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The total capacity of customers in the MMPP/GI/∞ queueing system

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Abstract. In the paper, the infinite-server queueing system with a random capacity of customers is considered. In this system, the total capacity of customers is analysed by means of the asymptotic analysis method with high-rate Markov Modulated Poisson Process arrivals. It is obtained that the stationary probability distribution of the total customer capacity can be approximated by the Gaussian distribution. Parameters of the approximation is also derived in the paper.

Keywords: infinite-server queueing system, customer with random capacity, Markovian Modulated Poisson Process.

1. Introduction

In the design of messages processing and transmission systems, determining the memory capacity required for information storage is a relevant open issue [1, 2]. The total capacity is a random quantity and in queueing theory it is given by the sum of the lengths of all messages, which are waiting in the buffer or currently processed by servers.

Since in real systems customers are heterogeneous (for instance in computer networks packet size may vary from a few tens of bytes to 1500 bytes in case of Ethernet links), this paper focuses on the analysis of queueing system (QS) with random customer capacity. The main classes of models used for such models and their applications in real information systems are given [1, 2].

There are several works on the study of such systems with Poisson arrival process and service time independent of the customer capacity. For example, in [3] for systems with limited total capacity the generalization of the Erlang problem is considered in stationary conditions. Moreover, in [4, 5], the stationary distribution of the customers number and the probability of losses are obtained for systems with limited memory. Finally, in [9], the authors consider systems with service time depended on the customers capacity or with waiting time restrictions, an assumption very relevant for real-time applications.

It is important to point out that most known results are obtained for queueing systems and networks with Poisson arrivals. Unfortunately, it has been proved that the Poisson model is suitable only for few cases of modern telecommunication streams [6] and, in general, the correlation among arrivals must be taken into account. Therefore, many researches use more complex models of arrivals, such as Markovian Arrival Processes (MAPs) [7] or semi-Markov processes [8].

The main contribution of this paper consists in extending previous works on random capacity customers to the case of correlated arrivals. In more detail, the problem statement is formally defined in section 2 and in section 3 the corresponding Kolmogorov differential equations are derived. Then section 4 presents the results of the asymptotic analysis, focusing on first and second order approximations. Finally, the main findings are summarized in section 5.

2. Problem statement

In this paper, the MMPP/GI/∞ QS with random capacity customers is studied. The arrival process is a Markov Modulated Poisson Process (MMPP), a widely-used special case of MAP [7]. The system has an unlimited number of servers and service times on each server are i.i.d. with distribution function $B(x)$. All customers have a random capacity $\nu > 0$ with the probability distribution $G(y) = P\{\nu < y\}$ and the customers capacity are independent. Moreover, we assume that service time and customers capacity are mutually independent. After the service, customers leave the system and carry out the capacity.

Let us denote the number of customers in the system and the total customers capacity at time t by $i(t)$ and $V(t)$, respectively. We consider two-dimensional stochastic process $\{i(t), V(t)\}$, which is not Markovian. Therefore, we propose the dynamic screening method for its investigation [10].

For the screened process construction, we fix some point in time T. We assume that the customer arrived in the system at time $t < T$ creates a point in the screened process with probability

$$S(t) = 1 - B(T - t)$$

or does not create it with probability $1 - S(t)$. We name the points occurred in the screened process before t as customers in the screened process at time t.

Let us denote the customers number in the screened process at the moment t by $n(t)$. Then, if at the initial moment $t_0 < T$ the system is empty, we have the following equality at the moment T:

$$P\{i(T) = m\} = P\{n(T) = m\}.$$

Note that this method exactly determines the characteristics of the process $V(t)$ since the screened process contains only customers which do not finish the service at the moment T.

3. The system of Kolmogorov differential equations

Let us consider the three-dimensional Markovian process $\{k(t), n(t), V(t)\}$, where $k(t)$ identifies the state of the modulating Markov chain of the MMPP input process at time t $(1 \le k(t) \le K)$, which is defined through the infinitesimal generator matrix $\mathbf{Q}$ and rate matrix $\mathbf{\Lambda}$:

$$\mathbf{Q} = \begin{bmatrix} q_{11} & q_{12} & \dots & q_{1K} \\ q_{21} & q_{22} & \dots & q_{2K} \\ \dots & \dots & \dots & \dots \\ q_{K1} & q_{K2} & \dots & q_{KK} \end{bmatrix}, \qquad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_K \end{bmatrix}.$$

