NUMERICAL ANALYSIS OF THE EFFECTIVE WIDTH OF THE SPECTRUM OF SYNCHROTRON RADIATION

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Abstract

For an exact quantitative description of spectral properties in the theory of synchrotron radiation, the concept of effective spectral width is introduced. In the classical theory, numeric calculations of effective spectral width (using an effective width not exceeding 100 harmonics) for polarization components of synchrotron radiation are carried out. The dependence of the effective spectral width and initial harmonic on the energy of a radiating particle is established.

INTRODUCTION

As one of the major quantitative characteristics of spectral distributions for electromagnetic radiation, one commonly uses the concept of spectral half-width. For spectral distributions having a sharp maximum, spectral half-width is the most informative physical characteristic.

However, once a spectral distribution has no pronounced maximum, spectral half-width ceases to be an adequate quantitative characteristic. In particular, this is exactly the case of spectral distributions for synchrotron radiation (SR), and therefore SR spectral half-width has neither been calculated theoretically, nor measured experimentally.

Instead of spectral half-width, the present study proposes to introduce a new precise quantitative characteristic of SR spectral distributions: effective spectral width. It is shown how this quantity can be calculated theoretically, and which physically relevant information can be obtained using this quantity.

In order to set up the problem, we now present some well-known expressions of the classical SR theory for the physical characteristics of synchrotron radiation, which can be found in [1–7].

The spectral-angular distribution for radiation power of SR polarization components can be written as

$$W_s = W \sum_{\nu=1}^{\infty} \int_0^{\pi} f_s(\beta;\, \nu,\, \theta) \sin\theta d\theta. \qquad (1)$$

Here, the following notation is used: θ is the angle between the control magnetic field strength and the radiation field pulse; ν is the number of an emitted harmonic; the charge orbital motion rate is $v = c\beta$, where c is the speed of light; W is the total radiated power of unpolarized radiation, which can be reveled in [1–7]. The index s numbers the polarization components: $s = 2$ corresponds to the σ-component of linear polarization; $s = 3$ corresponds to the π-component of linear polarization; $s = 1$ corresponds to right-hand circular polarization; $s = -1$ corresponds to left-hand circular polarization; $s = 0$ corresponds to the power of unpolarized radiation. The form of functions $f_s(\beta;\, \nu,\, \theta)$ can be founded in [1–7].

SPECTRAL DISTRIBUTION FOR POLARIZATION COMPONENTS OF SYNCHROTRON RADIATION IN THE UPPER HALF-SPACE

It is well known [1–7] that the angle range $0 \leqslant \theta < \pi/2$ (this range will be called the upper half-space) is dominated by right-hand circular polarization, and the angle range $\pi/2 < \theta \leqslant \pi$ (this range will be called the lower half-space) is dominated by left-hand circular polarization (exact quantitative characteristics of SR properties were first obtained in [8–11]). To reveal these features, the expressions (1) can be represented as

$$W_s = W \left[\Phi_s^{(+)}(\beta) + \Phi_s^{(-)}(\beta)\right],$$

$$\Phi_s^{(\pm)}(\beta) = \sum_{\nu=1}^{\infty} F_s^{(\pm)}(\beta;\, \nu),$$

$$F_s^{(\pm)}(\beta;\, \nu) = \int_{0\mp\pi/2}^{\pi/2\mp\pi/2} f_s(\beta;\, \nu,\, \theta) \sin\theta d\theta, \qquad (2)$$

and it suffices to study the properties of functions $F_s^{(+)}(\beta;\, \nu)$ (respectively, the properties of functions $\Phi_s^{(+)}(\beta)$), due to the evident relations

$$F_s^{(-)}(\beta;\, \nu) = F_s^{(+)}(\beta;\, \nu),\;\; \Phi_s^{(-)}(\beta) = \Phi_s^{(+)}(\beta)\; {}_{s=0,\,2,\,3};$$

$$F_{\pm1}^{(-)}(\beta;\, \nu) = F_{\mp1}^{(+)}(\beta;\, \nu),\;\; \Phi_{\pm1}^{(-)}(\beta) = \Phi_{\mp1}^{(+)}(\beta).$$

The exact form of the functions $F_s^{(\pm)}(\beta;\, \nu)$ and $\Phi_s^{(\pm)}(\beta)$ was revealed in [8–11].

