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Programme Schedule & Abstracts

Retrial queueing system under heavy load condition

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Our group of researchers from Tomsk State University develops the method of asymptotic analysis for different types of retrial queueing systems. Single-server and multi-server retrial queues with collision, priority or impatient customers, various protocols are studied. We consider systems under various asymptotic conditions: heavy load, long delay, long time of patience, etc. In the paper, we describe the asymptotic analysis method under a heavy load condition and present results obtained for some types of retrial queues.

Asymptotic analysis of retrial queue M/M/N. Let us consider retrial queueing system of M/M/N type. The arrival process is Poisson with a rate λ. There are N servers. Service times are exponentially distributed with a rate μ. If a call arrives when there is a free server, the call occupies it for the service. If all servers are busy, the call goes to the orbit. There, call is waiting during a random time distributed by exponential law with parameter σ. Then the call from the orbit makes an attempt to get a service again. If there is a free server, the call occupies it, otherwise the call instantly returns to the orbit. Let $i(t)$ be a number of calls in the orbit and $k(t)$ be a number of busy servers. Denote by $P(k,i,t) = P\{k(t) = k, i(t) = i\}$ the probability that there are k calls servicing and i calls waiting in the orbit at the time moment t.

We write Kolmogorov differential equations for partial characteristic functions $H_k(u) = \sum_i e^{jui}P(k,i)$, where $P(k,i) = \lim_{t\to\infty} P(k,i,t)$ is a probability distribution for stationary regime. Denote $\rho = \lambda/N\mu$ as a load parameter. We solve obtaining system by the method of asymptotic analysis under a heavy load condition $\rho \uparrow 1$ or $\varepsilon \downarrow 0$, where $\varepsilon = 1-\rho > 0$. First of all, we introduce notations:

$$u = \varepsilon w,\ H_0(u) = \varepsilon^N F_0(w,\varepsilon), \dots, H_k(u) = \varepsilon^{N-k} F_k(w,\varepsilon), \dots, H_N(u) = F_N(w,\varepsilon).$$

In the next step, using expansions $F_k(w,\varepsilon) = F_k(w) + \varepsilon f_k(w) + O(\varepsilon^2)$ and limits $\lim_{\varepsilon\to 0} F_k(w,\varepsilon) = F_k(w)$, we solve the system of equations for asymptotic functions. As a result, we derive the asymptotic characteristic function in the form $h(u) = \sum_{k=0}^{N} H_k(u) = F_N\left(\frac{u}{1-\rho}\right) + O(\varepsilon)$. Making inverse substitutions, we obtain that the required function is equal to $h(u) = \left(1 - \frac{ju}{1-\rho}\right)^{-\frac{\mu+\sigma}{\sigma}}$. So, we can make a conclusion that the probability distribution of number of calls in the orbit under a heavy load condition is a gamma distribution asymptotically with parameters $\alpha = (\mu+\sigma)/\sigma$ and $\beta = 1-\rho$.

Results for other types of retrial queues. Similar investigations were performed for retrial queues with Markovian modulated Poisson arrivals and general distribution of service time. It is interesting that for all considering systems, probability distributions of number of calls in the orbit are gamma distributions asymptotically under a heavy load condition. Parameters of these gamma distributions are showed in the Table 1. Here we use the following notations: b and b_2 are a mean and a second initial moment of the service time distribution, $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_n\}$, $\mathbf{V}$ is a solution of the equation $\mathbf{VQ} = \mathbf{R}(\mathbf{I}b^{-1} - \mathbf{\Lambda})$, $\mathbf{R}$ is a stationary distribution of states of the MMPP underlying chain, $\mathbf{e}$ is a vector of 1s, $\mathbf{I}$ is an identity matrix.

Table 1: Gamma distribution parameters

Type of RQ	$M\|M\|1$	$M\|GI\|1$	$MMPP\|M\|1$	$MMPP\|GI\|1$
α	$\dfrac{\mu+\sigma}{\sigma}$	$1+\dfrac{1}{b\sigma}\beta$	$1+\dfrac{\mu}{\sigma}\beta$	$1+\dfrac{1}{b\sigma}\beta$
β	1	$\dfrac{2b}{b_2}$	$\dfrac{\mu}{\mathbf{V\Lambda e} - \mu\mathbf{Ve} + \mu}$	$\dfrac{1}{b\left(\mathbf{V\Lambda e} - \mathbf{Ve}\frac{1}{b} + \frac{b_2}{2b^3}\right)}$

As a result of additional study, we have obtained that in the case of impatient calls (when waiting time has exponential distribution with a rate α/i) the asymptotic distribution of number of calls under a heavy load condition is also a gamma distribution with parameters $\alpha = S(\mu+\sigma)/\sigma - \alpha/\sigma$ and $\beta = S(S-\rho)$, where $S = 1 + \alpha/\mu$ is a system throughput value.

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