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Primordial Non-Gaussianity

and

Primordial Tensor Modes

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Abstract

The discoveries in observational cosmology of the last two decades led to a tremendous progress in cosmology. From a theory with mainly qualitative ideas about the expanding universe originated from a state with high density and temperature (Hot Big Bang cosmological model), cosmology rapidly evolved to a quantitative science that culminated with the formulation of the so-called ΛCDM model in which the composition and the evolution of the universe are known well enough to make very detailed predictions for a large number of observables on many different scales.

Nonetheless, the ΛCDM model lacks in giving a satisfactory explanation of the initial conditions that are necessary to explain the subsequent evolution of the universe. The inflationary paradigm elegantly solves this problem, furthermore it provides a solution for other issues that affect the standard cosmology such as the horizon and the flatness problems. Although the basic framework of inflationary cosmology is now well-established, the microphysical mechanism responsible for the accelerated expansion remains a mystery. In this thesis, we describe how the physics underlying inflation can be probed using the higher-order correlations of primordial density perturbations (non-Gaussianity). In particular, we focus on those correlation functions that involve primordial gravity waves (tensor modes).

The importance of primordial tensor modes lies in their theoretical robustness: while scalar perturbations are sensitive to many details, tensor modes are much more model independent. In this thesis we start by stressing this robustness, focussing on tensor non-gaussianity. We show that in single-field models of inflation (i.e. within the framework of the Effective Field Theory of Inflation) the predictions for the correlation functions that involve tensor modes are pretty model independent: tensor bispectra can assume very few shapes.

After having discussed the prediction of the simplest models we focus on the squeezed limit of the tensor-scalar-scalar 3-point function. The leading behaviour of this correlator is fixed by the so-called Tensor Consistency Relation in many inflationary theories. This model independent prediction is very robust and can be violated only in theories where there is an additional helicity-2 state besides the graviton or in models that enjoy a symmetry pattern different from the standard one.

In the last part of this thesis we explore both these possibilities. First we introduce a set of rule that allow us to include light particles with spin in the Effective Field Theory of Inflation, then focussing on the phenomenology that arises from an additional light spin-2 field. Finally, we describe a model of inflation which is very peculiar and cannot be incorporated in the context of the Effective Field Theory of Inflation: Solid Inflation. Here the “stuff” that drives inflation has the same symmetry as an ordinary solid. We show that even in solids some consistency relations among the non-gaussian correlators can still be derived.

Publications

Most of the material contained in this thesis has appeared previously in the following publications:

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Notation and Conventions

The flat metric is defined as $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$.
Greek indices $\mu, \nu \ldots$ are used for 4-vectors.
Latin indices $i, j, \ldots$ are used for spatial components.
Arrows (like this one $\vec{\ }$) are used for 3-vectors.
We use units in which $c = \hbar = 1$.
$M_{\rm Pl}$ is the Planck's mass $(8\pi G)^{-1/2}$.

Introduction

When we look up at the sky we see that the stars around us are grouped into a large-density concentration - the Milky Way Galaxy. On a slightly larger scale, we see that our Galaxy belongs to a small group of a galaxies (called the Local Group). On still larger scales we see that our Local Group sits on the outskirts of a giant supercluster of galaxies centred in the constellation of Virgo. Evidently, on small scales matter is distributed in a highly irregular way but, as we look on larger and larger scales, the matter distribution looks more and more uniform. In other words, our universe looks *homogeneous* and *isotropic* on the very largest scales.

In the past several decades, tremendous advances in observational cosmology were able to confirm this evidence. In fact, measurements of the cosmic microwave background (CMB) and large-scale structures (LSS) have revealed that our universe is isotropic to a very high accuracy [1]. Moreover, observations show that also homogeneity is satisfied even on the very largest scales, that were not supposed to be in causal contact. These observations can be explained if one postulates the existence of a phase of accelerated expansion in the early universe - called inflation - that smoothed away any inhomogeneity [2, 3, 4, 5]. Remarkably, not only inflation is able to explain the universe on very large scales but, it naturally predicts the generation of the primordial seeds for structure formations that arose from the quantum perturbations generated during the early epoch of expansion and that were stretched up to cosmological scales [6, 7, 8, 9]. The real power of inflation lies in the ability of making predictions on these primordial fluctuations. In fact, in any inflationary model, the perturbations are naturally correlated on super-horizon scales, and distributed with an almost scale invariant and nearly Gaussian statistics, in good agreement with many cosmological observations [10, 11, 12, 13].

Nonetheless, the physics that led to the inflationary epoch is still unexplained. In the standard picture the time-dependent vacuum energy of a scalar field - the *inflaton* - that dominates the energy density of the universe is responsible for the accelerated expansion. On top of the vacuum energy, quantum perturbations of the inflaton are generated and then subsequently stretched up to cosmological scales. They provide the initial conditions for the CMB temperature anisotropies and structures formation. Understanding the inflationary spectrum is thus very important to understand the physics of inflation. The minimal set of light degrees of freedom during inflation is given by a scalar mode, the Goldstone of broken time-translations, and the graviton.

Though the minimal model is enough to explain current observations, nothing prevents that additional fields are present and excited during inflation. In fact, the energy density of the inflaton could be as high as 10^{14} GeV and any particle lighter that this energy is generated during inflation. New particles shows up in the cosmological correlators: being coupled with the inflaton they modify the statistics of scalar and tensor perturbations giving rise to a non-vanishing signal in the non-gaussian correlators [14]. Therefore, understanding how to extract informations about the particle spectrum during the inflationary period would offer us a window to probe the laws of physics at energies much larger than what we can expect to achieve with particle accelerators in the foreseeable future.

Of course, even self interactions of the inflaton, or interactions of the inflaton with gravity, can give rise to non-gaussianities. In this thesis we want understand which specific non-gaussianities are a signature of new particles during inflation, as opposed to signatures that arise due to inflaton self interactions. So far, most of the effort in this direction has been focused on the study of additional scalar fields (see e.g. [15, 16, 17]). On the other hand, the phenomenological implications of particle with spin, have been less studied. Nonetheless, their phenomenology could be as rich as the scalar one. Understanding the features that these additional fields may leave on the primordial correlation functions of both scalar and tensor modes is the main goal of this thesis. This is motivated also by the huge experimental effort dedicated to the study of gravitational waves both of astrophysical and cosmological origin. In fact, the peculiar features due to spinning particles may show up also at interferometer scales, [18]. We will pursue these goals making use of two powerful tools at our disposal: *effective field theories* and *consistency relations.*

Effective Field Theory. Effective field theory (EFT) is the language in which modern physics is phrased. The basic idea lies in the fact that to describe a phenomenon which occurs at a given characteristic energy we do not need to know the physics at energies much higher than the considered energy. In other words, different scales are treated separately. The methods of the EFTs formalise this statement in a precise way. It allows us to study a physical phenomenon in a model-insensitive way and it also gives us some tools to parametrise our ignorance about UV physics, classifying higher order corrections in terms of the symmetries of the problem. The role of symmetries in the EFTs is crucial, also if broken they lead to model independent predictions for the low-energy spectrum of excitations and their interactions.

In this thesis we will make use of the EFT of Inflation (EFTI) [19]. It consists in an effective description of perturbations around the inflating background. A key step in building every EFT is to identify the relevant degrees of freedom. However, as we have already discussed, the particle spectrum at the inflationary energy scales is unknown. The EFTI makes the assumption of the minimal required setup: one scalar and two tensor helicity. The scalar mode arise from the inflaton, the field responsible for the accelerated expansion. Having a time dependent background, it introduces a preferred foliation of the spacetime that breaks time diffeomorphisms invariance. The resulting theory for perturbations is consequently not covariant: given the reduced symmetry pattern of the system many new operators are now allowed giving rise to a rich phenomenology. The effective action obtained using this procedure unifies all single-field inflationary models in a unique formalism, providing us a systematic way to characterise the dynamics of primordial perturbations, while at the same time remaining agnostic about the process that generates the background expansion. This approach is very useful in characterising the single-field model predictions for non-gaussianity in a model independent way. Despite the existence of a pletora of single-field models, very generic statements about the 3-point correlators that include tensor modes can be drawn: this confirms that tensor modes modes are much more independent than scalar ones.

In principle an infinite set of operators can contribute to the effective action. However we expect that the dominant contributions arise from the operators with the lowest number of derivatives acting on the metric. Identifying the minimal set of independent operators at a given order in a derivatives expansion is not an easy task. Besides integration by parts, one has also to take into account field redefinitions that could map an operator into another. In Chapter 2 we will discuss this issue, moreover we present the minimal set of operators that contribute to the quadratic and cubic dynamics of both scalar and tensor perturbations at lowest order in derivatives.

Consistency Relations. Together with the EFT tecniques, also the Consistency Relations (CRs) are extremely useful tools that can help us in probing the presence of additional particles during inflation [20, 21]. Inflationary CRs are recursive relations between the squeezed limit[1] of the $(n+1)$-point function and the n-point correlator of the primordial perturbations.

The most famous and studied is the *scalar* CR, that relates the squeezed bispectrum of scalar perturbations with the relative power spectrum:

$$\langle \zeta_{\vec{q}\to 0}\, \zeta_{\vec{k}}\, \zeta_{-\vec{q}-\vec{k}} \rangle' = -(n_s - 1)\, \langle \zeta_{\vec{q}}\, \zeta_{-\vec{q}} \rangle' \, \langle \zeta_{\vec{k}}\, \zeta_{-\vec{k}} \rangle' \,, \tag{1}$$

where $(n_s - 1)$ is the scalar tilt and the prime on the correlation functions indicates that the momentum conserving delta function has been removed. The scalar CR simply states that the effect of a long mode on smaller scales is equivalent to a coordinate rescaling and therefore it cannot affect the local physics. It has been shown that this CR holds for every single-field model of inflation [21] and therefore in the EFTI [22] [2]. In fact, if there is only one scalar which is active during inflation, different patches of the universe follow the same classical history and quantum fluctuations of the inflaton are locally equivalent to a small shift of this classical history in time. This shift in time is then equivalent to an additional expansion that is equivalent to a simple rescaling of coordinates. Of course, this is true once the long mode crosses the horizon, because after that it becomes frozen and remains constant. For short modes its effect is not physical because it is just a classical background whose effect can be removed just by a rescaling of coordinates. Notice that this argument is independent of the details of interactions, the form of the kinetic term, slow-roll assumptions or any other details of the theory. Things change if additional light fields were active during inflation. Now indeed, there is no longer a unique history and different patches experience a different evolution. The scalar CR is violated and in fact, one expects a non-vanishing local non-gaussianity in multi-field models of inflation. This well-known example highlights the power of the inflationary CRs: instead of trying to confirm them, we could look for their violation. For instance, a detection of $f_{NL} \gtrsim 1$ would be a smoking gun for multi-field models of inflation.

[1] With squeezed limit we mean the regime in which one of the modes has a wavelength much longer than the others.

[2] Notice that, in making this statement, we are also assuming that the scalar field sits on its attractor solution and far away in the past it was in Bunch-Davies vacuum.

CRs do not involve scalar modes only, but can be derived even in presence of tensor pertubations. In fact, also a long tensor mode can be traded with a rescaling of the coordinates. Then also CRs where the long mode is a tensor can be be derived [20]. Similarly to the scalar case, tensor CRs can be used to probe the inflationary particle content. However, as we will see explicitly in Chapter 3, the violation of the tensor CRs is sensible to the presence of particles with spin higher than zero. In this sense the tensor CR can be seen as a smoking gun for the presence of spinning particle during inflation.

Particles with spin can break also the scalar CR. In fact, the helicity-zero component of the new field (if present) might couple with the inflaton, thus changing the squeezed limit of the scalar 3-point function. However, the new shape that is generated by the exchange of this helicity-zero mode depends on the scalar product between long and short modes. Being angular dependent, it is orthogonal to the local shape. This fact lead us to conclude that in models where there is a spinned particle coupled with the inflaton field, the scalar CR still holds after one performs the angular average over all the possible directions of the long mode. This new CR will be discussed in Chapter 5.

Outline

The outline of the thesis is as follows: Chapter 1 focuses on effective theory of inflation. We briefly review the construction of the EFT for cosmological perturbations in a given inflationary background.

Chapter 2 is devoted to the study of the operators in the EFTI with at most two derivatives acting of the metric that contribute to both scalar and tensor perturbations up to cubic order. To identify the minimal set of independent operators we make use of all possible field redefinitions at our disposal in order to kill all the redundancies. Furthermore, we comment about the possible effects that can arise from terms which are higher order in derivatives that can contribute to the observables that involve tensor modes.

In Chapter 3 we focus on the tensor CR that involves one long graviton and two scalar modes. We show that this CR is very robust and hard to be violated. It holds not only in single-field, but also in multi-field models of inflation. It can be violated if consider exotic models with a symmetry pattern different from the standard one or, if additional light tensor fields were active during inflation. However, the presence of the Higuchi bound, severely constraints the existence of light spin-2 fields (beside the graviton) during inflation.

This sort of no-go theorem that forbids the existence of light spin-2 fields can be evaded already in the context of the EFTI. In fact, the coupling with the preferred foliation given by the inflaton can make light spin-2 fields (and in general any spin-s field) healthy. This argument is discussed in Chapter 4 where we first describe how to modify the EFTI in order to consistently include spin-s excitation. Then, as an example, we focus on the phenomenology that arises in the presence of an additional light spin-2 field.

In Chapter 5 we discuss a model that has a very peculiar symmetry pattern, it breaks spatial diffeomorphism invariance and hence it cannot be casted in the EFTI: *Solid inflation* [23]. After having briefly reviewed the model, we explicitly check that, because of the different symmetry pattern, both the scalar and tensor CRs are violated. However, a CR for scalar modes can still be derived: we will show that the squeezed scalar bispectrum still satisfies eq. (1) after one performs the angular average over the direction of the long mode.

Our conclusions appear in Chapter 6. We summarise the problems addressed in this thesis and the results we obtained, and give an outlook on potential future research directions.

Finally, a number of appendices contain technical details of the results presented in the main text. In Appendix A, we provide further details to the field redefinitions that are used in Chapter 2. In Appendix B, we make a few supplementary remarks to the results of Chapter 3 in regard of the duality that holds between the boundary of the de Sitter space and a 3-dimensional conformal field theory. Finally, Appendix C is devoted to make some clarifications on the calculations of Chapter 4.

Chapter 1

The Effective Field Theory of Inflation

In this chapter we present the construction of the *Effective Field Theory of Inflation*, i.e. the EFT for perturbations around the inflating background (Section (1.2)). We compute both action for the scalar mode that is the Goldstone boson of the non-linearly realised time-translational invariance and the action for the primordial gravitational waves. Before starting with the construction of the EFTI we briefly comment about the philosophy underlying every EFT and the use of the EFTs in Cosmology (Section 1.1).

1.1 EFT philosophy

Roughly speaking, the idea of the EFT is that any Lagrangian that we write is valid up to some energy scale. Consider, for instance, the following theory for two scalar fields in flat spacetime

$$\mathcal{L}(\phi,\Phi) = -\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \tfrac{1}{2}\partial_\mu\Phi\,\partial^\mu\Phi - \tfrac{1}{2}M^2\Phi^2 - \frac{1}{M_\star}\,\Phi\,(\partial_\mu\phi\partial^\mu\phi)\,, \tag{1.1}$$

where we assume $M_\star \gg M$. This Lagrangian contains a five dimensional operator that is clearly non-renormalisable however, as long as we consider energies below $M_\star$ we can treat the last operator of (1.1) as a perturbation and the theory still gives reasonably accurate predictions. Furthermore, if we are are interested in a physical process whose typical energy is $E \ll M$, the massive field does not propagate and it can be integrated out. The new Lagrangian for ϕ contains an infinite set of operators that can be organised according to their scaling behaviour in a derivative expansion

$$\mathcal{L}_{\text{eff}}\,[\phi] = -\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{(\partial_\mu\phi\,\partial^\mu\phi)^2}{2\Lambda^4} + \dots\,, \tag{1.2}$$

where $\Lambda \equiv \sqrt{M_\star M}$ and the ellipses stand for corrections of order E^2/M^2. This is the most general low-energy Lagrangian for ϕ that is compatible with symmetries. Notice, indeed, that eq. (1.2) is symmetric under the shift $\phi \to \phi + c$, as the initial theory (1.1) contains only derivatively coupled operators in ϕ.

In the above example we have used a top-down approach to construct a low-energy theory for ϕ. However, if the UV theory is not known a priori, still we can build an effective Lagrangian for the scalar field. All we have to do is to identify the relevant degrees of freedom and then write all the possible operators that are compatible with the symmetries at play. The operators are organised in a power series according to their scaling dimension.

Understanding the relevant degrees of freedom and the symmetries can be a hard task. For instance, during inflation, the spectrum of states is unknown. To construct the EFTI in the next section, we make the assumption of the minimal set of light degrees of freedom, i.e. a scalar mode and two graviton helicities. We will see in Chapter 4 how to relax this assumption in order to incorporate additional light fields.

EFTs in cosmology. On cosmological scales, the constituents of the universe - radiation, matter and dark energy - can be approximated as irrotational perfect fluids. The approximation is valid when dissipative phenomena are negligible. These fluids are also taken to be barotropic i.e., their pressure is a function of the energy density only, $p = p(\rho)$. Under these conditions, each fluid is characterised by a single scalar function, so that it is not surprising that its dynamics can be described in terms of a scalar field, ϕ, with a time-dependent background solution. At low energies, we have the following Lagrangian

$$\mathcal{L} = P(X)\,, \qquad X \equiv \, -\frac{\partial_\mu\phi\,\partial^\mu\phi}{2}\,, \tag{1.3}$$

where $P(X)$ is polynomial in X. By construction, this theory enjoys a shift symmetry, $\phi \to \phi + c$. Moreover, since we are interested in studying it around time-dependent solutions of the type $\bar{\phi}(t) \propto t$, time-translations are non-linearly realised. It is immediate to check that the stress-energy tensor of the field coincide with the stress-energy tensor of a superfluid with energy density, pressure and 4-velocity respectively,

$$\rho = 2\,X\,\partial_X P - P\,, \qquad p = P\,, \qquad u_\mu = \frac{\partial_\mu\phi}{\sqrt{X}}\,. \tag{1.4}$$

Given the ratio $w \equiv p/\rho$, from the above equations it is easy to see that the corresponding fluid Lagrangian is (see for instance [24])

$$\mathcal{L} \propto X^{\frac{1+w}{2w}}\,. \tag{1.5}$$

The inflationary epoch is well described by a fluid with $w \simeq 1$. Notice that in this regime that the above action approaches a constant. In a curved spacetime it therefore plays the role of a cosmological constant that inflates the spacetime.

This simple example shows us that Lagrangians in cosmology can be very different from what one is used in particle physics, both in their functional shape (e.g. $w = 1/3 \,\Rightarrow\, \mathcal{L} = X^2$) and in the underlying symmetries: in general time reparameterisation invariance is broken. Notice that is true also during inflation. We move now to the explicit construction of the EFTI.

1.2 EFT of Inflation

During inflation the spacetime is approximatively de Sitter, whose line element is

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)\,, \tag{1.6}$$

being H the Hubble rate and η the conformal time. Since the accelerated expansion must end at some point, we need a clock that smoothly connects the inflationary epoch to a decelerated hot Big Bang evolution. A scalar field - the inflaton - can serve as this clock, its time-dependent homogeneous solution keeps track of the duration of inflation, generating a preferred foliation that slices the quasi de Sitter spacetime. To work out the predictions for late time observables, we then need to study perturbations of this clock around a background that non-linearly realises time diffeomorphisms (diffs) invariance.

As we have outlined in the previous section, one way to build the effective Lagrangian consists in taking a scalar field ϕ, write the most general Lagrangian for ϕ and then expand it around its time dependent bacground $\phi = t + \pi$. [1] Instead of pursuing this route, we use a more geometrical approach. The procedure we outline follows references [19, 22].

Unitary Gauge. We set the gauge by choosing the time coordinate to be a function of ϕ, $t = t(\phi)$, in such a way that the clock sits in its unperturbed value everywhere. This is the so called *unitary gauge*, in which the perturbations of the clock are eaten by the metric. The constant-time slices generated by the inflaton foliate the spacetime, making the action for perturbations no longer invariant under time diffs. It follows that in the action, beside the genuinely 4-d covariant terms such as the Ricci scalar, there are objects which are constructed from the foliation. For instance, now we can consider separately the projection of tensors along the orthogonal and parallel directions to the surface. This is done by contracting tensors with the normal vector n_μ or with $h_{\mu\nu}$ the 3-metric, respectively given by

$$n_\mu = \frac{\partial_\mu \phi}{\sqrt{-\partial_\nu \phi\, \partial^\nu \phi}} \qquad \text{and} \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \,. \tag{1.7}$$

All geometrical objects built from the foliation can be defined using these two projectors. Examples are the extrinsic curvature tensor $K_{\mu\nu} = h^\rho_\mu\, h^\sigma_\nu \nabla_\rho\, n_\sigma$ or the "intrinsic" curvature of the 3-dimensional surfaces ${}^{(3)}R_{\mu\nu\rho\sigma}[h]$.

The most general effective action is constructed by writing down all possible operators that are compatible with the remaining symmetries. The reduced symmetry pattern of the system allows many terms in the action. They can be categorised as follows:

i. Terms which are invariant under all diffs: these are just polynomials of the 4-d Riemann tensor $R_{\mu\nu\rho\sigma}$ and of its covariant derivatives ∇_μ, contracted to give a scalar.

ii. Terms contracted with n_μ. Since in unitary gauge $n_\mu \propto \delta^0{}_\mu$, in every tensor we can leave free upper 0 indexes. For instance, we can use the 00 component of the metric or of the Ricci tensor. It is easy to check that these are scalars under spatial diffs.

iii. Terms derived from the foliation. This includes terms like $\mathcal{D}_\mu$, the derivatives of the induced 3-metric or the Riemann tensor ${}^{(3)}R_{\mu\nu\rho\sigma}$ that characterises the

[1] As an example, reference [25] uses this path.

3-d slices "intrinsically"; but one can use also objects that tell us how the hypersurfaces are embedded in the 4-d spacetime. These are the extrinsic curvature $K_{\mu\nu}$ and the acceleration vector $A_\mu = n^\rho \nabla_\rho n_\mu$. The action contains all the possible scalars made contracting these quantities.

iv. Since time diffs are broken all the couplings in front of the operators can be function of time.

The most general action constructed using these ingredients takes the form

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}\left[g_{\mu\nu},\, g^{00},\, R_{\mu\nu\rho\sigma},\, K_{\mu\nu},\, \nabla_\mu,\, t\right], \tag{1.8}$$

where the contractions are done with the metric $g_{\mu\nu}$, using the 3-metric does not lead to new interactions. Since the above action contains an infinite number of operators, we organise them in a derivative expansion: at lowest order in derivatives acting on the metric there are only polynomials in g^{00}, at first order we can use the trace of the extrinsic curvature K. The leading effective action is

$$\begin{aligned} S = \int d^4x \sqrt{-g}\Big[\frac{M_{\rm Pl}^2}{2} R - \Lambda(t) - c(t)\, g^{00} + \sum_{n=2}^{\infty} (g^{00}+1)^n \\ + \frac{\overline{M}_1(t)^3}{2}(g^{00}+1)\,\delta K - \frac{\overline{M}_2(t)^2}{2}\delta K^2 - \frac{\overline{M}_3(t)^2}{2}\delta K^\mu_\nu \delta K^\nu_\mu + \dots\Big], \end{aligned} \tag{1.9}$$

where $\delta K_{\mu\nu}$ is the variation of the extrinsic curvature with respect to the unperturbed FRW background, $\delta K_{\mu\nu} \equiv K_{\mu\nu} - a^2 H h_{\mu\nu}$, and the dots stand for terms with at least two derivatives on the metric [2]. In principle, one would expect that there is an infinite number of operators that contribute to the background solution, however, we defined the action above such that only the first three terms contribute to the background and start linearly in perturbations; all the others are explicitly quadratic or higher. This choice has been made since all the other operators that start linear in perturbations are higher order in derivatives and they can be integrated by parts to give a combination of the three linear terms we considered in eq. (1.8) plus covariant terms of higher order, [19]. This non-trivial fact has two immediate consequences: first of all it implies that we can write all the operators that start at perturbation level in a way that is compatible with the residual gauge symmetries; moreover it means that the coefficients $c(t)$ and $\Lambda(t)$ are fixed by the unperturbed history while the difference among different models is encoded in the higher order terms. By the requirement of having a given FRW evolution $H(t)$, i.e. requiring the tadpole terms cancellation, we get

$$\Lambda(t) = M_{\rm Pl}^2(3H^2 + \dot{H}) \quad \text{and} \quad c(t) \equiv -M_{\rm Pl}^2 \dot{H}\,. \tag{1.10}$$

We have previously stated that the coefficient in front of the operators can be time dependent. However we are interested in describing slow-roll inflation, that is when both H and $\dot{H}$ vary slowly in a Hubble time. It is then natural to assume that the same holds for all the coefficients. This requirement is realized when the inflaton

[2] In Chapter 2 we will identify the set of all the independent operators up to second order in derivatives, that contribute to both scalar and tensor perturbations up to cubic order.

ϕ sits in its attractor solution, i.e. when $\ddot{\phi} \ll \dot{\phi}$ together with all the higher time-derivatives.

Let us conclude this section by writing in the above language the action for the minimal inflationary model, i.e. scalar field with minimal kinetic term and slow-roll potential $V(\phi)$. This corresponds to taking the following limit: M_n, $\overline{M}_n \to 0$. To see this, notice that in unitary gauge, $\phi = \bar{\phi}(t)$ then, the slow-roll action becomes

$$\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\dot{\bar{\phi}}^2 g^{00} - V(\bar{\phi})\right], \tag{1.11}$$

By using Friedmann equations, $\dot{\bar{\phi}}^2 = -2M_{\rm pl}^2\dot{H}$ and $V(\bar{\phi}) = M_{\rm pl}^2\left(3H^2 + \dot{H}\right)$, we see that the above action corresponds to the first three terms of the action eq.(1.8).

1.3 Action for the Goldstone Boson

The unitary gauge Lagrangian, eq. (1.9) describes three degrees of freedom: two graviton helicities and a scalar mode. They affect the spatial part of the metric:

$$g_{ij} = a(t)^2 e^{2\zeta(t,\vec{x})}(e^\gamma(t,\vec{x}))_{ij}\,. \tag{1.12}$$

To make the dynamics of the theory more transparent, we introduce the Goldstone boson, π, associated with the spontaneous breaking of time translations. We do it via the so called Stückelberg trick that consists in performing the time diff $\tilde{t} \to t+\pi(t,\vec{x})$ and then promoting $\pi(t,\vec{x})$ to a scalar field that non-linearly realises time diffs and restores full diffs invariance of the action.

As an example, let us see how this procedure works in the following action,

$$\int d^4x\,\sqrt{-g}\,\left[A(t) + B(t)g^{00}(x)\right]\,. \tag{1.13}$$

Under a broken time diff. $t \to \tilde{t} = t + \pi(x)$, $\vec{x} \to \tilde{\vec{x}} = \vec{x}$, g^{00} transforms in the usual way, i. e.,

$$g^{00}(x) \to g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\,. \tag{1.14}$$

Applying this procedure to the action above we obtain,

$$\int d^4x\,\sqrt{-g}\,\left[A(t+\pi(x)) + B(t+\pi(x))\left(g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\right)\right]\,. \tag{1.15}$$

Decoupling Limit. As we see from the above example, after the reintroduction of the Goldstone boson the action looks very complicated. However, a great simplification occurs if we focus on sufficiently short distances. Then, in analogy with the equivalence theorem for the longitudinal components of a massive gauge boson [26], the physics of the Goldstone can be studied neglecting metric fluctuations. This regime correponds to the decoupling limit, $M_{\rm Pl} \to \infty$, $\dot{H} \to 0$ keeping $M_{\rm Pl}\dot{H}$ fixed. In this limit the transformation of g^{00} simplifies to

$$g^{00}(x) \to -1 - 2\dot{\pi} - \dot{\pi}^2 + a^2(\partial_i\pi)^2 \tag{1.16}$$

and one can show that the action (1.9) dramatically simplifies to

$$\begin{aligned} S_\pi &= \int d^4x \sqrt{-g} \Big[\frac{M_{\rm Pl}^2}{2} R - M_{\rm Pl}^2 \left(\dot{H} \dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) \\ &\quad + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + \dots \Big] , \end{aligned} \tag{1.17}$$

where we have neglected, for simplicity, terms that involve the extrinsic curvature. We see that M_2 induces a nontrivial sound speed for the Goldstone boson,

$$c_s^2 = \frac{M_{\rm Pl}^2 \dot{H}}{M_{\rm Pl}^2 \dot{H} - 2M_2^4} . \tag{1.18}$$

A small value of c_s is correlated with an enhanced cubic interaction through a non-linearly realized symmetry. The Planck constraints on primordial non-Gaussianity imply $c_s \geq 0.024$ [27].

The regime of validity of the decoupling limit can be estimated just looking at the mixing terms between the π field and the metric. In eq. (1.14) we see in fact that quadratic terms which mix π and $g_{\mu\nu}$ contain fewer derivatives than the kinetic term of π so that they can be neglected above some high energy scale. In general the answer will depend on which operators are present. Let us assume for simplicity that in the action (1.9) only the tadpole terms are relevant. Then it can be shown that the leading quadratic mixing between π and the metric perturbations can be neglected for energies above $E_{mix} \simeq \sqrt{|\dot{H}|} = \sqrt{\epsilon} H$. Since we are interested in computing observables at $E \sim H \gg \sqrt{\epsilon} H$, any corrections to taking the decoupling limit will be of order $E_{mix}/H \sim \sqrt{\epsilon}$, [19].

While the Goldstone boson is massless in the decoupling limit, it has a mass of order $\mathcal{O}(\epsilon)$ when the mixing with gravity is taken into account. A small mass for π means that it slightly evolves outside the horizon. To describe observable quantities, it is more convenient to use the curvature perturbation of spatially flat slices, ζ, which is the scalar degree of freedom in the unitary gauge. The field ζ is exactly massless and becomes constant outside the horizon [20]. As we are neglecting the mixing with gravity, the metric is unperturbed in the π gauge. To move to the unitary gauge the one has to set $\pi = 0$. This is done by performing the time diffeomorphism $t \to t - \pi(t, \vec{x})$ which gives a spatial metric of the form (1.12) with

$$\zeta(t, \vec{x}) = -H\, \pi(t, \vec{x}) + \mathcal{O}(\epsilon) . \tag{1.19}$$

Using the this relation the power spectrum of ζ is found to be

$$\langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_2} \rangle = H^2 \, \langle \pi_{\vec{k}_1} \pi_{-\vec{k}_2} \rangle = (2\pi)^3 \; \delta(\vec{k}_1 + \vec{k}_2) \; \frac{H_*^2}{4\epsilon_* M_{\rm Pl}^2} \frac{1}{k^3} , \tag{1.20}$$

where the $*$ means that the quantity has been evaluated at horizon crossing.

1.4 Action for Tensor Perturbations

It is important to stress that the approach of the EFTI does describe the most generic Lagrangian not only for the scalar mode, but also for gravity. In fact, in the action

(1.9) we can also systematically include terms containing tensor perturbations. In the minimal model of slow roll inflation, eq. (1.11), just the Einstein-Hilbert action contributes to the graviton Lagrangian. The quadratic action for γ_{ij} is

$$S_\gamma = \frac{M_{\rm Pl}^2}{8}\int d^4x\sqrt{-g}\left[(\dot\gamma_{ij})^2 - \frac{(\partial_k\gamma_{ij})^2}{a^2}\right] \tag{1.21}$$

and its power spectrum of γ_{ij} is given by

$$\langle\gamma_{ij}(\vec k_1)\gamma_{ij}(\vec k_2)\rangle = (2\pi)^3\delta(\vec k_1+\vec k_2)\frac{4H_*^2}{M_{\rm Pl}^2}\frac{1}{k^3}\,. \tag{1.22}$$

High energy effects will be encoded for example in operators containing the perturbations in the extrinsic curvature $\delta K_{\mu\nu}$ or in the Riemann tensor $\delta R_{\mu\nu\rho\sigma}$. The leading correction to the Einstein Hilbert action is the operator

$$\sqrt{-g}\;\widehat{M}_2^2\;\delta K_{\mu\nu}\delta K^{\mu\nu}\,. \tag{1.23}$$

In fact, the perturbed part of the extrinsic curvature is linear in γ_{ij} ,

$$\delta K_{ij} \supset \frac{1}{2}a^2\dot\gamma_{ij} + \mathcal{O}(\gamma^2)\,. \tag{1.24}$$

This operator can, in principle, change the speed of propagation,c_T of primordial tensor modes. However, as we will see in the Chapter 2 (see also [28]), this operator is redundant with the Ricci scalar and therefore we can be omitted. We will see indeed that c_T can always being brought to 1, during inflation.

Chapter 2

Simplifying the EFT of Inflation: Generalized Disformal Transformations and Redundant Couplings

In the previous chapter, we reviewed how the study of cosmological perturbations in single-field inflation can be embedded in an EFT framework. This involved first identifying the symmetries of the problem and then systematically writing down all possible interactions that are consistent with those symmetries. However, identifying the minimal set of operators that contributes to correlation functions at a given order is not a trivial task. In this Chapter we are going to address this problem, identifying all the possible field redefinitions that we can use to simplify the action at a given order in perturbations and derivatives, avoiding all possible redundancies.