EFFECTIVE SPECTRAL WIDTH FOR POLARIZATION COMPONENTS OF SYNCHROTRON RADIATION

As one of the quantitative characteristics of physical properties for spectral distributions of SR polarization components, it is proposed to introduce the concept of effective spectral width $\Lambda_s(\beta)$. Let us define $\Lambda_s(\beta)$ as follows.

For each fixed value of β, we examine the quantities of partial contributions $P_s(\beta;\nu)$ for individual spectral harmonics, introduced in [12].

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$$P_s(\beta;\nu) = \frac{F_s^{(+)}(\beta;\,\nu)}{\Phi_s^{(+)}(\beta)}\,. \tag{3}$$

Then (2) implies the property

$$\sum_{\nu=1}^{\infty} P_s(\beta;\nu) = 1\,. \tag{4}$$

We choose some values $\nu_s^{(1)}(\beta)$ and $\nu_s^{(2)}(\beta)$ such that the minimum difference $\nu_s^{(2)}(\beta) - \nu_s^{(1)}(\beta)$ should provide the minimum of the non-negative value

$$\sum_{\nu=\nu_s^{(1)}(\beta)}^{\nu_s^{(2)}(\beta)} P_s(\beta;\nu) - \frac{1}{2} \geqslant 0\,. \tag{5}$$

The effective spectral width $\Lambda_s(\beta)$ is defined by the expression

$$\Lambda_s(\beta) = \nu_s^{(2)}(\beta) - \nu_s^{(1)}(\beta) + 1,\ \ \Lambda_s(\beta) \geqslant 1\,. \tag{6}$$

we arrive at the following definition: effective spectral width is the minimum spectral range at which the sum of partial contributions for individual harmonics is not less than $1/2$.

In practice, the most interesting case is the ultra-relativistic limit ($\beta \approx 1$, equivalent to $\gamma \gg 1$). In this case, the analytical study of effective spectral width and other physically interesting quantitative characteristics for spectral distributions of SR polarization components can be significantly extended. This study was carried out in [13].

Given a particular value of β (or γ), it is a purely computational task to obtain the exact values of $\Lambda_s(\beta)$ and $\nu_s^{(1)}(\beta)$. In this article, we present a numerical study of the region $1 \leqslant \Lambda_s(\beta) \leqslant 100$. The effective width $\Lambda_s(\beta)$ is a positive integer, so there exists a range of β (corresponding to a range of γ; hereinafter, we only indicate γ) in which $\Lambda_s(\beta)$ is constant.

ANALYSIS OF NUMERICAL RESULTS FOR EFFECTIVE SPECTRAL WIDTH OF SYNCHROTRON RADIATION

The main results of a numerical study for effective spectral width of SR polarization components are given by our Table 1.

The numerical study is carried out as follows. For each type of polarization s, we examine the sequences of integers $\Lambda_s = 1, 2, 3\ldots$ and $\nu_s^{(1)} = 1, 2, 3\ldots$ (it is evident that $\nu_s^{(2)} = \nu_s^{(1)} + \Lambda_s - 1$) and determine the regions of values γ_s for which the condition (5) is satisfied. It is clear that the boundary points of possible regions for γ_s can be found, according to (5), as solutions of the equations

$$\sum_{l=0}^{\Lambda_s-1} P_s(\beta;\nu_s^{(1)} + l) - \frac{1}{2} = 0\,. \tag{7}$$

The roots $\beta_s = \beta_s(\Lambda_s, \nu_s^{(1)})$ of these equations determine the boundary points $\gamma_s = \gamma_s(\Lambda_s, \nu_s^{(1)})$.