Denoting the probability distribution of this process by $P(k,n,z,t) = P\{k(t) = k, n(t) = n, V(t) < z\}$, we can write the corresponding system of Kolmogorov differential equations taking into account the formula of total probability:

$$\begin{gathered} P(k,n,z,t+\Delta t) = P(k,n,z,t)(1-\lambda_k)(1-q_{kk}) + \\ P(k,n,z,t)\lambda_k \Delta t (1 - S(t)) + \\ \lambda_k \Delta t S(t) \int_0^z P(k,n-1,z-y,t)\, dG(y) + \\ \sum_{\nu \ne k} q_{\nu k} \Delta t P(\nu,n,z,t) + o(\Delta t), \end{gathered} \tag{1}$$

for $k = 1 \dots K$, $n = 0, 1, 2, \dots,$ $z > 0$.

From (1), we obtain the system of Kolmogorov differential equations

$$\begin{gathered} \frac{\partial P(k,n,z,t)}{\partial t} = \lambda_k S(t) \left[\int_0^z P(k,n-1,z-y,t)\, dG(y) \right] + \\ \sum_{\nu} q_{\nu k} P(\nu,n,z,t) \end{gathered}$$

for $k = 1 \dots K$, $n = 0, 1, 2, \dots,$ $z > 0$.

We introduce a partial characteristic function of the form:

$$\begin{gathered} H(k,u_1,u_2,t) = M\{\exp(ju_1 n(t) + ju_2 V(t))\} = \\ \sum_{n=0}^{\infty} e^{ju_1 n} \int_0^{\infty} e^{ju_2 z} P(k,n,z,t)\, dz \end{gathered}$$

$$\text{for } k = 1 \dots K,\ n = 0, 1, 2, \dots,\ z > 0.$$

Considering that

$$\sum_{n=0}^{\infty} e^{ju_1 n} \int_0^{\infty} e^{ju_2 z} \int_0^{z} P(k, n-1, z-y, t)\, dG(y)\, dz =$$

$$e^{ju_1} H(k, u_1, u_2, t)\, G^*(u_2),$$

where

$$G^*(u_2) = \int_0^{\infty} e^{ju_2 y} dG(y), \tag{2}$$

we can write the following system of equations:

$$\frac{\partial H(k, u_1, u_2, t)}{\partial t} = \lambda_k S(t)\, H(k, u_1, u_2, t) \left[e^{ju_1} G^*(u_2) - 1\right]$$

$$\text{for } k = 1 \dots K.$$

We write this system in the form of a matrix equation

$$\frac{\partial \mathbf{H}(u_1, u_2, t)}{\partial t} = \mathbf{H}(u_1, u_2, t) \left[\mathbf{\Lambda} S(t) \left(e^{ju_1} G^*(u_2) - 1\right) + \mathbf{Q}\right] \tag{3}$$

with the initial condition

$$\mathbf{H}(u_1, u_2, t_0) = \mathbf{r}, \tag{4}$$

where

$$\mathbf{H}(u_1, u_2, t) = [H(1, u_1, u_2, t), H(2, u_1, u_2, t), \dots, H(K, u_1, u_2, t)],$$

and

$$\mathbf{r} = [r(1), r(2), \dots, r(K)],$$

is the row vector of the stationary distribution of the modulating Markov chain:

$$\begin{cases} \mathbf{rQ} = 0, \\ \mathbf{re} = 1, \end{cases} \tag{5}$$

$\mathbf{e}$ being a column vector with all entries equal to 1.

4. The asymptotic analysis method

The exact solution of the equation (3) is, in general, not available, but it is possible to get asymptotic results in case of heavy loads. To this aim we will use the asymptotic analysis method under the condition of an infinitely growing arrival rate. Let us substitute $\mathbf{\Lambda} = N\mathbf{\Lambda^1}$ and $\mathbf{Q} = N\mathbf{Q^1}$ into the equation (3), where N is some parameter which will be used for the asymptotic analysis ($N \to \infty$ in theoretical studies).

Then, the equation (3) takes the form

$$\frac{1}{N}\frac{\partial \mathbf{H}(u_1, u_2, t)}{\partial t} = \mathbf{H}(u_1, u_2, t)\left[\mathbf{\Lambda^1} S(t)\left(e^{ju_1}G^*(u_2) - 1\right) + \mathbf{Q^1}\right] \tag{6}$$

with the initial condition

$$\mathbf{H}(u_1, u_2, t_0) = \mathbf{r}. \tag{7}$$

4.1. The first-order asymptotic analysis

The main result is summarized by the following lemma.

