|k|}{2m}\right]+\begin{pmatrix}\mu\leftrightarrow\nu\\ \lambda\leftrightarrow\rho\end{pmatrix}\end{aligned}\tag{D.11}$$

The final step is to return to the Lorentzian metric, $k^2\to -k^2$ and $\eta_{\mu\nu}\to-\eta_{\mu\nu}$, $\arctan\frac{|k|}{2m}\to i\,\mathrm{arctanh}\frac{|k|}{2m}$.

E An alternative method for Feynman integrals

An alternative method to calculate Feynman diagrams was introduced in a series of paper by A. I .Davydychev and collaborators, [22–24]. The basic integral to be computed in our case are

$$J_2(d;\alpha,\beta;m)=\int\frac{d^dp}{(2\pi)^d}\frac{1}{(p^2-m^2)^\alpha\,((p-k)^2-m^2)^\beta}\tag{E.1}$$

and

$$J_3(d;\alpha,\beta,\gamma;m)=\int\frac{d^dp}{(2\pi)^d}\frac{1}{(p^2-m^2)^\alpha((p-k_1)^2-m^2)^\beta((p-q)^2-m^2)^\gamma},\tag{E.2}$$

with $q=k_1+k_2$. Following [22–24] these can be expressed, via the Mellin-Barnes representation of the propagator, as

$$\begin{aligned}J_2(d;\alpha,\beta;m) &= \frac{i^{1-d}}{(4\pi)^{\frac{d}{2}}}\frac{(-m^2)^{\frac{d}{2}-\alpha-\beta}}{\Gamma(\alpha)\,\Gamma(\beta)}\\ &\quad\times\int\frac{du}{2\pi i}\left(-\frac{k^2}{m^2}\right)^u\Gamma(-u)\,\frac{\Gamma(\alpha+u)\,\Gamma(\beta+u)\,\Gamma\left(\alpha+\beta-\frac{d}{2}+u\right)}{\Gamma(\alpha+\beta+2u)}\end{aligned}\tag{E.3}$$

and

$$\begin{aligned}J_3(d;\alpha,\beta,\gamma;m) &= \frac{i^{1-d}}{(4\pi)^{\frac{d}{2}}}\frac{(-m^2)^{\frac{d}{2}-\alpha-\beta-\gamma}}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\\ &\quad\times\int\frac{ds}{2\pi i}\frac{dt}{2\pi i}\frac{du}{2\pi i}\left(-\frac{k_1^2}{m^2}\right)^s\left(-\frac{q^2}{m^2}\right)^t\left(-\frac{k_2^2}{m^2}\right)^u\Gamma(-s)\Gamma(-t)\Gamma(-u)\\ &\quad\times\frac{\Gamma(\alpha+\beta+\gamma-\frac{d}{2}+s+t+u)\Gamma(\alpha+s+t)\Gamma(\beta+s+u)+\Gamma(\gamma+t+u)}{\Gamma(\alpha+\beta+\gamma+2s+2t+2u)}.\end{aligned}\tag{E.4}$$

The integrals run from $-i\infty$ to $i\infty$ along vertical contours that separate the positive poles of the Γ's from the negative ones. Positive poles are those of $\Gamma(-u)$ in the case of J_2

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or those of $\Gamma(-s)\Gamma(-t)\Gamma(-u)$ in the case of J_3, negative poles are the others. It is clear that the contours of integration must cross the real axis just to the left of the origin. The contours close either to the left or to the right in such a way as to assure convergence of the series. Let us analyse more closely the case of J_2 to better understand how this works. Using the duplication formula of the gamma function, i.e.