In scalar-tensor theories one is used to conformal transformations of the metric and the possibility to describe physics in different frames. When dealing with backgrounds in which the scalar field is time-dependent, like in the case of inflation, one can consider disformal transformations [29] of the form

$$g_{\mu\nu} \to \mathcal{C}(\phi, X) g_{\mu\nu} + \mathcal{D}(\phi, X)\partial_\mu\phi\partial_\nu\phi \, , \qquad X \equiv -(\partial\phi)^2 \, . \tag{2.1}$$

In a gauge in which the inflaton perturbations are set to zero ($\phi = \phi_0(t)$), the so-called unitary gauge, these transformations are written as

$$g_{\mu\nu} \to C(t, N) g_{\mu\nu} + D(t, N) n_\mu n_\nu \, , \tag{2.2}$$

where $N \equiv (-g^{00})^{-1/2}$ is the lapse, n^μ is the unit vector perpendicular to the surfaces of constant inflaton defined in eq.(1.7), and $C(t, N)$ and $D(t, N)$ can be easily related to $\mathcal{C}(\phi, X)$ and $\mathcal{D}(\phi, X)$. However this is not the end of the story. More general transformations are possible if one considers objects with derivatives on the metric:

$$\begin{aligned} g_{\mu\nu} \to \; & C(t, N, V, K, {}^{(3)}\!R, \dots)\, g_{\mu\nu} + D(t, N, V, K, {}^{(3)}\!R, \dots)\, n_\mu n_\nu \\ & + E(t, N, V, K, {}^{(3)}\!R, \dots)\, K_{\mu\nu} + \dots \, , \end{aligned} \tag{2.3}$$

where V is defined in eq. (2.10), $K_{\mu\nu}$ is the extrinsic curvature of the surfaces of constant inflaton, $K = g^{\mu\nu}K_{\mu\nu}$, and ${}^{(3)}\!R$ their scalar curvature. The purpose of this chapter is to study these general transformations in the context of the EFTI and to understand to which extent they can be used to simplify the original action. This generalises the results of [28], where disformal transformations with time dependent coefficients $C(t)$ and $D(t)$ were used to remove, without loss of generality, the time-dependence of the Planck mass and a non-trivial speed of tensor modes in the EFTI. The Planck mass and the tensor speed are couplings that can be changed without affecting observables: in QFT these are called *redundant couplings*.

When calculating S-matrix elements, field redefinitions cannot change the final result. In cosmology one is interested in correlation functions and, contrarily to S-matrix elements, these are not invariant under field redefinitions. However, *late-time* correlation functions—the ones that are relevant for observations—are left invariant by the transformations discussed above. Indeed, at late time all derivatives of metric perturbations decay to zero and the lapse gets its background value, $N \to 1$. We are left with a transformation of the form $g_{\mu\nu} \to C(t)g_{\mu\nu} + D(t)n_\mu n_\nu$. This redefines the scale factor and the cosmic time of the background FRW solution, but scalar and tensor perturbations are not changed. Therefore, *the general transformation eq.* (2.3) *modifies the form of the action, without changing late-time correlators.* The identification of the minimal set of non-redundant operators in the context of inflation was carried out in [30], albeit with some different assumptions.

The effect of general disformal transformations on the EFTI operators have been also studied in the context of dark energy and modified gravity in [31, 32, 33]. In this case one is interested also in the way matter couples with the metric and this coupling is modified by the redefinition of the metric. On the other hand, in single field models of inflation the coupling with matter does not enter in the inflationary predictions and therefore we will not consider it in the following.

Since we are talking about an infinite set of possible field redefinitions, an organization principle is needed. In the EFTI (and in all other EFTs!) one organises operators in terms of order of perturbations (tadpoles, quadratic terms, cubic terms, etc.) and in a derivative expansion. The same can be done for the transformations. Transformations involving derivatives of the metric, such as $K_{\mu\nu}$ and ${}^{(3)}R$, will increase the number of derivatives in the action. For instance, starting from the Einstein-Hilbert term, they will generate operators with three or more derivatives. Therefore, let us start with transformations without derivatives on the metric. An additional simplification comes from expanding the transformations in powers of perturbations. Since we dropped all terms with more derivatives, this boils down to an expansion in powers of δN. If one is interested in correlation functions up to cubic order—like we are in this chapter—one can truncate the transformations to quadratic order in perturbations. Indeed transformations which are cubic in perturbations will only modify the action with terms that are at least quartic, since the field redefinition multiplies the equations of motion, which vanish at zeroth order in perturbations. Therefore, one is left with

$$g_{\mu\nu} \to \left(f_1(t) + f_3(t)\delta N + f_5(t)\delta N^2\right) g_{\mu\nu} + \left(f_2(t) + f_4(t)\delta N + f_6(t)\delta N^2\right) n_\mu n_\nu \,. \tag{2.4}$$

This set of transformations will change the coefficients of the operators in the EFTI. In particular one can consider all the operators with at most two derivatives on the metric (and up to cubic order in perturbations) and study which simplifications are allowed by the six free functions $f_i(t)$: we are going to do that in Section 2.1

and Appendix A.1. One can use the free functions to set to zero the coefficients of the operators in the EFTI. The choice of which operator should be set to zero is clearly arbitrary. However, since only a few operators enter in calculations involving tensor modes, a natural choice is to try and simplify as much as possible the tensor couplings. The functions f_1 and f_2 can be used to have a time-independent Planck mass and to set to unity the speed of tensor modes [28]. This procedure also fixes the correlator $\langle\gamma\gamma\gamma\rangle$ at leading order in derivatives. In Section 2.1.1 we are going to see that the functions f_3 and f_4 can similarly be used to simplify the coupling $\gamma\gamma\zeta$ in such a way that the correlator $\langle\gamma\gamma\zeta\rangle$ is only modified by changes in the scalar sector. In particular we are going to verify that different actions, related by eq. (2.4), give the same result for $\langle\gamma\gamma\zeta\rangle$. In Appendix A.1 we explicitly give the effect of a general transformation, eq. (2.4), on operators up to two derivatives on the metric. The transformations f_5 and f_6 cannot be used to standardize any coupling involving tensors, so that their use to set to zero some operators remains, to some extent, arbitrary.

While the field redefinitions of Section 2.1 do not change the number of derivatives in the operators, in Section 2.2 we consider transformations that add one or more derivatives. In particular, in Section 2.2.1 we study a subset of the field redefinitions (2.3) that act on the metric as diffeomorphisms and leave the Einstein-Hilbert action invariant. Provided that higher derivatives can be treated perturbatively, we can use six of these transformations to further reduce the number of independent operators. In Sections 2.2.2 and Appendix A.2 we study higher-derivative corrections to tensors. We show that there is a single operator with three derivatives that modifies $\langle\gamma\gamma\gamma\rangle$ and a single operator that modifies the tensor power spectrum at 4-derivative order. We calculate the contribution of this operator to $\langle\gamma\gamma\zeta\rangle$: it is not slow-roll suppressed and therefore potentially relevant. Finally, in Section 2.3 we discuss additional field redefinitions that one can perform in the decoupling limit. These are not necessarily constrained by the nonlinear realization of Lorentz invariance and we show that they cannot be generally recovered by field transformations in unitary gauge. Conclusions are drawn in Section 2.4.

2.1 Operators up to two derivatives

We want to consider the action of the EFTI up to second order in derivatives and cubic in perturbations [34, 19]. In order to do so, we introduce the ADM decomposition of the metric [35], i.e.

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \, , \tag{2.5}$$

in terms of the lapse N, the shift N^i and the spatial metric h_{ij}. For later use we decompose the latter as [20]

$$h_{ij} = a^2(t)e^{2\zeta}(e^\gamma)_{ij} \, , \qquad \gamma_{ii} = 0 \, , \tag{2.6}$$

and we define the Hubble rate, $H \equiv \dot{a}/a$. The unitary gauge can be fixed by choosing the time coordinate to coincide with constant inflaton hypersurfaces and by imposing $\partial_i\gamma_{ij} = 0$ on these slices. In this gauge, ζ and γ_{ij} respectively represent the scalar and tensor propagating degrees of freedom.

The unitary gauge EFTI action (1.9) can be organised as follow:

$$S = S_0 + \int d^4x \sqrt{-g} \left(\mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \dots \right) , \tag{2.7}$$

where S_0, that consists in the first three terms of eq. (1.9) is the minimal canonical action [1]

$$S_0 \equiv \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[{}^{(4)}R - \frac{2\dot{H}}{N^2} - 2(3H^2 + \dot{H}) \right] , \tag{2.8}$$

and

$$\mathcal{L}^{(2)} \equiv M_{\rm Pl}^2 \sum_{I=0}^{8} a_I(t) \mathcal{O}_I^{(2)} , \qquad \mathcal{L}^{(3)} \equiv M_{\rm Pl}^2 \sum_{I=1}^{8} b_I(t) \mathcal{O}_I^{(3)} , \tag{2.9}$$

are, respectively, linear combinations of 9 quadratic operators, $\mathcal{O}_I^{(2)}$ $(I = 0, \dots, 8)$, and 8 cubic operators $\mathcal{O}_I^{(3)}$ $(I = 1, \dots, 8)$. The list of quadratic operators is given in Table 2.1, while the cubic ones in Table 2.2. The operators in these tables are constructed by combining the perturbation of the lapse, $\delta N \equiv N - 1$, of the extrinsic curvature, $\delta K_{\mu\nu} \equiv K_{\mu\nu} - a^2 H h_{\mu\nu}$, and of its trace $\delta K \equiv K - 3H$. The 3-dimensional Ricci scalar curvature ${}^{(3)}R$ is already a perturbed quantity, because we are assuming a flat FRW background. Moreover, the "acceleration" vector A^μ is given by $A^\mu \equiv n^\nu \nabla_\nu n^\mu$: it is projected on the surfaces of constant inflaton, i.e. $A^\mu n_\mu = 0$, and can also be written as $A_\mu = N^{-1} h^\nu{}_\mu \partial_\nu N$. With V we denote the covariant derivative of the lapse projected along n^μ,

$$V \equiv n^\mu \nabla_\mu N = \frac{1}{N} \left(\delta\dot{N} - N^i \partial_i N \right) , \tag{2.10}$$

which is a 3-dimensional scalar. Indeed, using the unitary gauge relation $n^\mu = -N g^{\mu 0}$, V is proportional to the upper time derivative of the lapse, i.e. $V = -N \partial^0 N$. Operators like $\delta N V$ and $\delta N^2 V$ can be written in terms of δN and δK after integrations by parts, and we can always get rid of R^{00} using the Gauss-Codazzi relation of eq. (2.13b) below. In Table 2.1 and Table 2.2 we also indicate the number of derivatives of each operator and whether an operator modifies a given coupling: only a few operators modify couplings that include gravitons.[2] For the time being, we do not assume any hierarchy among these operators (while in Sec. 2.2 we will).

Let us now study how one can use the transformations eq. (2.4) to simplify the action. In [28] the transformations f_1 and f_2 were used to make the quadratic action for gravitons canonical. This boils down to eliminate the first two operators in Table 2.1 in such a way that the spatial and time kinetic term of the graviton only arise from the standard Einstein-Hilbert action with time-independent $M_{\rm Pl}$. Since the transformations f_1 and f_2 do not contain perturbations, they cannot be done perturbatively. They also modify the background FRW and the definition of cosmic time. The details are spelled out in [28] and in Appendix A.1. The bottom line is that there is no loss of generality in setting to zero the first two operators in Table 2.1. Notice that, since only these two operators modify the coupling $\gamma\gamma\gamma$, one concludes

[1]Notice that in the ADM formalism $g^{00} = -1/N^2$.

[2]Notice that scalar operators such as δN, δK and V cannot contain γ_{ij} at linear order in perturbations.

Coeff.	$\mathcal{O}^{(2)}$	$\#\partial_\mu$	$\gamma\gamma$	$\gamma\gamma\gamma$	$\gamma\gamma\zeta$	$\gamma\zeta\zeta$	$\zeta\zeta\zeta$	$\to 0$
a_0	${}^{(3)}R$	2	✓	✓	✓	✓	✓	$f_{1,2}$
a_1	$\delta K_{\mu\nu}\delta K^{\mu\nu}$	2	✓	✓	✓	✓	✓	$f_{1,2}$
a_2	${}^{(3)}R\,\delta N$	2			✓	✓	✓	$f_{3,4}$
a_3	$A_\mu A^\mu$	2				✓	✓	
a_4	$H^2\delta N^2$	0					✓	
a_5	$H\delta N\delta K$	1					✓	
a_6	δK^2	2					✓	
a_7	V^2	2					✓	
a_8	$V\delta K$	2					✓	

Table 2.1: Quadratic operators up to second order in derivatives, together with the list of the couplings they affect. The last column shows which transformation can be used to set to zero the corresponding operator.

Coeff.	$\mathcal{O}^{(3)}$	$\#\partial_\mu$	$\gamma\gamma\gamma$	$\gamma\gamma\zeta$	$\gamma\zeta\zeta$	$\zeta\zeta\zeta$	$\to 0$
b_1	$\delta N\delta K_{\mu\nu}\delta K^{\mu\nu}$	2		✓	✓	✓	$f_{3,4}$
b_2	${}^{(3)}R\,\delta N^2$	2				✓	$f_{5,6}$
b_3	$\delta N A_\mu A^\mu$	2				✓	f_5
b_4	$H^2\delta N^3$	0				✓	
b_5	$H\delta N^2\delta K$	1				✓	$f_{5,6}$
b_6	$\delta N\delta K^2$	2				✓	
b_7	$\delta N V^2$	2				✓	
b_8	$\delta N V\delta K$	2				✓	f_5

Table 2.2: Cubic operators up to second order in derivatives, together with the list of the couplings they affect. The last column shows which transformation can be used to set to zero the corresponding operator. Two of the operators among b_2, b_3, b_5 and b_8 can be se to zero using the transformations f_5 and f_6.

that the correlator $\langle\gamma\gamma\gamma\rangle$ cannot be modified at leading order in derivatives. We come back to corrections at higher order in derivatives in Section 2.2.2.

Consider now the transformations of order $\mathcal{O}(\delta N)$, i.e. f_3 and f_4. At leading order in perturbations the field redefinitions multiply the variation of the action with respect to the metric, i.e. the equations of motion. In particular, the variation of the Einstein-Hilbert action under the transformation $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$ gives

$$\delta S_{\rm EH} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g}\, G_{\mu\nu} \delta g^{\mu\nu} \,, \tag{2.11}$$

which for the transformations f_3 and f_4 becomes

$$\delta S_{\rm EH} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left(f_3 {}^{(4)}\!R\, \delta N - f_4 \delta N G_{\mu\nu} n^\mu n^\nu \right) \,. \tag{2.12}$$

Using the geometric Gauss-Codazzi relations [36]

$$^{(4)}\!R = {}^{(3)}\!R - K^2 + K_{\mu\nu}K^{\mu\nu} + 2\nabla_\mu (K n^\mu - A^\mu) \,, \tag{2.13a}$$

$$G_{\mu\nu}n^\mu n^\nu = \frac{1}{2}\left({}^{(3)}\!R + K^2 - K_{\mu\nu}K^{\mu\nu} \right) \,, \tag{2.13b}$$

one can write the variation of the action in the EFTI form. We postpone all details to the next section: here it suffices to notice that one can use f_3 and f_4 to set the operators ${}^{(3)}\!R\,\delta N$ and $\delta N \delta K_{\mu\nu}\delta K^{\mu\nu}$ to zero. This choice can be convenient since these are the only (remaining) operators that modify the coupling $\gamma\gamma\zeta$. In the following section we are going to verify explicitly the invariance of the correlator $\langle\gamma\gamma\zeta\rangle$ in doing the transformations f_3 and f_4.

The logic is the same for the two functions f_5 and f_6. Since the operators they generate are proportional to δN^2 and there is no scalar that one can build at linear order that contains γ, f_5 and f_6 do not affect anything that has to do with tensor modes at this order. Therefore, the choice of which operator to set to zero with f_5 and f_6 is, to some extent, arbitrary. In Appendix A.1 we explicitly calculate the variation of the operators under the transformations f_i and we find which ones can be set to zero, see last column of Tables 2.1 and 2.2. In particular, f_5 can set to zero one of the following 2-derivative operators: ${}^{(3)}\!R\,\delta N^2$, $\delta N V \delta K$, $H\delta N^2 \delta K$ or $\delta N A_\mu A^\mu$, while f_6 only ${}^{(3)}\!R\,\delta N^2$ or $H\delta N^2\delta K$.

Although in this chapeter we focus on terms that are up to cubic in perturbations, one can easily see what happens at higher order in δN. At each new order one gets a table similar to Table 2.2 with more powers of δN. These are 8 new operators at each order. At the same time one has 2 new possible field redefinitions of the same form of f_5 and f_6 but with more powers of δN. One can use these two free functions to set to zero two of the new operators. In conclusion, one remains with 6 non-redundant operators at each order in perturbations.

2.1.1 Simplifying $\langle\gamma\gamma\zeta\rangle$

As an explicit application and check, in this section we will show how to exploit the field redefinitions (2.4) to set to zero the operators involving two gravitons and a scalar. As shown in the tables, these operators are

$${}^{(3)}\!R \,, \qquad \delta K_{\mu\nu}\delta K^{\mu\nu} \,, \qquad {}^{(3)}\!R\,\delta N \,, \qquad \delta N \delta K_{\mu\nu}\delta K^{\mu\nu} \,. \tag{2.14}$$

All these operators already appear in the Einstein-Hilbert action. As explained in Section 2.1, transformations f_1 and f_2 can be used to remove ${}^{(3)}R$ and $\delta K_{\mu\nu}\delta K^{\mu\nu}$ [28]. We will verify that the redefinitions

$$g_{\mu\nu} \to g_{\mu\nu} + f_3 \delta N g_{\mu\nu} \ , \tag{2.15a}$$

$$g_{\mu\nu} \to g_{\mu\nu} + f_4 \delta N n_\mu n_\nu \ , \tag{2.15b}$$

can be used to set to zero, respectively, ${}^{(3)}R\,\delta N$ and $\delta N \delta K_{\mu\nu}\delta K^{\mu\nu}$. In particular, we will show that the action S_0, eq. (2.8), changes under the transformations (2.15), but the late-time correlation functions do not.[3]

It is important to stress that, although the coupling $\gamma\gamma\zeta$ can be brought back to the standard Einstein-Hilbert form, the correlator $\langle\gamma\gamma\zeta\rangle$ is also sensitive to the solution of the scalar constraints. As such the correlator is modified by the quadratic scalar operators and it is not completely fixed at the 2-derivative level, contrarily to what happens for $\langle\gamma\gamma\gamma\rangle$.

Before proceeding, it is convenient to remind how the ADM components of the metric (2.5) change under the metric transformation of eq. (2.2) [37]:

$$h_{ij} \to C(t,N) h_{ij} \ , \qquad N^2 \to \left[C(t,N) - D(t,N)\right] N^2 \ , \qquad N^i \to N^i \ . \tag{2.16}$$

Transformation $\mathbf{f}_4$. We start by considering the disformal transformation f_4, eq. (2.15b), which is the simplest to treat. We can work at linear order in the metric transformation, because higher orders carry two or more powers of δN and hence do not contribute to $\langle\gamma\gamma\zeta\rangle$. To keep calculations simple we assume $|f_{3,4}| \ll 1$ and constant in time.

The logic will be the following. By eq. (2.16), f_4 only affects the lapse: the action for ζ and γ_{ij} has to be invariant under this transformation once the Hamiltonian and momentum constraints are solved. This is not obvious, because the intermediate action that explicitly contains the lapse changes. However, also the relation between the lapse and ζ given by the solution of the constraints changes accordingly. We will check that once the lapse is replaced in terms of ζ, the action for ζ and γ_{ij} remains unchanged by the transformation.

To do this, let us study the variation of the action S_0, eq. (2.8), under the transformation f_4. For the Einstein-Hilbert part of the action, one can use the Gauss-Codazzi relation (2.13b) in eq. (2.12). Adding the variation of the scalar part, one finds

$$\delta S_0 = -\frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g}\, f_4 \delta N \left[\frac{1}{2}\left({}^{(3)}R + K^2 - K_{ij}K^{ij} \right) + \dot{H}\left(\frac{1}{N^2} - 1 \right) - 3H^2 \right] . \tag{2.17}$$

The coefficient f_4 can be used, for example, to set to zero the operator ${}^{(3)}R\,\delta N$. To verify that the action in terms of ζ and γ does not change, we need to solve the lapse in terms of ζ only at linear order. This is because its second-order part does not contribute to the cubic action, as it multiplies the background equations of motion

[3] We assume that the time dependence of the parameters f_3 and f_4 is mild enough. More precisely, f_3 and f_4 must not grow faster than η^{-2} for $\eta \to 0$.

[20]. Thus, we can focus on the quadratic action. To do so, it is convenient to define $E_{ij} \equiv NK_{ij}$, whose explicit components are

$$E_{ij} \equiv \frac{1}{2}\left(\dot{h}_{ij} - \mathcal{D}_i N_j - \mathcal{D}_i N_j\right) , \tag{2.18}$$

where $\mathcal{D}_i$ denotes the covariant derivative with respect to the spatial metric h_{ij}. With this notation and using that $\sqrt{-g} = N\sqrt{h} = a^3 e^{3\zeta} N$, we can expand S_0 and δS_0 above at quadratic order. They read, respectively,

$$\begin{aligned} S_0^{(2)} = \frac{M_{\rm Pl}^2}{2} \int dt\, d^3x\, a^3 \Big[&-2(3H^2 + \dot{H})\delta N^2 + 4H\delta N \delta E + \delta E^i{}_j \delta E^j{}_i \\ &- \delta E^2 + {}^{(3)}\!R\, \delta N + 3\, {}^{(3)}\!R\, \zeta \Big] , \end{aligned} \tag{2.19}$$

where δE denotes the trace of $\delta E_{ij} \equiv E_{ij} - Hh_{ij}$, and

$$\delta S_0^{(2)} = -\frac{M_{\rm Pl}^2}{2} \int dt\, d^3x\, a^3\, f_4 \left[\frac{1}{2} {}^{(3)}\!R\, \delta N - 2(3H^2 + \dot{H})\delta N^2 + 2H\delta N \delta E + 18H^2 \delta N \zeta \right] . \tag{2.20}$$

Varying $S_0^{(2)} + \delta S_0^{(2)}$ with respect to the shift yields the momentum constraint,

$$\mathcal{D}_i \left[(E^i{}_j - E\delta^i{}_j) N^{-1} - f_4 H N \delta N \delta^i{}_j \right] = 0 . \tag{2.21}$$

Solving this equation for the lapse, gives

$$\delta N = \frac{\dot{\zeta}}{H}\left(1 + \frac{f_4}{2}\right) . \tag{2.22}$$

At this point, it is straightforward to verify that plugging the above expression for δN in the original action S_0, the term proportional to f_4 which is generated exactly cancels the action variation (2.17). We have also checked that the expression of the shift in terms of ζ, which can be obtained from the Hamiltonian constraint, is not modified by the disformal transformation.

Transformation $\mathbf{f}_3$. The conformal transformation f_3, eq. (2.15a), is more complicated than the previous one, because not only does it changes the solution for δN but it also redefines ζ. Indeed, working again at linear order in the metric transformation, from eq. (2.16) we find the following transformations for ζ and δN:

$$\zeta \to \zeta + f_3 \frac{\delta N}{2} , \qquad \delta N \to \delta N \left(1 + \frac{f_3}{2}\right) , \tag{2.23}$$

while the scalar component of the shift, defined as $\psi \equiv \partial^{-2}\partial_i N^i$, remains unchanged. The solutions of the Hamiltonian and momentum constraints change accordingly. Using these transformations in the usual solutions for δN and ψ derived from action S_0 and assuming for simplicity a constant f_3, these are given respectively by

$$\delta N = \frac{\dot{\zeta}}{H} - \frac{f_3}{2}\left(\frac{\dot{\zeta}}{H} - \frac{1}{H}\frac{d}{dt}\frac{\dot{\zeta}}{H}\right) , \tag{2.24}$$

$$\psi = -\frac{\zeta}{a^2 H} + \epsilon \partial^{-2}\dot{\zeta} - \frac{f_3}{2}\left[\frac{1}{a^2 H}\frac{\dot{\zeta}}{H} - \epsilon\partial^{-2}\left(\frac{d}{dt}\frac{\dot{\zeta}}{H}\right)\right] , \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} . \tag{2.25}$$

Let us also derive these two relations by solving the constraints of the new action. For the Einstein-Hilbert part of the action, one can use again the Gauss-Codazzi relation (2.13a) in eq. (2.12). Integrating by parts, using the definitions of $A_i = N^{-1}\partial_i N$ and V (eq. (2.10)), and adding the variation of the scalar part, one finds

$$\delta S_0 = -\frac{M_{\rm Pl}^2}{2}\int d^4x\,\sqrt{-g}\, f_3 \Big\{ \delta N \Big[K^2 - K_{ij}K^{ij} - {}^{(3)}\!R + 2\dot H\left(\frac{1}{N^2}+2\right) + 12H^2\Big] + KV - A_i A^i N^{-1}\Big\} . \tag{2.26}$$

Here the lapse appears with a time derivative in V, which makes δN dynamical. This can be also seen by varying the action $S_0 + \delta S_0$ with respect to the shift. One obtains

$$\mathcal{D}_i\left\{ (E^i{}_j - E\delta^i{}_j)N^{-1} - f_3\left[-\delta N (E^i{}_j - E\delta^i{}_j)N^{-1} + V\delta^i{}_j\right]\right\} - f_3 E A_j = 0\,, \tag{2.27}$$

which is a dynamical equation for δN and not a constraint. However, since V comes only at first order in f_3 it can be treated perturbatively. Indeed, solving this equation perturbatively in f_3 one recovers eq. (2.24). Moreover, the Hamiltonian constraint equation derived from this action gives the solution of eq. (2.25) for the shift.

The transformation f_3 changes the quadratic action for scalar perturbations (but not the one of tensors). This implies that the correlation functions $\langle\zeta\zeta\rangle$ and $\langle\gamma\gamma\zeta\rangle$ change when evaluated inside the horizon. Only at late times, the correlation functions will not depend on f_3 as we are now going to show. Let us first look at the quadratic action for scalar perturbations to verify that this is the case for the two-point function of ζ. The second-order expansion of the action S_0 is given by eq. (2.19). Expanding the action (2.26) at second order yields, after some integrations by parts,

$$\delta S_0^{(2)} = M_{\rm Pl}^2 \int d^4x\, f_3\, a^3 \Big[\frac{{}^{(3)}\!R\,\delta N}{2} - \frac{5}{2}H^2(3-\epsilon)\delta N^2 - 2H\delta N\delta E + 3HN^i\partial_i\delta N - \delta\dot N\delta E + \frac{(\partial_i\delta N)^2}{a^2} - 9(3H^2+\dot H)\delta N\zeta - 3H\delta\dot N\zeta\Big] . \tag{2.28}$$

Expressing the action above as function of the curvature perturbation ζ using eqs. (2.24) and (2.25), the second-order expansion of $S_0 + \delta S_0$ gives

$$S_\zeta^{(2)} = M_{\rm Pl}^2\left(1 - \frac{3f_3}{2} + \mathcal{O}(f_3^2)\right)\int d^4x\, a^3\epsilon\left[\dot\zeta^2 - c_{\rm s}^2\frac{(\partial_i\zeta)^2}{a^2}\right] , \tag{2.29}$$

with

$$c_{\rm s} = 1 + \frac{f_3}{2} + \mathcal{O}(f_3^2)\,. \tag{2.30}$$

Therefore, both the normalization and the speed of propagation of ζ are affected by the transformation f_3. This is reflected in a change of the wavefunction for ζ, which becomes

$$\zeta(\eta,k) = \frac{-iH}{2\sqrt{\epsilon}\,M_{\rm Pl}(1-3f_3/4)}\frac{1}{(c_{\rm s}k)^{3/2}}(1+ic_{\rm s}k\eta)e^{-ic_{\rm s}k\eta}\,. \tag{2.31}$$

However, the late-time two-point function of ζ does not change. Indeed, this is proportional to $(1+3f_3/2)c_{\rm s}^{-3}=1$ at leading order in f_3.

Let us now move to the computation of the cubic action $\gamma\gamma\zeta$. After many integrations by parts that show that the action is slow-roll suppressed [20], we obtain

$$S^{(3)}_{\gamma\gamma\zeta}=\frac{M_{\rm Pl}^2}{4}\int d^4x\,a^3\epsilon\bigg\{2\zeta\bigg(\dot\gamma_{ij}^2+\frac{(\partial_k\gamma_{ij})^2}{a^2}\bigg)-\dot\gamma_{ij}\partial_k\gamma_{ij}\frac{\partial_k}{\partial^2}\dot\zeta \\ -f_3\bigg[-\frac{1}{4}\frac{\dot\zeta}{H}\bigg(\dot\gamma_{ij}^2+\frac{(\partial_k\gamma_{ij})^2}{a^2}\bigg)+\frac{1}{2}\dot\gamma_{ij}\partial_k\gamma_{ij}\frac{\partial_k}{\partial^2}\bigg(\frac{d}{dt}\frac{\dot\zeta}{H}\bigg)\bigg]\bigg\}\ . \tag{2.32}$$

We can thus compute the $\langle\gamma\gamma\zeta\rangle$ three-point function. To do this, we need to use the wavefunctions of eq. (2.31) in the in-in calculation. The final result is independent of f_3 up to $\mathcal{O}(f_3^2)$ corrections, thus confirming that late-time correlation functions are insensitive to the transformation of eq. (2.15a).

2.2 Transformations of higher order in derivatives

So far, we have considered only field redefinitions without derivatives. These do not change the number of derivatives of the operators. In this section we consider more general transformations (2.3), involving one or two derivatives on the metric.

2.2.1 Diff-like transformations

In general, field redefinitions which include derivatives will generate, starting from the Einstein-Hilbert action operators with 3 or more derivatives absent from Tables 2.1 and 2.2. However there is a particular set of higher-derivatives transformations which do not change the Einstein-Hilbert action but only the inflaton one. Indeed, consider the transformation

$$g_{\mu\nu}\to g_{\mu\nu}+\nabla_\mu\xi_\nu+\nabla_\nu\xi_\mu\ , \tag{2.33}$$

where ξ_μ is a vector field starting linear in perturbations. This is analogous to a linearized diffeomorphism generated by ξ_μ (notice, however, that we are *not* reintroducing the Stueckelberg π). The Einstein-Hilbert action does not change under this transformation because it is invariant under 4-dimensional diffs. Since the EFTI action (2.7) is invariant under spatial diffs, in the following we will consider only transformations along n_μ, which are associated to time-diffs. Note that eq. (2.33) is a particular case of the general transformation (2.3), as it can be checked by replacing in the above equation the general expression for ξ_μ in unitary gauge, i.e.,

$$\xi_\mu=F(t,N,V,K,\dots)\,n_\mu\ . \tag{2.34}$$

We can thus focus on how the rest of the action transforms. We assume that the inflaton dynamics is dominated by δN^2 and δN^3 with the other operators (that we list in Table 2.3) being suppressed by negative powers of some energy scale Λ. (Our assumptions do not apply to cases in which the quadratic action of the inflaton is dominated by higher-derivative operators, such as, for instance, Ghost Inflation

[38] and Galileons [39].) In this case the size of the operators is parametrically governed by a derivative expansion in ∂_μ/Λ and the coefficients in front of the higher-derivative operators are suppressed by positive powers of H/Λ. More specifically, if operators with no derivatives are of the order of the slow-roll parameter $\epsilon = -\dot{H}/H^2$, $a_4, b_4 \sim \mathcal{O}(\epsilon)$, those with one derivative are suppressed by a single power of H/Λ, $a_5, b_5 \sim \mathcal{O}(\epsilon H/\Lambda)$, while those with two derivatives are suppressed by $(H/\Lambda)^2$. Since the above transformation generates at least one more derivative in the action, the variation of the operators with two derivatives is suppressed by at least $(H/\Lambda)^3$ and we neglect it here.

We first focus on transforming the operators with no derivatives. In this case, it is straightforward to compute the variation of the action (2.7) under eq. (2.33). In particular, we use that the transformation of the lapse is given by

$$\delta_\xi N = -N n_\mu n_\nu \nabla^\mu \xi^\nu \,. \tag{2.35}$$

Assuming that the operator coefficients a_I and b_I are time independent and neglecting slow-roll corrections, the linear variation of the action (2.7) reads, up to third order in perturbations,

$$\begin{aligned}\delta_\xi S = -M_{\rm Pl}^2 \int d^4x \sqrt{-g} \Big\{ 2\dot{H} \left(\frac{3H}{N} + \frac{V}{N^3} - \frac{K}{N^2} \right) + 2a_4 H^2 \Big[3H(1+\delta N)\delta N \\ + (1+3\delta N)V + \delta N \delta K \Big] + 3b_4 H^2 (3H\delta N + 2V)\delta N \Big\} F \,,\end{aligned} \tag{2.36}$$

with F defined by eq. (2.34).