Lemma. *The first-order asymptotic characteristic function of the probability distribution of the process* $\{k(t), n(t), V(t)\}$ *has the form*

$$\mathbf{H}(u_1, u_2, t) = \mathbf{r}\exp\left\{N\lambda\left[ju_1 + ju_2a_1\right]\int\limits_{t_0}^{t} S(\tau)\,d\tau\right\},$$

where the row vector $\mathbf{r}$ *is defined by the system of linear equations* (5), λ *denotes the average rate*

$$\lambda = \mathbf{r\Lambda e}$$

and a_1 *is the mean of the random variable defining the customer capacity*

$$a_1 = \int\limits_0^{\infty} y\,dG(y).$$

Proof.

Let us perform the substitutions

$$\varepsilon = \frac{1}{N}, u_1 = \varepsilon w_1, u_2 = \varepsilon w_2, \mathbf{H}(u_1, u_2, t) = \mathbf{F_1}(w_1, w_2, t, \varepsilon) \tag{8}$$

in the expressions (5) and (6).

Then the problem (5) – (6) takes the form

$$\varepsilon\frac{\partial \mathbf{F_1}(w_1,w_2,t,\varepsilon)}{\partial t}=$$
$$\mathbf{F_1}(w_1,w_2,t,\varepsilon)\left[\mathbf{\Lambda^1}S(t)\left(e^{j\varepsilon w_1}G^*(\varepsilon w_2)-1\right)+\mathbf{Q^1}\right] \quad (9)$$

with the initial condition

$$\mathbf{F_1}(w_1,w_2,t,\varepsilon)=\mathbf{r}.$$

Let us find the asymptotic solution (where $\varepsilon\to 0$) of the problem (8) – (9), i.e. the $\mathbf{F_1}(w_1,w_2,t)=\lim\limits_{\varepsilon\to 0}\mathbf{F_1}(w_1,w_2,t,\varepsilon)$.

Step 1. Letting $\varepsilon\to 0$ in (9), we obtain

$$\mathbf{F_1}(w_1,w_2,t)\,\mathbf{Q^1}=0.$$

Comparing this equation with the first one in (5), we can conclude that $\mathbf{F_1}(w_1,w_2,t)$ can be expressed as

$$\mathbf{F_1}(w_1,w_2,t)=\mathbf{r}\Phi_1(w_1,w_2,t), \quad (10)$$

where $\Phi_1(w_1,w_2,t)$ is some scalar function which satisfies the condition

$$\Phi_1(w_1,w_2,t_0)=1.$$

Step 2. Let us multiply (9) by vector $\mathbf{e}$, substitute (10), divide the results by ε and perform the asymptotic transition $\varepsilon\to 0$. Then, taking into account that $\mathbf{Q^1e}=0$ and $\mathbf{re}=1$, we obtain the following differential equation for the function $\Phi_1(w_1,w_2,t)$

$$\frac{\partial\Phi_1(w_1,w_2,t)}{\partial t}=\Phi_1(w_1,w_2,t)\left[\lambda S(t)(jw_1+jw_2a_1)\right]. \quad (11)$$

The solution of (11) with the initial condition gives

$$\Phi_1(w_1,w_2,t)=\exp\left\{\lambda(jw_1+jw_2a_1)\int\limits_{t_0}^{t}S(\tau)\,d\tau\right\}.$$

Substituting this expression into (10), we obtain

$$\mathbf{F_1}(w_1,w_2,t)=\mathbf{r}\exp\left\{\lambda(jw_1+jw_2a_1)\int\limits_{t_0}^{t}S(\tau)\,d\tau\right\}.$$

Using substitutions (8), we can write the asymptotic (as $\varepsilon \to 0$) equality:

$$\mathbf{H}(u_1, u_2, t) = \mathbf{F_1}(w_1, w_2, t, \varepsilon) \approx \mathbf{F_1}(w_1, w_2, t) = \mathbf{r}\Phi_1(w_1, w_2, t) =$$

$$= \mathbf{r}\exp\left\{\lambda\left[j\frac{u_1}{\varepsilon} + j\frac{u_2}{\varepsilon}a_1\right]\int\limits_{t_0}^{t} S(\tau)\,d\tau\right\} =$$

$$\mathbf{r}\exp\left\{N\lambda\left[ju_1 + ju_2a_1\right]\int\limits_{t_0}^{t} S(\tau)\,d\tau\right\}.$$

The proof is complete.