$$\Gamma(2z) = 2^{2z-1}\pi^{-\frac{1}{2}}\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) \tag{E.5}$$

we are able to recast (E.3) into the form

$$\frac{i^{1-d}}{(4\pi)^{\frac{d}{2}}}(-m^2)^{\frac{d}{2}-\alpha-\beta}\frac{\Gamma\left(\frac{\alpha+\beta}{2}\right)\Gamma\left(\frac{\alpha+\beta+1}{2}\right)}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(\alpha+\beta)}\int\frac{du}{2\pi i}\left(-\frac{k^2}{4m^2}\right)^u \times\Gamma(-u)\,\frac{\Gamma(\alpha+u)\,\Gamma(\beta+u)\,\Gamma\left(\alpha+\beta-\frac{d}{2}+u\right)}{\Gamma\left(\frac{\alpha+\beta}{2}+u\right)\Gamma\left(\frac{\alpha+\beta+1}{2}+u\right)}. \tag{E.6}$$

Assuming $\left|\frac{k^2}{4m^2}\right| < 1$ (IR region), we must close the contour of integration on the right ($\mathrm{Re}\,(u) > 0$) in order to guarantee convergence of the result and by doing so we will pick-up the poles of $\Gamma(-u)$. For $\alpha=\beta=1$ and $d=3$ we obtain

$$J_2^{\mathrm{IR}}(3;1,1;m) = \frac{i}{8\pi|m|}\sum_{j=0}^{\infty}\left(\frac{k^2}{4m^2}\right)^j\frac{1}{2j+1} = \frac{i}{4\pi|k|}\mathrm{arctanh}\left(\sqrt{\frac{k^2}{4m^2}}\right). \tag{E.7}$$

On the other hand, assuming $\left|\frac{k^2}{4m^2}\right| > 1$ (UV region), we need to close the integration contour on the left. For $\alpha=\beta=1$ and $d=3$, we will have poles at $u=-\frac{1}{2}$ and at $u=-1,-2,-3,\ldots$, hence

$$\begin{aligned} J_2^{\mathrm{UV}}(3;1,1;m) &= \frac{i}{8\pi|m|}\left(i\pi\frac{|m|}{|k|}+\sum_{j=1}^{\infty}\left(\frac{4m^2}{k^2}\right)^j\frac{1}{(2j-1)}\right) \\ &= -\frac{1}{8|k|}+\frac{i}{4\pi|k|}\mathrm{arctanh}\left(\sqrt{\frac{4m^2}{k^2}}\right). \end{aligned} \tag{E.8}$$

As far as (E.4) is concerned, in this paper we are interested in particular in the IR region, which is the one where m^2 is much larger than k_1^2, k_2^2, q^2 in the case of J_3. This requires that the relevant powers s,t,u in the integrands be positive, and, so, the contours must close around the poles of the positive real axis, that is the poles of $\Gamma(-s)\Gamma(-t)\Gamma(-u)$. An easy calculation gives

$$J_3(d;\alpha,\beta,\gamma;m) = \frac{i^{1-d}}{(4\pi)^{\frac{d}{2}}}(-m^2)^{\frac{d}{2}-\alpha-\beta-\gamma}\frac{\Gamma(\alpha+\beta+\gamma-\frac{n}{2})}{\Gamma(\alpha+\beta+\gamma)} \times\Phi_3\left[\begin{matrix}\alpha+\beta+\gamma-\frac{n}{2},\alpha,\beta,\gamma\\ \alpha+\beta+\gamma\end{matrix}\middle|\frac{k_1^2}{m^2},\frac{q^2}{m^2},\frac{k_2^2}{m^2}\right], \tag{E.9}$$

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where Φ_3 is a generalized Lauricella function:

$$\Phi_3\left[\begin{matrix}a_1,a_2,a_3,a_4\\ c\end{matrix}\middle| z_1,z_2,z_3\right] = \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{z_1^{j_1}}{j_1!}\frac{z_2^{j_2}}{j_2!}\frac{z_3^{j_3}}{j_3!}\frac{(a_1)_{j_1+j_2+j_3}(a_2)_{j_1+j_2}(a_3)_{j_1+j_3}(a_4)_{j_2+j_3}}{(c)_{2j_1+2j_2+2j_3}} \tag{E.10}$$

where $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$ is the Pochammer symbol. The leading term in the IR is clearly the one given by $j_1 = j_2 = j_3 = 0$, i.e. by setting $\Phi_3 = 1$ in (E.9).

