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ESTIMATION OF THE DEAD TIME PERIOD DURATION IN THE MODULATED SEMI-SYNCHRONOUS GENERALIZED FLOW OF EVENTS USING THE MAXIMUM LIKELIHOOD TECHNIQUE

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In the recent literature the problem of studying the doubly stochastic Poisson processes has been of a great interest, since these processes have found applications in many fields such as network theory, peer-to-peer streaming networks and adaptive data streaming, optical communication systems, statistical modeling, quantitative finance, spatial epidemiology, etc. [1, 2]. In this paper we study the modulated semi-synchronous generalized flow of events, which is related to the class of doubly stochastic Poisson processes, in conditions of its incomplete observability, when the dead time period of a constant duration T is generated after every registered event. This paper is devoted to the maximum likelihood estimation of the dead time period duration on monitoring the time moments of the flow events occurrence.

1. Problem Statement

We consider the modulated semi-synchronous generalized flow of events, which intensity process is a piecewise constant stationary random process $\lambda(t)$ with two states λ_1 and λ_2 $(\lambda_1 > \lambda_2 \geq 0)$. During the time interval of a random duration when the process $\lambda(t)$ is in state λ_i $(\lambda(t) = \lambda_i)$ a Poisson flow of events with intensity λ_i, $i = 1,2$, arrives. The transition of the process $\lambda(t)$ from the first state to the second state is possible at any moment of a Poisson event occurrence in state 1 of the process $\lambda(t)$, herewith the process $\lambda(t)$ can change its state to the second one with probability p $(0 \leq p \leq 1)$ or continue to stay in state 1 with complementary probability $1-p$. The transition of the process $\lambda(t)$ from state 1 to state 2 is also possible at any moment that does not coincide with the moment of a Poisson event occurrence, herewith the duration of the process $\lambda(t)$ staying in the first state is distributed according to the exponential law with parameter β: $F(\tau) = 1 - e^{-\beta\tau}$, $\tau \geq 0$. Then the duration of the process $\lambda(t)$ staying in the first state is distributed according to the exponential law $F_1(\tau) = 1 - e^{-(p\lambda_1 + \beta)\tau}$, $\tau \geq 0$. The transition of the process $\lambda(t)$ from the second state to the first state at the moment of a Poisson event occurrence in state 2 is impossible and can be done only at a random time moment. In this case the duration of the process $\lambda(t)$ staying in state 2 is distributed according to the exponential law with parameter α: $F_2(\tau)=1-e^{-\alpha\tau}$, $\tau \geq 0$. At the moment when the state changes from the second to the first one, an additional event is assumed to be initiated with probability δ $(0 \leq \delta \leq 1)$. Such flows with additional events initiation are called generalized flows. Under the made assumptions we can assert that $\lambda(t)$ is a Markovian process.

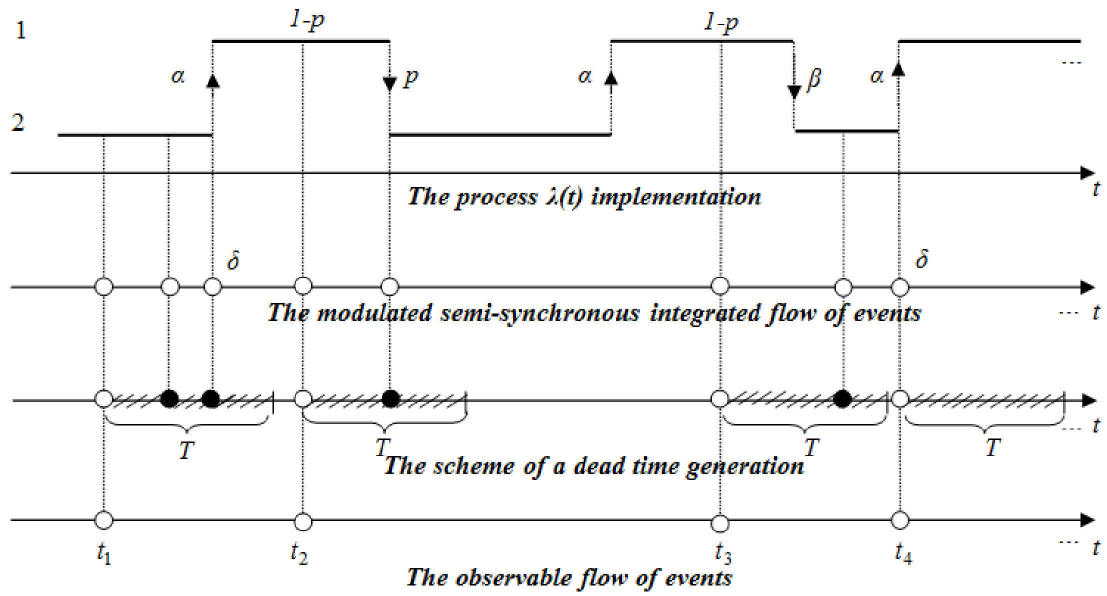

Fig. 1. The formation of the observable flow of events

The registration of the flow events is considered in conditions of a constant (unextendable) dead time. The dead time period of a constant duration T begins after every registered at the moment t_k event. During this period no other events are observed. When the dead time period is over, the first coming event causes the next interval of a dead time of duration T and so on. Figure 1 shows the possible variant of the flow operation and observation. Here 1, 2 are the states of the process $\lambda(t)$; additional events, that may occur at the moment of the process $\lambda(t)$ transition from state 2 to state 1, are marked with letter δ; dead time periods of duration T are marked with hatching; unobservable events are displayed as black circles, observable events t_1, t_2,... are shown as white circles.