Corollary. *When $t = T$ we obtain the characteristic function of the process $\{i(t), V(t)\}$ in the steady state regime*

$$H(u_1, u_2, t) = \exp\{N\lambda b_1[ju_1 + ju_2a_1]\},$$

where

$$b_1 = \int\limits_{-\infty}^{T} S(\tau)\,d\tau = \int\limits_{-\infty}^{T}(1 - B(T-\tau))\,d\tau = \int\limits_{0}^{\infty}(1 - B(\tau))\,d\tau$$

denotes the mean service time.

4.2. The second-order asymptotic analysis

The main result is summarized by the following theorem.

Theorem. *The second-order asymptotic characteristic function of the probability distribution of the process $\{k(t), n(t), V(t)\}$ has the form*

$$\mathbf{H}(u_1, u_2, t) = \mathbf{r}\exp\left\{N\lambda(ju_1 + ju_2a_1)\int\limits_{t_0}^{t} S(\tau)\,d\tau + \right.$$

$$\frac{(ju_1)^2}{2}\left(N\lambda\int\limits_{t_0}^{t} S(\tau)\,d\tau + N\kappa\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right) +$$

$$\frac{(ju_2)^2}{2}\left(N\lambda a_2\int\limits_{t_0}^{t} S(\tau)\,d\tau + N\kappa a_1^2\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right) +$$

$$j^2 u_1 u_2 \left(N\lambda a_1 \int\limits_{t_0}^{t} S(\tau)\, d\tau + N\kappa a_1 \int\limits_{t_0}^{t} S^2(\tau)\, d\tau \right) \Bigg\},$$

where

$$\kappa = 2\mathbf{g}\left(\mathbf{\Lambda^1} - \lambda \mathbf{I}\right)\mathbf{e},$$

and the row vector $\mathbf{g}$ *satisfies the linear matrix system*

$$\mathbf{g}\mathbf{Q^1} = \mathbf{r}\left(\lambda \mathbf{I} - \mathbf{\Lambda^1}\right),$$

$$\mathbf{g}\mathbf{e} = 1.$$

Proof.

Denote by $\mathbf{H_2}(u_1, u_2, t)$ a multi-dimensional function that satisfies the equation

$$\mathbf{H}(u_1, u_2, t) = \mathbf{H_2}(u_1, u_2, t) \exp\left\{ N\lambda (ju_1 + ju_2 a_1) \int\limits_{t_0}^{t} S(\tau)\, d\tau \right\}. \quad (12)$$

Substituting this expression into (6) and (7), we obtain the following problem:

$$\frac{1}{N} \frac{\partial \mathbf{H_2}(u_1, u_2, t)}{\partial t} + \lambda (ju_1 + ju_2 a_1) S(t) \mathbf{H_2}(u_1, u_2, t) = \quad (13)$$

$$= \mathbf{H_2}(u_1, u_2, t) \left[\mathbf{\Lambda^1} S(t) \left(e^{ju_1} G^*(u_2) - 1\right) + \mathbf{Q^1}\right],$$

with the initial condition

$$\mathbf{H_2}(u_1, u_2, t_0) = \mathbf{r}. \quad (14)$$

Let us perform the following substitutions

$$\varepsilon^2 = \frac{1}{N}, u_1 = \varepsilon w_1, u_2 = \varepsilon w_2, \mathbf{H_2}(u_1, u_2, t) = \mathbf{F_2}(w_1, w_2, t, \varepsilon). \quad (15)$$

Using these notations the problem (13) – (14) can be rewritten in the form

$$\varepsilon^2 \frac{\partial \mathbf{F_2}(w_1, w_2, t, \varepsilon)}{\partial t} + \mathbf{F_2}(w_1, w_2, t, \varepsilon) \lambda (j\varepsilon w_1 + j\varepsilon w_2 a_1) S(t) = \quad (16)$$

$$= \mathbf{F_2}(w_1, w_2, t, \varepsilon) \left[\mathbf{\Lambda^1} S(t) \left(e^{j\varepsilon w_1} G^*(\varepsilon w_2) - 1\right) + \mathbf{Q^1}\right],$$

with the initial condition

$$\mathbf{F_2}(w_1, w_2, t_0, \varepsilon) = \mathbf{r}. \quad (17)$$

Let us find the asymptotic solution (as $\varepsilon \to 0$) of this problem, i.e. the function $\mathbf{F_2}(w_1, w_2, t) = \lim\limits_{\varepsilon \to 0} \mathbf{F_2}(w_1, w_2, t, \varepsilon)$.