In general, we need to evaluate not only (E.2) but more general integrals

$$J_{3,\mu_1\dots\mu_M}(d;\alpha,\beta,\gamma;m) = \int\frac{d^dp}{(2\pi)^d}\frac{p_{\mu_1}\dots p_{\mu_M}}{(p^2-m^2)^\alpha((p-k_1)^2-m^2)^\beta((p-q)^2-m^2)^\gamma} \tag{E.11}$$

One can prove by induction that the following formula holds in general

$$\begin{aligned}J_{3,\mu_1\dots\mu_M}(d;\alpha,\beta,\gamma;m) = &\sum_{\substack{\lambda,\kappa_1,\kappa_2,\kappa_3\\ 2\lambda+\sum\kappa_i=M}}\left(-\frac{1}{2}\right)^\lambda(4\pi)^{M-\lambda}\left\{[\eta]^\lambda[q_1]^{\kappa_1}[q_2]^{\kappa_2}[q_3]^{\kappa_3}\right\}_{\mu_1\dots\mu_M}\\ &\times(\alpha)_{\kappa_1}(\beta)_{\kappa_2}(\gamma)_{\kappa_3}J_3(d+2(M-\lambda);\alpha+\kappa_1,\beta+\kappa_2,\gamma+\kappa_3;m),\end{aligned} \tag{E.12}$$

where the symbol $\{[\eta]^\lambda[q_1]^{\kappa_1}\dots[q_N]^{\kappa_N}\}_{\mu_1\dots\mu_M}$ stands for the complete symmetrization of the objects inside the curly brackts, for example

$$\{\eta q_1\}_{\mu_1\mu_2\mu_3} = \eta_{\mu_1\mu_2}q_{1\mu_3} + \eta_{\mu_1\mu_3}q_{1\mu_2} + \eta_{\mu_2\mu_3}q_{1\mu_1}.$$

F Third order gravity CS and 3-point e.m. correlator

In this appendix we collect the result concerning the odd parity 3-point function of the e.m. tensor and its relation to the third order term in gravitational CS action.

F.1 The third order gravitational CS

From the action term (4.6), by differentiating three times with respect to $h_{\mu\nu}(x)$,$h_{\lambda\rho}(y)$ and $h_{\alpha\beta}(z)$ and Fourier-transforming the result one gets the sum of the following local terms in momentum space (they feature in the same order they appear in (4.6),

$$\begin{aligned}&\frac{\kappa}{4}\frac{i}{4}k_1^\sigma k_2^\tau\Big(\epsilon_{\mu\sigma\tau}(q_\alpha\eta_{\nu\lambda}\eta_{\rho\beta}-q_\rho\eta_{\nu\alpha}\eta_{\lambda\beta})+\epsilon_{\lambda\sigma\tau}(k_{1\alpha}\eta_{\mu\rho}\eta_{\nu\beta}-k_{1\nu}\eta_{\mu\alpha}\eta_{\rho\beta})\\ &\qquad+\epsilon_{\alpha\sigma\tau}(k_{2\nu}\eta_{\mu\rho}\eta_{\lambda\beta}-k_{2\lambda}\eta_{\mu\beta}\eta_{\nu\rho})\Big)\end{aligned} \tag{F.1}$$

$$\begin{aligned}&\frac{\kappa}{4}\frac{i}{4}\epsilon_{\mu\lambda\alpha}\Big(-k_1\cdot k_2\,(k_{1\rho}\eta_{\beta\nu}-k_{2\beta}\eta_{\rho\nu}+(k_2-k_1)_\nu\eta_{\beta\rho})\\ &\qquad+k_2^2\,(\eta_{\beta\rho}k_{1\nu}-\eta_{\nu\rho}k_{1\beta})+k_1^2\,(\eta_{\beta\nu}k_{2\rho}-\eta_{\beta\rho}k_{2\nu})\Big)\end{aligned} \tag{F.2}$$

$$\frac{\kappa}{4}\frac{i}{4}\epsilon_{\mu\lambda\alpha}\,(k_{1\beta}q_\rho k_{2\nu}-k_{1\nu}q_\beta k_{2\rho}) \tag{F.3}$$