Restricting it to be at most first order in derivatives, we have

$$F = \frac{1}{H} \left[g_1(t)\delta N + g_2(t)\delta N^2 + g_3(t)\frac{V}{H} + g_4(t)\delta N \frac{V}{H} + g_5(t)\frac{\delta K}{H} + g_6(t)\delta N \frac{\delta K}{H} \right] \,, \tag{2.37}$$

where, typically, g_1 and g_2 are suppressed by H/Λ while $g_{3,4,5,6}$ carry a $(H/\Lambda)^2$ suppression. From eq. (2.36) one sees that the transformations g_1 and g_2 generate one-derivative operators suppressed by H/Λ. Thus, they can be used to set to zero the operators $a_5 \delta N \delta K$ and $b_5 \delta N^2 \delta K$, leaving us with four independent transformations. Making use of eq. (2.36), the latter can be employed to set to zero four of the coefficients a_I and b_I for $I = 6, 7, 8$, up to corrections that are at least third order in derivatives. This is summarized by Table 2.3.

In conclusion, the higher-derivative corrections to the leading order dynamics δN^2 and δN^3 start quadratic in H/Λ and there are only 3 two-derivative corrections. This is a major simplification: out of 17 operators, 12 are redundant and one is left with only 5 of them. It is again straightforward to consider higher-order operators. At each order one has 8 new operators, 2 f-like field redefinitions and 3 new g's: only 3 couplings are not redundant.

2.2.2 Higher-derivative operators for tensors

We now consider operators with more than 2 derivatives on the metric. Instead of remaining general we focus on operators that modify the tensor dynamics, which cannot be changed at the two-derivative level.

Coeff.	$\mathcal{O}^{(2)}$	$\#\partial_\mu$	$\to 0$	$\to 0$
a_0	${}^{(3)}R$	2	$f_{1,2}$	
a_1	$\delta K_{\mu\nu}\delta K^{\mu\nu}$	2	$f_{1,2}$	
a_2	${}^{(3)}R\,\delta N$	2	$f_{3,4}$	
a_3	$A_\mu A^\mu$	2		
a_4	$H^2\delta N^2$	0		
a_5	$H\delta N\delta K$	1		g_1
a_6	δK^2	2		g_5
a_7	V^2	2		g_3
a_8	$V\delta K$	2		$g_{3,5}$

Coeff.	$\mathcal{O}^{(3)}$	$\#\partial_\mu$	$\to 0$	$\to 0$
b_1	$\delta N\delta K_{\mu\nu}\delta K^{\mu\nu}$	2	$f_{3,4}$	
b_2	${}^{(3)}R\,\delta N^2$	2	$f_{5,6}$	
b_3	$\delta N A_\mu A^\mu$	2	f_5	
b_4	$H^2\delta N^3$	0		
b_5	$H\delta N^2\delta K$	1	$f_{5,6}$	g_2
b_6	$\delta N\delta K^2$	2		$g_{5,6}$
b_7	$\delta N V^2$	2		$g_{3,4}$
b_8	$\delta N V\delta K$	2	f_5	$g_{3,4,5,6}$

Table 2.3: Quadratic (left panel) and cubic (right panel) operators up to second order in derivatives. The fourth column shows which operator can be set to zero by the transformation (2.4), which is exact in the derivative expansion, see Section 2.1. Treating higher derivatives perturbatively, the fifth column shows which operator can be set to zero by the transformation (2.33). The transformations g_1 and g_2 are used to set to zero the one-derivative operators a_5 and b_5, while $g_{3,4,5,6}$ can be used to set to zero four of the two-derivative operators $a_{6,7,8}$ and $b_{6,7,8}$.

Operators with three derivatives. Possible three-derivative operators for tensors up to cubic order are

$$ {}^{(3)}R_{\mu\nu}\delta K^{\mu\nu}\,, \quad \delta R^{\mu 0\nu 0}\delta K_{\mu\nu} \quad \text{and} \quad \delta K_{\mu\nu}\delta K^{\mu}_{\ \rho}\delta K^{\nu\rho}\,. \tag{2.38} $$

(Here we are assuming parity. For a discussion about parity violating operators one can see [28].) However, using the relation [40]

$$ \lambda(t)\,{}^{(3)}R_{\mu\nu}K^{\mu\nu} = \frac{\lambda(t)}{2}\,{}^{(3)}RK + \frac{\dot\lambda(t)}{2N}\,{}^{(3)}R + \text{boundary terms}\,, \tag{2.39} $$

one can get rid of the first operator. Moreover, using the Gauss-Codazzi relation, one can show that

$$ N^2\,K_{\alpha\gamma}R^{\alpha 0\gamma 0} = -K^{\alpha\gamma}K_\gamma^{\ \rho}K_{\rho\alpha} + K_\alpha^{\ \gamma}D_\gamma A^\alpha + K_\alpha^{\ \gamma}A_\gamma A^\alpha - K_\alpha^{\ \gamma}n^\delta\nabla_\delta K_\gamma^{\ \alpha}\,. \tag{2.40} $$

The second and third operators contain scalar perturbations, while the last one can be integrated by parts. In this way one can also dispose of the second operator in eq. (2.38). One can then wonder whether it is possible to set also the third operator to zero with a suitable field redefinition: as we are now going to show, this is not possible.

To see this, one has to find all the possible field redefinitions that carry one derivative on the metric. Since the only scalars that satisfy this requirement are K and V, and the only symmetric tensors that we can add to $g_{\mu\nu}$ are $K_{\mu\nu}$ and $n_{(\mu}A_{\nu)}$, we see that eq. (2.3) reduces to

$$ g_{\mu\nu} \to C(t,N,K,V)g_{\mu\nu} + D(t,N,K,V)n_\mu n_\nu + E(t,N)\delta K_{\mu\nu} + F(t,N)n_{(\mu}A_{\nu)}\,, \tag{2.41} $$

where we have considered $\delta K_{\mu\nu}$ instead of $K_{\mu\nu}$ on the r.h.s., without loss of generality. Since A_μ does not contain tensor modes, the term $\propto n_{(\mu}A_{\nu)}$ cannot affect the cubic action for three gravitons. Therefore, the only way to possibly induce the operator $\delta K_{\mu\nu}\delta K^{\mu}_{\ \rho}\delta K^{\nu\rho}$ is a transformation of the form

$$g_{\mu\nu} \to g_{\mu\nu} + c_K \delta K_{\mu\nu} \,. \tag{2.42}$$

It is now straightforward to see that we cannot generate $\delta K_{\mu\nu}\delta K^{\mu}_{\ \rho}\delta K^{\nu\rho}$ through eq. (2.42). When written in terms of γ_{ij}, the transformation becomes

$$\gamma_{ij} \to \gamma_{ij} + c_K \dot{\gamma}_{ij} \,, \tag{2.43}$$

i.e. a linear shift at all orders in perturbations. Therefore, the only effect it has is to change $S_\gamma^{(2)}$ and $S_\gamma^{(3)}$ separately: since in the Einstein-Hilbert action $S_\gamma^{(3)}$ comes only from ${}^{(3)}\!R$ [20, 41], at leading order in c_K we will just have generated terms with two spatial derivatives and one time derivative, and no terms of the form $\dot{\gamma}_{ij}^3$.

Operators with four derivatives. There are no parity-conserving corrections to the tensor power spectrum with three derivatives. The first correction is at fourth order in derivatives, [42]. Up to integration by parts, there are four operators with four derivatives that modify the tensor power spectrum:

$${}^{(3)}\!R_{\mu\nu}^2 \,, \quad (\nabla^0 \delta K_{\mu\nu})^2 \,, \quad {}^{(3)}\!R_{\mu\nu}\nabla^0\delta K^{\mu\nu} \quad \text{and} \quad (\mathcal{D}_\rho \delta K_{\mu\nu})^2 \,. \tag{2.44}$$

The corresponding modifications in the quadratic action for tensors are of the form $(\partial^2\gamma_{ij})^2$, for the first, $\ddot{\gamma}_{ij}^2$ for the second and $(\partial_k\dot{\gamma}_{ij})^2$ for the last two. One has the freedom to perform field redefinitions, but there are not enough of them to get rid of all the three operators. Indeed, there are only two possible field redefinitions at second order in derivatives that affect tensor modes:

$$g_{\mu\nu} \to g_{\mu\nu} + c_R\, {}^{(3)}\!R_{\mu\nu} + c_0 \nabla^0 \delta K_{\mu\nu} \,. \tag{2.45}$$

They correspond to $\gamma_{ij} \to c_R\, \partial^2\gamma_{ij} + c_0\, \ddot{\gamma}_{ij}$. We conclude that we cannot eliminate all the corrections to the tensor power spectrum at this order.

The modification of the power spectrum is only possible because of the preferred foliation provided by the inflaton. In the absence of a preferred foliation one is forced to write only operators that are fully diffeomorphism-invariant. Since the Gauss-Bonnet term, $R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$, is a total derivative in four dimensions, the only allowed operator is $\delta R_{\mu\nu}^2$. However, one can dispose of it by using the field redefinition $g_{\mu\nu} \to g_{\mu\nu} + \delta R_{\mu\nu}$.

Let us now compute the correction to the tensor power spectrum due to the new couplings. To simplify things, we use the free parameters c_R and c_0 in eq. (2.45) to set to zero the coupling in front of $(\nabla^0\delta K_{\mu\nu})^2$ and in front of the quadratic part of the combination $c_1\,{}^{(3)}\!R_{\mu\nu}\nabla^0\delta K^{\mu\nu} + c_2(D_\rho\delta K_{\mu\nu})^2$. Therefore, we just need to expand ${}^{(3)}\!R_{\mu\nu}^2$ at quadratic order in γ. We get

$$S_4^{(2)} = \frac{1}{4}\frac{M_{\rm Pl}^2}{\Lambda^2}\int d\eta\, d^3x\, (\partial^2\gamma_{ij})^2 \,. \tag{2.46}$$

It is straightforward to study the effect of this term in the usual in-in formalism. The interaction Hamiltonian $H_{\rm int}$ in Fourier space is

$$H_{\rm int} = -\frac{1}{4}\frac{M_{\rm Pl}^2}{\Lambda^2}\int \frac{d^3k}{(2\pi)^3}\, k^4 \sum_{s_1,s_2} \gamma^{s_1}_{\vec{k}_1}\gamma^{s_2}_{\vec{k}_2}\, \epsilon^{s_1}_{ij}(\hat{k}_1)\epsilon^{s_2}_{ij}(\hat{k}_2) \,. \tag{2.47}$$

Then, the correction to the power spectrum is given by

$$\delta\langle\gamma^s_{\vec{k}}\gamma^{s'}_{-\vec{k}}\rangle' = \frac{5}{4}\frac{H^2}{\Lambda^2}\frac{H^2}{2M_{\rm Pl}^2k^3}\delta_{ss'} \ . \tag{2.48}$$

In general, one expects the above correction to be small, being suppressed by a factor H^2/Λ^2. However it could become sizable if the suppression scale Λ is not too large.

When the power spectrum is modified, we also expect sizable non-Gaussianities. More precisely, we expect an enhancement if we consider three-point functions involving scalars. In fact, the operators (2.44) are constructed from the foliation, i.e. they entail a direct coupling with the inflaton. For instance, we expect that their contribution to the cubic action $\gamma\gamma\zeta$ will not be suppressed by slow-roll parameters. On the other hand, a slow-roll suppression is present at the 2-derivative level where, as we discussed in Section 2.1.1, the only freedom comes by the modification of the scalar constraint equations.

We can estimate these non-Gaussianities in the following way. At cubic order in perturbations, given that we have used c_R and c_0 to put the quadratic action in the form of eq. (2.46), there will be ${}^{(3)}R^2_{\mu\nu}$ and the cubic part of $c_1{}^{(3)}R_{\mu\nu}\nabla^0\delta K^{\mu\nu} + c_2(D_\rho\delta K_{\mu\nu})^2$ that will contribute. For our estimation, however, it is enough to consider ${}^{(3)}R^2_{\mu\nu}$. In Appendix A.2 we show that, as expected, for two gravitons and a scalar we have $S_4^{(3)} \sim \frac{H^2}{\Lambda^2}\times\epsilon^0$. Since $S_0^{(3)} \sim \epsilon$ [20], we see that the bispectrum coming from these 4-derivative operators could dominate the standard slow-roll result for $\frac{H^2}{\Lambda^2} \gtrsim \epsilon$.

2.3 Field redefinitions in the decoupling limit

In many cases inflationary correlation functions can be calculated, at leading order in slow-roll, in the so-called decoupling limit. This means concentrating on the Goldstone field π, which is introduced in the action when we depart from the unitary gauge, and neglecting the fluctuations of the metric (see Section 1.3). In this limit one can consider field redefinitions of π that decay at late times and thus do not change the asymptotic correlation functions. A natural question is whether these field redefinitions of π are simply the decoupling limit of the ones we discussed before or they are different in nature. We want to argue that, in general, these two kinds of field redefinitions are different and cannot be simply related.

First of all, notice that the interactions of π are constrained by the non-linear realization of Lorentz invariance. Indeed if we neglect metric perturbations and we go to short scales, we have a theory with spontaneously broken Lorentz symmetry: the combination $t+\pi(t,\vec{x})$ transforms as a scalar under Lorentz and this defines the non-linear transformation of π. Starting from a generic unitary gauge action and reintroducing π with the usual Stückelberg procedure, one ends up in the decoupling limit with an action of π with this well-defined non-linear realization of the Lorentz symmetry. In particular, this will remain true even when one performs a metric field redefinition in unitary gauge and considers two equivalent actions in the sense discussed above: in the decoupling limit of either theories one has the same non-linear realization of the Lorentz symmetry. This, however, does not happen when one considers a general field redefinition of π in the decoupling limit:

$$\tilde{\pi} = \pi + f(\pi, \dot{\pi}, \partial_i \pi, \ldots) \, . \tag{2.49}$$

Given the transformation rules of π, one sees that $\tilde{\pi}$ will transform in a different way for a generic f. This is enough to show that the action for $\tilde{\pi}$ cannot generically be obtained as the decoupling limit of an action in unitary gauge via Stückelberg.

Let us focus on a concrete example where a π field redefinition is useful. In single-field inflation the leading operators giving a potentially large 3-point function for ζ are $\dot{\pi}^3$ and $\dot{\pi}(\partial\pi)^2$. At subleading order in derivatives one should look at cubic operators with four derivatives. It is straightforward to realize that (up to integration by parts) there are only two 4-derivative operators [43, 44]: $\partial^2\pi(\partial\pi)^2$ and $\partial^2\pi\,\dot{\pi}^2$. They arise from the unitary gauge operators $\delta N\delta K$ and $\delta N^2\delta K$. The action in the decoupling limit is given by

$$S_\pi = \int d^4x\, a^3\,(-M_{\rm Pl}^2\dot{H})\Big[(1+\alpha_1)\left(\dot{\pi}^2 - c_{\rm s}^2\frac{(\partial\pi)^2}{a^2}\right) + (\alpha_2-\alpha_1)\,\dot{\pi}\frac{(\partial\pi)^2}{a^2} \\ - 2(\alpha_1+\alpha_3)\dot{\pi}^3 + 2\frac{\alpha_2-\alpha_4}{H}\,\dot{\pi}^2\frac{\partial^2\pi}{a^2} + \frac{\alpha_2}{H}\,\frac{(\partial\pi)^2\partial^2\pi}{a^4}\Big]\,, \tag{2.50}$$

with $c_{\rm s}^2 \equiv (1+\alpha_2)/(1+\alpha_1)$. We use here the notation of [45]; the α's are related to our a's as $\alpha_1 = -a_4/\epsilon\,,\ \alpha_2 = -a_5/2\epsilon\,,\ \alpha_3 = -b_4/2\epsilon\,,\ \alpha_4 = b_5/2\epsilon\,.$ Naively, the last two operators in the equation above give 3-point functions whose shape is different from the standard operators with three derivatives. However, it is straightforward to check that the field redefinitions $\pi \to \pi + c_1(\partial\pi)^2$ and $\pi \to \pi + c_2\dot{\pi}^2$ can be used to remove both these operators, while changing the coefficient of the 3-derivative operators $\dot{\pi}^3$ and $\dot{\pi}(\partial\pi)^2$. This shows that, in the decoupling limit, operators with one extra derivative do not give rise to new shapes.[4] The removal of the 4-derivative term does not mean the theory is equivalent to one with only δN^2 and δN^3. This can be seen noting that the operator $\dot{\pi}(\partial\pi)^2$, after the field redefinition, has a coefficient $\alpha_2 - \alpha_1 + 2\alpha_2(1+\alpha_1)/(1+\alpha_2)$, which is not related to $c_{\rm s}$ in the standard way as dictated by the non-linear realization of Lorentz invariance.

In Section 2.2.1 we showed that the operators $\delta N\delta K$ and $\delta N^2\delta K$ can be removed by a unitary-gauge field redefinition, provided their coefficients are small so that one can neglect quadratic corrections. The corresponding statement in the decoupling limit should be that there is a field redefinition of π which preserves the usual Lorentz transformation of π and gets rid of the 4-derivative terms at linear order in their coefficient. Since the combination $\psi \equiv t + \pi$ transforms as a Lorentz scalar, also $(\partial_\mu\psi)^2 + 1 = -2\dot{\pi} - \dot{\pi}^2 + (\partial\pi)^2$ transforms as a scalar. This means that the field redefinitions

$$\tilde{\pi} = \pi + c_1(2\dot{\pi} + \dot{\pi}^2 - (\partial\pi)^2)\,, \qquad \tilde{\pi} = \pi + c_2(2\dot{\pi} + \dot{\pi}^2 - (\partial\pi)^2)^2 = \pi + 4c_2\dot{\pi}^2 + \ldots \tag{2.51}$$

preserve the Lorentz transformations of π. Notice that there is now a linear piece in the first transformation: this means one has to restrict the transformation to linear

[4] The same argument can be run at any order in π to argue that there are no genuine new terms with one extra derivative at any order in π.

order in c_1 to avoid the proliferation of higher derivatives and that the quadratic action, and in particular the speed of sound, will be modified. It is straightforward to check that using these field redefinition one can eliminate the 4-derivative terms in eq. (2.50) *at linear order in* α_2 *and* α_4 and that the resulting theory has the usual relation between the speed of sound and the coefficient of the operator $\dot{\pi}(\partial\pi)^2$. This is the decoupling limit of a unitary gauge action in which $\delta N \delta K$ and $\delta N^2 \delta K$ are removed.

In conclusion, the π field redefinitions in the decoupling limit is an extra freedom that one is not able to trace in the unitary gauge theory. This should not be surprising after all: in the case of a spontaneously broken non-abelian gauge theory, one has freedom to parametrize the coset of the Goldstones in various way. This freedom has no obvious analogy in unitary gauge where the Goldstones are eaten by the massive gauge fields.

2.4 Discussion and Outlook

In this chapter we have explored the effect of generalized disformal transformations in the *Effective Field Theory of Inflation*. These transformations do not change the predictions for the late-time observables and can thus be used to simplify the action. They can be organized in an expansion in derivatives and perturbations. These are the main results we obtained.

- If one considers (unitary gauge) operators with up to two derivatives and up to n-th order in perturbations ($n \geq 2$), one has $8(n-1)+1$ independent operators (taking into account integration by parts). $2n$ of these can be set to zero by conformal and disformal transformations, which carry powers of δN up to δN^{n-1}.

- Using these transformations, it is easy to show that the predictions for the tensor power spectrum and the correlator $\langle\gamma\gamma\gamma\rangle$ cannot be modified at leading order in derivatives [28]. Also all the couplings contributing to $S^{(3)}_{\gamma\gamma\zeta}$ beyond the Einstein-Hilbert action can be removed. Even so, $\langle\gamma\gamma\zeta\rangle$ will still be affected by the possible changes in the scalar sector through the constraint equations, therefore we cannot conclude that $\langle\gamma\gamma\zeta\rangle$ is fully fixed.

- Among the additional transformations that contain derivatives, some do not affect the Einstein-Hilbert action but only the inflaton part. These can be used to reduce the number of higher-derivative corrections. For instance, if one starts from a theory where the dominant terms in the inflaton action are those with zero derivatives, one has six additional transformations (up to cubic order in perturbations) that can be used to further simplify the action. One is left with only three higher-derivative corrections up to 2-derivative order.

- At three-derivative order, there are no corrections to the tensor power spectrum and only one independent operator contributing to $S^{(3)}_{\gamma\gamma\gamma}$ after integration by parts.

- At 4-derivative order, there is only one independent operator that affects the tensor power spectrum. This is due to the coupling with the inflaton and as such can give a large bispectrum $\langle\gamma\gamma\zeta\rangle$.

- In the decoupling limit, one can perform field redefinitions of the Goldstone π to simplify the action. In general this kind of transformations does not preserve the way π transforms under Lorentz and cannot be seen as the decoupling limit of the unitary gauge transformations discussed above.

It would be interesting to understand how to phenomenologically identify the few higher-derivative corrections we are left with after the field redefinitions. Also the potentially large bispectrum $\langle\gamma\gamma\zeta\rangle$ due to four-derivative operators deserves further studies.

Chapter 3

Tensor Squeezed Limits and the Higuchi Bound

The dynamics of spin-2 particles is theoretically very constrained. While General Relativity (GR) is the only consistent theory of an interacting massless spin-2 particle [46, 47], there are tight theoretical constraints on the physics of a massive spin-2 [48] and in general on modifications of GR. This theoretical robustness is particularly appealing and motivates the huge experimental effort dedicated to the study of gravitational waves (GWs) both of astrophysical and cosmological origin. The robustness of GR allows to predict in terms of few parameters the production of GWs by binary black holes. The same robustness shows up in the predictions for primordial tensor modes, which are way more model-independent than their scalar counterpart. In fact, we have seen in the previous chapter that, in the context of the EFTI, the tensor power spectrum cannot be modified at leading order in derivatives (see also [28, 49]) and that non-gaussian correlators are quite well constrained. The graviton bispectrum is indeed fixed by the Einstein-Hilbert term, moreover there is only one possible shape for $\langle\zeta\gamma\gamma\rangle$ and just two for $\langle\gamma\zeta\zeta\rangle$ [1].

In this chapter we explore another aspect of this robustness: the tensor consistency relations (CRs) [20]. Usually these relations are associated to single-field models [21] and, indeed, scalar CRs are in general violated when more than one field is relevant: a large $f_{\rm NL}^{\rm local}$ is generated in many multifield models and violates the CR for the 3-point function. On the other hand, it is easy to realize that tensor CRs still hold in multi-field models. As we will discuss in Section 3.1, the argument for which a long-wavelength GW can be locally removed by a suitable anisotropic change of coordinates (an adiabatic mode in the terminology of Weinberg [50]) holds even in the presence of multiple scalar fields[2]. More generally tensor CRs are violated only when there are light tensor perturbations which are not adiabatic, which means anisotropic perturbations are not efficiently damped. Therefore, while the violation of scalar CRs is a smoking gun of the presence of additional scalars, *the violation of tensor CRs would show that the Universe does not quickly evolve towards an anisotropic attractor during inflation.* (A similar conclusion about Solid Inflation was reached in [52, 53].) The usefulness of such a signature becomes clear in view of the fact that at the level of background cosmology the isotropy of the observed universe puts an extremely weak constraint on the degree of anisotropy in the early

[1] Notice that the theoretical predictions becomes less and less constraining as we add scalar perturbations in the primordial correlators.

[2] Scalar CRs can also be seen as a consequence of the equivalence principle [51]. Notice however that scalar violation of the equivalence principle cannot spoil tensor CRs.

universe. Background anisotropy rapidly dilutes during the thermal history.

In the same way extra light scalars can violate scalar CRs, extra light spin-2 particles can violate tensor CRs by introducing long-lived anisotropies. Here, however, the theoretical constraints on spin-2 particles come into play. The Higuchi bound [54], as we will discuss in Section 3.2 and in the Appendix B, forbids the existence of a spin-2 field in de Sitter (dS) space with a mass $m^2 < 2H^2$, where H is the Hubble constant of de Sitter. More generally, the Higuchi bound ensures that all perturbations with non-zero spin (and hence anisotropic) dilute faster than $\exp(-Ht)$. Therefore, although dS is allowed to have scalar hair, it cannot support curly hair. We will see that this is in some sense a stronger statement than Wald's no-hair theorem [55] which assumes strong energy condition on matter fields: a condition that is violated by an innocuous light scalar field. Using the terminology of conformal field theory, we will discuss how primary composite operators are constrained by the Higuchi bound, while non-primary ones can evade it at the expense of introducing tachyonic instabilities.

Since inflation occurs in a space-time which is approximately de Sitter, the bound should also apply to this case as long as de Sitter isometries are approximately respected by the active degrees of freedom. This ensures tensor CRs to hold in this subclass of models. Nevertheless, there are many inflationary models in which part of dS isometries are fully broken. Among those, tensor CRs are often violated in models with a symmetry pattern different from the one of the EFTI, for example in Solid Inflation [56, 57, 58, 23, 59, 60, 42, 61]. In these cases tensor fluctuations, even on long wavelength, deform the background and are therefore not adiabatic.

In Section 3.3 we discuss the various observables sensitive to a violation of tensor CRs. The CMB correlator $\langle BTT \rangle$, recently studied in [62], is directly sensitive to the correlation function $\langle \gamma\zeta\zeta \rangle$. However, there is another slightly indirect probe of this correlation. In contrast to scalar entropy modes (such as extra light scalar fields), where the super-horizon fluctuations just redefine the observed homogeneous background in the observable Universe, non-adiabatic tensors leave a local imprint in the form of a quadrupolar anisotropy of the power spectrum. The anisotropy depends on the particular place in the Universe and we can only study its statistics: we find that the eigenvalues of the tensor which describes the anisotropy tend to be different. As such they are easy to distinguish from models with a preferred direction in the sky, which induce an axisymmetric quadrupolar modulation.

Finally, the violation of tensor CRs shows up indirectly in the countercollinear limit of scalar correlation functions: the exchange of a light helicity-2 state generates a scalar 4-point function with a particular angular dependence, very different from the one usually parameterized by $\tau_{\rm NL}$. This 4-point function can be observed both in the CMB $\langle TTTT \rangle$ correlator or in future Large Scale Structure (LSS) surveys. Conclusions and future directions are discussed in Section 3.4.

3.1 Tensor consistency relations

Squeezed limit CRs among cosmological correlation functions are in one to one correspondence with the adiabatic modes of Weinberg [50]. Below we will give a brief derivation (details can be found in [63, 64]). Then we will focus on potential violations of tensor CR as an indicator of whether during inflation the background converges to an isotropic solution exponentially fast.

Adiabatic modes in cosmology are super-horizon physical perturbations which are locally indistinguishable from a gauge mode. As such they do not affect short distance dynamics and therefore their correlation with the short distance perturbations is trivially related to a coordinate transformation. To find them Weinberg introduced the following trick

i. First fix the gauge. Using ADM parametrization,

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{3.1}$$

the spatial part of the metric can be factorized as

$$g_{ij} = a^2 e^{2\zeta} \left(e^{\gamma}\right)_{ij}, \qquad \text{with} \quad \gamma_{ii} = 0. \tag{3.2}$$

The gauge can be fixed by imposing $\partial_i \gamma_{ij} = 0$, and fixing time-reparametrization by choosing time-slices to coincide with constant energy density ρ slices. This completely fixes the reparametrization freedom at finite wavelength. There are still asymptotic (non-vanishing at spatial infinity) spatial diffeomorphisms $x^i \to x^i + \xi^i(t, \vec{x})$ which preserve the gauge condition. They satisfy [63]

$$\nabla^2 \xi_i + \frac{1}{3}\partial_i \partial_j \xi^j = 0. \tag{3.3}$$

ii. Except for translations and rotations, applying these infinitesimal transformations to the FRW background excites linear metric perturbations:

$$\zeta = \frac{1}{3}\partial_i \xi^i, \quad N^i = \dot{\xi}^i, \quad \gamma_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3}\delta_{ij}\partial_k \xi^k, \tag{3.4}$$

where the dot denotes d/dt and spatial indices are raised and lowered by δ_{ij} and its inverse. So one obtains a family of (trivial) infinite wavelength solutions to the equations of motion.

iii. The adiabatic modes are identified as the subfamily of solutions that can be deformed to finite wavelength. This requirement fixes the time-dependence of $\xi^i(t, \vec{x})$.

If adiabatic modes exist (that is if the last requirement can be satisfied), there is generically an infinite number of them. They can be organized by Taylor expanding the generating asymptotic diffeomorphisms at a fixed time-slice

$$\xi^i(t_0, \vec{x}) = \sum_{n=1}^{\infty} \frac{1}{n!} M_{i\, i_1 \cdots i_n} x^{i_1} \cdots x^{i_n}. \tag{3.5}$$

The condition (3.3) translates into a trace condition on the matrices $M_{i\, i_1 \cdots i_n}$.

The n^{th} order adiabatic mode in (3.5) leads to a CR that constrains the $\mathcal{O}(q^{n-1})$ term in the cosmological correlation functions with one soft momentum $\vec{q} \to 0$. Here we concentrate on the leading tensor CR for which

$$\xi^i = \omega^i_j x^j, \qquad \omega^i_i = 0, \tag{3.6}$$

under which $\gamma_{ij} \to \gamma_{ij} + 2\omega_{ij}$. If super-horizon tensor fluctuations become adiabatic, in the sense that to zeroth order in their wavenumber q they can be locally removed by (3.6) at all times, then the CR can be derived as follows. Violations of this condition will be discussed in Section 3.1.1.

Consider the change of a short distance correlator under the transformation (3.6), $\delta_\omega \langle O \rangle$. For instance, O can be the product of two scalar fluctuations at small separation compared to q: $\langle \zeta(\vec{x}_1)\zeta(\vec{x}_2)\rangle$ with $q|\vec{x}_1 - \vec{x}_2| \ll 1$ and

$$\delta_\omega \langle \zeta(\vec{x}_1)\zeta(\vec{x}_2)\rangle = \langle \zeta((\delta^i_j + \omega^i_j)x^j_1)\ \zeta((\delta^i_j + \omega^i_j)x^j_2)\rangle - \langle \zeta(\vec{x}_1)\zeta(\vec{x}_2)\rangle. \qquad (3.7)$$

If the short wavelength modes are in Bunch-Davies vacuum, which ensures that they are not excited until the mode $\vec{q}$ crosses the horizon, this can be related to the correlation function in the presence of a long wavelength tensor fluctuation:

$$\delta_\omega \langle O \rangle = 2\omega_{ij} \frac{\delta}{\delta \gamma_{ij}} \langle O \rangle_\gamma \Big|_{\gamma=0} \simeq \lim_{q\to 0} \sum_s \epsilon^s_{ij}\omega_{ij} \frac{1}{P_\gamma(s,q)} \langle \gamma^s_{\vec{q}} O \rangle, \qquad (3.8)$$

where we introduced the polarization vectors by expanding $\gamma^{ij}_{\vec{q}} = \sum_{s=1,2} \epsilon^s_{ij} \gamma^s_{\vec{q}}$, normalized to $\epsilon^s_{ij}\epsilon^{s'}_{ij} = 2\delta^{ss'}$. The tensor power spectrum is defined

$$\langle \gamma^s_{\vec{q}} \gamma^{s'}_{-\vec{q}} \rangle' = \delta^{ss'} P_\gamma(s,q)\,. \qquad (3.9)$$

In writing (3.8) we have used the fact that super-horizon fluctuations of γ_{ij} can be treated as a classical background up to corrections of order q^3 (see e.g. [64] for a derivation). Taking O to be the product of two scalar modes as in (3.7) and transforming them to momentum space, we obtain the following expression for the squeezed tensor-scalar-scalar correlation function [20]

$$\lim_{\vec{q}\to 0} \frac{1}{P_\gamma(s,\vec{q})} \langle \gamma^s_{\vec{q}}\, \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' = -\epsilon^s_{ij} k^i_1 k^j_1 \frac{\partial}{\partial k^2_1} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' \simeq \frac{3}{2} \epsilon^s_{ij} \hat{k}^i_1 \hat{k}^j_1 \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle'\,, \qquad (3.10)$$

where in the last passage we neglected the small deviation from scale invariance of the scalar spectrum.

It is worthwhile stressing that in the derivation above we did not assume that inflation is single-field: the argument applies to cases with multiple scalars, like the curvaton scenario [65] or quasi-single field models [16]. We do not have to assume that the short-wavelength scalar perturbations will eventually become adiabatic: the operator O above could include isocurvature perturbations. Also we did not have to assume that inflation is an attractor for scalar perturbations: the argument works for non-attractor models [66] as well. The tensor consistency relation is simply equivalent to the adiabaticity of tensor perturbations.

Of course we never observe the $\vec{q} = 0$ mode. So the above relation is useful because it applies at finite $q \ll k$ up to corrections of order q, as long as the vacuum state is Bunch-Davies. That is, the sub-horizon modes are not excited until their momentum redshifts to $k/a \sim H$. On the other hand, if the short wavelength modes are excited at a physical frequency $\omega > H$, then the above CR holds for sufficiently squeezed correlators: $q/k \ll H/\omega$. This ensures that the mode q crosses the horizon and becomes adiabatic well before the short modes are excited [67]. Two concrete examples are: (1) Models with significant scalar and tensor emission from secondary sources (see for example [68, 69, 70, 71]). Here one would expect most of the observed modes to be excited at $\omega \sim H$ because of the rapid expansion and redshift [72]. (2) Inflationary models with periodic features in their potential [73], where $\omega \gg H$ is possible. For all these models the tensor CRs hold for sufficiently small q.