In general, for given column s (given polarization component) of the Table 1 indicates for each Λ_s the smallest possible value $\nu_s^{(1)} = \nu_s^{(1)}(\Lambda_s)$ and the corresponding largest value $\gamma_s = \gamma_s(\Lambda_s, \nu_s^{(1)}(\Lambda_s))$, as well as the largest possible value $\tilde{\nu}_s$ for this Λ_s and the corresponding smallest value $\tilde{\gamma}_s$. Possible intermediate values $\nu_s^{(1)}$ between the smallest $\nu_s^{(1)}(\Lambda_s)$ and the largest $\tilde{\nu}_s$, as well as the respective intermediate values γ_s, are not specified. The intermediate values of γ_s always satisfy relations (8). Besides, for a given width Λs following inequalities are true

$$\gamma_s(\Lambda_s\,, k) < \gamma_s(\Lambda_s\,, n)\,,\ \ k > n. \tag{8}$$

Consequently, in the column for given s we indicate the regions of values γ

$$\gamma_s(\Lambda_s - 1, \nu_s^{(1)}(\Lambda_s - 1)) < \gamma \leqslant \gamma_s(\Lambda_s, \nu_s^{(1)}(\Lambda_s)), \tag{9}$$

for which the effective spectral width for the s-component of polarization for SR equals to Λ_s. We also indicate the initial points of the effective spectral width. These points are not determined uniquely. For the smallest initial value $\nu_s^{(1)}$, the range of values (9) taken by γ is the largest one, while this region is the smallest one for the largest possible value $\tilde{\nu}_s$.

For the other polarization components, the results of calculation are given in the respective columns of the Table 1. In particular, the Table 1 shows that for equal values Λ_s the corresponding values γ_s obey the inequalities

$$\gamma_3 > \gamma_1 > \gamma_0 > \gamma_2 > \gamma_{-1}\,. \tag{10}$$

At a fixed energy γ, the corresponding values of Λ_s are restricted by

$$\Lambda_3 < \Lambda_1 < \Lambda_0 < \Lambda_2 < \Lambda_{-1}\,. \tag{11}$$

In this way, for each polarization component of synchrotron radiation we have found energy regions at which the effective spectral width equals to Λ_s, and the initial harmonic of this effective width is determined. Numeric calculations have been carried out in the case $\Lambda_s \leqslant 100$.

In the ultrarelativistic case, the corresponding results have been obtained in [13].

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Table 1: Bondary harmonics and respective energy of effective spectral width for polarization component of SR