Step 1. Letting $\varepsilon \to 0$ in (16) – (17), we obtain the following system of equations:

$$\begin{cases} \mathbf{F_2}(w_1, w_2, t)\,\mathbf{Q^1} = \mathbf{0}, \\ \mathbf{F_2}(w_1, w_2, t_0) = \mathbf{r}, \end{cases}$$

Then, using (4), we can write

$$\mathbf{F_2}(w_1, w_2, t) = \mathbf{r}\Phi_2(w_1, w_2, t), \tag{18}$$

where $\Phi_2(w_1, w_2, t)$ is some scalar function which satisfies the condition

$$\Phi_2(w_1, w_2, t_0) = 1.$$

Step 2. Using (18), the function $\mathbf{F_2}(w_1, w_2, t)$ can be represented in the expansion form

$$\mathbf{F_2}(w_1, w_2, t, \varepsilon) =$$

$$\Phi_2(w_1, w_2, t)\left[\mathbf{r} + \mathbf{g}\left(j\varepsilon w_1 + j\varepsilon w_2 a_1\right) S(t)\right] + \mathbf{O}\left(\varepsilon^2\right), \tag{19}$$

where $\mathbf{g}$ is some row vector which satisfying the condition $\mathbf{ge} = 1$ and $\mathbf{O}\left(\varepsilon^2\right)$ is row vector whose elements are infinitesimals of the same order as ε^2.

Let us use the substitution (19) and the Taylor-Maclaurin expansions

$$e^{j\varepsilon w_1} = 1 + j\varepsilon w_1 + O\left(\varepsilon^2\right), e^{j\varepsilon w_2} = 1 + j\varepsilon w_2 + O\left(\varepsilon^2\right)$$

in (16). Considering the (2), we perform in the obtained equality of the limiting transition $\varepsilon \to 0$, we obtain matrix equation for the vector $\mathbf{g}$

$$\mathbf{gQ^1} = \mathbf{r}\left(\lambda\mathbf{I} - \mathbf{\Lambda^1}\right),$$

where $\mathbf{I}$ is diagonal unit matrix.

Step 3. We multiply the (16) by the $\mathbf{e}$, using (19) and the second-order expansions

$$e^{j\varepsilon w_1} = 1 + j\varepsilon w_1 + \frac{\left(j\varepsilon w_1\right)^2}{2} + O\left(\varepsilon^2\right),$$

$$e^{j\varepsilon w_2} = 1 + j\varepsilon w_2 + \frac{\left(j\varepsilon w_2\right)^2}{2} + O\left(\varepsilon^2\right).$$

As a result of simple transformations with the notation

$$\kappa = 2\mathbf{g}\left(\mathbf{\Lambda^1} - \lambda\mathbf{I}\right)\mathbf{e},$$

we obtain the following differential equation for the function $\Phi_2(w_1, w_2, t)$

$$\frac{\partial \Phi_2(w_1,w_2,t)}{\partial t} = \Phi_2(w_1,w_2,t)\Bigg\{\frac{(jw_1)^2}{2}\left(\lambda S(t) + \kappa S^2(t)\right) +$$

$$\frac{(jw_2)^2}{2}\left(\lambda a_2 S(t) + \kappa a_1^2 S^2(t)\right) + j^2 w_1 w_2 \left(\lambda a_1 S(t) + \kappa a_1 S^2(t)\right)\Bigg\},$$

where $a_2 = \int\limits_0^\infty y^2 dG(y)$ is the second moment of the random customer capacity ν.

The solution of the latter equation with the available initial condition $\Phi_2(w_1, w_2, t_0) = 1$ gives the expression $\Phi_2(w_1, w_2, t)$

$$\Phi_2(w_1,w_2,t) = \exp\Bigg\{\frac{(jw_1)^2}{2}\left(\lambda\int\limits_{t_0}^{t} S(\tau)\,d\tau + \kappa\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right) +$$

$$\frac{(jw_2)^2}{2}\left(\lambda a_2\int\limits_{t_0}^{t} S(\tau)\,d\tau + \kappa a_1^2\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right) +$$

$$j^2 w_1 w_2\left(\lambda a_1\int\limits_{t_0}^{t} S(\tau)\,d\tau + \kappa a_1\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right)\Bigg\},$$

and substituting in (18) we obtain

$$\mathbf{F_2}(w_1,w_2,t) = \mathbf{r}\exp\Bigg\{\frac{(jw_1)^2}{2}\left(\lambda\int\limits_{t_0}^{t} S(\tau)\,d\tau + \kappa\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right) +$$

$$\frac{(jw_2)^2}{2}\left(\lambda a_2\int\limits_{t_0}^{t} S(\tau)\,d\tau + \kappa a_1^2\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right) + \qquad (20)$$

$$j^2 w_1 w_2\left(\lambda a_1\int\limits_{t_0}^{t} S(\tau)\,d\tau + \kappa a_1\int\limits_{t_0}^{t} S^2(\tau)\,d\tau\right)\Bigg\}.$$