The process $\lambda(t)$ is considered in a steady-state conditions, that is why we may neglect transient processes at the interval of observation $(t_0,t]$, where t_0 is an instant of beginning the observations, t is an instant of ending the observations (the moment of a decision making). In a steady-state conditions we may take $t_0=0$. We should note that the process $\lambda(t)$ and possible events are basically unobservable; we register only time moments $t_1,t_2,...,t_k$ of the events occurrence in observable flow during the interval of observation $(t_0,t]$. We assume that the flow parameters $\lambda_1>\lambda_2\geq 0$, $0\leq p\leq 1$, $\beta>0$, $\alpha>0$, $0\leq\delta\leq 1$ are known and the duration of the dead time period T is not known. In that way, the main problem is to obtain the estimate $\hat{T}$ of the dead time period duration at the moment t of ending the observations on monitoring the time moments $t_1, t_2, ..., t_k$ of the events occurrence in observable flow using the maximum-likelihood technique.

2. The construction of likelihood function

Denote by $\tau_k = t_{k+1} - t_k$, $k = 1,2,...$, the value of the k interval length between two consecutive flow events ($\tau_k > 0$). In a steady-state conditions we may take that the probability density function of the k interval length is $p_T(\tau_k) = p_T(\tau)$, $\tau \ge 0$, for any k. Thereby we may also take that the time moment t_k is equal to zero, i.e. the moment of the event occurrence is $\tau = 0$. Then the probability density function $p_T(\tau)$, $\tau \ge 0$, can be written as [3, 4]

$$p_T(\tau) = 0, \; 0 \le \tau < T; \tag{1}$$

$$p_T(\tau) = \frac{z_1}{z_2 - z_1}\left[z_2 - \frac{1}{\beta_1 + \beta_2} f(T)\right] e^{-z_1(\tau - T)} - \frac{z_2}{z_2 - z_1}\left[z_1 - \frac{1}{\beta_1 + \beta_2} f(T)\right] e^{-z_2(\tau - T)}, \; \tau \ge T;$$

$$f(T) = \lambda_1\alpha + (p\lambda_1 + \beta)(\lambda_2 + \alpha\delta) + \alpha(\lambda_1 - \lambda_2 - \alpha\delta)\{(p\lambda_1 + \beta)[\lambda_1(1 - p + p\delta) -$$

$$-\lambda_2 + \delta\beta] - p\lambda_1\alpha\}\frac{e^{-(\beta_1 + \beta_2)T}}{F(T)}, \quad F(T) = z_1 z_2 - q e^{-(\beta_1 + \beta_2)T},$$

$$\beta_1 = p\lambda_1 + \beta, \; \beta_2 = \alpha, \; q = \lambda_1[\lambda_2 - p(\lambda_2 + \alpha\delta)],$$

$$z_{1,2} = \frac{1}{2}\left(\lambda_1 + \lambda_2 + \alpha + \beta \mp \sqrt{(\lambda_1 - \lambda_2 - \alpha + \beta)^2 + 4\alpha\beta(1 - \delta)}\right), \quad 0 < z_1 < z_2.$$

Now let $\tau_1 = t_2 - t_1$, $\tau_2 = t_3 - t_2$, …, $\tau_k = t_{k+1} - t_k$, $\tau_1 \ge 0$, $\tau_2 \ge 0$, …, $\tau_k \ge 0$, be the sequence of values of the intervals lengths between consecutive flow events measured during the interval of observation $(0, t]$. Arrange the variables τ_1, …, τ_k in ascending order: $\tau_{\min} = \tau^{(1)} < \tau^{(2)} < ... < \tau^{(k)}$. Under the made assumptions we can assert that the sequence of the time moments t_1, t_2,…, t_k,… corresponds to an embedded Markov chain $\{\lambda(t_k)\}$, i.e. the flow has the Markov property if the evolution of the flow is considered from the time moment t_k, $k = 1,2,...$, of the event occurrence. As the main problem is to estimate the dead time period duration on assumption that the flow parameters $\lambda_1 > \lambda_2 \ge 0$, $0 \le p \le 1$, $\beta > 0$, $\alpha > 0$, $0 \le \delta \le 1$ are known, then according to the maximum-likelihood technique we should maximize the likelihood function $L(T \mid \tau^{(1)}, ..., \tau^{(k)})$ and solve the following task of optimization:

$$L(T \mid \tau^{(1)}, ..., \tau^{(k)}) = \prod_{j=1}^{k} p_T(\tau^{(j)}) \Rightarrow \max_T, \quad 0 \le T \le \tau_{\min}, \quad \tau_{\min} > 0, \tag{2}$$

where $p_T(\tau^{(j)})$ is defined in (1) for $\tau = \tau^{(j)}$. The value of T, when the likelihood function (2) reaches its global maximum, will be the estimate $\hat{T}$ of the dead time period duration.