Λ	$\nu_2^{(1)}$	γ_2	$\tilde{\nu}_2$	$\tilde{\gamma}_2$	$\nu_3^{(1)}$	γ_3	$\tilde{\nu}_3$	$\tilde{\gamma}_3$	$\nu_0^{(1)}$	γ_0	$\tilde{\nu}_0$	$\tilde{\gamma}_0$	$\nu_{-1}^{(1)}$	γ_{-1}	$\tilde{\nu}_{-1}$	$\tilde{\gamma}_{-1}$	$\nu_1^{(1)}$	γ_1	$\tilde{\nu}_1$	$\tilde{\gamma}_1$
1	1	1.1434	1	1.1434	1	1.2363	1	1.2363	1	1.1592	1	1.1592	1	1.1062	1	1.1062	1	1.1712	1	1.1712
2	1	1.2955	1	1.2955	1	1.4519	1	1.4519	1	1.3204	1	1.3204	1	1.2348	1	1.2348	1	1.3411	1	1.3411
3	1	1.4179	2	1.3237	1	1.6126	1	1.6126	1	1.4476	1	1.4476	1	1.3440	2	1.3233	1	1.4737	1	1.4737
10	1	1.9566	3	1.9286	1	2.2680	2	2.2161	1	1.9986	3	1.9482	2	1.8655	4	1.8455	1	2.0416	2	2.0225
11	1	2.0120	3	1.9906	1	2.3329	2	2.2868	1	2.0550	3	2.0131	3	1.9197	5	1.8721	1	2.0995	3	2.0448
12	2	2.0657	4	2.0129	1	2.3949	2	2.3537	1	2.1084	3	2.0731	3	1.9711	5	1.9353	1	2.1542	3	2.1072
20	2	2.4168	5	2.3807	1	2.7998	2	2.7779	2	2.4603	4	2.4313	4	2.3049	7	2.2738	1	2.5141	3	2.4967
21	2	2.4538	5	2.4215	1	2.8422	2	2.8217	2	2.4980	4	2.4716	4	2.3402	7	2.3139	1	2.5522	4	2.5154
22	2	2.4895	5	2.4608	1	2.8831	2	2.8636	2	2.5345	4	2.5104	4	2.3744	7	2.3521	1	2.5891	4	2.5556
30	3	2.7443	6	2.7193	1	3.1728	3	3.1407	2	2.7937	5	2.7685	5	2.6176	9	2.5933	2	2.8530	4	2.8350
31	3	2.7731	6	2.7499	1	3.2053	3	3.1740	2	2.8227	5	2.7992	5	2.6449	9	2.6233	2	2.8828	4	2.8659
32	3	2.8012	6	2.7797	1	3.2374	3	3.2078	2	2.8512	5	2.8292	5	2.6716	9	2.6525	2	2.9119	4	2.8960
40	3	3.0078	7	2.9869	1	3.4723	3	3.4511	2	3.0601	6	3.0371	6	2.8684	11	2.8465	2	3.1258	5	3.1063
41	3	3.0316	7	3.0121	1	3.4998	3	3.4793	2	3.0842	6	3.0625	6	2.8911	11	2.8712	2	3.1505	5	3.1319
42	3	3.0550	7	3.0367	1	3.5270	3	3.5074	2	3.1079	6	3.0875	6	2.9133	11	2.8953	2	3.1748	5	3.1572
50	4	3.2309	8	3.2126	1	3.7286	3	3.7123	3	3.2864	6	3.2728	7	3.0811	12	3.0670	2	3.3567	6	3.3363
51	4	3.2515	8	3.2342	1	3.7528	3	3.7383	3	3.3074	6	3.2944	8	3.1007	13	3.0814	2	3.3781	6	3.3585
52	4	3.2720	8	3.2555	1	3.7757	3	3.7619	3	3.3281	7	3.3078	8	3.1202	13	3.1022	2	3.3992	6	3.3805
60	4	3.4267	9	3.4098	1	3.9502	3	3.9402	3	3.4849	7	3.4705	9	3.2675	14	3.2529	3	3.5590	6	3.5458
61	4	3.4450	9	3.4289	1	3.9700	4	3.9502	3	3.5035	7	3.4897	9	3.2851	14	3.2714	3	3.5781	6	3.5653
62	4	3.4634	9	3.4480	1	3.9911	4	3.9704	3	3.5219	7	3.5087	9	3.3024	14	3.2896	3	3.5969	6	3.5846
70	5	3.6025	10	3.5866	1	4.1489	4	4.1321	3	3.6625	8	3.6477	10	3.4347	16	3.4193	3	3.7409	7	3.7258
71	5	3.6191	10	3.6039	1	4.1678	4	4.1513	4	3.6794	8	3.6651	10	3.4505	16	3.4361	3	3.7581	7	3.7434
72	5	3.6356	10	3.6210	1	4.1844	4	4.1703	4	3.6961	8	3.6823	10	3.4662	16	3.4526	3	3.7751	7	3.7610
80	5	3.7624	11	3.7472	1	4.3263	4	4.3122	4	3.8246	8	3.8140	11	3.5868	17	3.5749	3	3.9061	7	3.8952
81	5	3.7776	11	3.7631	1	4.3432	4	4.3295	4	3.8400	9	3.8249	11	3.6013	17	3.5901	3	3.9219	7	3.9114
82	5	3.7931	11	3.7788	1	4.3601	4	4.3466	4	3.8554	9	3.8406	11	3.6157	17	3.6051	3	3.9375	8	3.9220
90	6	3.9105	11	3.9002	1	4.4884	4	4.4763	4	3.9736	9	3.9622	12	3.7268	19	3.7140	3	4.0582	8	4.0459
91	6	3.9247	12	3.9107	1	4.5039	4	4.4919	4	3.9879	9	3.9768	12	3.7403	19	3.7281	3	4.0727	8	4.0609
92	6	3.9386	12	3.9253	1	4.5192	4	4.5074	4	4.0021	9	3.9914	12	3.7536	19	3.7420	4	4.0872	8	4.0757
98	6	4.0214	12	4.0105	1	4.6089	4	4.5976	5	4.0851	10	4.0723	13	3.8317	20	3.8202	4	4.1722	8	4.1623
99	6	4.0349	12	4.0244	1	4.6232	4	4.6121	5	4.0987	10	4.0861	13	3.8444	20	3.8334	4	4.1860	8	4.1764
100	6	4.0482	12	4.0381	1	4.6374	4	4.6263	5	4.1121	10	4.0999	13	3.8570	20	3.8465	4	4.1997	9	4.1860

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