3.1.1 Violation of tensor CR and cosmic no-hair theorem

The Universe we observe is extremely isotropic at large scales. However, unlike the large scale homogeneity and flatness, the present isotropy implies an extremely weak constraint on the degree of anisotropy at the onset of the hot phase of cosmology. Anisotropies quickly dilute during the thermal history (and indeed we never asked inflation to solve the "anisotropy problem" !). Thus we cannot rule out inflationary models that are not strong (i.e. exponential) isotropic attractors. However, the decay of anisotropies during the thermal history seems to imply that cosmological observables are insensitive to this possibility.

This conclusion is not necessarily true. During inflation quantum fluctuations probe all possible small deformations of the background history, including anisotropic deformations, and the super-horizon correlation functions are a recorded memory of how those deformations evolve. In particular, suppose fluctuations of γ_{ij} do not become adiabatic exponentially fast after horizon crossing. Then local observers would be able to detect the effect of the super-horizon γ_{ij}. They experience living in an anisotropic quasi-de Sitter Universe, which to their surprise does not isotropise exponentially fast.

As discussed in the previous section, a violation of tensor CRs implies the existence of light non-adiabatic tensor perturbations: in this case inflation is not a strong isotropic attractor, or in other words, it supports anisotropic hair. This appears to be in contradiction to Wald's no-hair theorem, which states that homogeneous cosmologies with cosmological constant (CC) and any type of matter that satisfies strong and dominant energy conditions (respectively SEC and DEC) approach isotropic de Sitter space exponentially fast [36]. However, the assumption of SEC is too strong to be of interest in the present discussion. Let us denote the components of the stress-energy tensor of the perturbations by an index X. When the back-reaction of perturbations on geometry is negligible DEC ($\rho_X > 0$) and SEC ($\rho_X + 3p_X > 0$) imply

$$\dot{\rho}_X = -3H(\rho_X + p_X) < -2H\rho_X, \tag{3.11}$$

which implies an exponentially fast decay $\rho_X \propto a^{-2}$ of the underlying perturbations. However, even the superhorizon fluctuations of a light scalar field σ violate SEC, because

$$\rho_X + 3p_X = 2\dot{\sigma}^2 - \tilde{m}_\sigma^2\sigma^2 \simeq -\tilde{m}_\sigma^2\sigma^2 < 0, \tag{3.12}$$

where we used the fact that in the limit $\tilde{m}^2 = m_\sigma^2 + 2H^2 \ll H^2$ the time derivative behaves as

$$\dot{\sigma} = -\Delta_\sigma H\sigma, \qquad \Delta_\sigma \simeq \frac{\tilde{m}_\sigma^2}{3H^2} \tag{3.13}$$

and hence the kinetic term is negligible. (Note that m_σ is the mass of a conformally coupled field. We use this for consistency with the following discussion about particles with spin.) In this model the deviation from dS,

$$\rho + p = \dot{\sigma}^2 = \Delta_\sigma^2 H^2 \sigma^2, \tag{3.14}$$

decays arbitrarily slowly for sufficiently small $\tilde{m}_\sigma^2 > 0$. The fact that in the presence of a light scalar field dS is not an exponential attractor is the perturbative manifestation of why slow-roll inflation can occur. This shows that both de Sitter space and inflation can have scalar hair. But can they support anisotropic hair?

A necessary condition for a no-hair theorem to exist is for it to hold for small perturbations. To study them it is useful to distinguish two qualitatively different cases

I. **dS isometries are approximately respected:** In this case the degrees of freedom furnish the representation of dS isometry group, which at super-horizon scales coincide with those of a $3d$ conformal field theory. This is an especially interesting case because the limit of exact dS is continuous. A long lived tensor degree of freedom (and in general any degree of freedom with nonzero spin) except for an adiabatic γ_{ij} would constitute an anisotropic hair. However, as we will discuss in detail in Section 3.2, all such degrees of freedom are either forbidden by the Higuchi bound, if they are fundamental fields, or they are composite operators made of tachyonic scalar fields. So at least perturbatively there exists a no-hair theorem for particles with spin: we can say *de Sitter does not have curly hair* [3].

II. **Some dS isometries are fully broken:** Inflation has a preferred time, so dS isometries do not have to be respected by perturbations. We do not attempt to classify all possibilities but only list some of the known examples:

   a. **Effective Field Theory of Inflation:** applicable when there is no preferred spatial frame [19]. Scalar fields cannot have anisotropic stress at super-horizon scales [74] so that tensor modes become adiabatic. However, it is possible to consider vector or tensor degrees of freedom which evade dS bounds: while light particles with spin are pathological in de Sitter (see Section 3.2), the coupling with the preferred foliation can make them healthy. For instance it is well known that one can change the 2-point function of vector perturbations adding a suitable function of the inflaton in front of the vector kinetic term $f(\phi)F^2$ [75, 76]: in this case the correlation can decay slower than the Higuchi bound outside the horizon. In Chapter 4 we generalise the EFTI allowing for the existence of new particles with spin coupled with the foliation. In particular the phenomenology arising from an additional light spin-2 field besides the graviton is studied.

   b. **Preferred spatial frame:** which exists when there is space-dependent background fields. Most of all the existing examples of violation of tensor CR like Solid Inflation [23, 77] (see also [56]), Gauge-flation [58] or Cromo-natural inflation [59, 60] fall in this category. As illustrated in the example of Chapter 5 the long wavelength γ_{ij} are not adiabatic. Moreover, additional

[3] Since an adiabatic tensor mode is not locally observable, for us it does not constitute a genuine hair.

tensor degrees of freedom often arise in these scenarios and they are not constrained by dS symmetries because of the presence of the additional background fields. [4] [5]

3.2 Exact-de Sitter limit and the Higuchi bound

As mentioned in the previous Section to have an isotropic attractor, all degrees of freedom with nonzero spin except for the graviton have to decay exponentially outside the horizon. This guarantees that the tensor CRs hold. The situation is analogous to scalar CRs which could be violated in the presence of additional light scalar degrees of freedom, also known as entropy perturbations. Different patches of the Universe with different entropy fluctuations experience different histories. If the entropy fluctuations mix with the adiabatic fluctuations during the cosmic evolution the absence of a unique history leads to a violation of scalar CRs.

Non-vanishing scalar fluctuations do not lead to anisotropy. In this Section we will review the Higuchi bound and show that non-pathological higher-spin fields indeed decay as a^{-1} or faster in dS. Our focus will be on spin-1 fields and the more important case of spin-2 fields where the Higuchi bound forbids the mass range $0 < m^2 < 2H^2$ [54].

The pathology of long-lived spin-1 and spin-2 degrees of freedom can be seen in the 2-point function of the fields, which is fixed by the de Sitter symmetries [14]. De Sitter is a maximally symmetric space with 10 isometries. Apart from spatial translations and rotations, it is invariant under the following two transformations

$$\begin{aligned} D &= -i\,(\eta\partial_\eta + x^i\partial_i) && (3.15)\\ K_i &= 2i\,x_i(\eta\partial_\eta + \vec{x}\partial_{\vec{x}}) + i(\eta^2 - |\vec{x}|^2)\partial_i\,. && (3.16)\end{aligned}$$

In de Sitter, an elementary field ϕ with mass m and spin s has two eigenmodes which at late times go as powers of the conformal time $\phi_\pm \sim \eta^{\Delta_\pm}$, where $\Delta_\pm$ are given by

$$\Delta_\pm = \frac{3}{2} \pm \sqrt{\left(s - \frac{1}{2}\right)^2 - \frac{m^2}{H^2}}\,. \tag{3.17}$$

If Δ is real, the solution Δ_- dominates at late times. So D and K_i act on the fields (taking $\Delta = \Delta_-$) as

$$D \to -i(\Delta + x^i\partial_i)\,, \qquad K_i \to -i(2\Delta x_i + 2x_i\,(x^i\partial_i) - |\vec{x}|^2\partial_i)\,. \tag{3.18}$$

These transformations are the same as conformal transformations in 3 spatial dimensions, with the conformal dimension of the fields determined by their late-time behavior (3.17), which is fixed by the mass and spin.

The universality of dS results then follows from the fact that the 2-point correlation functions of primary fields in a conformal field theory are fixed by the

[4] Another example in which the dS isometries are broken is massive gravity when the fiducial metric is not the inflationary dS: in this case one has an additional background with different symmetries and the Higuchi bound does not apply straightforwardly (for a recent discussion in the context of the ghost-free massive gravity see [78]).

[5] It is worth noting that even in the latter case where there is long-lived or growing anisotropy during inflation the anisotropic expansion rate cannot exceed $\mathcal{O}(\epsilon H)$ [74].

symmetries, up to the overall normalization. The 2-point function of a spin-1 field A_i takes the form [14]

$$\left\langle A^i(\vec{x})A^j(0)\right\rangle \propto \frac{1}{|\vec{x}|^{2\Delta}}(\delta^{ij}-2\hat{x}^i\hat{x}^j), \qquad \text{with} \quad \hat{x}\equiv\frac{\vec{x}}{|\vec{x}|}\,, \tag{3.19}$$

and that of a spin-2 field S^{ij}

$$\left\langle S^{ij}(\vec{x})S^{kl}(0)\right\rangle \propto \frac{1}{|\vec{x}|^{2\Delta}}(\delta^{ik}-2\hat{x}^i\hat{x}^k)(\delta^{jl}-2\hat{x}^j\hat{x}^l)+(k\leftrightarrow l)\,. \tag{3.20}$$

Going to Fourier space one gets

$$\langle \epsilon.A_{\vec{k}}\,\tilde{\epsilon}.A_{-\vec{k}}\rangle' \propto e^{i\psi}+2\frac{(2-\Delta)}{(\Delta-1)}+e^{-i\psi}\,, \tag{3.21}$$

$$\langle \epsilon^2.S_{\vec{k}}\,\tilde{\epsilon}^2.S_{-\vec{k}}\rangle' \propto e^{2i\psi}+4\frac{3-\Delta}{\Delta}e^{i\psi}+6\frac{(3-\Delta)(2-\Delta)}{(\Delta-1)\Delta}+4\frac{3-\Delta}{\Delta}e^{-i\psi}+e^{-2i\psi}\,, \tag{3.22}$$

where $\vec{\epsilon}$ and $\vec{\tilde{\epsilon}}$ are polarization vectors which (following [14]) are chosen to be

$$\vec{\epsilon}=(\cos\psi,\sin\psi,i)\,, \qquad \vec{\tilde{\epsilon}}=(1,0,-i) \qquad \text{for} \quad \vec{k}=(0,0,k). \tag{3.23}$$

When Δ goes below 1 (corresponding to $m^2<0$ for $s=1$ and $m^2<2H^2$ for $s=2$), the helicity-0 component becomes negative: it becomes a ghost.[6] For fields of higher spin s the Higuchi bound is always at $m^2=s(s-1)H^2$ corresponding to $\Delta_-=1$. Thus at a perturbative level all anisotropic hair decay at least as a^{-1}, and as a consequence geometric anisotropies decay as a^{-2}.

As we saw, the bound is simply a consequence of dS invariance: in particular, it does not require that the spin-2 state is described by an effective field theory with a parametric separation between the mass and the cutoff. For example it applies to the tower of Kaluza-Klein gravitons. One can evade the bound considering departures from exact de Sitter invariance given that during inflation the metric is not exactly de Sitter. However, by continuity one does not expect a significant change of the bound due to this.

The Higuchi bound comes from the relation among the different helicities, which is a consequence of the full dS isometry group. Inflation is usually associated with a preferred foliation of dS (the case IIa of the previous Section) and this breaks the conformal isometries of de Sitter. Only dilation invariance is approximately respected. In the absence of those there is no relation among helicities and no Higuchi bound. Therefore, as we show in Chapter 4 a particle with a large coupling with the preferred foliation can evade the Higuchi bound.

[6] The singularity at the threshold is not necessarily a pathology. It signifies an enhanced gauge symmetry which renders the longitudinal mode non-dynamical. Similarly, the case $\Delta=0$, $s=2$ is an exception, since it corresponds to a massless particle for which only the helicity-2 components are physical, the others being only gauge artifacts.

3.2.1 Composite operators

Phrased in these general terms, the Higuchi bound looks very powerful, since it looks one can apply it to any spin-2 operator, and not only to elementary spin-2 particles. For example it seems it applies also to a composite operator built out of scalars $\partial_i\phi\partial_j\phi - \frac{1}{3}(\partial\phi)^2\delta_{ij}$. Actually this conclusion is too quick and it is straightforward to verify that this operator does *not* have a 2-point function of the form of eq. (3.20). Indeed eq. (3.20) only applies to *primary* operators of a CFT and the operator $\partial_i\phi\partial_j\phi - \frac{1}{3}(\partial\phi)^2\delta_{ij}$ is not a primary. At first, the distinction between primaries and descendants in de Sitter seems odd: the transformation properties of a field in de Sitter is fixed by its indeces, independently of whether it is the derivative of another field or not. Why should there be difference between the spatial part of a field A_μ and the one of $\partial_\mu\phi$? The difference stems from the different time dependence of the time components, A_0 and $\partial_0\phi$ respectively. For A_μ all components will asymptotically behave in the same way: $A_\mu(\eta,\vec{x}) \sim \bar{A}_\mu(\vec{x})\eta^\Delta$. In this case under a de Sitter isometry, A_0 does not affect the transformation of A_i which behave like a CFT primary. On the other hand if $\partial_i\phi \propto \eta^\Delta$, then $\partial_0\phi \propto \eta^{\Delta-1}$. Now the time component grows faster for $\eta \to 0$ and one cannot neglect, for $\eta \to 0$, the first term on the RHS of eq. (3.16). One can check that taking this into account, $\partial_i\phi$ transforms differently than A_i.

In a CFT a generic operator is a sum of primaries and descendants. Thus $\partial_i\phi\partial_j\phi - \frac{1}{3}(\partial\phi)^2\delta_{ij}$ can be made primary by adding suitable descendant fields. To find them one can impose that the variation under a special conformal transformation vanishes at the origin: this is the definition of a primary field, while descendants change even at $x = 0$. In particular:

$$\delta_K\partial_i\phi = \Delta b_i\phi + \mathcal{O}(x) \qquad \delta_K\partial_i\partial_j\phi = (\Delta+1)(b_i\partial_j + b_j\partial_i)\phi - \delta_{ij}b^k\partial_k\phi + \mathcal{O}(x)\ , \tag{3.24}$$

where Δ is the dimension of ϕ. For the particular case at hand we find the following primary operator quadratic in ϕ (for simplicity we multiplied by $(2\Delta+1)$):

$$S_{ij} = (2\Delta+1)(\partial_i\phi\partial_j\phi - \frac{1}{3}\delta_{ij}(\partial\phi)^2) - \Delta\left[\partial_i(\phi\partial_j\phi) - \frac{1}{3}\delta_{ij}\partial_k(\phi\partial_k\phi)\right] \tag{3.25}$$

$$= (\Delta+1)(\partial_i\phi\partial_j\phi - \frac{1}{3}\delta_{ij}(\partial\phi)^2) - \Delta(\phi\partial_i\partial_j\phi - \frac{1}{3}\delta_{ij}\phi\nabla^2\phi)\,. \tag{3.26}$$

We verify explicitly in Appendix B.1 that the 2-point function of this operator is of the form (3.20) with dimension $\Delta_t = 2\Delta + 2$.

The operator S_{ij} is now a primary and the Higuchi bound tells us that its longitudinal part should become ghost-like for sufficiently small Δ_t. But how can this happen if we start from a scalar with a manifestly positive 2-point function in momentum space? The best way to understand what happens as we approach the bound is to think about the wavefunction of the Universe for S_{ij}. In the Gaussian approximation it is given by

$$\Psi[S_{ij}] \sim \exp\left[-\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\, S_{ij}S_{kl}\langle\sigma_{ij}(\vec{k})\sigma_{kl}(-\vec{k})\rangle'\right]\,. \tag{3.27}$$

Here σ_{ij} represents the "dual" operator in the putative CFT dual. As a consequence of conformal invariance $\langle\sigma_{ij}(\vec{k})\sigma_{kl}(-\vec{k})\rangle$ has the same form as eq. (3.22), but choosing the other branch of eq. (3.17), i.e. $\Delta = \Delta_+$ (this is checked explicitly in Appendix B.2). In approaching the Higuchi bound $\Delta_- \to 1$ and thus $\Delta_+ \to 2$, we see that the wavefunction becomes broader and broader and it becomes non-normalizable at the Higuchi bound. In the explicit calculation in Fourier space of the spectrum of the composite operator, one always gets an IR divergence at the Higuchi bound as we verify explicitly in the Appendix B.1 for the case of a spin-1 composite operator. This IR divergence is cut off by the first (and hence longest) modes that exit the horizon during inflation. However, the cutoff dependence breaks conformal symmetry of the correlator so the general form (3.22) is no longer expected, neither is the negativity of the helicity-0 correlation function.[7]

In summary, the conformal symmetry relates the various helicities as in eq. (3.22), so that the pathology of the helicity-0 part becomes a pathology of the full operator. However we saw that IR divergences modify eq. (3.22) in the case of composite operators. Moreover, the contributions of descendants will change the ratio among the different helicities and in particular one can have a non-primary spin-2 operator with an arbitrarily small Δ_t. However, such composite operators are made of tachyonic primary fields with negative dimension which grow exponentially fast at super-horizon scales. For example, for the operator $\partial_i\phi\partial_j\phi - \frac{1}{3}(\partial\phi)^2\delta_{ij}$, we need the scalars to have $\Delta \simeq -1$ for the composite operator to have Δ_t close to zero. Thus long lived anisotropic hair can be obtained at the expense destabilizing dS by growing scalar hair.

3.3 Observational prospects

Tensor modes with a wavelength much longer than the present Hubble radius are unobservable if the consistency relation holds [79]. In this case, the only observable effects arise when the tensor enters the Hubble radius: it induces tides which result in a quadrupolar modulation of the density field [80, 81, 82, 83, 84]. In the following we are going to neglect this "standard" effect, since we are interested in possible *additional* signatures due to the violation of the consistency relation.

We parametrize the squeezed limit of $\langle\gamma\zeta\zeta\rangle$ as

$$\langle\gamma^s_{\vec{q}}\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle' \sim \left(f^{\gamma}_{\rm NL} + \frac{3}{2}\right) \langle\gamma^s_{\vec{q}}\gamma^s_{-\vec{q}}\rangle' \, \langle\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle' \, \epsilon^s_{ij}(\vec{q})\hat{k}_{1,i}\hat{k}_{1,j} \, . \tag{3.30}$$

[7] One must be careful with the real-space computations involving tachyons. Indeed if one uses the scalar 2-point function

$$\langle\phi(x)\phi(0)\rangle = \frac{1}{|x|^{2\Delta}} \, , \qquad \Delta < 0 \, , \tag{3.28}$$

in the expression of the primary eq. (3.25), one gets the real-space expression eq. (3.20) even below the Higuchi bound. But again the appearance of "negative probabilities" is fictitious. In fact the usual quantization in momentum space guarantees that the momentum space correlators are positive definite. The problem with tachyons is that the Fourier transform from momentum to real space,

$$\xi(\vec{x}) = \int d^3\vec{k}\; k^{2\Delta-3} e^{i\vec{k}\cdot\vec{x}}, \tag{3.29}$$

is IR divergent and hence the real space correlator is *not* (3.28). Indeed eq. (3.28) cannot come from a positive Fourier-space spectrum, since $\xi(\vec{x}=0) = \int d^3\vec{k} P(k)$ must be positive while (3.28) vanishes at coincidence point for $\Delta < 0$. The correct 2-point function contains an IR divergent constant which physically describes the growth of the tachyon field in an eternal de Sitter.

In this way $f_{\rm NL}^{\gamma}$ parametrizes deviations from the consistency relation: as we discussed, when the tensor is way out of the Hubble radius the only physical effects are $\propto f_{\rm NL}^{\gamma}$.

The tensor-scalar-scalar correlation function can be directly tested by measuring the correlation between temperature and B-mode polarization in the CMB, $\langle BTT\rangle$ [62]. A very rough estimation of the signal-to-noise of such three-point function, when noise is dominated by cosmic variance, is

$$\begin{aligned}(S/N)^2 &= \Omega\int\frac{d^2l_B}{(2\pi)^2}\frac{d^2l_T}{(2\pi)^2}\frac{\langle B_{l_B}T_{l_T}T_{l_T}\rangle'^{\,2}}{\langle B_{l_B}B_{l_B}\rangle'\,\langle T_{l_T}T_{l_T}\rangle'^{\,2}}\\ &\simeq {f_{\rm NL}^{\gamma}}^2 r\,\Delta_\zeta^2\left(\frac{l_{T,max}}{l_{T,min}}\right)^2\log\left(\frac{l_{B\,max}}{l_{B\,min}}\right)\,,\end{aligned}\qquad(3.31)$$

where the angular size of the survey and $l_{T,min}$ are related as $\Omega\simeq\frac{(2\pi)^2}{l_{T,min}^2}$. Δ_ζ^2 is the amplitude of scalar power spectrum, $\Delta_\zeta^2=2.2\times10^{-9}$ [85]. We see that the signal is proportional to the combination ${f_{\rm NL}^{\gamma}}^2\,r$. Note also that this depends on the range of scales over which B-modes are observed. The reader may refer to [62] for a detailed analysis of the signal-to-noise of the $\langle\gamma\zeta\zeta\rangle$ 3-point function. The authors find that a futuristic experiment with $l_{T\,max}\sim4500$ and $l_{B\,max}\sim500$ should be able to reach [8]

$${f_{\rm NL}^{\gamma}}^2 r\lesssim 6\times10^3\,;\qquad(3.32)$$

an experiment of this kind will be cosmic variance limited unless $r\lesssim10^{-3}$.

Even if the tensor modes are not directly measured, the violation of the tensor CR can be observed looking at the statistics of scalar perturbations only. We are now going to study the effect on the scalar 2-point function and 4-point function.

3.3.1 Modulation of the scalar 2-point function

As discussed above, if the consistency relation is violated, the effect of a super-horizon tensor mode is physical and can be observed locally (see for example [86, 87, 88, 89]). Here we focus on the modulation it induces on the scalar 2-point function[9]

$$\langle\zeta_{\vec k_1}\zeta_{\vec k_2}\rangle_\gamma\simeq\langle\zeta_{\vec k_1}\zeta_{\vec k_2}\rangle+\gamma^s_{\vec q}\,\frac{\langle\gamma^s_{\vec q}\zeta_{\vec k_1}\zeta_{\vec k_2}\rangle'}{P_\gamma(q)}\,,\qquad(3.33)$$

which would be observed as a quadrupole in the power spectrum proportional to the average amplitude of the super-horizon spin-2 modes

$$P_\zeta(\vec k)=P_\zeta(k)\left[1+f_{\rm NL}^{\gamma}\epsilon^s_{ij}(\vec q)\hat k_i\hat k_j\gamma^s_q\right]\equiv P_\zeta(k)\left[1+\mathcal{Q}_{ij}\hat k_i\hat k_j\right]\,.\qquad(3.34)$$

The (squared) amplitude of $\mathcal{Q}_{ij}$ is obtained averaging over all the super-horizon modes:

$$\mathcal{Q}^2=\frac{8\pi}{15}\langle\mathcal{Q}_{ij}\mathcal{Q}_{ij}\rangle=\frac{8}{15\pi^2}\,{f_{\rm NL}^{\gamma}}^2\int_{q<H_0}dq\;q^2\langle\gamma^s_{\vec q}\gamma^s_{-\vec q}\rangle'\approx\frac{4}{15}{f_{\rm NL}^{\gamma}}^2\,r\,\Delta_\zeta^2\,\Delta N\,;\qquad(3.35)$$

[8]Notice that ref. [62] uses a different notation compared to ours: $f_{\rm NL}^{here}\sqrt{r}=24f_{\rm NL}^{there}$.

[9]Superhorizon tensor modes also induce short-scale correlation among scalar and tensor perturbations (see e.g. [53]).

where r is the tensor-to-scalar ratio and ΔN is the number of e-folds of all modes outside the present Hubble radius. Experimental limits come from the CMB: $\mathcal{Q} \lesssim 10^{-2}$ [1].[10] This gives:

$$f_{\rm NL}^{\gamma}{}^2\, r = \frac{75}{4}\,\frac{g_2^2}{\Delta_\zeta^2}\frac{1}{\Delta N} \lesssim 8.5\times 10^5\,\frac{1}{\Delta N}\ . \tag{3.36}$$

Notice that the measurable quantity is always the combination $f_{\rm NL}^{\gamma}{}^2 r$, as in eq. (3.32). In this case, since we do not observe the tensor mode directly, the measurement is also sensitive to additional helicity-2 states even if they are not correlated with gravitational waves. In this case the effect is proportional to the power spectrum of this extra state, instead of r, and its coupling with scalar perturbations, instead of $f_{\rm NL}^{\gamma}$.

Let us comment on the quantity ΔN which describes the cumulative effect of all super-horizon modes [90]. If the tensor (or the additional helicity-2 field) has a (small) blue spectrum, $n_T > 0$—this is the case of solid inflation—the integral converges in the IR. In this case, assuming inflation is sufficiently long, the sum over all modes gives $\Delta N \simeq n_T^{-1}$. For example, taking $n_T = 0.03$, the experimental bound above gives $f_{\rm NL}^{\gamma}{}^2\, r \lesssim 2.5\times 10^4$. In models with a red spectrum, the integral is IR divergent and therefore the result depends on the duration of inflation. Notice also that if the tensor mass during inflation is negative, the model does not evolve towards isotropy but (slowly) away from it: in this case the initial condition for inflation must be carefully chosen to satisfy the experimental limits on $\mathcal{Q}$.

Future constraints on the amplitude $\mathcal{Q}^2$ will arise from the new generation of experiments. For instance new LSS surveys may greatly improve the current limits. In this case a very rough estimate of the signal to noise is given by

$$\begin{aligned}(S/N)^2 &= \frac{V}{2}\int \frac{d^3k}{(2\pi)^3}\left(\frac{\Delta P(\vec{k})}{P(k)}\right)^2 \\ &\simeq f_s\frac{2}{45\pi}\Delta_\zeta^2\left(\frac{k_{max}}{k_{min}}\right)^3 f_{\rm NL}^{\gamma}{}^2 r\,\Delta N\ ,\end{aligned} \tag{3.37}$$

where f_s is the fraction of the sky covered by the survey. An improvement of (k_{max}/k_{min}) of order 10 in the future experiments will put constraints on $f_{\rm NL}^{\gamma}{}^2 r$ roughly of order $\mathcal{O}(100/\Delta N)$. Note, however, that LSS measurements of the quadrupole could be complicated by the fact that both gravitational non-linearities and redshift-space distortions induce a quadrupole. One can also look for this effect in the 21 cm power spectrum. A detailed analysis on the bounds one could get on $\mathcal{Q}^2$ is given in [91].

The statistics of Q_{ij}

A quadrupolar modulation of the power spectrum is not only induced by tensor perturbations but also, in models like solid inflation, by scalars and vectors [90]. The anisotropy in short-scale power spectrum depends on $(\partial_i\partial_j/\partial^2 - 1)\zeta$ in the case of scalars and on $\partial_i/\sqrt{\partial^2}V_j$ in the case of a vector. Naively one may expect that the statistical distribution of the matrix Q_{ij}, i.e. the distribution of its eigenvalues,

[10] For comparison, notice that the quantity g_2 used in [1] is given by: $g_2 \equiv \mathcal{Q}/\sqrt{5}$.

depends on whether the origin of the quadrupolar modulation is a scalar, a vector or a tensor. Indeed for a single Fourier mode the matrix is quite different comparing scalars with tensors: for instance the scalar one has two equal eigenvalues and it is thus axially symmetric. However once we average over all nearly Gaussian super-horizon modes, this difference is lost: rotational invariance and Gaussianity imply that the distribution is uniquely fixed in terms of the variance

$$\langle Q_{ij} Q_{kl} \rangle \propto \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) - \frac{1}{3}\delta_{ij}\delta_{kl} \,. \tag{3.38}$$

Since in all cases Q_{ij} is Gaussian (it is always linear in an approximately free field), all correlation functions just reduce to the previous one and everything can be written in terms of the variance. Therefore there is no way to distinguish whether Q_{ij} comes from long-wavelength fluctuations of a scalar, a vector or a tensor.

Let us now address a different question: whether one can distinguish a quadrupole generated by long-wavelength perturbations from the case in which the quadrupole is exactly axially symmetric [92]: $P_\zeta(\vec{k}) = P_\zeta(k)\left[1 + c(\hat{k}\cdot\hat{n})^2\right]$. This happens when there is a preferred direction $\hat{n}$ in the sky, for example due to the presence of a vector field in the background solution, see e.g. [93, 94]. In this case two eigenvalues of the matrix Q_{ij} are the same, while in general this will not hold for a Q_{ij} with the Gaussian statistical distribution discussed above. If the observed Q_{ij} is axially symmetric within the experimental uncertainties, this will disfavor a quadrupole generated by super-horizon fluctuations. We can easily quantify this statement and ask how well one should measure the quadrupole modulation before being able to rule out the Gaussian statistics discussed above.

A similar problem arises when studying the statistics of the shear tensor in the Large Scale Structure [95]. Given a symmetric traceless 3×3 matrix we want to find the probability density function (PDF) of the two independent eigenvalues. As we discussed, the entries of the matrix are Gaussian. Since the distribution must be rotationally invariant it can only depend on $Q_{ij}Q_{ij}$:

$$P(Q_{11}, Q_{22}, Q_{12}, Q_{13}, Q_{23}) = \mathcal{N} e^{-\frac{Q_{11}^2 + Q_{22}^2 + Q_{33}^2 + 2Q_{12}^2 + 2Q_{13}^2 + 2Q_{23}^2}{2\sigma^2}} d[Q_{11}\, Q_{22}\, Q_{12}\, Q_{13}\, Q_{23}], \tag{3.39}$$

where $Q_{33} = -(Q_{11} + Q_{22})$ and $\mathcal{N}$ a normalization constant. We want to perform a change of variables writing the PDF in terms of the eigenvalues of the matrix, respectively a, b and $c = -(a+b)$, plus the three Euler angles, α, β, γ. The Gaussian now depends on $a^2 + b^2 + c^2 = 2(a^2 + ab + b^2)$, while the Jacobian of the trasformation is

$$|J| = (a-b)(a-c)(b-c)\sin\beta = (a-b)(2a+b)(a+2b)\sin\beta. \tag{3.40}$$

The eigenvalues are assumed to be in decrescent order, i.e. $a \geq b \geq c$, this implies

$$b \leq a, \quad b \geq -a/2.$$

After integrating over the Euler angles we get (with a suitable change of the normalization $\mathcal{N}$)

$$P(a,b) = \mathcal{N}(a-b)(2a+b)(a+2b)e^{-\frac{a^2+ab+b^2}{\sigma^2}}. \tag{3.41}$$

This PDF agrees with the one found by Doroshkevich [95] after one sets $\mathrm{Tr}(Q) = 0$. Notice that the Jacobian suppresses the PDF when two eigenvalues are similar and this makes the distinction from the axially symmetric case easier.

We can now compute the probability of having two nearly degenerate eigenvalues. One has to integrate the PDF (3.41) in the region

$$\left(\frac{|a-b|}{|a|} \leq \epsilon \bigcup \frac{|b-c|}{|b|} \leq \epsilon\right) \bigcap \left(b \leq a, \quad b \geq -\frac{a}{2}\right) .$$

The explicit integration gives

$$\begin{aligned} P(\epsilon) &= 1 + \frac{3\sqrt{3}}{4\sqrt{(3-3\epsilon+\epsilon^2)^3}} - \frac{3\sqrt{3}}{4\sqrt{3-3\epsilon+\epsilon^2}} \\ &\quad + \frac{3\sqrt{3}}{4\sqrt{(3+3\epsilon+\epsilon^2)^3}} - \frac{3\sqrt{3}}{4\sqrt{3+3\epsilon+\epsilon^2}} \\ &= \frac{3}{8}\epsilon^2 + \mathcal{O}(\epsilon^4). \end{aligned} \quad (3.42)$$

The above formula tells us that the probability of having two eigenvalues that differ less than 10% from each other is $\sim 0.4\%$. Therefore if the errors on the quadrupolar modulation are reasonably small, one can rule out that Q_{ij} is generated by many superhorizon Gaussian modes. Conversely, if the eigenvalues are observed to be different one can rule out all models with a preferred direction.