Performing in (20) the substitutions inverse to (15) and (12), we obtain the following expression for the asymptotic characteristic function of the number of customers of screened process and total capacity of customers at the moment t:

$$\mathbf{H}(u_1,u_2,t)=\mathbf{r}\exp\left\{N\lambda\left(ju_1+ju_2a_1\right)\int\limits_{t_0}^{t}S(\tau)\,d\tau+\right.$$

$$\frac{\left(ju_1\right)^2}{2}\left(N\lambda\int\limits_{t_0}^{t}S(\tau)\,d\tau+N\kappa\int\limits_{t_0}^{t}S^2(\tau)\,d\tau\right)+$$

$$\frac{\left(ju_2\right)^2}{2}\left(N\lambda a_2\int\limits_{t_0}^{t}S(\tau)\,d\tau+N\kappa a_1^2\int\limits_{t_0}^{t}S^2(\tau)\,d\tau\right)+$$

$$\left.j^2u_1u_2\left(N\lambda a_1\int\limits_{t_0}^{t}S(\tau)\,d\tau+N\kappa a_1\int\limits_{t_0}^{t}S^2(\tau)\,d\tau\right)\right\},$$

where

$$\kappa=2\mathbf{g}\left(\mathbf{\Lambda^1}-\lambda\mathbf{I}\right)\mathbf{e},$$

and the row vector $\mathbf{g}$ satisfies the linear matrix system

$$\mathbf{gQ^1}=\mathbf{r}\left(\lambda\mathbf{I}-\mathbf{\Lambda^1}\right),$$

$$\mathbf{ge}=1.$$

The proof is complete.

Corollary 1. *When $t=T$ we obtain the characteristic function of the process $i(t),V(t)$ in the steady state regime*

$$H(u_1,u_2,t)=\exp\left\{N\lambda\left(ju_1+ju_2a_1\right)b_1+\frac{\left(ju_1\right)^2}{2}\left(N\lambda b_1+N\kappa b_2\right)+\right.$$

$$\left.\frac{\left(ju_2\right)^2}{2}\left(N\lambda a_2b_1+Na_1^2\kappa b_2\right)+j^2u_1u_2\left(N\lambda a_1b_1+N\kappa a_1b_2\right)\right\},\qquad(21)$$

where

$$b_2=\int\limits_{-\infty}^{T}S^2(\tau)\,d\tau.$$

From the form of function (21) it is clear that the two-dimensional process $i(t), V(t)$ is asymptotically Gaussian with the vector of mathematical expectations

$$\mathbf{a} = [N\lambda b_1, N\lambda a_1 b_1]$$

and the covariance matrix

$$\mathbf{A} = \begin{bmatrix} \sigma_1^2 & r\sigma_1\sigma_2 \\ r\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} N\lambda b_1 + N\kappa b_2 & N\lambda a_1 b_1 + N\kappa a_1 b_2 \\ N\lambda a_1 b_1 + N\kappa a_1 b_2 & N\lambda a_2 b_1 + N\kappa a_1^2 b_2 \end{bmatrix}.$$

Corollary 2. *The asymptotic characteristic function of the total customer capacity in the steady-state regime is given by a Gaussian characteristic function*

$$H(u,t) = \exp\left\{ juN\lambda a_1 b_1 + \frac{(ju)^2}{2}\left(N\lambda a_2 b_1 + N a_1^2 \kappa b_2\right)\right\},$$

with parameters $a = N\lambda a_1 b_1$ *and* $\sigma^2 = N\lambda a_2 b_1 + N a_1^2 \kappa b_2$.

5. Conclusions

In the paper, a queueing system with random customers capacity and service time independent of its capacity is considered in case of correlated arrivals, described by an MMPP process. For such system, the total customers capacity is derived by using the asymptotic analysis method in case of heavy loads. It is obtained that the stationary probability distribution of total capacity can be approximated by a Gaussian distribution and the parameters of the approximation are derived in the paper.

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