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$$\frac{\kappa}{4}\frac{i}{4}\Big(\epsilon_{\mu\alpha\sigma}q^\sigma(q_\beta\eta_{\nu\lambda}-q_\lambda\eta_{\beta\nu})k_{2\rho}+\epsilon_{\mu\lambda\sigma}q^\sigma(q_\rho\eta_{\nu\beta}-q_\beta\eta_{\nu\rho})k_{1\alpha}$$
$$+\epsilon_{\lambda\alpha\sigma}k_1^\sigma(k_{1\beta}\eta_{\mu\rho}-k_{1\mu}\eta_{\beta\rho})k_{2\nu}+\epsilon_{\alpha\lambda\sigma}k_2^\sigma(k_{2\rho}\eta_{\mu\beta}-k_{2\mu}\eta_{\beta\rho})k_{1\nu}$$
$$+\epsilon_{\mu\lambda\sigma}k_1^\sigma(k_{1\nu}\eta_{\beta\rho}-k_{1\beta}\eta_{\rho\nu})q_\alpha+\epsilon_{\mu\alpha\sigma}k_2^\sigma(k_{2\nu}\eta_{\rho\beta}-k_{2\rho}\eta_{\beta\nu})q_\lambda\Big) \tag{F.4}$$

$$-\frac{\kappa}{4}\frac{i}{8}\Big(\epsilon_{\nu\rho\sigma}\left(q^\sigma q_\alpha(k_{2\mu}-k_{1\mu})\eta_{\lambda\beta}-k_1^\sigma k_{1\alpha}(k_{2\lambda}+q_\lambda)\eta_{\mu\beta}\right)$$
$$+\epsilon_{\nu\beta\sigma}\left(q^\sigma q_\lambda(k_{1\mu}-k_{2\mu})\eta_{\alpha\rho}-k_2^\sigma k_{2\lambda}(k_{1\alpha}+q_\alpha)\eta_{\mu\rho}\right)$$
$$+\epsilon_{\beta\rho\sigma}\left(k_1^\sigma k_{1\mu}(k_{2\lambda}+q_\lambda)\eta_{\alpha\nu}-k_2^\sigma k_{2\mu}(k_{1\alpha}+q_\alpha)\eta_{\lambda\nu}\right)\Big) \tag{F.5}$$

$$-\frac{\kappa}{4}\frac{i}{8}\Big(\epsilon_{\sigma\lambda\alpha}(\eta_{\beta\mu}\eta_{\rho\nu}(k_1^\sigma k_1\cdot k_2-k_2^\sigma k_2\cdot q)+\eta_{\beta\nu}\eta_{\rho\mu}(k_1^\sigma k_1\cdot q-k_2^\sigma k_1\cdot k_2))$$
$$+\epsilon_{\sigma\mu\alpha}(\eta_{\beta\lambda}\eta_{\nu\rho}(q^\sigma q\cdot k_2+k_2^\sigma k_1\cdot k_2)+\eta_{\beta\rho}\eta_{\nu\lambda}(-q^\sigma k_1\cdot q+k_2^\sigma q\cdot k_2))$$
$$+\epsilon_{\sigma\mu\lambda}(\eta_{\nu\beta}\eta_{\rho\alpha}(q^\sigma q\cdot k_1+k_1^\sigma k_1\cdot k_2)+\eta_{\nu\alpha}\eta_{\rho\beta}(-q^\sigma k_2\cdot q+k_1^\sigma q\cdot k_1))\Big) \tag{F.6}$$