3.3.2 The scalar 4-point function

The exchange of a soft graviton gives a 4-point scalar correlator in the limit in which the sum of two momenta is small: $q \equiv |\vec{k}_1 + \vec{k}_2| \ll k_i$. One gets, neglecting the terms which respect the tensor CR,

$$\begin{aligned} \langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle' &\simeq \left\langle\langle\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle\,\langle\zeta_{\vec{k}_3}\zeta_{-\vec{k}_3}\rangle\right\rangle' \quad (3.43) \\ &= \sum_s \langle\gamma^s_{\vec{q}}\gamma^s_{-\vec{q}}\rangle' \,\frac{\langle\gamma^s_{\vec{q}}\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle'}{P_\gamma(q)}\,\frac{\langle\gamma^s_{\vec{q}}\zeta_{\vec{k}_3}\zeta_{-\vec{k}_3}\rangle'}{P_\gamma(q)} \\ &= {f^\gamma_{\rm NL}}^2 P_\gamma(q)P_\zeta(k_1)P_\zeta(k_3)\sum_s \epsilon^s_{ij}(\vec{q})\epsilon^s_{kl}(\vec{q})\hat{k}_{1,i}\hat{k}_{1,j}\hat{k}_{3,k}\hat{k}_{3,l}\,. \end{aligned}$$

The angular dependence of (3.43) can be cast in the form [96]

$$\sum_s \epsilon^s_{ij}(\vec{q})\epsilon^s_{kl}(\vec{q})\hat{k}_{1,i}\hat{k}_{1,j}\hat{k}_{3,k}\hat{k}_{3,l} \equiv \cos 2\chi_{12,34}\,, \quad (3.44)$$

where $\chi_{12,34} \equiv \phi_1 - \phi_3$ is the angle between the projection of k_1 and k_3 on the plane orthogonal to q. The final expression of the trispectrum due to a graviton exchange is

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle' = {f^\gamma_{\rm NL}}^2 P_\gamma(q)P_\zeta(k_1)P_\zeta(k_3)\cos 2\chi_{12,34}. \quad (3.45)$$

One can look for this kind of 4-point function directly in the CMB temperature map. The momentum dependence is similar to the standard τ_{NL} trispectrum shape [97],

$$\langle\zeta_{\vec{k}_1}\,\zeta_{-\vec{k}_1+\vec{q}}\zeta_{\vec{k}_3}\,\zeta_{-\vec{k}_3-\vec{q}}\rangle' = \frac{5}{3}\tau_{NL}P_\zeta(q)P_\zeta(k_1)P_\zeta(k_3)\,. \quad (3.46)$$

Notice, however, that the additional angular dependence of eq. (3.45) is such that the two shapes are effectively orthogonal and no constraint on eq. (3.45) can be obtained from the τ_{NL} bounds: a dedicated analysis must be performed. It is easy to estimate what kind of constraints (or detection!) one should be able to get with a dedicated analysis. The bound on $\tau_{NL} < 2800$ (95% CL) [12] can be roughly converted in

$$ {f_{\rm NL}^{\gamma}}^2 r \lesssim 4 \times 10^4 \; , \tag{3.47} $$

where we wrote $P_\gamma = P_\zeta \, r/4$ and we took into account a factor of $\langle \cos^2 \chi_{12,34} \rangle = 1/2$ which arises when one integrates eq. (3.45) over all configurations. It is important to stress that this bound is not so much worse than the futuristic limit using B-modes of eq. (3.32). This strongly motivates a dedicated analysis of this shape of the 4-point function: a signal of tensors could be already in Planck temperature data, even without observing B-modes! Moreover this signal has the advantage of being sensitive to extra helicity-2 states which induce a 4-point function of the form (3.46), even in the absence of mixing with gravitational waves.

One can also attempt to measure the scalar 4-point function by looking at galaxy number counts. By correlating the amplitudes of a pair of 2-point functions, reference [98] studies whether future surveys will be able to observe this effect. It is straightforward to translate their results in our notation. The variance of the effect induced by a violation of the consistency relation for future surveys is

$$ \left(\frac{S}{N}\right)^2 \simeq \frac{\pi}{6075} {f_{\rm NL}^{\gamma}}^4 r^2 \Delta_\zeta^4 \left(\frac{k_{max}}{k_{min}}\right)^6 . \tag{3.48} $$

For a survey like Euclid, for which $k_{min} \simeq 10^{-3}\,{\rm Mpc}^{-1}$, the expected constraints from this observable are ${f_{\rm NL}^{\gamma}}^2 r \lesssim 2 \times 10^4 \,(0.1\,{\rm Mpc}^{-1}/k_{max})^3$, comparable to the limits that would be obtained by analyzing the CMB 4-point function. Note that in the event of a positive detection, the above estimate of the signal-to-noise ratio in the 4-point function breaks down due to the non-Gaussian contribution to the noise. Beyond this point, an improved estimator (analogous to the one introduced in [99]) is necessary to decrease the error-bars as $1/\sqrt{N_{\rm data}}$.

3.4 Discussion and Outlook

We showed that tensor CRs are very robust and that their violation would contain a lot of insight into the physics of inflation, showing that anisotropic perturbations are not quickly redshifted away. We know few explicit models which violate tensor CRs, but it would be nice in the future to study systematically this violation. In particular one could explore the connection between the general analysis of tensor mass terms done in [42, 100] and the violation of tensor CRs. Since we are discussing non-adiabatic tensor perturbations, one would like to understand what happens at reheating and in particular if additional contributions which violate tensor CRs can arise at the end of inflation, similarly to what happens in the case of scalars. Since the Higuchi bound applies also to higher-spin states, much of what we said could be generalized to these cases as well. From the experimental side, it would be extremely interesting to get explicit constraints on the helicity-2 mediated scalar trispectrum, eq. (3.45), using Planck data and start thinking about future improvements from LSS data. We will come back to some of these issues in the next chapters.

Chapter 4

Light Particles with Spin in Inflation

The addition of extra light scalars in the inflationary spectrum has been studied for many years. On the other hand, the study of particles with spin is much more recent [14, 101, 102]. One of the reasons lies in the Higuchi bound [54] that states that the mass m of a particle with spin s must satisfy $m^2 > s(s-1)H^2$. As we have extensively discussed in the previous chapter for spin-2 particles, this consequence of the (approximate) de Sitter symmetries implies that particles with spin decay outside the horizon faster than $1/a$, and hence they leave a small effect in the squeezed limit of correlation functions. The effect of even more massive particles, $m^2 > \left(s-\frac{1}{2}\right)^2 H^2$, is exponentially suppressed but very peculiar, with an oscillatory pattern in the squeezed limit [14].

In this chapter we show explicitly that the coupling with the preferred foliation can make a light particle with spin healthy and we study the physics of these light states. The construction of the action for these states follows rather straightforwardly from the rules of the EFTI, as we will discuss in Section 4.1. The Higuchi bound implies that these states only exist in the presence of the inflating background, similarly to excitations of a fluid more than elementary particles. (In this sense what we propose is cosmological condensed matter rather than cosmological collider physics [14].) In the language on non-linearly realised symmetries, they are matter fields coupled to the Goldstones. As such their action can also be constructed following the usual Coleman-Callan-Wess-Zumino (CCWZ) rules for non-linearly realised spacetime symmetries, as we will discuss in Subsection 4.1.1. These light states have prescribed couplings with the inflaton. Indeed the "boost" isometries of de Sitter are spontaneously broken by the foliation and thus non-linearly realised. This gives rise to consistency relations: the 2-point function of light particles with spin is not de Sitter invariant (since it violates the Higuchi bound) and this variation fixes the coupling with the Goldstone of time-translations, i.e. the inflaton fluctuations, in the squeezed limit. This relation will be explicitly verified in the simplest example of these theories: spin-1 (Section 4.2).

To study the phenomenological implications of these states, we are going to focus on the most interesting example: the one of helicity-2 states. The reason is two-fold. First of all, a simple parity argument suppresses the contribution of vectors in the squeezed limit which, as we discussed, is a most prominent signatures of light states. This makes the simplest case, the one of spin-1, not so interesting. Particles of spin-2 are unsuppressed in the squeezed limit and moreover they can mix with the graviton and modify the predictions related to tensor modes. These signatures are studied in

Section 4.4, leaving some details of the calculations to Appendix C. A light particle of spin-2 modifies both the scalar and the tensor power spectrum. Depending of the parameters one of the spectra (or both) can be dominated by the exchange of the extra state. We study the effect of the light spin-2 state on $\langle\zeta\zeta\zeta\rangle$ and $\langle\zeta\zeta\zeta\zeta\rangle$, where an angle dependent non-Gaussianity is induced, and on $\langle\gamma\zeta\zeta\rangle$ where the limit of soft graviton momentum shows a violation of the tensor consistency relation [103, 104].

The effect of these light states is enhanced when they have a small speed of propagation. In Section 4.5 we study the experimental and theoretical constraints on this speed of propagation. Conclusions are drawn in Section 4.6.

Before starting, let us comment on the relation with other works in the literature. Light spin states may appear below the Higuchi bound in the form of partially massless states [105], whose possible phenomenology in inflation was studied in [106, 107]. In this chapter we insisted on keeping the approximate shift-symmetry of the inflaton, which is behind the observed approximate scale-invariance of the scalar power-spectrum. A strong breaking of this symmetry can also efficiently violate the Higuchi bound as discussed in [108]. Another way to get light states with spin is to start with a symmetry pattern of inflation that is different from the standard one: two examples are gauge-flation [109, 60, 110] and gaugid inflation [111].

4.1 Particles with spin in the EFTI

We want to understand how to describe matter fields, i.e. fields in addition to the clock of inflation π, in the framework of the EFTI. For scalars, the procedure is straightforward: one just writes an action for an extra scalar σ with the usual rules of the EFTI [15]. (For instance in unitary gauge a term of the form $(g^{00}+1)(g^{0\mu}\partial_\mu\sigma)$ describes the mixing between the inflaton and σ.) Things are somewhat different for particles with spin. Let us concentrate for concreteness on particles with spin-1. One might think to start with a four-vector Σ^μ and write an action with the usual rules of the EFTI; however this is not the most correct procedure. If we concentrate on scales much shorter than Hubble and forget about gravity, the inflaton background spontaneously breaks the Lorentz group to rotations. The usual logic of non-linearly realised symmetries tells us that one should classify fields as representations of the unbroken group, in this case rotations. Therefore we should start from a 3-vector, not a 4-vector [1]. It is irrelevant to know to which Lorentz representation this 3-vector belongs: to build a Lagrangian which non-linearly realises the broken group one just needs to know the transformation properties under the unbroken group. Actually the question about the Lorentz representation is ill-defined: one in general will not be able to recombine the fields to form Lorentz multiplets in the same sense as in the chiral Lagrangian one cannot combine states under representations of the axial group.

Let us see what is the procedure to build an action in terms of a 3-vector.

- In a generic gauge, the slices of constant inflaton are $\psi \equiv t + \pi(x^i, t) = c$. Given the preferred foliation there is a natural way to split the tangent space

[1] The breaking of de Sitter isometries induces a separation among the different helicities of a particle with spin. However, one cannot completely disentangle one helicity from the others since the selection of one helicity is an intrinsically non-local operation.

at each point, and thus tensors, in the projections parallel and orthogonal to the surface (see Chapter 1). Fields are classified as three-tensors.

- It is also natural to parametrise the 3-surfaces of constant inflaton with the spatial coordinates x^i that we use for the whole spacetime. This gives a basis of the tangent space on the submanifold: $\partial/\partial x^i$. Objects living on the slice can be written in terms of this basis. A vector will have only three components: $\Sigma^i \cdot \partial/\partial x^i$.

- So far the fields live on the three-dimensional slices embedded in spacetime, but in order to describe their couplings to the four-dimensional fields including gravity one needs to "push forward" them to objects living in four dimensions. Of course the mapping will depend on the particular configuration of the inflaton slices described by π

$$\Sigma^\mu(\Sigma^i, \pi) = \Sigma^i \left.\frac{\partial x^\mu}{\partial x^i}\right|_\psi \, . \tag{4.1}$$

  This is a four-vector tangent to the surfaces of constant ψ and it is specified by its three independent components Σ^i.

- Given that

$$\left.\frac{\partial t}{\partial x^i}\right|_\psi = -\frac{\partial \psi}{\partial x^i} \Big/ \frac{\partial \psi}{\partial t} = -\frac{\partial_i \pi}{1+\dot{\pi}} \tag{4.2}$$

  one can write an explicit expression for the components of the corresponding four-vector

$$\Sigma^\mu(\Sigma^i, \pi) = \left(-\frac{\partial_i \pi \, \Sigma^i}{1+\dot{\pi}} \, , \quad \Sigma^i \right) \, . \tag{4.3}$$

  This is the object we will use to build the action: one can explicitly check that it transforms as a vector under all diffs.

This method can be applied to build an action for a field transforming in any tensor representation of three dimensional rotations, and therefore representing particles of arbitrary spin. In particular, a spin-2 particle would be described by a traceless rank-2 symmetric tensor Σ^{ij} with five independent components. The diffeomorphism-invariant action for it can be written in terms of the traceless symmetric four-tensor $\Sigma^{\mu\nu}$ with the four extra components given in terms of π and Σ^{ij}:

$$\Sigma^{00} = \frac{\partial_i \pi \partial_j \pi}{(1+\dot{\pi})^2} \Sigma^{ij} \, , \qquad \Sigma^{0j} = -\frac{\partial_i \pi}{1+\dot{\pi}} \Sigma^{ij} \, . \tag{4.4}$$

The above construction can also be understood if one starts in unitary gauge, i.e. with $\pi = 0$. In this gauge, since the slices of constant inflaton coincide with the ones of constant time, a vector on the surface has only spatial components. It transforms as a vector under time-dependent spatial diffs, so one can use it in this gauge provided spatial indexes are contracted. If one goes out from the unitary gauge, doing a time diff. , one has to perform the usual Stückelberg procedure introducing π. This gives the expression (4.1) albeit from a somewhat different perspective.

4.1.1 Matter fields in the CCWZ approach

Since the presence of the foliation introduces a natural split of the tangent space, it breaks the local Lorentz invariance to the rotation subgroup. This suggests that one can employ the CCWZ approach [112, 113] for writing an action for the Goldstone and matter fields, which would non-linearly realises the broken symmetries. A formulation of single-clock inflation in the presence of an approximate shift-symmetry of the inflaton was presented in [114] directly in the CCWZ language, including the coupling with gravity. The system has the same symmetries of a superfluid. The full symmetry group is taken to consist of the internal shift symmetry generated by Q and local $ISO(1,3)$ Poincare group on the tangent space generated by translation P_a and Lorentz transformation J_{ab} operators.[2] The superfluid phase corresponds to a finite density of the global charge Q with chemical potential μ. The state breaks local boosts, time translations, and the global shift symmetry, but preserves a combination of time translations and shifts generated by $\overline{P}_0 = P_0 + \mu\, Q$. Moreover, in order to recover gravity in this approach one assumes that all the local Poincare shifts are non-linearly realised. The only linearly realised symmetries are local rotations generated by J_{mn}. The corresponding coset element is parameterised by eight Goldstone fields: y^a for local translations, π for the internal shift, and η^m for boosts, and can be chosen to be

$$\Omega = e^{i\, y^a\, \bar{P}_a}\, e^{i\,\pi\, Q}\, e^{i\,\eta^m J_{0m}} \ . \tag{4.5}$$

All the building blocks that are allowed to be used in the action can be read off from the Maurer-Cartan form

$$\Omega^{-1} D_\mu \Omega \equiv i\, \nabla_\mu y^a \left(\bar{P}_a + \nabla_a \pi\, Q + \nabla_a \eta^m\, J_{0m} + \frac{1}{2} \mathcal{A}^{mn}{}_a\, J_{mn} \right) \ . \tag{4.6}$$

The coefficients in front of the broken generators correspond to the CCWZ covariant derivatives of the Goldstone fields and can be used in the action in the combinations that preserve unbroken local rotations. The coefficients of the unbroken generators give a connection that defines a covariant derivative needed to construct higher derivative terms and to act on non-Goldstone matter fields. All these objects are calculated in reference [114] where it was also shown that they are the same as the building blocks of the EFTI action. In what follows we briefly review these building blocks and show that adding matter fields in the CCWZ language is equivalent to the EFTI construction introduced in the previous Section.

The covariant derivative $\nabla_\mu y^a$ of the translation Goldstones gives the “coset vierbein"

$$\nabla_\mu y^a \equiv E^a{}_\mu = \Lambda_b{}^a\, e^b{}_\mu \ , \tag{4.7}$$

where we have introduced the boost matrix in the vector representation

$$\Lambda^a{}_b(\eta) \equiv (e^{i\,\eta^m J_{0m}})^a{}_b \ , \tag{4.8}$$

and $e^b{}_\mu$ is the space-time vierbein, which defines the metric as $e^a{}_\mu e^b{}_\nu\, \eta_{ab} = g_{\mu\nu}$. The coset vierbein transforms covariantly under the unbroken $SO(3)$ rotations: $E^0{}_\mu$ is

[2] Here we use the latin indices $a, b, \dots$ and $m, n, \dots$ for the four- and three-dimensional tangent spaces respectively in order to distinguish them from the coordinate indices $\mu, \nu, \dots$ and $i, j, \dots$.

a singlet and $E^m{}_\mu$ is a triplet. It can be used to construct an invariant integration measure $d^4x\ \det E = d^4x\ \det e$.

The covariant derivatives of the Goldstones π and η^m read

$$\nabla_a\pi \equiv e_b{}^\mu\,\Lambda^b{}_a\,\partial_\mu\psi - \mu\,\delta^0_a\,,\quad \nabla_a\eta^m \equiv e_b{}^\mu\,\Lambda^b{}_a\,\left(\Lambda_c{}^0\partial_\mu\Lambda^{cm} + \omega^c{}_{d\,\mu}\Lambda_c{}^0\Lambda^{dm}\right)\,,\quad (4.9)$$

where we have introduced the field $\psi \equiv \pi + \mu\,y^0$, and $\omega^c{}_{d\,\mu}$ is the spin-connection that corresponds to the vierbein $e^b{}_\mu$. These covariant derivatives can also be used to impose additional constraints on the effective theory. If these constraints allow one to reduce the number of the Goldstone fields by expressing some of the Goldstone fields in terms of the others in a local manner then they are called inverse Higgs constraints. In the case at hand we can impose the condition $\nabla_m\pi = 0$, which allows to express η^m in terms of the derivatives of π:

$$\nabla_m\pi = \Lambda^a{}_m\,e_a{}^\mu\partial_\mu\psi = 0\,. \quad (4.10)$$

It means that the transformation $\Lambda^a{}_b$ corresponds to a boost with velocity

$$\beta_m \equiv \frac{\eta_m}{\eta}\ \tanh\eta = -\frac{e_m{}^\mu\partial_\mu\psi}{e_0{}^\mu\partial_\mu\psi}\,, \quad (4.11)$$

where $\eta \equiv \sqrt{\eta_m\eta_n\delta_{mn}}$. Therefore, the first row of the boost matrix $\Lambda^a{}_b$ gives the unit normal to the constant ψ slices in the orthonormal basis and the other three rows are three orthonormal vectors lying in the tangent space to a slice:

$$\Lambda^a{}_0 = n^a \equiv -\frac{e^a{}_\mu\partial^\mu\psi}{\sqrt{-\partial_\nu\psi\partial^\nu\psi}}\,,\qquad n_a\Lambda^a{}_m = 0\,,\qquad \eta_{ab}\,\Lambda^a{}_m\Lambda^b{}_n = \delta_{mn}\,. \quad (4.12)$$

Four vectors $\Lambda^a{}_m$ give an embedding of the space tangent to the slice in the tangent space of the space-time. In particular, any vector tangent to a constant ψ slice can be written in terms of three components Σ^m in this orthonormal basis as

$$\Sigma^\mu(\Sigma^m,\pi) = e_a{}^\mu\,\Lambda^a{}_m\,\Sigma^m = \Sigma^m\,E_m{}^\mu\,, \quad (4.13)$$

and the normal to a slice is given by the remaining forth tetrad $n^\mu = E_0{}^\mu$. Comparing this embedding to equation (4.1) we can write an explicit transformation between the three components of a tangent vector in the coordinate and orthonormal bases respectively:

$$\Sigma^m = E^m{}_\mu\,\frac{\partial x^\mu}{\partial x^i}\Big|_\psi\,\Sigma^i\,. \quad (4.14)$$

The transformation matrix is nothing else but a dreibein for the induced metric on constant ψ slices.

After fixing the boost Goldstones η, the remaining covariant derivatives (4.9) give the familiar objects of the EFTI:

$$\nabla_0\pi = \sqrt{-\partial_\mu\psi\partial^\mu\psi} - \mu = -\mu(\sqrt{-g^{00}} + 1)\,, \quad (4.15)$$

$$\nabla_0\eta_m = E_m{}^\mu\,n^\nu\nabla_\nu n_\mu = -e_m{}^\mu\partial_\mu\log\sqrt{-g^{00}}\,, \quad (4.16)$$

$$\nabla_n\eta_m = E_m{}^\mu\,E_n{}^\nu\nabla_\nu n_\mu = e_m{}^\mu\,e_n{}^\nu\,K_{\nu\mu}\,, \quad (4.17)$$

where ∇_μ is the usual four-dimensional covariant derivative associated with the metric $g_{\mu\nu}$ and the last equalities give the corresponding unitary gauge objects. Higher order terms in the derivative expansion are obtained by acting on the above terms by the CCWZ covariant derivative.

Additional matter fields in the CCWZ language should belong to the representations of the unbroken subgroup, the $SO(3)$ rotations in our case. Under the broken symmetry transformations these fields transform with Goldstone-dependent, and thus space-time dependent rotations. In order to preserve the non-linearly realised symmetries one has to use the CCWZ covariant derivatives with the Goldstone-dependent connection:

$$\mathcal{A}^m{}_{n\,a} = E_a{}^\mu \left(\Lambda_c{}^m \partial_\mu \Lambda^c{}_n + \omega^c{}_{d\,\mu} \Lambda_c{}^m \Lambda^d{}_n \right) , \tag{4.18}$$

with parameters η_m of the boost matrix Λ fixed in terms of the time translations Goldstone field π by equation (4.11). For a vector field under $SO(3)$ rotations Σ^m the covariant derivative is thus

$$\begin{aligned} \nabla_a \Sigma^m &\equiv E_a{}^\mu \partial_\mu \Sigma^m + \mathcal{A}^m{}_{n\,a}\, \Sigma^n \\ &= E_a{}^\mu \left(\partial_\mu \Sigma^m + (\Lambda_c{}^m \partial_\mu \Lambda^c{}_n + \omega^c{}_{d\,\mu} \Lambda_c{}^m \Lambda^d{}_n)\, \Sigma^n \right) \\ &= E_a{}^\mu\, \Lambda_c{}^m \left(\partial_\mu (\Lambda^c{}_n \Sigma^n) + \omega^c{}_{d\,\mu} \Lambda^d{}_n\, \Sigma^n \right) \\ &= E_a{}^\mu\, E^m{}_\nu \nabla_\mu (\Sigma^m\, E_m{}^\nu) \,. \end{aligned} \tag{4.19}$$

It is nothing else but the projection to the tangent space of a slice of the usual four-dimensional covariant derivative of corresponding four-vector $\Sigma^\mu(\Sigma^m, \pi)$ defined in equation (4.13). This can be readily extended to a matter field that transforms in any other representation of the unbroken rotations. We have thus shown that the CCWZ construction is equivalent to the natural embedding prescription introduced before up to a π-dependent field redefinition (4.14). In what follows we will use the latter since it can be easily combined with the usual EFT of Inflation routine.

4.2 Spin-one example

In order to warm up let us consider the theory of a massive spin-1 particle in inflation and discuss the connection to the relativistic Proca theory. The reader interested in the phenomenology of light spinning particles can skip straight to Section 4.4. In order to write a general quadratic action for the vector field Σ^i we apply the EFTI prescription to the four-vector Σ^μ . The most general quadratic Lagrangian for Σ^μ can be written as

$$\begin{aligned} \mathcal{L}_2 = \frac{1}{2}(1-c_1^2)\, n^\nu n^\lambda \nabla_\nu \Sigma^\mu \nabla_\lambda \Sigma_\mu - \frac{1}{2} c_1^2\, \nabla_\mu \Sigma^\nu \nabla^\mu \Sigma_\nu \\ - \frac{1}{2}(c_0^2 - c_1^2)\, \nabla_\mu \Sigma^\mu \nabla_\nu \Sigma^\nu - \frac{1}{2}(m^2 + c_1^2 H^2) \Sigma^\mu \Sigma_\mu \,. \end{aligned} \tag{4.20}$$

There are three independent kinetic terms one can write. We have fixed the overall normalisation to have a canonical time kinetic term and chosen the remaining two parameters c_0^2 and c_1^2 to be the propagation speeds for the helicity-0 and -1 modes respectively. Given that $\Sigma^\mu n_\mu \equiv 0$, all the possible kinetic terms where n_μ is contracted with the vector field index contribute to the quadratic action only by changing the mass term: $n_\mu \nabla_\nu \Sigma^\mu = -\Sigma^\mu \nabla_\nu n^\mu$. They do, however, contribute to the interactions with π and have to be considered for the phenomenological applications. The quadratic action for Σ^i then reads

$$S_2 = \frac{1}{2}\int d^3x\, dt\, a^3 \left((\dot\sigma^i)^2 - c_1^2\, a^{-2}\, (\partial_j \sigma^i)^2 - (c_0^2 - c_1^2)\, a^{-2}\, (\partial_i \sigma^i)^2 - m^2\, (\sigma^i)^2 \right) , \quad (4.21)$$

where we have defined a new field $\sigma^i \equiv a\,\Sigma^i$ which has a time kinetic term of a canonical scalar field. The Latin indices here and in what follows are summed with δ_{ij}. Note that, at variance with the Lorentz-invariant Proca action for the massive spin-1 field the sign of the helicity-0 kinetic term is not related to the mass parameter. It shows that the scaling dimension of the spin-1 field in the inflationary background is not bounded by unitarity requirements (Higuchi Bound) and can be chosen to be arbitrary, even when the space-time metric is exactly de Sitter.

In Fourier space we decompose σ_i in terms of its helicites, $\sigma_i(\vec{k},\eta) \equiv \sum_h \sigma_k^{(h)}(\eta) \times \epsilon_i^{(h)}(\hat{k})$, where $\sum_i \epsilon_i^{(h)}(\hat{k})\, (\epsilon_i^{(h')}(\hat{k}))^* = \delta^{hh'}$. The mode functions for a given comoving momentum k and helicity h that correspond to the Minkowski-like vacuum for the deep subhorizon modes are given by

$$\sigma_k^{(h)}(\eta) = H\, \frac{\sqrt{\pi}}{2}\, e^{\frac{1}{2}i\pi(\nu+\frac{1}{2})}\, (-\eta)^{3/2}\, H_\nu^{(1)}(-c_h\, k\, \eta) \ , \qquad \text{with} \qquad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \ . \quad (4.22)$$

The two-point function at late times takes the form

$$\langle \sigma^i(\eta,\vec{k}) \sigma^j(\eta,-\vec{k}) \rangle' \simeq \frac{2^{2\nu-2}\, \Gamma^2(\nu)\, H^2}{\pi\, k^{2\nu}\, (-\eta)^{2\nu-3}} \left((\delta_{ij} - \hat{k}_i \hat{k}_j)\, c_1^{-2\nu} + \hat{k}_i \hat{k}_j\, c_0^{-2\nu} \right) , \quad (4.23)$$

which corresponds to an operator with scaling dimension $\Delta = \frac{3}{2} - \nu$. The scaling dimension is determined by the mass parameter in the quadratic action and can be made to lie below the Higuchi bound $0 \le \Delta \le 1$. Negative scaling dimensions for a massive field would mean that the stress energy tensor of the fluctuations is growing at late time signalling the breakdown of perturbation theory.

Apart from the quadratic action (4.21) the Lagrangian (4.20) leads also to cubic interactions of order $\mathcal{O}(\sigma^2\,\pi)$ given by the following Hamiltonian:

$$H_{\rm int}^{\sigma\sigma\pi} = -\int d^3x\, a^2 \Big(2c_1^2\, H\, \partial_i\pi\, \dot\sigma^i\, \partial_j\sigma^j - (c_0^2 - c_1^2)\, \partial_i\pi\, \sigma^i\, (\partial_t - H)\partial_j\sigma^j + (c_1^2 - 1)\, \partial_i\pi\, \partial_i\sigma^j\, \dot\sigma^j \Big) \ . \quad (4.24)$$

To perform one more check that this geometric construction reproduces the correct couplings to the Goldstone field π in the next subsection we shall check the conformal consistency relation for the vector field σ^i. We calculate the $\pi\sigma\sigma$ three-point function sourced by this interactions in the limit of the soft π mode and confirm that is related to the action of the special conformal transformation on the two-point function of σ. But before we proceed let us comment on the relation of the developed formalism to the standard Proca action for a massive vector field.

Proca field. Since in our formalism the spin-1 field is embedded in a four-vector with only three independent components, it might not be obvious how, in the same formalism, we can discuss the ordinary Proca theory, which has four components. Let us see how to do this. Let us consider a Proca four-vector, B^μ, and let us decompose it in a component perpendicular to the slicing of uniform physical clock and three components parallel to it:

$$B^\mu = \Sigma^\mu + n^\mu \phi \ , \quad \text{with} \quad n_\mu \Sigma^\mu = 0 \ . \tag{4.25}$$

To describe a Proca field, B^μ, we need an additional scalar field, ϕ, on top of $\Sigma^\mu = \Sigma^\mu(\Sigma^i, \pi)$, which is the spin-1 field we defined in (4.1). B^μ is a combination of Σ^i, ϕ and π:

$$B^\mu = \begin{pmatrix} \phi - \partial_j \pi \, \Sigma^j \\ \Sigma^i - a^{-2} \, \partial_i \pi \, \phi \end{pmatrix} \ . \tag{4.26}$$

The dependence on π appears because our splitting in Σ^i and ϕ is done with reference to the time-slicing induced by the physical clock, while the splitting $B^{\{0,1,2,3\}}$ is done with respect to a clock-independent slicing. One can check that the dependence on Σ^i, ϕ and π indeed guarantees that B^μ transforms as a four-vector (as obvious from (4.25)).

Now, a simplification occurs if B^μ appears in the unitary-gauge Lagrangian always contracted in a fully diff. invariant way, for example as it appears in the combination of a diff. invariant mass term: $B^\mu B_\mu$. In this case, we expect that we should not need π to describe the field $B^\mu(\Sigma^i, \phi, \pi)$. Indeed, the problem is solved by performing a somewhat obvious field redefinition

$$\begin{pmatrix} \Sigma^i \\ \phi \\ \pi \end{pmatrix} \quad \to \quad \begin{pmatrix} B^\mu \\ \pi \end{pmatrix} \ . \tag{4.27}$$

If $B^\mu(\Sigma^i, \phi, \pi)$ appears in the action only in such a way that B^μ is contracted in a diff. invariant way, then the action in the Proca sector will depend only on the four components B^μ, reproducing the familiar approach. However, as it made clear by our formalism, given the spontaneous breaking of time diff.s provided by inflation, there is no need to introduce a scalar component ϕ to describe a spin-1 field.

We notice that in the case of Proca, the component B^0 of the vector field, which in our formalism is described by ϕ, does not have a time-kinetic term. Therefore, a Proca Lagrangian describes the propagation of three helicities, i.e. the same number as in a theory described solely in terms of Σ^i. However, the physics associated to the two Lagrangians is completely different, as it is evident from the fact that the spectrum of the theory built with just Σ^i can violate the Higuchi bound.

This observation highlights a freedom in our construction: the possibility of having auxiliary fields. Lacking a time-kinetic term, these fields do not propagate additional degrees of freedom. They can therefore be integrated out by solving for them and plugging back the resulting solution. This will lead to a non-local-looking Lagrangian, as it will contain several factors of inverse-Laplacians. This can lead to instantaneous propagation which is not compatible with a standard Lorentz invariant UV completion. Therefore one should impose microcausality, i.e. the commutativity of field outside the lightcone, as a restriction on the model parameters. Due to the abundance of auxiliary fields and terms that we can add, the process can become quite cumbersome, and it would be nice to find a straightforward way to add these auxiliary fields automatically preserving microcausality.[3]

4.2.1 Consistency relation for special conformal transformations

The Higuchi bound [54] is a consequence of the de Sitter isometries, or equivalently of the conformal symmetry of late-time correlators. The 2-point function is fixed by these symmetries and for small masses the longitudinal component becomes a ghost. The coupling with the foliation breaks the de Sitter isometries[4]: the 2-point functions of the various helicities are not anymore related to each other and thus the Higuchi bound can be avoided. However, the breaking induced by the inflaton background is spontaneous and not explicit, so that the symmetry is still there albeit non-linearly realised. The 2-point function of a particle with spin is now not invariant under the symmetries, but its variation is related to the coupling with soft Goldstone modes. The consistency relations associated with special conformal transformations (or boost isometries of de Sitter) were studied in [115, 116, 117].

These relations state that the effect of the gradient of the soft mode $\partial_i \pi_L$ is equivalent to a special conformal transformation with parameter $b_i = -\frac{1}{2} H\, \partial_i \pi_L$:

$$\langle \pi(\vec{q})\, \sigma^i(\vec{k})\, \sigma^j(-\vec{k}-\vec{q})\rangle' \underset{\vec{q}\to 0}{=} \frac{1}{2} H\, P_\pi(q) \left(\vec{q}\cdot\vec{K}\right) \langle \sigma^i(\vec{k})\, \sigma^j(-\vec{k})\rangle' + \mathcal{O}(q/k)^2 \;, \quad (4.28)$$

where $\vec{K}$ is the generator of the special conformal transformations in momentum space.[5] Particles with spin with masses below the Higuchi bound do not have a conformal invariant 2-point function: the RHS of the equation above is therefore non-zero and it implies a prescribed coupling with the soft inflaton fluctuations. It is worthwhile stressing that this consistency relation implies a coupling with π that remains perturbative, i.e. suppressed by Δ_ζ, even when we are well below the Higuchi bound and the 2-point function is very far from conformal invariance. This remains true even when one gives very different speeds of propagations to the various helicities.

Let us now verify eq. (4.28). In order to calculate the special conformal transformation of the two-point function it is useful to contract the tensor indices of the vectors σ^i with the polarization vectors ϵ_i, $\tilde{\epsilon}_j$. The late time limit of the two-point

[3] Working directly in terms of the field B^μ does not seem to help. While in the Lorentz invariant case of Proca one can easily verify that the Lagrangian obtained after integrating out B^0 is local, notwithstanding the non-local-looking factors, microcausality is not generically preserved as we move away from the Lorentz invariant case by adding couplings of B^μ to the foliation.