3. The solution of optimization problem

Since the likelihood function (2) is different from zero when $0 \le T \le \tau_{\min}$, assume that $p_T(\tau^{(j)}) = 0$, $j = \overline{2,k}$, $T > \tau_{\min}$ ($\tau_{\min} > 0$). The situation further, when $\tau_{\min} = 0$, refers to the extension of definition the functions being studied at the boundary points. Let us study the behavior of the function $p_T(\tau_{\min})$ as a function of a variable T, $0 \le T \le \tau_{\min}$. We shall note that $p_T(\tau_{\min}) \ge 0$, as $p_T(\tau_{\min})$ is the probability density function.

Lemma 1. The derivative $p'_T(\tau_{\min})$ is a positive function of a variable $\tau_{\min}$ for $T = 0$, $0 \le \tau_{\min} < \infty$ ($p'_{T=0}(\tau_{\min}) > 0$).

Lemma 2. The derivative $p'_T(\tau_{\min})$ is strictly greater than zero for $T = \tau_{\min}$, $0 \le \tau_{\min} < \infty$ ($p'_{T=\tau_{\min}}(\tau_{\min}) > 0$).

Theorem 1. The derivative $p'_T(\tau_{\min})$ is a positive function ($p'_T(\tau_{\min}) > 0$) of a variable T, $0 \le T \le \tau_{\min}$, $0 \le \tau_{\min} < \infty$, for any set of parameters $\lambda_1 > \lambda_2 \ge 0$, $0 \le p \le 1$, $\beta > 0$, $\alpha > 0$, $0 \le \delta \le 1$.

Theorem 2. The probability density function $p_T(\tau_{\min})$ is an increasing function of a variable T, $0 \le T \le \tau_{\min}$, $0 \le \tau_{\min} < \infty$, for any set of parameters $\lambda_1 > \lambda_2 \ge 0$, $0 \le p \le 1$, $\beta > 0$, $\alpha > 0$, $0 \le \delta \le 1$, and reaches its global maximum at the point $T = \tau_{\min}$, $0 \le \tau_{\min} < \infty$.

Corollary 1. By Theorem 1, functions $p_T(\tau^{(j)})$, $j = \overline{2,k}$, are increasing functions of a variable T, $0 \le T \le \tau^{(j)}$, $0 \le \tau^{(j)} < \infty$, $j = \overline{2,k}$, for any set of parameters $\lambda_1 > \lambda_2 \ge 0$, $0 \le p \le 1$, $\beta > 0$, $\alpha > 0$, $0 \le \delta \le 1$.

Corollary 2. By Theorem 2, the likelihood function $L(T \mid \tau^{(1)},...,\tau^{(k)})$ reaches its global maximum at the point $\hat{T} = \tau_{\min}$, i.e. the solution of optimization task (2) is the estimate of the dead time period duration $\hat{T} = \tau_{\min}$.

Conclusion

The obtained results provide the possibility to solve the problem of parameters estimation of the modulated semi-synchronous generalized flow of events in conditions of a constant dead time without using numerical methods: during the observation of the flow (during the time interval $(0,t]$) the values τ_k, $k = \overline{1,n}$, are calculated; $\tau_{\min} = \min \tau_k$ ($k = \overline{1,n}$), is found and the estimate of the dead time period duration is assumed to be equal $\hat{T} = \tau_{\min}$.

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OPTIMAL CONTROL OF A TWO-SERVER HETEROGENEOUS QUEUEING SYSTEM WITH BREAKDOWNS AND CONSTANT RETRIALS

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Heterogeneous servers which can differ in service speed and reliability are getting more popular in modelling of modern communication systems. For a two-server queueing system with one non-reliable server and constant retrial discipline we formulate an optimal allocation problem for minimizing a long-run average cost per unit of time. Using a Markov decision process formulation we prove the optimality of the two-level threshold control policy. We derive expressions for the stationary state probabilities and provide also a heuristic solution for the optimal threshold levels in explicit form.

1. Introduction

In modern communication systems the speed of data transmission in a link can interact with its reliability. In many cases this interaction is differently directed. The complementary properties of different links lead engineers to idea to combine them in such a way that the advantages of high speed links could guarantee acceptable values of the cost and reliability characteristics. As example of the system where the data transmission links differ in speed and reliability is a Radio Frequency/Free Space Optic (RF/FSO) channel [15]. Another example of a system with combined technologies is a modern call-centre where the human operators are working together with a speech recognition self-service facility [4].