$$\frac{\kappa}{4}\frac{i}{8}\Big[\epsilon_{\sigma\beta\nu}\eta_{\mu\rho}k_2^\sigma\left(\eta_{\alpha\lambda}k_2^2-k_{2\lambda}k_{2\alpha}\right)+\epsilon_{\sigma\beta\lambda}\eta_{\mu\rho}k_2^\sigma\left(\eta_{\alpha\nu}k_2^2-k_{2\nu}k_{2\alpha}\right)$$
$$+\epsilon_{\sigma\rho\nu}\eta_{\mu\beta}k_1^\sigma\left(\eta_{\alpha\lambda}k_1^2-k_{1\lambda}k_{1\alpha}\right)+\epsilon_{\sigma\rho\alpha}\eta_{\mu\beta}k_1^\sigma\left(\eta_{\lambda\nu}k_1^2-k_{1\nu}k_{1\lambda}\right)$$
$$-\epsilon_{\sigma\nu\beta}\eta_{\alpha\rho}\eta_{\mu\lambda}q^\sigma q^2-\epsilon_{\sigma\nu\rho}\eta_{\alpha\mu}\eta_{\beta\lambda}q^\sigma q^2+\epsilon_{\sigma\mu\rho}\eta_{\alpha\lambda}q^\sigma q_\beta q_\nu+\epsilon_{\sigma\mu\alpha}\eta_{\beta\rho}q^\sigma q_\lambda q_\nu\Big] \tag{F.7}$$

These terms must be simmetrized under $\mu\leftrightarrow\nu,\lambda\leftrightarrow\rho,\alpha\leftrightarrow\beta$. They are expected to correspond to odd-parity 3-point e.m. tensor correlator.

F.2 The IR limit of the 3-point e.m. correlator

The 0-th order term, after adding the cross contribution, is given (up to an overall multiplicative factor of $\frac{1}{128\cdot 32\pi}$) by

$$\tilde{T}^{(\mathrm{odd,IR})}_{\mu\nu\alpha\beta\lambda\rho}(k_1,k_2)=\frac{1}{256\pi}\sum_{i=1}^{4}\mathcal{T}^{(i)}_{\mu\nu\lambda\rho\alpha\beta}(k_1,k_2) \tag{F.8}$$

where

$$\begin{aligned}\mathcal{T}^{(1)}_{\mu\nu\lambda\rho\alpha\beta}(k_1,k_2)=&-\epsilon_{\sigma\beta\nu}k_2^\sigma\Big[\frac{4}{3}k_1\cdot k_2\left(\eta_{\rho\lambda}\eta_{\alpha\mu}+\eta_{\rho\alpha}\eta_{\lambda\mu}+\eta_{\rho\mu}\eta_{\lambda\alpha}\right)+\frac{4}{3}q_\alpha k_{2\mu}\eta_{\rho\lambda}-\frac{4}{3}k_{1\alpha}k_{2\lambda}\eta_{\rho\mu}\\&-\frac{2}{3}\eta_{\lambda\mu}(q_\alpha k_{1\rho}+k_{1\alpha}q_\rho+k_{1\alpha}k_{2\rho})+\frac{2}{3}\eta_{\lambda\alpha}(2q_\rho k_{1\mu}+k_{1\rho}(k_1-k_2)_\mu)\\&+\frac{4}{3}k_{1\mu}q_\lambda\eta_{\alpha\rho}+\frac{2}{3}\eta_{\mu\alpha}(2q_\rho q_\lambda+k_{1\rho}q_\lambda+q_\rho k_{2\lambda}+k_{2\rho}k_{2\lambda})\Big]\\&+\frac{2}{3}\epsilon_{\sigma\beta\nu}k_1^\sigma k_{2\rho}\Big[(k_1-k_2)_\mu\eta_{\lambda\alpha}+(q+k_2)_\lambda\eta_{\mu\alpha}-(q+k_1)_\alpha\eta_{\lambda\mu}\Big]\end{aligned} \tag{F.9}$$