[4] The foliation breaks both the dilation and the boost isometries of de Sitter. We are here interested in the boosts since these relate the different helicities and therefore must be broken to violate the Higuchi bound. Furthermore in this thesis we assume that the inflaton is endowed with an approximate shift symmetry, so that a residual diagonal symmetry—dilation combined with an inflaton shift—is linearly realised. This is the usual slow-roll assumption and it is the origin of the observed approximate scale-invariance of the scalar power spectrum.

[5] An explicit expression for $\mathbf{K}$ can be found in the equation (A.157) of reference [14].

function (4.23) then takes the form:

$$\langle \epsilon_i \,\sigma^i(\eta,\vec{k})\,\tilde{\epsilon}_j\,\sigma^j(\eta,-\vec{k})\rangle' \underset{\eta\to 0}{=} \frac{2^{2\nu-2}\,\Gamma^2(\nu)\,H^2}{\pi\,k^{2\nu}\,(-\eta)^{2\nu-3}}\left(\frac{(\epsilon\cdot\tilde{\epsilon})}{c_1^{2\nu}}+\left(\frac{1}{c_0^{2\nu}}-\frac{1}{c_1^{2\nu}}\right)(\epsilon\cdot\hat{k})(\tilde{\epsilon}\cdot\hat{k})\right). \tag{4.29}$$

For a particular choice of polarisation vectors and the soft momentum:

$$\vec{q}\cdot\vec{k}=\vec{q}\cdot\vec{\epsilon}=0\,, \qquad\qquad \vec{q}\cdot\vec{\tilde{\epsilon}}\neq 0\,, \tag{4.30}$$

the special conformal transformation of a two-point function was calculated in [14] and for the 2-point function (4.29) reads

$$(\vec{q}\cdot\vec{K})\,\langle \epsilon_i \,\sigma^i(\eta,\vec{k})\,\tilde{\epsilon}_j\,\sigma^j(\eta,-\vec{k})\rangle' = -\left(\frac{2\nu+1}{c_1^{2\nu}}+\frac{2\nu-1}{c_0^{2\nu}}\right)\frac{H^2\,2^{2\nu-2}\,\Gamma^2(\nu)\,(\epsilon\cdot\hat{k})(\tilde{\epsilon}\cdot\vec{q})}{k^{2\nu+1}(-\eta)^{2\nu-3}}\,. \tag{4.31}$$

Notice that the 2-point function is not conformal invariant even for masses above the Higuchi Bound. This is compatible with the fact our formalism is intrisically related to the foliation and does not simply interpolate with that de Sitter invariant Proca action. The perturbative contribution to the three point function $\langle \pi\,\sigma\,\sigma\rangle$ due to the interaction (4.24) is given by

$$\langle \pi(\mathbf{q})\,\epsilon_i\,\sigma^i(\mathbf{k})\,\tilde{\epsilon}_j\,\sigma^j(\mathbf{p})\rangle_{\eta_0} = -i\int_{-\infty}^{\eta_*} d\eta \langle\Big[\pi(\vec{q})\,\epsilon_i\,\sigma^i(\vec{k})\,\tilde{\epsilon}_j\,\sigma^j(\vec{p}),\,H^{\pi\sigma\sigma}_{\text{int}}(\eta)\Big]\rangle\,. \tag{4.32}$$

For the consistency relation we need only the leading order in q/k, and it is possible to obtain an explicit expression for the three-point function at late times $\eta_*\to 0$:

$$\begin{aligned}\langle \pi(\vec{q})\,\epsilon_i\,\sigma^i(\vec{k})\,\tilde{\epsilon}_j\,\sigma^j(\vec{p})\rangle'_{\eta_*\to 0} &=\\ -\,i\,H\,P_\pi(q)\,\Big[(\epsilon\cdot\vec{k})(\tilde{\epsilon}\cdot\vec{q})-(\epsilon\cdot\vec{q})(\tilde{\epsilon}\cdot\vec{k})\Big]&\left(\sigma_k^{(0)}(\eta_*)\,\sigma_k^{(1)}(\eta_*)\right)_{\eta_*\to 0}\\ \int_{-\infty}^{0} d\eta\,\Big\{\Big((c_1^2-c_0^2)\,(-\eta)\sigma_k'^{(0)}(\eta)\sigma_k^{(1)}(\eta)&+2c_0^2\,\sigma_k^{(0)}(\eta)\sigma_k^{(1)}(\eta)\Big)-\text{c.c.}\Big\}\,,\end{aligned} \tag{4.33}$$

where the mode functions are given in (4.22). Taking the integral one can check that the consistency relation holds for all values of the sound speeds and scaling dimension.

4.3 Minimal spin-s theory

As discussed, a minimal description of a spin-s particle during inflation is given by a traceless[6] rank-s tensor field $\Sigma^{i_1\ldots i_s}$. The action that explicitly preserves all space-time symmetries can be written by using its four-dimensional version $\Sigma^{\nu_1\ldots\nu_s}$, and contracting with the vector n^μ:

$$\begin{aligned}S=\frac{1}{2s!}\int a^3\,d^3x\,dt\,\Big((1-c_s^2)\,n^\mu n^\lambda\,\nabla_\mu\Sigma^{\nu_1\ldots\nu_s}\nabla_\lambda\Sigma_{\nu_1\ldots\nu_s}-c_s^2\,\nabla_\mu\Sigma^{\nu_1\ldots\nu_s}\nabla^\mu\Sigma_{\nu_1\ldots\nu_s}\\ -\delta c_s^2\,\nabla_\mu\Sigma^{\mu\nu_2\ldots\nu_s}\nabla_\lambda\Sigma^\lambda{}_{\nu_2\ldots\nu_s}-(m^2+s\,c_s^2H^2)\,\Sigma^{\nu_1\ldots\nu_s}\Sigma_{\nu_1\ldots\nu_s}\Big)\,.\end{aligned} \tag{4.34}$$

[6]The trace of $\Sigma^{i_1\ldots i_s}$ should be taken using the induced metric on the constant inflaton slices: $h_{ij}=g_{\mu\nu}\frac{\partial x^\mu}{\partial x^i}\Big|_\psi\frac{\partial x^\nu}{\partial x^j}\Big|_\psi$. This implies that the four-dimensional field $\Sigma^{\nu_1\ldots\nu_s}$ is also traceless.

For a canonically normalized field there are only three free parameters in the quadratic action at leading order in derivatives, independently of the spin: two speed parameters c_s^2 and δc_s^2 fixing the propagation speeds for all the helicity modes, and the mass m.

Expanding the action in powers of the Goldstone field one obtains a free action for the spinning particle and its leading interactions with π. The quadratic action reads

$$S_2 = \frac{1}{2s!}\int a^3\, d^3x\, dt\Big((\dot{\sigma}^{i_1\ldots i_s})^2 - c_s^2\, a^{-2}(\partial_j \sigma^{i_1\ldots i_s})^2 - \delta c_s^2\, a^{-2}(\partial_j \sigma^{j i_2\ldots i_s})^2 - m^2\, (\sigma^{i_1\ldots i_s})^2\Big)\,, \tag{4.35}$$

where we have defined a new field $\sigma^{i_1\ldots i_s} \equiv a^s\, \Sigma^{i_1\ldots i_s}$, which has the same time-kinetic term as a canonical scalar field with mass m^2. Note that there are only two independent operators that constitute the spatial kinetic term for a traceless tensor field. This means that although the action propagates $2s+1$ different helicity modes, all propagation speeds c_h^2 for $h = 0, \ldots, s$ can be expressed as some linear combinations of c_s^2 and δc_s^2. The coefficients in these combinations are not universal and depend on the spin. It is easy to see, however, that the speed of the highest helicity mode $h = s$ is given by c_s for any spin.[7] In particular, for a spin-1 field $c_1^2 = c_s^2$ and $c_0^2 = c_s^2 + \delta c_s^2$, and for a spin-2 field

$$c_2^2 = c_s^2\,, \qquad c_0^2 = c_s^2 + \frac{2}{3}\delta c_s^2\,, \qquad c_1^2 = c_s^2 + \frac{1}{2}\delta c_s^2 = \frac{1}{4}c_2^2 + \frac{3}{4}c_0^2\,. \tag{4.36}$$

In particular, in order to avoid gradient instabilities and superluminal propagation for all the modes of a speed-2 particle, the speed parameters have to be in the range $0 \le c_s^2 \le 1$ and $0 \le c_s^2 + \frac{2}{3}\delta c_s^2 \le 1$. Notice that the mass term is the same for all the helicities and this implies that the time dependence on super-horizon scales is common to the whole multiplet.

Given that the transformation of σ under boosts is π-dependent, the action (4.34) has to contain interactions between σ and the Goldstone field π. The structure of these minimal interactions is completely fixed by symmetries and all the couplings are determined in terms of the parameters of the free Lagrangian. In order to study the phenomenological and theoretical consequences of these interactions, we need to consider the structure of the cubic $\sigma^2\pi$ and quartic $\sigma^2\pi^2$ interactions. The cubic action is given by

$$S_3 = \frac{1}{s!}\int dt\, d^3x\, a^3\Big((c_s^2 - 1)\, \partial_j \pi\, \partial_j \sigma^{i_1\ldots i_s}\, \dot{\sigma}^{i_1\ldots i_s} + 2c_s^2\, sH\, \partial_i \pi\, \sigma^{i i_2\ldots i_s}\, \partial_j \sigma^{j i_2\ldots i_s} - \delta c_s^2\, \partial_i \pi\, \sigma^{i i_2\ldots i_s}\, (\partial_t - sH)\, \partial_j \sigma^{j i_2\ldots i_s}\Big)\,, \tag{4.37}$$

[7] One can also show that $c_h^2 = a\, c_s^2 + (1-a)\, c_0^2$ with $0 \le a \le 1$ for any helicity h.

while the quartic interactions are given by the following action:

$$
\begin{aligned}
S_4 = \frac{1}{2s!}\int & d^3x\, dt\, a^3 \Big(\big(s\, m^2 - (5s + (5s^2 - 4s)\, c_s^2 + (s-1)(s+4)\, \delta c_s^2)\, H^2\big)\, a^{-2}\, (\partial_i \pi\, \sigma^{ii_2\ldots i_s})^2 \\
&+ 2(2c_s^2 + \delta c_s^2)\, sH\, a^{-2} \dot\pi\, \partial_i \pi\, \sigma^{ii_2\ldots i_s}\, \partial_j \sigma^{ji_2\ldots i_s} + 2(1 - c_s^2)\, sH\, a^{-2}\, \partial_i \dot\pi\, \partial_j \pi\, \sigma^{ii_2\ldots i_s}\, \sigma^{ji_2\ldots i_s} \\
&- (s + \delta c_s^2 (s-1))\, a^{-2}\, (\partial_t (\partial_i \pi\, \sigma^{ii_2\ldots i_s}))^2 + \delta c_s^2\, sH\, a^{-2}\, \dot\pi\, \partial_i \pi\, \sigma^{ii_2\ldots i_s}\, \partial_j \dot\sigma^{ji_2\ldots i_s} \\
&+ c_s^2\, s\, a^{-4}\, (\partial_j (\partial_i \pi\, \sigma^{ii_2\ldots i_s}))^2 + \delta c_s^2\, (s-1)\, a^{-4}\, (\partial_j (\partial_i \pi\, \sigma^{iji_3\ldots i_s}))^2 \\
&+ (1 - c_s^2)\, \big[a^{-4}\, (\partial_j \pi\, \partial_j \sigma)^2 + 2\, a^{-2}\, \dot\pi\, \partial_j \pi\, \dot\sigma\, \partial_j \sigma + a^{-2}\, (\partial_j \pi)^2\, (\dot\sigma)^2\big]\Big)\,. \qquad (4.38)
\end{aligned}
$$

As expected, the fact that the matter field σ have to realise non-linearly the full Poincare symmetry fixes the form and the coefficients of its interactions with the Goldstone field π.[8] These interaction will induce non-Gaussianities in π correlation functions that are inevitable consequences of the presence of the extra field σ and its non-relativistic structure. In Section 4.5 we provide an estimate for these non-Gaussianities, where we also take into account the constraints induced by the radiative stability of the theory. We will find that radiative stability and observational constraints do not qualitatively limit the observational consequences associated to the presence of spinning particles coupled to the inflaton, that we elaborate in detail in the next section for the particularly interesting case of a spin-2 particle.

4.4 Phenomenology of a light spin-two

In this section we focus on the phenomenology of a light spin-2 field Σ^{ij} during inflation. There are two main reasons to skip the simpler spin-1 case. First, the contributions to the squeezed limit of the bispectra $\langle \zeta_{\vec q \to 0} \zeta_{\vec k} \zeta_{-\vec k}\rangle$ and $\langle \gamma^{(s)}_{\vec q \to 0}\, \zeta_{\vec k}\, \zeta_{-\vec k}\rangle$ mediated by a field with an odd spin have an extra q/k suppression [101]. Second, the helicity-2 mode of the spin-2 field can mix with the tensor metric fluctuations: this affects the phenomenology of gravitational waves (besides the scalar sector).

In our construction, a spin-2 field is embedded in a four-dimensional tensor $\Sigma^{\alpha\beta}$ which is traceless and orthogonal to n^α. In this case the general action (4.34) reads

$$
\begin{aligned}
S[\Sigma] = \frac{1}{4}\int & d^4x\, \sqrt{-g}\Big((1 - c_2^2) n^\mu n^\nu \nabla_\mu \Sigma^{\alpha\beta}\, \nabla_\nu \Sigma_{\alpha\beta} - c_2^2 \nabla_\mu \Sigma^{\alpha\beta}\, \nabla^\mu \Sigma_{\alpha\beta} \\
&- \frac{3}{2}(c_0^2 - c_2^2)\, \nabla_\mu \Sigma^{\mu\alpha}\, \nabla^\nu \Sigma_{\nu\alpha} - (m^2 + 2\, c_2^2 H^2)\, \Sigma^{\alpha\beta} \Sigma_{\alpha\beta}\Big)\,. \qquad (4.39)
\end{aligned}
$$

The parameters c_0 and c_2 give the sound speeds for the helicity-0 and helicity-2 modes respectively. The sound speed for the helicity-1 mode $c_1^2 = \frac{1}{4}(3c_0^2 + c_2^2)$ is always positive and less then unity given that $0 \le c_0^2,\, c_2^2 \le 1$. The quadratic action reads

$$
S[\sigma] = \frac{1}{4}\int dt\, d^3x\, a^3 \Big((\dot\sigma^{ij})^2 - c_2^2\, a^{-2}\, (\partial_i \sigma^{jk})^2 - \frac{3}{2}(c_0^2 - c_2^2)\, a^{-2}\, (\partial_i \sigma^{ij})^2 - m^2\, (\sigma^{ij})^2 \Big)\,, \qquad (4.40)
$$

[8] Some of these interaction terms are degenerate with other interactions that one is allowed to add to the effective field theory action and that start at higher order in Goldstone fields. The coefficients of such terms are not fixed by symmetries.

where, as before, we have defined $\sigma^{ij} \equiv a^2\,\Sigma^{ij}$. Apart from the quadratic part, the covariant action (4.39) includes also interactions of σ^{ij} with π and γ_{ij} dictated by the non-linearly realised diffeomorphism invariance. For a systematic study of the phenomenology of the spin-2 field we also have to include other interaction and mixing terms allowed by the symmetries. At the leading order in fields and derivatives there are three independent operators[9]

$$S_{\rm int} = \int d^4x\,\sqrt{-g}\Big(M_{\rm Pl}\,\rho\,\delta K_{\alpha\beta}\Sigma^{\alpha\beta} + M_{\rm Pl}\,\tilde\rho\,\delta g^{00}\delta K_{\alpha\beta}\Sigma^{\alpha\beta} - \mu\,\Sigma^{\alpha\beta}\Sigma_\alpha{}^\gamma\Sigma_{\gamma\beta}\Big)\,, \quad (4.41)$$

where $\delta K_{\alpha\beta} \equiv K_{\alpha\beta} - a^2 H h_{\alpha\beta}$ is the fluctuation of the extrinsic curvature of constant ψ surfaces. The term proportional to ρ is responsible of the mixing of σ both with scalar and tensor perturbations. Going to the decoupling limit and in terms of the canonically normalized fields, $\pi_c \equiv (2\epsilon H^2 M_{\rm Pl}^2)^{1/2}\pi$ and $\gamma_{ij}^{(c)} \equiv M_{\rm Pl}\gamma_{ij}$ one has, up to cubic order:

$$\begin{aligned} S_{\rm int} = \int d^4x\,\sqrt{-g}\Big[-\frac{\rho}{\sqrt{2\epsilon}\,H}\,a^{-2}\,\partial_i\partial_j\pi_c\,\sigma^{ij} + \frac{1}{2}\,\rho\,\dot\gamma_{c\ ij}\sigma^{ij} - \frac{\rho}{2\epsilon H^2 M_{\rm Pl}}\,a^{-2}\times \\ \big(\partial_i\pi_c\partial_j\pi_c\,\dot\sigma^{ij} + 2H\,\partial_i\pi_c\partial_j\pi_c\,\sigma^{ij}\big) + \frac{\tilde\rho}{\epsilon H^2 M_{\rm Pl}}\,a^{-2}\,\dot\pi_c\partial_i\partial_j\pi_c\sigma^{ij} - \mu(\sigma^{ij})^3\Big]\,. \end{aligned} \quad (4.42)$$

In the following we are going to study the phenomenology associated with the action above when σ is light, $m \ll H$. Notice that a background for the field σ would induce a certain amount of anisotropy [118, 119]. In the presence of a small mass for σ the background (slowly) redshifts away. Here we assume that the field has a negligible background value.

Let us comment on the radiative stability of our setup. Since the interactions involve σ without derivatives, one expects loop corrections to generate a mass for σ. The interactions ρ and $\tilde\rho$ would not generate a σ mass in flat space since they preserve the shift symmetry in σ up to the terms proportional to H. One can therefore estimate the radiative corrections to the mass induced by the operator in ρ to be

$$\delta m_\rho^2 \sim H^2\left(\frac{\rho}{\sqrt{\epsilon}\,H}\right)^2\frac{\Lambda^4}{\epsilon\,H^2\,M_{\rm Pl}^2}, \quad (4.43)$$

where Λ is a cutoff at which the loop is cut. As we will discuss later $\rho/(\sqrt{\epsilon}H) \lesssim 1$ is the condition for the stability of the system. In this case, requiring $\delta m^2 \ll H^2$ gives $\Lambda^4 \ll \epsilon H^2 M_{\rm Pl}^2$. The loop must be cut at a scale somewhat below $\epsilon H^2 M_{\rm Pl}^2$, which is the unitarity cut-off associated to the ρ interaction itself. The same estimate eq. (4.43) works for the $\tilde\rho$ interaction. The cubic μ interaction gives a logarithmic divergent contribution to the mass, so that for naturalness one needs $\mu \ll H$. Notice that the smallness of the couplings ρ, $\tilde\rho$ and μ is radiatively stable since these couplings are odd for $\sigma \to -\sigma$. In conclusion, it is technically natural to have a light σ with sizeable mixing with γ and ζ.

[9] For simplicity we omit the operator $\tilde m^2\delta g^{00}\Sigma^2$. This induces a $\pi\sigma\sigma$ coupling that is subdominant to the ones in of eq. (4.37) in the regime $\tilde m^2 \ll H^2$.

4.4.1 Estimates of the effects

In this Section we will estimate the effects of the σ field, described by the action (4.40)-(4.41), while in the following we will make explicit calculations in some specific cases. The spin-2 field σ affects the observables in particular through the term in the action $\propto \rho$; this induces a mixing of σ both with scalar and tensor perturbations. In terms of the canonically normalized scalar and tensor perturbations π_c and γ_c the mixings are schematically of the form

$$\sim \frac{\rho}{\sqrt{\epsilon}H}\,\partial\partial\pi_c\,\sigma_c\,, \qquad \sim \rho\,\dot{\gamma}_c\,\sigma_c\,. \tag{4.44}$$

The mixing with the scalar perturbations is enhanced with respect to the tensor one since $\epsilon \ll 1$. This suggests that the effects of σ should be searched in the statistics of scalar perturbations only. However, as we discussed, the different helicity components of σ will have in general different propagation speeds.[10] (One expects the speeds to be also different from the speed of light, since σ is an intrinsically non-Lorentz-invariant object.) If the speed of propagation of the helicity-2 component, c_2, is smaller than the one of the helicity-0, c_0, then the helicity-2 power spectrum is boosted and this can easily overcome the smaller mixing. Additionally, the mixing between γ and σ does not turn off outside the horizon: as we will see, this induces, for sufficiently small mass of σ, an enhancement of the effect of the mixing of order N^2, where N is the number of e-folds of observable inflation. We assume $\rho/(\sqrt{\epsilon}H) \ll c_0$. In the opposite case the speed of propagation of σ is dominated by the mixing term, this leads to gradient instabilities.

Power spectra. It is easy to realise that the contribution of σ to the scalar and tensor power spectra is of the form (see Figure 4.1)

$$P_\zeta \sim \frac{H^2}{\epsilon M_{\rm Pl}^2}\frac{1}{k^3}\left(\frac{\rho}{\sqrt{\epsilon}H}\right)^2\frac{1}{c_0^3}\,, \qquad P_\gamma \sim \frac{H^2}{M_{\rm Pl}^2}\frac{1}{k^3}\left(\frac{\rho}{H}\right)^2\frac{N^2}{c_2^3}\,, \tag{4.45}$$

where we assumed σ massless for simplicity. Depending on the parameters, one can get sizeable modification of either the tensor or scalar power spectrum, or both. Notice also that, for sufficiently small sound-speeds $c_{0,2}$, both power spectra may be dominated by the σ exchange, while remaining in the weak mixing regime $\rho/(\sqrt{\epsilon}H) \ll 1$.

3-point functions. For the purpose of estimates, we will focus on the squeezed limit of the 3-point function. As it is well-known, this limit is sensitive to the content of light fields during inflation. Since we now understand that fields with masses below the Higuchi bound can exist, it is natural to look for their signatures in the squeezed limit. By symmetry arguments one can easily write the behaviour in the squeezed limit of the 3-point functions up to an overall factor

$$\langle\zeta_{\vec{q}\to 0}\zeta_{\vec{k}}\zeta_{-\vec{k}}\rangle' = B_\zeta\left(\frac{q}{k}\right)^{\frac{3}{2}-\nu}P_\zeta(q)\,P_\zeta(k)\left((\hat{q}\cdot\hat{k})^2-\frac{1}{3}\right)\,, \tag{4.46}$$

[10] In this chapter we assume, for simplicity, that the speed of the scalar perturbations π is unity. The speed of propagation of tensor perturbations can always be taken to be one without loss of generality [28].

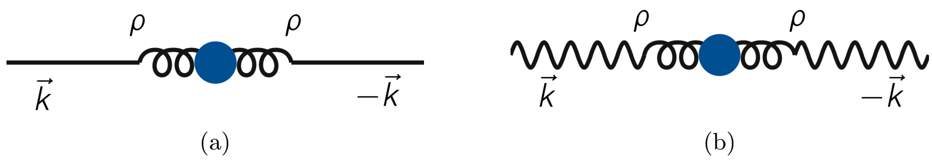

Figure 4.1: Contributions to the scalar and tensor power spectra due to the exchange of a σ field. The dots indicate a contraction between a pair of free fields, i.e. the insertion of a power spectrum. Notice that the external lines cannot be connected without going through a dot (contraction), and each dot is connected to external lines from both sides [64].

$$\langle \gamma^{(s)}_{\vec{q}\to 0}\, \zeta_{\vec{k}}\, \zeta_{-\vec{k}} \rangle' = B_\gamma \left(\frac{q}{k}\right)^{\frac{3}{2}-\nu} P_\gamma(q)\ P_\zeta(k)\ \epsilon^{(s)}_{ij}\,\hat{k}_i \hat{k}_j\,, \tag{4.47}$$

where $\nu \equiv \sqrt{\frac{9}{4}-\frac{m^2}{H^2}}$ and where $\epsilon^{(s)}_{ij}(\hat{q})$ is the polarization tensor relative to the s-th helicity of σ. (The present experimental limit on B_ζ, for $\nu \simeq \frac{3}{2}$, from Planck is: $|B_\zeta| = |-\frac{144}{5} f^{L=2}_{\rm NL}| \lesssim 200$ at 2σ [120]. For future constraints see [121, 118, 122].) It is also quite easy to give a parametric estimate of the prefactors in the various cases

$$\begin{aligned}
&\rho\rho \text{ interaction. Fig 4.2a} \qquad B_\zeta \sim \left(\frac{\rho}{H\sqrt{\epsilon}}\right)^2 \frac{1}{c_0^{2\nu}}\,,\\
&\rho\rho \text{ interaction. Fig 4.2b} \qquad B_\zeta \sim \left(\frac{\rho}{H\sqrt{\epsilon}}\right)^2 \frac{1}{c_0^{4\nu}}\,,\\
&\rho\tilde{\rho} \text{ interaction. Fig 4.2a} \qquad B_\zeta \sim \left(\frac{\rho}{H\sqrt{\epsilon}}\right)\left(\frac{\tilde{\rho}}{H\sqrt{\epsilon}}\right) \frac{1}{c_0^{2\nu}}\,,\\
&\mu \text{ interaction. Fig 4.2c} \qquad B_\zeta \sim \frac{\mu}{H}\left(\frac{\rho}{H\sqrt{\epsilon}}\right)^3 \Delta_\zeta^{-1}\frac{1}{c_0^{4\nu}}\,.
\end{aligned} \tag{4.48}$$

The dependence of c_0 is obtained looking at the σ power spectra in the graphs, taking into account that $P_{\sigma^{(s)}} \propto 1/(c_s\, q)^{2\nu}$, and that σ freezes before π crosses the Hubble horizon.

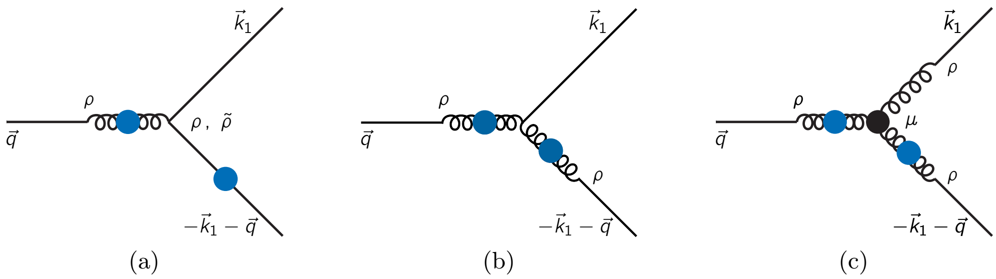

Figure 4.2: Leading contributions to the $\langle\zeta\zeta\zeta\rangle$ 3-point function.

The correlator $\langle \gamma^{(s)}_{\vec q}\, \zeta_{\vec k}\, \zeta_{-\vec k}\rangle'$ can be estimated[11] to be

$$\begin{aligned} &\rho\rho \text{ interaction. Fig 4.3a} && B_\gamma \sim \left(\frac{\rho}{H\sqrt{\epsilon}}\right)^2 \frac{1}{c_2^{2\nu}}\,, \\ &\rho\rho \text{ interaction. Fig 4.3b} && B_\gamma \sim \left(\frac{\rho}{H\sqrt{\epsilon}}\right)^2 \frac{1}{c_0^{2\nu} c_2^{2\nu}}\,, \\ &\rho\tilde\rho \text{ interaction. Fig 4.3a} && B_\gamma \sim \left(\frac{\rho}{H\sqrt{\epsilon}}\right)\left(\frac{\tilde\rho}{H\sqrt{\epsilon}}\right) \frac{1}{c_2^{2\nu}}\,, \\ &\mu \text{ interaction. Fig 4.3c} && B_\gamma \sim \frac{\mu}{H}\left(\frac{\rho}{H\sqrt{\epsilon}}\right)^3 \Delta_\zeta^{-1} \frac{1}{c_0^{2\nu} c_2^{2\nu}}\,. \end{aligned} \tag{4.49}$$

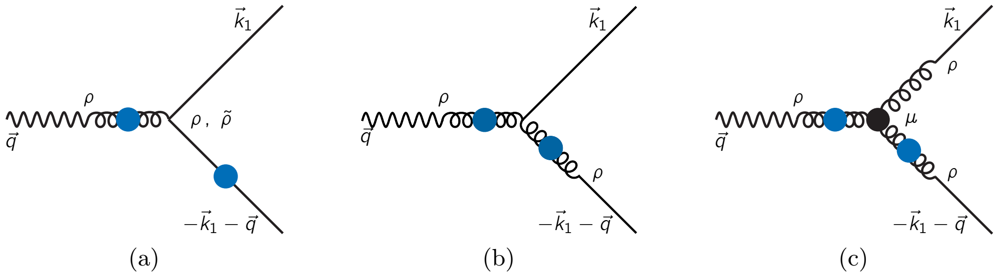

Figure 4.3: Leading contributions to the $\langle\gamma\zeta\zeta\rangle$ 3-point function.

Consistency relations and isocurvature perturbations. In our setup, besides the inflaton, one has the additional field σ_{ij}. Since this field is traceless, it cannot mix with scalar perturbations in the long wavelength limit, i.e. when derivatives are negligible.[12] One can thus conclude, even without knowing the details of reheating, that isocurvature perturbations cannot be generated. Following the same logic one can conclude that the usual Maldacena consistency relation holds after an angular average over the orientation of the long mode:

$$\int \frac{d^2\hat q}{4\pi} \langle \zeta_{\vec q}\zeta_{\vec k}\zeta_{-\vec k-\vec q}\rangle'_{q\ll k} = -\frac{d\log k^3 P_\zeta(k)}{d\log k} P_\zeta(q) P_\zeta(k)\,. \tag{4.50}$$

(This situation is similar to what happens in the case of Solid Inflation as we will see in detail in the next chapter.) Notice that this consistency relation will hold even when the power spectrum of ζ is dominated by the σ exchange. Eq. (4.50) implies that there is no way to generate angle-independent local non-Gaussianities in this setup. There is no sense in which tensor consistency relations are preserved. Moreover, since σ mixes with γ also in the long-wavelength limit, one in general expects that this mixing also occurs during reheating: both the tensor power spectrum and

[11] Notice that the estimates computed in eqs. (4.48) and (4.49) of B_ζ and B_γ are valid even away from the squeezed limit with the only possible exception of the last line of eq. (4.49), Fig 4.3c. Away from the squeezed limit there is another contribution to $\langle\gamma\zeta\zeta\rangle$ besides Fig 4.3c. It is proportional to $c_0^{-4\nu}$ instead of $(c_0\, c_2)^{-2\nu}$.

[12] At quadratic order $\sigma_{ij}\sigma_{ij}$ is scalar so the mixing is possible.

"local" tensor non-Gaussianities can be generated at reheating, similarly to what happens for scalar perturbations when one has more than one scalar field. It would be interesting to understand the most general tensor bispectrum generated in this way.

4-point functions. The 4-point function simplifies in the countercollinear limit, when couples of external momenta are almost equal and opposite. It receives independent contributions from all helicities of σ. In the countercollinear limit the behaviour of the correlator is fixed by symmetries. The exchange of the helicity states gives

$$\langle\zeta_{\vec{k}_1-\vec{q}}\zeta_{-\vec{k}_1}\zeta_{\vec{k}_2+\vec{q}}\zeta_{-\vec{k}_2}\rangle' = T_0\left(\frac{q}{\sqrt{k_1k_2}}\right)^{3/2-\nu}P_\zeta(q)\,P_\zeta(k_1)\,P_\zeta(k_2) \times\sum_{s\,s'=-2}^{+2}\left(\epsilon_{ij}^{(s)}(\hat{q})\,\hat{k}_{1\,,i}\hat{k}_{1\,j}\right)\left(\epsilon_{ij}^{(s')}(\hat{q})\,\hat{k}_{2\,,i}\hat{k}_{2\,j}\right)\,, \tag{4.51}$$

where $\epsilon_{ij}^{(s)}(\hat{q})$ is the helicity-s polarization tensor. It is easy to estimate the prefactors, for example

$$\begin{aligned} &\mu\mu \text{ interaction. Fig 4.4a} \qquad T_0\sim\left(\frac{\mu}{H}\right)^2\left(\frac{\rho}{H\sqrt{\epsilon}}\right)^4\Delta_\zeta^{-2}\frac{1}{c_i^{2\nu}\,c_0^{4\nu}}\,,\\ &\tilde{\rho}\tilde{\rho} \text{ interaction. Fig 4.4b} \qquad T_0\sim\left(\frac{\tilde{\rho}}{H\sqrt{\epsilon}}\right)^2\frac{1}{c_i^{2\nu}}\,, \end{aligned} \tag{4.52}$$

where the c_i stands for the speed of propagation of the helicity exchanged in the horizontal propagator. Notice that the $\tilde{\rho}\tilde{\rho}$ diagram is present even in the absence of the mixing $\propto \rho$.