$$\begin{aligned}\mathcal{T}^{(2)}_{\mu\nu\lambda\rho\alpha\beta}(k_1,k_2)=&-\epsilon_{\sigma\rho\nu}k_1^\sigma\Big[\frac{4}{3}k_1\cdot k_2\left(\eta_{\beta\lambda}\eta_{\alpha\mu}+\eta_{\alpha\beta}\eta_{\lambda\mu}+\eta_{\beta\mu}\eta_{\lambda\alpha}\right)+\frac{4}{3}q_\lambda k_{1\mu}\eta_{\beta\alpha}-\frac{4}{3}k_{1\alpha}k_{2\lambda}\eta_{\beta\mu}\\&+\frac{2}{3}\eta_{\lambda\mu}(2q_\alpha q_\beta+k_{1\alpha}q_\beta+q_\alpha k_{2\beta}+k_{1\alpha}k_{1\beta})+\frac{2}{3}\eta_{\lambda\alpha}(2q_\beta k_{2\mu}+k_{2\beta}(k_2-k_1)_\mu)\\&+\frac{4}{3}k_{2\mu}q_\alpha\eta_{\lambda\beta}-\frac{2}{3}\eta_{\mu\alpha}(q_\beta k_{2\lambda}+k_{2\beta}q_\lambda+k_{2\lambda}k_{1\beta})\Big]\\&-\frac{2}{3}\epsilon_{\sigma\rho\nu}k_2^\sigma k_{1\beta}\Big[(k_2-k_1)_\mu\eta_{\lambda\alpha}-(q+k_2)_\lambda\eta_{\mu\alpha}+(q+k_1)_\alpha\eta_{\lambda\mu}\Big]\end{aligned} \tag{F.10}$$

$$\mathcal{T}^{(3)}_{\mu\nu\lambda\rho\alpha\beta}(k_1,k_2) = \epsilon_{\rho\beta\nu}\Big[\frac{74}{15}k_1\cdot k_2(k_1-k_2)_\mu\eta_{\alpha\lambda} - \frac{1}{3}k_1\cdot k_2(15k_2+44k_1)_\alpha\eta_{\lambda\mu} \tag{F.11}$$
$$+\frac{1}{3}k_1\cdot k_2(44k_2+15k_1)_\lambda\eta_{\alpha\mu} - \frac{1}{15}k_{1\alpha}k_{1\lambda}(11k_1+47k_2)_\mu$$
$$+\frac{1}{15}k_{2\alpha}k_{2\lambda}(4k_2+7k_1)_\mu+\frac{1}{5}k_{1\alpha}k_{2\lambda}(k_2-k_1)_\mu+\frac{1}{15}k_{2\alpha}k_{1\lambda}(37k_1+3k_2)_\mu\Big]$$
$$\mathcal{T}^{(4)}_{\mu\nu\lambda\rho\alpha\beta} = -\eta_{\rho\nu}\epsilon_{\sigma\beta\tau}k_1^\sigma k_2^\tau\left(\frac{2}{3}\eta_{\mu\alpha}(k_1+2k_2)_\lambda+\frac{2}{3}\eta_{\lambda\alpha}(k_1-k_2)_\mu-\frac{2}{3}\eta_{\mu\lambda}(2k_1+k_2)_\alpha\right)$$
$$-\eta_{\beta\nu}\epsilon_{\sigma\rho\tau}k_2^\sigma k_1^\tau\left(-\frac{2}{3}\eta_{\mu\alpha}(k_1+2k_2)_\lambda+\frac{2}{3}\eta_{\lambda\alpha}(k_2-k_1)_\mu+\frac{2}{3}\eta_{\mu\lambda}(2k_1+k_2)_\alpha\right). \tag{F.12}$$

This must be simmetrized under $\mu\leftrightarrow\nu, \lambda\leftrightarrow\rho, \alpha\leftrightarrow\beta$. The IR limit is entirely local.

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