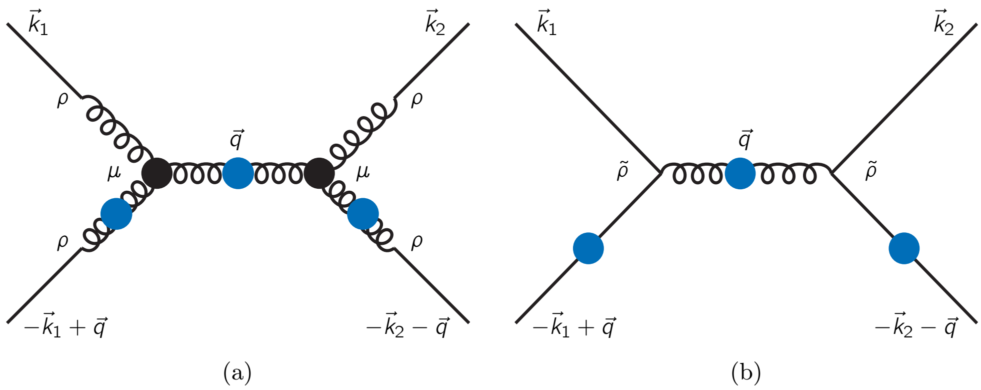

Figure 4.4: Leading contribution to the $\langle\zeta\zeta\zeta\zeta\rangle$.

Large tensor non-Gaussianities. The presence of a light helicity-2 state during inflation changes the prediction for tensor modes. In particular one can consider a regime where the tensor spectrum is dominated by the mixing with σ, while the correction to the scalar power spectrum remains small. If the σ sector is quite non-Gaussian, one will have large non-Gaussianity for tensors, while scalar perturbations may remain close to Gaussian, as required by experiments. For concreteness let us focus on the cubic term proportional to μ and see how it affects the various 3-point functions. Schematically, the deviations from a Gaussian statistics are given by the following dimensionless estimates:

$$\begin{aligned} \frac{\langle\gamma\gamma\gamma\rangle}{\Delta_\gamma^3} &\sim \frac{\mu}{Hc_2^{3/2}}\,, \\ \frac{\langle\gamma\zeta\zeta\rangle}{\Delta_\gamma\,\Delta_\zeta^2} &\sim \frac{\mu}{Hc_2^{3/2}}\left(\frac{\rho}{\sqrt{\epsilon}H}\right)^2\frac{1}{c_0^3}\,, \\ \frac{\langle\zeta\zeta\zeta\rangle}{\Delta_\zeta^3} &\sim \frac{\mu}{Hc_2^{3/2}}\left(\left(\frac{\rho}{\sqrt{\epsilon}H}\right)^2\frac{1}{c_0^3}\right)^{3/2}\left(\frac{c_2}{c_0}\right)^{3/2}\,. \end{aligned} \tag{4.53}$$

In these estimates we assumed that the tensor power spectrum is dominated by the σ exchange (while in eq. (4.49) we assumed that the σ exchange was only a correction to the standard prediction). When $\mu/(Hc_2^{3/2})$ approaches unity, tensor fluctuations become strongly non-Gaussian. This can happen keeping the other correlators involving ζ close to a Gaussian statistics. Indeed $(\rho/\sqrt{\epsilon}H)^2\cdot 1/c_0^3 \ll 1$ if one wants the scalar mixing with σ not to change significantly the scalar power spectrum (see eq. (4.45)) and this makes the non-Gaussianity in $\langle\gamma\zeta\zeta\rangle$ subdominant. The same will happen for $\langle\zeta\zeta\zeta\rangle$ in the regime $c_2 \lesssim c_0$.

In the rest of this Section we are going to confirm some of these estimates by explicit calculations.

4.4.2 Power Spectra

Let us start explicitly computing the leading corrections to both the power spectra of curvature perturbation and of gravitational waves. For, simplicity we will do the computations only in the limit c_0, $c_2 \ll 1$. As we will show this is the most interesting limit since in both cases the corrections can dominate the power spectra, being proportional to $c_0^{-2\nu}$ and $c_2^{-2\nu}$ respectively. Denoting with X the perturbation ζ or γ, the leading correction to their power spectra is given, using the in-in formalism, by

$$\begin{aligned} \langle X_{\vec{k}}\, X_{-\vec{k}}\rangle &= \int_{-\infty}^{0}\!d\eta\int_{-\infty}^{\eta}\!d\eta'\,\langle\Big[H_{\rm int}^{X\sigma}(\eta),\,\Big[H_{\rm int}^{X\sigma}(\eta'),\,X_{\vec{k}}(0)\,X_{-\vec{k}}(0)\Big]\Big]\rangle \\ &= \int_{-\infty}^{0}\!d\eta\int_{-\infty}^{0}\!d\eta'\,\langle H_{\rm int}^{X\sigma}(\eta)\,X_{\vec{k}}X_{-\vec{k}}H_{\rm int}^{X\sigma}(\eta')\rangle \\ &\qquad -2{\rm Re}\left[\int_{-\infty}^{0}\!d\eta\int_{-\infty}^{\eta}\!d\eta'\,\langle X_{\vec{k}}X_{-\vec{k}}H_{\rm int}^{X\sigma}(\eta)\,H_{\rm int}^{X\sigma}(\eta')\rangle\right]. \end{aligned} \tag{4.54}$$

$H_{\rm int}^{X\sigma}(\eta)$ denotes the interaction Hamiltonian between X and σ. This interaction is proportional to ρ both for scalars and tensors: see the first line of eq. (4.42).

Power spectrum of curvature perturbations. Let us compute the contribution to $\langle\zeta\zeta\rangle$. Substituting the wavefunctions in eq. (4.54) we find

$$P_\zeta(k) = \frac{H^2}{4M_{\rm Pl}^2\epsilon\, k^3}\left(1+\frac{\mathcal{C}_\zeta(\nu)}{\epsilon\, c_0{}^{2\nu}}\left(\frac{\rho}{H}\right)^2\right) \tag{4.55}$$

where,

$$\mathcal{C}_\zeta(\nu) \equiv \mathcal{C}_{\zeta,1}(\nu)+\mathcal{C}_{\zeta,2}(\nu)\,, \tag{4.56}$$

$$\mathcal{C}_{\zeta,1}(\nu) \equiv \frac{\pi}{6}\, c_0{}^{2\nu}\left|\int_0^\infty dx\; \frac{e^{-ix}\,(1+ix)\,H_\nu^{(2)}(c_0\,x)}{\sqrt{x}}\right|^2, \tag{4.57}$$

$$\mathcal{C}_{\zeta,2}(\nu) \equiv -\frac{\pi}{3}\, c_0{}^{2\nu}\,\mathrm{Re}\left[\int_0^\infty dx\; \frac{e^{-ix}\,(1+ix)\,H_\nu^{(1)}(c_0\,x)}{\sqrt{x}}\int_x^\infty dy\; \frac{e^{-iy}\,(1+iy)\,H_\nu^{(2)}(c_0\,y)}{\sqrt{y}}\right]. \tag{4.58}$$

For $c_0 \ll 1$ the integrals can be computed analytically. We get

$$\mathcal{C}_\zeta = \frac{2^{2\nu-3}\,(3-2\nu)^2\,\Gamma\left(\frac{1}{2}-\nu\right)^2\Gamma(\nu)^2\,(1-\sin(\pi\nu))}{3\pi}\,. \tag{4.59}$$

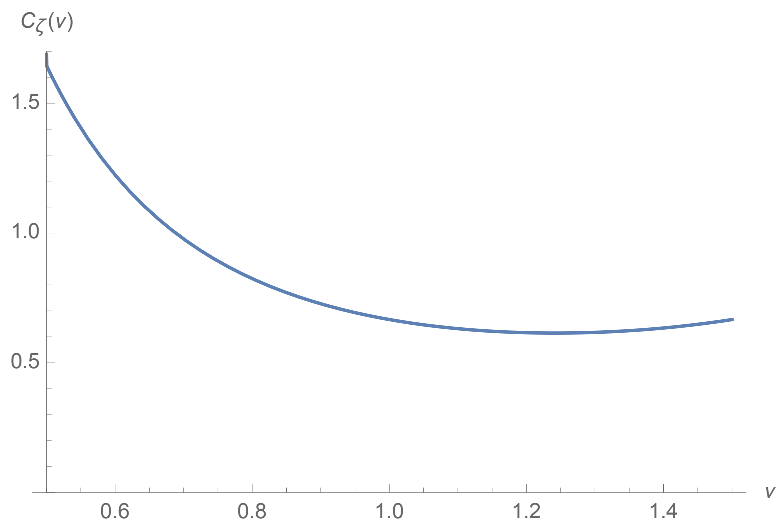

Figure 4.5: $\mathcal{C}_\zeta$ as a function of $\nu=\sqrt{\frac{9}{4}-\left(\frac{m}{H}\right)^2}$, in the range of masses below the Higuchi bound: $\nu\in[\frac{1}{2},\frac{3}{2}]$.

Figure 4.5 shows the plot of C_ζ as a function of ν in the mass range below the Higuchi bound. In the massless case one gets $\mathcal{C}_\zeta(\nu=3/2)=2/3$. The expression in eq. (4.59) receives relative corrections of order $c_0^{2\nu}$ so it should not be trusted for small ν. More details about the computation can be found in App. C.1.

Power spectrum of tensor perturbations. Let us move now to the computation of the tensor power spectrum. The contribution to $\langle\gamma\gamma\rangle$ is due to an exchange of a helicity-2 mode $\sigma_{ij}^{(2)}$. Substituting the wavefunctions in eq. (4.54) we find

$$P_\gamma(k) = \frac{4H^2}{M_{\rm Pl}^2\, k^3}\left(1 + \frac{\mathcal{C}_\gamma(\nu)}{{c_2}^{2\nu}}\left(\frac{\rho}{H}\right)^2\right)\,, \tag{4.60}$$

where,

$$\mathcal{C}_\gamma(\nu) \equiv \mathcal{C}_{\gamma,1}(\nu) + \mathcal{C}_{\gamma,2}(\nu)\,, \tag{4.61}$$

$$\mathcal{C}_{\gamma,1}(\nu) \equiv \frac{\pi}{2}\,{c_2}^{2\nu}\left|\int_0^\infty dx\,\frac{e^{-ix}\,H_\nu^{(2)}(c_2\,x)}{\sqrt{x}}\right|^2\,, \tag{4.62}$$

$$\mathcal{C}_{\gamma,2}(\nu) \equiv -\pi\,{c_2}^{2\nu}\,\mathrm{Re}\left[\int_0^\infty dx\,\frac{e^{-ix}\,H_\nu^{(1)}(c_2\,x)}{\sqrt{x}}\int_x^\infty dy\,\frac{e^{-iy}\,H_\nu^{(2)}(c_2\,y)}{\sqrt{y}}\right]\,. \tag{4.63}$$

Even in this case the integrals can be computed analytically in the limit $c_2 \ll 1$. For some details about the computation we refer the reader to App. C.1. The final result is

$$\mathcal{C}_\gamma = \frac{2^{2\nu-1}\,\Gamma\left(\frac{1}{2}-\nu\right)^2\,\Gamma(\nu)^2(1-\sin(\pi\nu))}{\pi}\,. \tag{4.64}$$

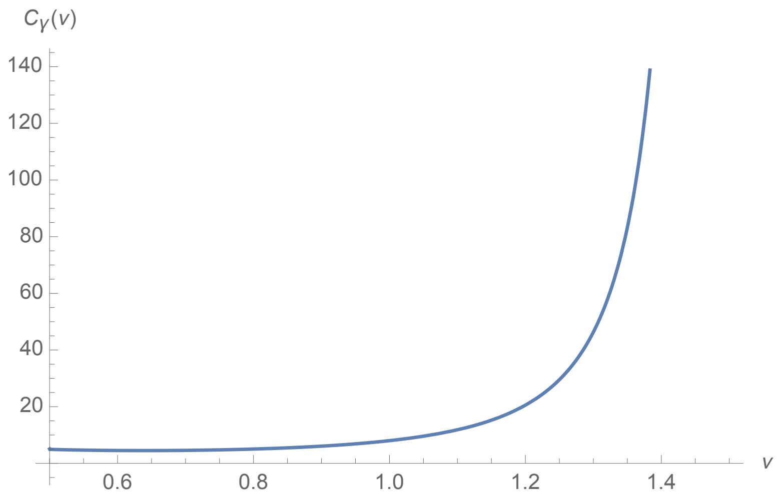

Figure 4.6: $\mathcal{C}_\gamma$ as a function of $\nu = \sqrt{\frac{9}{4} - \left(\frac{m}{H}\right)^2}$, in the range of masses below the Higuchi bound: $\nu \in [\frac{1}{2}, \frac{3}{2}]$.

In figure 4.6 we plot C_γ as function of ν in the mass range below the Higuchi Bound. Notice that the result in this case diverges in the massless limit since the mixing does not turn off outside the horizon. The divergence will be eventually regulated by the finite duration of inflation. For small mass the time evolution of σ outside the horizon is $\sigma \propto (-\eta)^{m^2/3H^2} \sim e^{-Nm^2/3H^2}$, where N is the number of e-folds to the end of inflation. Therefore the result (4.64) is accurate only for $(m/H)^2 \gg 1/N$. One can take into account the finite duration of inflation cutting off the integrals eqs. (4.62) and (4.63). In the massless case $\nu = 3/2$ one gets

$$P_\gamma(k) = \frac{4H^2}{M_{\rm Pl}^2\, k^3}\left(1 + \frac{2}{{c_2}^3}\left(\frac{N\,\rho}{H}\right)^2\right) . \qquad (4.65)$$

The enhancement in the massless limit can be quite sizeable: $2N^2 \sim 10^4$.

If P_γ is dominated by the σ exchange, while the correction to the scalar spectrum is small, the tensor-to-scalar ratio r is given by[13]

$$r \equiv \frac{P_\gamma}{P_\zeta} \simeq 32\left(\frac{c_0}{c_2}\right)^3\left(\left(\frac{\rho}{\sqrt{\epsilon}H}\right)^2\frac{1}{c_0^3}\right)(\epsilon\, N)^2\ . \qquad (4.67)$$

Notice that the tilt of the tensor power spectrum is given by the usual formula, $n_t = -2\epsilon$, in the regime $(m/H)^2 \gg 1/N$ (although the tensor to scalar ratio is now not fixed in term of ϵ). In the regime of smaller masses one has the extra N dependence of eq. (4.65), which gives an additional negative contribution to the tilt:

$$n_t = -2\epsilon - \frac{2}{N}\ . \qquad (4.68)$$

This formula neglects the possible time dependence of c_2: this is arbitrary and can make tensor modes large on the short scales observed by interferometers. This opens the possibility of studying the statistics of primordial gravitational waves on short scales, see e.g. [18].

As it is discussed in Appendix C.2, one can exactly solve the coupled equations of γ and σ instead of treating the mixing pertubatively. The mixing term effectively gives rise to an additional mass term of order ρ^2, however this effect is never relevant since we are always in the regime $(\rho/H)^2 \ll \epsilon \lesssim 1/N$.

4.4.3 Bispectra

Squeezed limit. In this section we compute the squeezed limit of $\langle\zeta\zeta\zeta\rangle$ and $\langle\gamma\zeta\zeta\rangle$. For simplicity we assume $\rho \ll \tilde\rho$ and take $\mu = 0$. In this way we can just focus on the contributions of Fig. 4.2a for $\langle\zeta\zeta\zeta\rangle$ and of Fig. 4.3a for $\langle\gamma\zeta\zeta\rangle$. In the squeezed limit, the long mode leaves the horizon much earlier than the short ones. We can then split the computation of the bispectrum in two: first we look at the effect of the long mode on the small scales and then we compute the mixing between σ and the soft field, ζ or γ_{ij}.

Let $X_{\vec q}$ be the long mode that mixes with σ. Leaving the horizon much earlier than the other two modes, $X_{\vec q}$ acts on the smaller scales as a classical background. Working in spatially flat gauge we have

$$\langle X_{\vec q}\,\pi_{\vec k}\,\pi_{-\vec k-\vec q}\rangle_{\vec q\to 0} = \langle X_{\vec q}\,\langle\pi_{\vec k}\,\pi_{-\vec k}\rangle_{\sigma_b}\rangle\,, \qquad (4.69)$$

[13] In the regime in which also P_ζ is dominated by the σ exchange we have

$$r \simeq 48\,(\epsilon N)^2\left(\frac{c_0}{c_2}\right)^3 . \qquad (4.66)$$

where $\langle \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle_{\sigma_b}$ is the power spectrum of π modulated by the long σ mode. It can be expanded in a power series in terms of the long mode,

$$\langle \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle_{\sigma_b} \simeq \langle \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle + \sum_s \sigma^{(s)}_{\vec{q}}\, \frac{\langle \sigma^{(s)}_{\vec{q}}\, \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle}{P_{\sigma^{(s)}}(q)}\,. \tag{4.70}$$

Bracketing eq. (4.70) with the soft $X_{\vec{q}}$ we get the leading contribution to the squeezed 3-point function,

$$\langle X_{\vec{q}}\, \zeta_{\vec{k}}\, \zeta_{-\vec{k}-\vec{q}} \rangle'_{\vec{q}\to 0} = H^2 \frac{\langle X_{\vec{q}}\, \sigma^{(s)}_{-\vec{q}} \rangle' \langle \sigma^{(s)}_{\vec{q}}\, \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle'}{P_{\sigma^{(s)}}(q)}\,. \tag{4.71}$$

where we used the fact that $\zeta = -H\pi$. We use the in-in formalism to compute the correlator $\langle X\sigma^{(s)} \rangle$ and the 3-point function $\langle \sigma^{(s)} \pi\pi \rangle$, we get

$$\begin{aligned} \langle \sigma^{(s)}_{\vec{q}}\, \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle &= -i \int_{-\infty}^{0} d\eta' \langle \Big[\hat{\sigma}^{(s)}_{\vec{q}}\, \hat{\pi}_{\vec{k}}\, \hat{\pi}_{-\vec{k}}\,,\ H^{\pi\pi\sigma}_{\text{int}}(\eta') \Big] \rangle \\ &= -4\, \tilde{\rho}\, M_{\rm Pl}\, k^2\, \Big(\sigma^{(s)}_{q}(\eta_*)\, \pi_k(\eta_*)\, \pi_k(\eta_*) \Big) \\ &\qquad \times \text{Im} \Big[\int_{-\infty}^{\eta_*} d\eta\, a\, \sigma^{(s)*}_{q}(\eta)\, \pi_k{}^{*\prime}(\eta)\, \pi_k{}^{*}(\eta) \Big]\, \epsilon^{(s)}_{ij}(\hat{q})\, \hat{k}_i \hat{k}_j\,. \end{aligned} \tag{4.72}$$

and

$$\langle X_{-\vec{q}}\, \sigma^{(s)}_{\vec{q}} \rangle = -i \int_{-\infty}^{\eta_*} d\eta \langle \Big[\hat{\sigma}^{(s)}_{\vec{q}}\, \hat{\pi}_{-\vec{q}}\,,\ H^{X\sigma}_{\text{int}}(\eta) \Big] \rangle\,. \tag{4.73}$$

Here η_* is the conformal time at the end of inflation. In the next two subsections we report the expression for both the scalar and the tensor squeezed bispectra as a function of the mass and the speed of propagation of σ. The details of the calculations can be found in App. C.3.

Scalar bispectrum in the squeezed limit. Only the helicity-0 component of σ can mix with the soft ζ. Therefore we need to evaluate the mixing $\langle \pi_{\vec{q}}\, \sigma^{(0)}_{-\vec{q}} \rangle$ and then the 3-point function $\langle \sigma^{(0)}_{\vec{q}}\, \pi_{\vec{k}}\, \pi_{-\vec{k}-\vec{q}} \rangle$.

The expression for the mixing $\langle \pi_{\vec{q}}\, \sigma^{(0)}_{-\vec{q}} \rangle$ cannot be written analytically for generic c_0. In the limit $c_0 \ll 1$ one has

$$\langle \sigma^{(0)}_{\vec{q}}\, \pi_{-\vec{q}} \rangle' = \frac{d_\pi(\nu)}{c_0^{2\nu}}\Bigg|_{\vec{q}\to 0} M_{\rm Pl}\, \rho\, (-q\,\eta_*)^{3/2-\nu}\, P_\pi(k)\,, \tag{4.74}$$

with $d_\pi(\nu)$ given in eq. (C.29). The 3-point correlation function $\langle \sigma^{(0)}_{\vec{q}}\, \pi_{\vec{k}}\, \pi_{-\vec{k}-\vec{q}} \rangle$ is given by

$$\langle \sigma^{(0)}_{\vec{q}}\, \pi_{\vec{k}}\, \pi_{-\vec{k}} \rangle'_{\vec{q}\to 0} = \frac{\sqrt{3}\, c(\nu)}{\epsilon\, M_{\rm Pl}}\, \frac{\tilde{\rho}}{H}\, (-k\,\eta_*)^{-\frac{3}{2}+\nu}\, P_{\sigma^{(0)}}(q)\, P_\pi(k) \left((\hat{q}\cdot\hat{k})^2 - \frac{1}{3} \right)\,. \tag{4.75}$$

The coefficient $c(\nu)$ is given in eq. (C.31) and the power spectrum of σ is (see eq. (C.7))

$$P_{\sigma^{(s)}}(q) = \frac{2^{2\nu-2}\,\Gamma(\nu)^2\, H^2}{\pi}\, \frac{(-\eta_*)^{3-2\nu}}{(c_s\, q)^{2\nu}}\,, \qquad s = 0\,,1\,,2\,. \tag{4.76}$$

The final expression of the curvature bispectrum is then,

$$\langle \zeta_{\vec{q}}\zeta_{\vec{k}}\zeta_{-\vec{k}}\rangle'_{\vec{q}\to 0} = \sqrt{3}\,\frac{\mathcal{F}_\pi(\nu)}{c_0^{2\nu}}\,\frac{\rho\,\tilde{\rho}}{\epsilon\, H^2}\,\left(\frac{q}{k}\right)^{\frac{3}{2}-\nu} P_\zeta(q)\, P_\zeta(k)\left((\hat{q}\cdot\hat{k})^2-\frac{1}{3}\right), \qquad (4.77)$$

where again we used the leading relation among ζ and π, $\zeta = -H\pi$. Notice that, even if $\langle\sigma\pi\rangle$ and $\langle\sigma\pi\pi\rangle$ decay in time, their product has the exact time dependence of $P_{\sigma^{(0)}}$ so that the contribution due to an exchange of σ to the squeezed scalar bispectrum is time-independent, as expected. The function $\mathcal{F}_\pi(\nu)$ is plotted in Fig. 4.7 in the mass range below the Higuchi bound. Its expression is given by

$$\mathcal{F}_\pi(\nu) = d_\pi(\nu)\, c(\nu)\,. \qquad (4.78)$$

The bispectrum $\langle\zeta_{\vec{q}\to 0}\zeta\zeta\rangle$ can be exactly calculated when σ is massless for any value

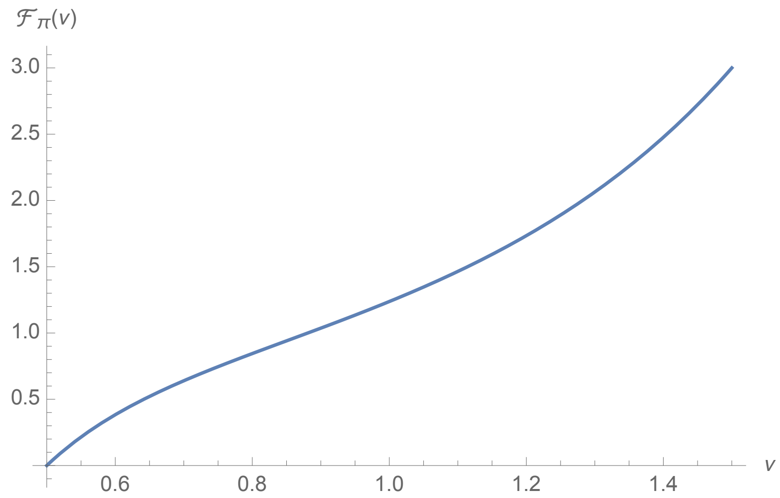

Figure 4.7: $\mathcal{F}_\pi$ as a function of $\nu = \sqrt{\frac{9}{4}-\left(\frac{m}{H}\right)^2}$, in the range of masses below the Higuchi bound: $\nu\in[\frac{1}{2},\frac{3}{2}]$.

of the sound speed c_0:

$$\langle \zeta_{\vec{q}}\,\zeta_{\vec{k}}\,\zeta_{-\vec{k}}\rangle'_{\vec{q}\to 0} = 3\,\frac{1+c_0+c_0^2}{c_0^3\,(1+c_0)}\,\frac{\rho\,\tilde{\rho}}{\epsilon\, H^2}\,P_\zeta(q)P_\zeta(k)\,\left((\hat{q}\cdot\hat{k})^2-\frac{1}{3}\right). \qquad (4.79)$$

Tensor-scalar-scalar bispectrum in the squeezed limit. Let us now compute the squeezed tensor-scalar-scalar bispectrum. The expression of $\langle\sigma\gamma\rangle$, in the limit $c_2 \ll 1$ is

$$\langle \sigma^{(2)}_{\vec{q}}\,\gamma_{-\vec{q}}\rangle' = \frac{d_\gamma(\nu)}{c_2^{2\nu}}\,M_{\rm Pl}\,\frac{\rho}{H}\,(-q\,\eta_*)^{\frac{3}{2}-\nu}\,P_\gamma(q)\,. \qquad (4.80)$$

The coefficient $d_\gamma(\nu)$ is given in eq. (C.33). Notice that, although the computation of the above expression has been carried out explicitly assuming $c_2 \ll 1$, one can check that eq. (4.80) is a good approximation i.e. ($\mathcal{O}(1)$) even if $c_2 \simeq 1$ in the range

$1/2 < \nu < 3/2$, i.e. the most interesting mass range, since it corresponds to the masses below the Higuchi Bound. The correlator $\langle \sigma^{(\pm 2)} \pi\pi \rangle$ is,

$$\langle \sigma^{(\pm 2)}_{\vec{q}} \, \pi_{\vec{k}} \, \pi_{-\vec{k}} \rangle'_{\vec{q}\to 0} = \frac{c(\nu)}{\epsilon \, M_{\rm Pl}} \, \frac{\tilde{\rho}}{H} \, (-k\,\eta_*)^{-\frac{3}{2}+\nu} \, P_{\sigma^{(2)}}(q) \, P_\pi(k) \, \epsilon^{(\pm 2)}_{ij}(\hat{q}) \, \hat{k}_i \hat{k}_j \,, \quad (4.81)$$

with $c(\nu)$ given in eq. (C.31). The expression of the squeezed $\langle \gamma\zeta\zeta \rangle$ bispectrum for a generic mass of the field σ_{ij} is

$$\langle \gamma^{(s)}_{\vec{q}} \, \zeta_{\vec{k}} \, \zeta_{-\vec{k}} \rangle'_{\vec{q}\to 0} = \frac{\mathcal{F}_\gamma(\nu)}{c_2^{2\nu}} \, \frac{\rho\,\tilde{\rho}}{\epsilon\, H^2} \, \left(\frac{q}{k}\right)^{\frac{3}{2}-\nu} P_\gamma(q) \; P_\zeta(k) \; \epsilon^{(s)}_{ij}(\hat{q}) \, \hat{k}_i \hat{k}_j \,, \quad (4.82)$$

being,

$$\mathcal{F}_\gamma(\nu) = -d_\gamma(\nu)\, c(\nu)\,. \quad (4.83)$$

The function $\mathcal{F}_\gamma(\nu)$ is plotted in Fig. 4.8 in the mass range below the Higuchi bound.

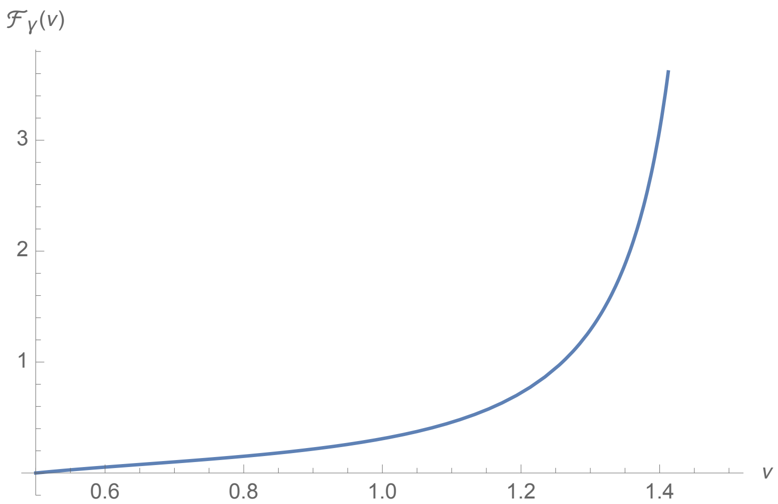

Figure 4.8: $\mathcal{F}_\gamma$ as a function of $\nu = \sqrt{\frac{9}{4} - \left(\frac{m}{H}\right)^2}$, in the range of masses below the Higuchi bound: $\nu \in [\frac{1}{2}, \frac{3}{2}]$.

The squeezed bispectrum diverges in the limit $\nu \to 3/2$. The situation is analogous to what we have already discussed at the end of sec. 4.4.2. The divergence should be trusted up to $m^2/H^2 \lesssim 1/N$. For smaller masses one must keep the number of e-folds finite. In the massless limit the computation of $\langle \gamma^{(s)}_{\vec{q}\to 0} \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle'$ can be done substituting the massless wavefunctions directly into eqs. (4.72) and (4.73).

$$\langle \gamma^{(s)}_{\vec{q}} \, \zeta_{\vec{k}} \, \zeta_{-\vec{k}} \rangle'_{\vec{q}\to 0} = \frac{3}{4} \, \frac{N_q}{c_2^3} \, \frac{\rho\tilde{\rho}}{\epsilon\, H^2} \, P_\gamma(q) \; P_\zeta(k) \; \epsilon^{(s)}_{ij} \hat{k}_i \hat{k}_j \,. \quad (4.84)$$

Scalar three-point function in any configuration. The contribution to the scalar bispectrum, due to a spinning particle, can be computed for any configuration of the momenta $\vec{k}_i$ without much effort if σ is massless and in the limit

$c_0 \ll 1$. Using the in-in formalism, the expression for $\langle\zeta\zeta\zeta\rangle$ is given by

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle' = H^3\int_{-\infty}^{0} d\eta \int_{-\infty}^{\eta} d\tilde{\eta}\left[\langle H_{\rm int}(\tilde{\eta})\, H_{\rm int}(\eta)\, \pi_{\vec{k}_1}\,\pi_{\vec{k}_2}\,\pi_{\vec{k}_3}\rangle + \langle \pi_{\vec{k}_1}\,\pi_{\vec{k}_2}\,\pi_{\vec{k}_3}\, H_{\int}(\eta)\, H_{\rm int}(\tilde{\eta})\rangle\right]$$
$$-H^3\int_{-\infty}^{0} d\eta \int_{-\infty}^{0} d\tilde{\eta}\, \langle H_{\rm int}(\eta)\, \pi_{\vec{k}_1}\,\pi_{\vec{k}_2}\,\pi_{\vec{k}_3}\, H_{\rm int}(\tilde{\eta})\rangle\,, \tag{4.85}$$

where $H_{\rm int} \equiv H_{\rm int}^{\pi\sigma} + H_{\rm int}^{\pi\pi\sigma}$ denotes the interaction hamiltonian (see eq. (4.42)). Performing the time integrals we get

$$B_\zeta(k_1,k_2,k_3) = -\frac{8\pi^4}{c_0{}^3}\,\frac{\rho\,\tilde{\rho}}{\epsilon H^2}\,\frac{\mathcal{I}(k_1,k_2,k_3)}{\Delta_\zeta{}^4}\,, \tag{4.86}$$

with $\Delta_\zeta^2 \equiv k^3 P_\zeta(k)/2\pi^2$ and $\mathcal{I}(k_1,k_2,k_3)$ given by

$$\mathcal{I}(k_1,k_2,k_3) = \frac{k_2+2k_3}{k_1^3\,k_2\,k_3\,(k_2+k_3)^2}\,\left((\hat{k}_1\cdot\hat{k}_3)^2 - \frac{1}{3}\right) + 5\ perms\,. \tag{4.87}$$

To analyze the shape of the bispectrum for general momentum configurations it is convenient to define the dimensionless shape function

$$S(k_1,k_2,k_3) \equiv (k_1\,k_2\,k_3)^2 \times \left(\mathcal{I}(k_1,k_2,k_3) + 5\ perms\right)\,. \tag{4.88}$$

The characteristic feature of this shape is its angular dependence due to the exchange of the higher spin particle: we expect a modulation that approaches the Legendre polynomial $P_2(\vec{k}_1\cdot\vec{k}_3)$ as we approach the squeezed configuration $k_1 \ll k_3$. One natural question that arises is how much the triangle has to be squeezed in order to see this behaviour. Figure 4.9 shows the shape of the total signal as a function of the angle between the modes $\vec{k}_1$ and $\vec{k}_3$, $\theta \equiv \cos^{-1}(\vec{k}_1\cdot\vec{k}_3)$, for a range of momentum configurations with fixed k_1/k_3. Notice that the signal does not deviate much from the $P_2(\cos\theta)$ in the range $k_1/k_3 \lesssim 0.5$. As the triangle approaches the equilateral shape ($k_1/k_3 \lesssim 1$) the angular dependence deviates from the pure Legendre behaviour : the peak around $\theta = 180°$ becomes prominent while its width shrinks. This happens because for $k_1/k_3 \simeq 1$ and $\theta \simeq 180°$ the triangle squeezes since $k_2 \to 0$ making $S(\vec{k}_1,\vec{k}_2,\vec{k}_3)$ diverge.

4.4.4 Trispectrum

Let us compute the trispectrum $\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle$ in the counter-collapsed limit assuming $\rho \ll \tilde{\rho}$ and $\mu = 0$, in such a way that the dominant contribution is given by Fig. 4.4b. In the counter-collapsed configuration, the four-point function simply expresses the correlation between a pair of two-point functions, which is induced by a low frequency $\sigma^{(s)}$ mode of momentum $q \ll k_1 \approx k_2,\ k_3 \approx k_4$. This long mode crossed its sound horizon much earlier than any of the k_a modes crossed the Hubble radius and can be considered as a fixed classical background, [96]. This allows us to use eq. (4.70) to compute the trispectrum:

$$\langle\zeta_{\vec{k}_1}\,\zeta_{-\vec{k}_1-\vec{q}}\zeta_{\vec{k}_3}\,\zeta_{-\vec{k}_3+\vec{q}}\rangle'_{\vec{q}\to 0} \simeq H^4\sum_{s=-2}^{+2}\frac{\langle\sigma_{\vec{q}}^{(s)}\,\pi_{\vec{k}_1}\,\pi_{-\vec{k}_1-\vec{q}}\rangle'\langle\sigma_{-\vec{q}}^{(s)}\pi_{\vec{k}_3}\,\pi_{-\vec{k}_3+\vec{q}}\rangle'}{P_{\sigma^{(s)}}(q)}\,. \tag{4.89}$$

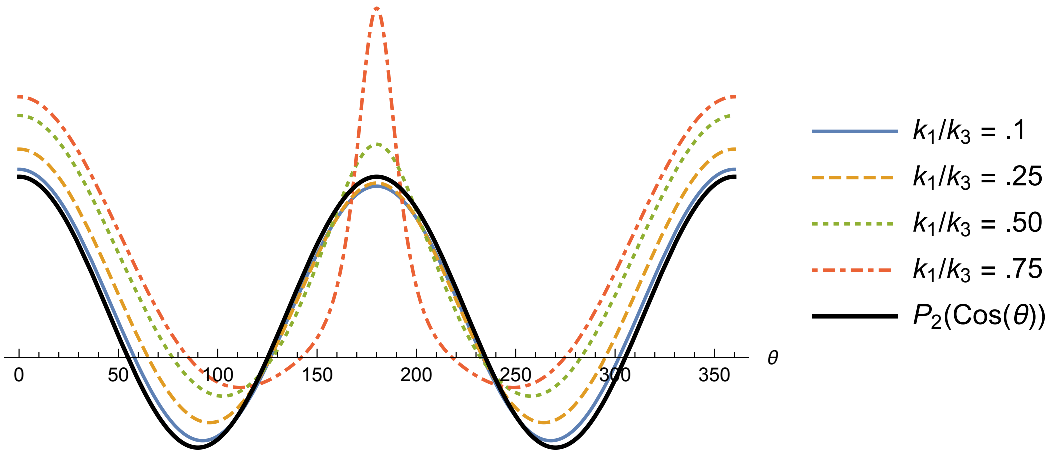

Figure 4.9: Bispectrum shape function $S(k_1, k_2, k_3)$ as a function of the angle $\theta = \cos^{-1}(\hat{k}_1 \cdot \hat{k}_3)$ for fixed ratio k_1/k_3. For easy comparison, the plot has been normalized such that the height difference between $\theta = 0°$ and $\theta = 90°$ of each curve is fixed to $\frac{3}{2}$.

The squeezed $\langle \sigma^{(s)} \pi\pi \rangle'$ 3-point function is given by, (see eqs. (4.75) and (4.81)),

$$\langle \sigma^{(s)}_{\vec{q}} \, \pi_{\vec{k}} \, \pi_{-\vec{k}-\vec{q}} \rangle'_{\vec{q}\to 0} = \frac{c(\nu)\, \tilde{\rho}}{\epsilon\, M_{\rm Pl}\, H} \, (-k\,\eta)^{-3/2+\nu} \; P_{\sigma^{(s)}}(q)\, P_\pi(k)\; \epsilon^{(s)}_{ij}(\hat{q}) \hat{k}_i \hat{k}_j \,, \qquad (4.90)$$

Plugging eqs. (4.90) and (4.76) into eq. (4.89), we get

$$\langle \zeta_{\vec{k}_1}\, \zeta_{-\vec{k}_1-\vec{q}} \zeta_{\vec{k}_3}\, \zeta_{-\vec{k}_3+\vec{q}} \rangle'_{\vec{q}\to 0} = \sum_{s=-2}^{+2} \frac{\mathcal{T}(\nu)}{c_s^{2\nu}} \left(\frac{\tilde{\rho}}{H\sqrt{\epsilon}} \right)^2 \left(\frac{q}{k} \right)^{3-2\nu} P_\zeta(q) P_\zeta(k_1) P_\zeta(k_3) \qquad (4.91)$$
$$\times \left(\epsilon^{(s)}_{ij}(\hat{q}) \hat{k}_{1,i} \hat{k}_{1,j} \right) \left(\epsilon^{(s)}_{ij}(\hat{q}) \hat{k}_{2,i} \hat{k}_{2,j} \right) .$$

The function $\mathcal{T}(\nu)$ is plotted in Fig. 4.10 in the mass range below the Higuchi bound and its expression is

$$\mathcal{T}(\nu) = \frac{2^{2\nu}\, \Gamma(\nu)^2\; c(\nu)^2}{\pi} \,, \qquad (4.92)$$

with $c(\nu)$ given by eq. (C.31). If σ is massless, $\nu = 3/2$ then the expression for the trispectrum simplifies to

$$\langle \zeta_{\vec{k}_1}\, \zeta_{-\vec{k}_1-\vec{q}} \zeta_{\vec{k}_3}\, \zeta_{-\vec{k}_3+\vec{q}} \rangle'_{\vec{q}\to 0} = \frac{9}{8\, c_2^3} \left(\frac{\tilde{\rho}}{H\sqrt{\epsilon}} \right)^2 P_\zeta(q)\, P_\zeta(k_1)\, P_\zeta(k_3) \times$$
$$\sum_{s=+}^{\times} \left(\epsilon^{(s)}_{ij}(\hat{q}) \hat{k}_{1,i} \hat{k}_{1,j} \right) \left(\epsilon^{(s)}_{ij}(\hat{q}) \hat{k}_{2,i} \hat{k}_{2,j} \right) , \qquad (4.93)$$

where we have also assumed that $c_2 \ll c_0 \approx c_1$, so that neglected the contributions from the helicity 0 and 1 modes.

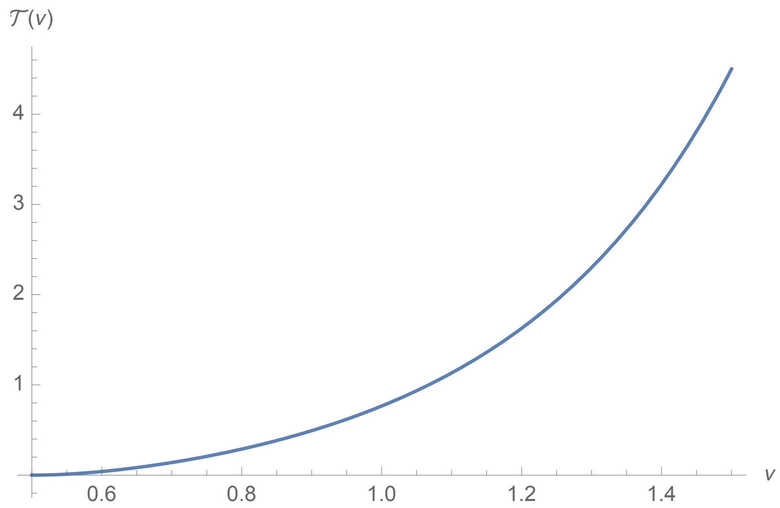

Figure 4.10: Trispectrum amplitude $\mathcal{T}(\nu)$ as a function of $\nu = \sqrt{\frac{9}{4} - \left(\frac{m}{H}\right)^2}$, in the range of masses below the Higuchi bound: $\nu \in [\frac{1}{2}, \frac{3}{2}]$.

Quadrupolar modulation of the scalar power spectrum. The same effect that enhances the scalar trispectrum induces, in the presence of super-horizon modes of σ, a quadrupolar modulation on the 2-point function of scalar perturbations (see Section 3.3),

$$P_\zeta(\vec{k}) = P_\zeta(k)\left[1 + \mathcal{Q}_{ij}\hat{k}_i\hat{k}_j\right] . \tag{4.94}$$

The matrix $\mathcal{Q}_{ij}$ can be computed using eqs. (4.70) and (4.90). We get

$$\mathcal{Q}_{ij} \simeq \frac{c(\nu)}{\epsilon M_{\rm Pl}}\,\frac{\tilde{\rho}}{H}\,(-k\eta)^{-3/2+\nu}\,\sum_s \sigma^{(s)}(\vec{q})\,\epsilon^{(s)}_{ij}(\hat{q})\,. \tag{4.95}$$

By averaging over all the super-horizon modes we get the expected (squared) amplitude $\mathcal{Q}^2$:

$$\begin{aligned}\mathcal{Q}^2 &= \frac{8\pi}{15}\langle \mathcal{Q}_{ij}\mathcal{Q}_{ij}\rangle \\ &= \frac{16}{15\pi}\mathcal{T}(\nu)\left(\frac{\tilde{\rho}}{\sqrt{\epsilon}H}\right)^2 \Delta_\zeta^2 \int_{q<H_0} dq\,\frac{q^2}{k^3}\left(\frac{k}{c_2\,q}\right)^{2\nu} \\ &= \frac{16}{15\pi}\frac{\mathcal{T}(\nu)}{3-2\nu}\left(\frac{\tilde{\rho}}{\sqrt{\epsilon}H}\right)^2 \frac{\Delta_\zeta^2}{c_2^{2\nu}}\left(\frac{H_0}{k}\right)^{3-2\nu} ,\end{aligned} \tag{4.96}$$

where in the last two lines we considered, for simplicity, only the contribution of the helicity-2 modes. This is justified if $c_2 \ll c_0 \approx c_1$. This result can be compared with the experimental limits that are set by the CMB: $\mathcal{Q} \lesssim 10^{-2}$ [1]. For future constraints see [123] (notice that the model studied in this paper differs from ours since the modulation of the scalar power spectrum is quadratic, and not linear, in the higher-spin field).

It is worthwhile stressing that in this chapter we never considered the possibility of a large background for σ_{ij}. This is justified provided σ has a non-zero mass squared so that the background anisotropy is diluted away and one is just left with the background due to quantum fluctuations.

4.5 Minimal non-Gaussianities and constraints on c_s

Since the minimal action (4.34) does not include any mixing between σ and π there are no tree level contributions to the π correlation functions from σ. However, non-Gaussianities in the π correlation functions are induced at loop level. Due to the smallness of the scalar fluctuation amplitude Δ_ζ, it is relevant to consider only the lowest order correlation functions—the power spectrum, the bispectrum and the trispectrum. The one-loop contributions to the three- and four-point functions of π due to the interactions with σ are represented by the diagrams on figure 4.11, where the cubic and quartic vertices are given by the interactions (4.37) and (4.38) respectively (while the quintic and sixtic vertices can be easily derived). In terms of the canonically normalized scalar field $\pi_c \sim \frac{H^2}{\Delta_\zeta}\pi$ these interactions are proportional to the small factors Δ_ζ and Δ_ζ^2 respectively, up to the factors of H needed to take care of dimensions. The three- and four-point functions generated by these loop diagrams can be, therefore, estimated as $NG_3 \equiv f_{NL}\Delta_\zeta \sim \Delta_\zeta^3$ and $NG_4 \equiv \tau_{NL}\Delta_\zeta^2 \sim \Delta_\zeta^4$. This would make them essentially unobservable[14] unless the speed of propagation of σ is parametrically small. In the latter case the loop contributions are enhanced by inverse powers of c_s, which could compensate for the smallness of Δ_ζ and lead to observably large non-Gaussianities. The experimental upper bounds on NG_3 and NG_4 thus translate to lower bounds on c_s. We are interested in the lower bound on c_s because the power spectrum of σ fluctuations and hence all the observational effects of the extra spinning field are enhanced when c_s is parametrically smaller than unity. Perturbativity of the theory and the observed Gaussianity of the scalar perturbations put limits on all other effects from the fields with spin, which we discussed in the previous section.

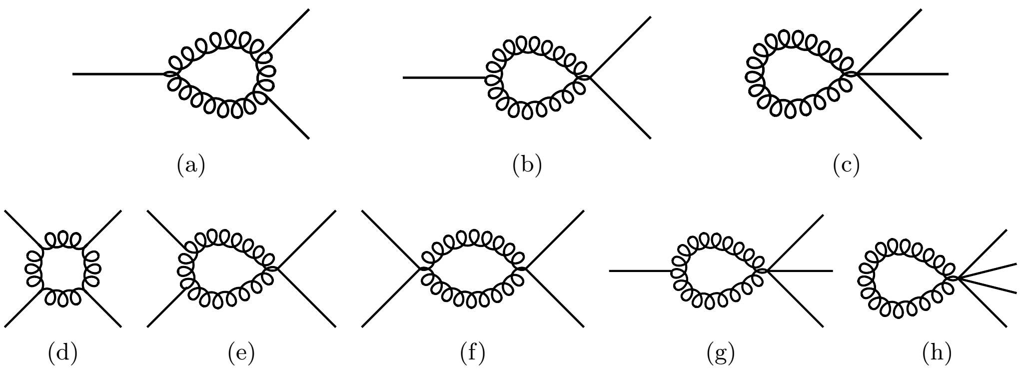

Figure 4.11: One-loop corrections to the three- and four-point function of π from the field σ. The straight lines stand for π while the curly ones represent the exchange of a σ field.

We can split every loop contribution in the IR and UV parts depending on the value of the loop momentum. We start with the UV contributions that correspond to the part of the loop integral where the loop momentum is parametrically higher than the external legs momenta. These contributions are UV divergent and are

[14] The normalisation of the scalar perturbations power spectrum is $\Delta_\zeta^2 \approx 2\cdot 10^{-9}$ and the current observations show the curvature perturbations to be Gaussian with precision $NG_3, NG_4 \lesssim 10^{-3}$ [120].

dominated by the modes with energies at the loop UV cutoff Λ. For the EFT description to make sense at energies of order Hubble the energy cutoff scale Λ has to be larger than H. Therefore it should be possible to represent such contributions by local self-interactions of the π field in flat space. The coefficients of these self-interaction operators can be estimated by evaluating the divergent part of the loop diagram in Minkowski space and cutting it off at the energy scale Λ. Given that the value of Λ depends on the UV completion of the theory we only demand that the non-Gaussianites induced by the Hubble scale modes do not violate observational constraints, i.e., we evaluate the loop contributions using the lowest possible cut-off, $\Lambda \sim H$. We also require the loop contribution to the power spectrum to be subdominant with respect to the tree level one in order for the loop expansion to make sense.

It is important to note that the minimal action (4.34) employs only the building blocks that are invariant under reparameterisation of the inflaton field $\psi \mapsto f(\psi)$ and hence possesses an extra symmetry with respect to the standard EFT of Inflation. The subclass of the inflationary theories with this symmetry is called khronon inflation [124]. All the π operators generated by loops of σ have to respect this symmetry and this has important consequences for the loop-induced non-Gaussianities by enforcing cancellations of the leading (and subleading) UV divergencies. Naive dimensional analysis predicts the loop diagrams on figure 4.11 to diverge quartically and they would generate operators like $\dot{\pi}^3 \Lambda^4$ and $\dot{\pi}^4 \Lambda^4$. However all khronon-inflation operators have in total more than one derivative per field. The $\dot{\pi}^3$ and $\dot{\pi}^4$ operators with one derivative per π field can not arise from these loops and we have checked that the leading quartic divergences do indeed cancel. Furthermore, in khronon inflation the self-interactions of π which is leading in the derivative expansion arise from the expansion in π of two covariant operators: $M_\alpha^2 (\nabla_\mu n^\mu - 3H)^2 \sim M_\alpha^2 (\partial\dot{\pi})^2$ and $M_\lambda^2 n^\mu n^\nu \nabla_\mu n^\rho \nabla_\nu n_\rho \sim M_\lambda^2 (\partial^2\pi)^2$. The coefficients M_α^2 and M_λ^2 of these operators have dimensions mass squared, hence these operators can be generated as the remaining quadratic divergence of the loop diagrams. Both these operators start quadratically in the Goldstone field. It means that the coefficients M_α^2 and M_λ^2 of these cubic and quartic operators are fixed in terms of the one-loop corrections to the two-point function of π.

Figure 4.12: One-loop corrections to the two-point function of π.

The one-loop corrections to the two-point function of π come from the two diagrams on figure 4.12. The absence of the usual two-derivative kinetic term for π in khronon inflation means that the leading quartic divergence in these diagrams cancels out. The remaining quadratically divergent contribution can indeed be represented by the higher derivative khronon operators

$$\frac{\Lambda^2}{c_s^3} (\partial^2\pi)^2 \qquad \text{and} \qquad \frac{\Lambda^2}{c_s^3} (\partial\dot{\pi})^2 \,, \tag{4.97}$$

with coefficients that are enhanced by a factor c_s^{-3} in the small c_s limit, $M_\alpha^2 \sim M_\lambda^2 \sim \Lambda^2/c_s^3$. For the perturbativity of the EFT we require these operators to be subdominant with respect to the tree level kinetic term at horizon crossing, i.e. when the derivatives are of order H, at least with $\Lambda \sim H$:

$$\frac{H^2}{c_s^3}\,(\partial^2\pi)^2\Big|_{\partial\sim H} \lesssim \frac{H^4}{\Delta_\zeta^2}\,(\partial\pi)^2\,. \tag{4.98}$$

Perturbativity imposes a lower bound on the sound speed of the σ field

$$c_s \gtrsim \Delta_\zeta^{2/3} \approx 10^{-3}\,. \tag{4.99}$$

Non-Gaussianities generated in the khronon inflation case were extensively studied in ref. [124]. The corresponding cubic and quartic operators can be schematically represented as $\frac{\Lambda^2}{c_s^3}\,(\partial^2\pi)^2\,\partial\pi$ and $\frac{\Lambda^2}{c_s^3}\,(\partial^2\pi)^2\,(\partial\pi)^2$, where some ∂ can stand for time derivatives. The loop contributions to the three- and four-point functions of π from the σ modes with energies around $\Lambda \sim H$ can be thus estimated as $NG_3 \sim \frac{\Delta_\zeta^3}{c_s^3}$ and $NG_4 \sim \frac{\Delta_\zeta^4}{c_s^3}$. Using the perturbativity bound (4.99) we see that these contributions can be at most of order $f_{\rm NL} \sim 1$ and $\tau_{\rm NL} \sim 1$ and are below the experimental constrains.

Given that the khronon symmetry constraints the coefficients of the quadratic divergencies, it turns out that the leading UV contribution in the small c_s regime is given by the sub-sub-leading logarithmically divergent terms. The local parts of these terms can be represented by the cubic and quartic π self-interactions with seven and eight derivatives respectively, which can be schematically written as

$$\frac{\log\left(\Lambda^2/H^2\right)}{c_s^5}\,\partial^2\dot\pi\,(\partial^2\dot\pi)^2 \qquad \text{and} \qquad \frac{\log\left(\Lambda^2/H^2\right)}{c_s^7}\,(\partial^2\dot\pi)^4\,. \tag{4.100}$$

The corresponding contributions to the bi- and tri-spectrum from the modes with frequencies around H are given by

$$NG_3 \sim \frac{\Delta_\zeta^3}{c_s^5} \qquad \text{and} \qquad NG_4 \sim \frac{\Delta_\zeta^4}{c_s^7}\,. \tag{4.101}$$

Let us turn to the IR contributions to the loop diagrams on figure 4.11. They can be estimated by taking the loop momentum comparable to the momentum of the external legs. It is easy to count the c_s dependence of the one-loop graphs. Each σ line contributes a factor of c_s^{-3} and each time derivative $\dot\sigma$ in the vertices gives an extra factor of c_s^2. The extra symmetry of the minimal action (4.34) also impacts the IR part of the loop contributions. Note that all the cubic interactions in the action (4.37) have an extra c_s^2 suppression either as an explicit factor or coming from the time derivative of σ. Since this is not the case for the quartic interactions (4.38) all the diagrams with quartic vertices are enhanced in the small c_s regime with respect to the corresponding cubic diagrams, where one quartic vertex is substituted by two cubic ones. In particular, the leading contribution to the bispectrum is given by diagram (4.11b) and can be estimated as

$$NG_3 \sim \frac{\Delta_\zeta^3 c_s^2}{c_s^6} = \frac{\Delta_\zeta^3}{c_s^4}\,. \tag{4.102}$$

Similarly, the leading loop contribution to the four-point function is given by the diagram (4.11f) and can be estimated as

$$NG_4 \sim \frac{\Delta_\zeta^4}{c_s^6} \,. \tag{4.103}$$

We see that these IR contributions are smaller than the UV contributions (4.101) and we shall use the latter in order to compare with the observational limits on the non-Gaussianity. The experimental upper bounds on NG_3 and NG_4 can be translated into the lower limit on the sound speed of σ field:

$$c_s \gtrsim \left(\frac{\Delta_\zeta^3}{NG_3}\right)^{1/5} \qquad \text{and} \qquad c_s \gtrsim \left(\frac{\Delta_\zeta^4}{NG_4}\right)^{1/7} \,. \tag{4.104}$$

These constraints have different parametric behaviour, nevertheless the resulting numerical bound on c_s is approximately the same (and it is stronger than the bound (4.99) from perturbativity alone):

$$c_s \gtrsim 10^{-2} \,. \tag{4.105}$$

Note that the bounds above apply to the propagation speed c_h of every polarisation, where we assumed that there is no splitting of the speeds. In the presence of splitting δc_s^2 the situation is somewhat more involved. Some of the cubic interactions (4.37) carry an explicit factor of c_s^2 and therefore are not suppressed if $c_s \approx 1$. This implies that for the models with $c_0^2 \lesssim c_s^2 \approx 1$ the non-Gaussianities induced by the helicity-0 mode running in the loops are larger than the estimates above. This translates into a stronger lower bound on c_0. We are, however not interested in such a regime, since the phenomenology is not qualitatively different with respect to the case where all c_hs are degenerate. Therefore, we focus on the cases when all the c_h are of the same order or when c_s is the smallest one.

The extra symmetry of the minimal action (4.34) means that it is possible to describe particles with arbitrary spin, scaling dimension, and propagation speeds in the context of the symmetries of khronon inflation. However, we have no reason to expect this additional symmetry to be present during inflation, and we indeed assume that the action for the Goldstone field π does not preserve it. It is therefore consistent and, in the absence of the khronon reparameterisation symmetry, even compulsory to add more interactions between σ and π in the EFT. At variance with the cubic and quartic terms in (4.37) and (4.38), the coefficients of these interactions are not predicted by the non-linear realisation of the Lorentz symmetry but are free parameters in the EFT approach. Therefore, comparing the induced non-Gaussianities with the experimental bounds does not allow one to constrain c_s, it only leads to constraints on the size of these new couplings. However, given the strong impact of the khronon symmetry of the minimal action on the loop-induced three- and four-point functions, one can worry that even reasonably weak σ-π interactions, which break the accidental khronon symmetry, induce loop non-Gaussianities, which are parametrically larger than the estimates (4.101). In this case, one should consider such interactions in order to put a realistic lower bound on c_s. For example, it is natural to expect that at order $\pi\sigma^2$ there are interactions that are given by the operators already present in the quadratic action (4.35) multiplied by δg^{00}:

$$\begin{aligned} m^2\, \delta g^{00}\, \Sigma^{\nu_1 \ldots \nu_s} \Sigma_{\nu_1 \ldots \nu_s} &\sim m^2\, \dot{\pi}\, (\dot{\sigma}^{i_1 \ldots i_s})^2 \,, \\ \delta g^{00}\, n^\mu n^\lambda\, \nabla_\mu \Sigma^{\nu_1 \ldots \nu_s} \nabla_\lambda \Sigma_{\nu_1 \ldots \nu_s} &\sim \dot{\pi}\, (\dot{\sigma}^{i_1 \ldots i_s})^2 \,, \\ \delta g^{00}\, c_s^2\, \nabla_\mu \Sigma^{\nu_1 \ldots \nu_s} \nabla^\mu \Sigma_{\nu_1 \ldots \nu_s} &\sim c_s^2\, \dot{\pi}\, (\partial_j \sigma^{i_1 \ldots i_s})^2 \,. \end{aligned}$$

It is straightforward to check that loop diagrams constructed with these cubic vertices contribute in the same way at comparable level when the loop frequency is taken to be of order H. They induce a correction to the quadratic action for π of the order $H^2\, c_s^{-3}\, \dot{\pi}^2$ and three- and four-point functions of size

$$NG_3 \sim \frac{\Delta_\zeta^3}{c_s^3} \qquad \text{and} \qquad NG_4 \sim \frac{\Delta_\zeta^4}{c_s^3} \,. \tag{4.106}$$

The correction to the quadratic action is subleading when the perturbativity constraint (4.99) holds, and the induced non-Gaussianities happen to be smaller than the ones induced by the minimal interactions that are given in equation (4.101). This shows that despite the cancellations the lower bound (4.105) is sufficient to render the theory perturbative and to satisfy observational constraints on non-Gaussianity even when the accidental khronon symmetry is broken by the interactions.

We see that in the regime of small c_s the effects of the spinning field σ are boosted due to enhancement in the amplitude of its fluctuations. In the previous section we discussed the main observational signatures of σ. Both the main signatures—changing the tensor mode power spectrum by mixing and inducing spin-dependent angular dependence in the squeezed limits of the scalar correlation functions—are also naturally enhanced by inverse powers of c_s. The constraints obtained in this section limit the extent of the parametric range where those observational signatures hold, but leave a rather large one.

Note that for any reasonable UV completion the loop cut-off is somewhat higher than H and the UV part of the loop dominates over the IR part. Hence, if during inflation there was a spinning field with parametrically small c_s the non-Gaussianities induced by its loops will first manifest themselves as the effects of the local self-interactions of π and will not carry any distinctive features. In order to see some features of the spinning field σ one should introduce a mixing between σ and observable tensor and scalar modes γ and π. These are the theories whose phenomenology we studied in the previous section.

4.6 Discussion and Outlook

In this chapter we have shown that the coupling with the inflaton foliation allows to violate the Higuchi bound: one can have particles with spin that do not decay outside the Hubble radius, while preserving the approximate shift symmetry of the inflaton. This opens a window on new phenomenology that we have only started to explore focussing on the case of spin 2. It should be straightforward to extend our analysis in various directions. One could include a non-trivial speed for the inflaton perturbations, consider the case of higher spin, or look at fermions. It would be interesting to understand whether it is possible to distinguish the effect of fermionic fields, although they cannot mix with scalar and tensors perturbations and play a role only at loop level. A more speculative investigation would be to violate the relation spin/statistics using the spontaneous breaking of Lorentz invariance. It would also be interesting to understand how one can observationally distinguish our setup from constructions based on different symmetry patterns [23, 61, 125], where unconventional light spin-2 particles do exist.

In this chapter we did not consider the possibility that higher spin fields have a sizeable background: in this case isotropy is broken as studied in [118, 119]. It would be interested to explore this possibility since it does not rely on any mixing with ζ and γ, but only on the interactions of σ with the foliation required to violate the Higuchi bound.

On the more theoretical side, it would be interesting to explore the possibility of modifying our Lagrangian by adding auxiliary fields that do not change the number of propagating degrees of freedom, but that can change and potentially enlarge the resulting phenomenological consequences. Microcausality, i.e. the commutativity of fields outside the lightcone, is not automatically preserved after these additions (while it is automatic without them) and it would be nice to have a straightforward procedure to include auxiliary fields.

Chapter 5

Solid Consistency

Consistency relations in single-field inflation are a consequence of adiabaticity: a long mode is locally unobservable and its effect can be removed by a coordinate redefinition [20, 21]. In the presence of additional fields, long-wavelength relative fluctuations (entropy modes) can be locally observed and CRs are violated. This common lore is challenged when one considers models of inflation with a different symmetry structure that cannot be described in the framework of the EFTI [19]. In this chapter we focus on Solid Inflation [56, 23] where the "stuff" that drives inflation has the same symmetry as an ordinary solid. Here the situation is different from the usual case. In solids there are five excitations: a scalar mode, the longitudinal phonon, two vector and two tensor perturbations. However, none of these modes is adiabatic. The absence of adiabaticity suggests at first sight that one cannot derive any CR.

This conclusion is too quick. As we are going to show, a CR for scalar modes can still be derived. The existence of CRs also with this different symmetry structure can be seen in this way. If one considers an isotropic perturbation of the solid, i.e. a dilation or compression, this will be adiabatic since in solids there is a unique relation between the pressure and the energy density, $p(\rho)$. Since the solid experiences all states of compression as the universe expands, this perturbation cannot be locally distinguished from the unperturbed evolution. We are going to verify this statement in Section 5.2 showing that an isotropic superposition of linear scalar modes is indeed adiabatic. This adiabatic mode is not standard: the two variables ζ and $\mathcal{R}$ do not coincide and they are both time dependent. This stems from the fact that the solids do not admit curved FRW solution, but only flat ones.

The existence of adiabatic modes imply CRs for the variable ζ. This does not happen for $\mathcal{R}$ since the diffeomorphism which removes the long mode cannot be written in terms of $\mathcal{R}$ in a model-independent way. In Section 5.3 we are going to verify the CRs in various cases, both when the short modes are inside the Hubble radius and outside. The conclusion is that, in models with the symmetry pattern of solid inflation, after reheating correlation functions satisfy the usual CRs once an average over the relative orientation between long and short modes has been done. We discuss the implications of this and open questions in Section 5.5.

Before proceeding, we briefly review the model in section 5.1 following references [23, 77], and we also show that the usual CRs are violated.

5.1 The model

The dynamics of the solid is described in terms of three scalar fields ϕ^I which parametrise the position of the elements of the solid: $\phi^I = x^I + \pi^I$. The action can be written in terms of SO(3)-invariant objects built out of the matrix $B^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J$. One can choose these invariants to be ([...] indicates a trace)

$$X \equiv [B]\,, \quad Y \equiv \frac{[B^2]}{[B]^2}\,, \quad Z \equiv \frac{[B^3]}{[B]^3}\,. \tag{5.1}$$

So the action, at lowest order in derivatives and including gravity, is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R + F(X,Y,Z)\right]. \tag{5.2}$$

Inspecting the stress-energy tensor of the solid we see that, on the background, T^μ_ν reduces to the standard $T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p)$ with

$$\rho = -F\,, \quad p = F - \frac{2}{3}XF_X \tag{5.3}$$

where the subscript X stands for partial derivative, and F and F_X are evaluated at the background values for our invariants: $X \to 3/a^2$, $Y \to 1/3$, $Z \to 1/9$. Therefore, to have near exponential inflation, we need

$$\epsilon = \frac{X\,F_X}{F} \ll 1\,. \tag{5.4}$$

The requirement of having a sufficiently long period of inflation, i.e. $\eta \ll 1$, forces also the second derivative F_{XX} to be small. One can show indeed that

$$\frac{F_{XX}X^2}{F_X X} = -1 + \epsilon - \frac{\eta}{2}\,. \tag{5.5}$$

The above equations imply that the solid action has a very weak dependence on X. This is not surprising, since X is the only invariant that is sensitive to the volume of the universe: for inflation to happen, the solid's energy should not change much if we dilate the solid by $\sim e^{60}$; this is only possible if the solid's dynamics do not depend much on X.

5.1.1 Action for Perturbations

We choose to work in the so-called Spatially Flat Slicing Gauge (SFSG) which is defined as the gauge where the spatial part of the metric is only perturbed by tensor modes. In this gauge perturbations consist in three phonons π^I which are the excitations of the solid and two transverse and traceless graviton helicities $\gamma_{ij}^{(s)}$. The fields π^I are part of a $SO(3)$ triplet, they can therefore be decomposed in terms of its longitudinal scalar and transverse vector component:

$$\pi^I = \frac{\partial_I}{\sqrt{-\partial^2}}\,\pi_L + \pi_T^I\,. \tag{5.6}$$

The action for perturbations is derived expanding eq. (5.2) following the standard procedure: working in the ADM parametrisation of the metric, eq. (2.5), one has to perturb the function F, solve for the lapse and shift and then plug them back into the action. At quadratic order in perturbations one gets

$$S^{(2)} = S^{(2)}_\gamma + S^{(2)}_T + S^{(2)}_L \tag{5.7}$$

$$S^{(2)}_\gamma = \frac{M_{\rm Pl}^2}{8}\int dt d^3x\, a^3\left[\dot\gamma_{ij}^2 - \frac{(\partial_k\gamma_{ij})^2}{a^2} - 4\epsilon H^2 c_T^2\gamma_{ij}^2\right] , \tag{5.8}$$

$$S^{(2)}_T = M_{\rm Pl}^2\int dt \int_{\vec k} a^3\left[\frac{k^2/4}{1+k^2/4\epsilon H^2a^2}|\dot\pi^i_T|^2 - \epsilon H^2 c_T^2 k^2|\pi^i_T|^2\right] , \tag{5.9}$$

$$S^{(2)}_L = M_{\rm Pl}^2\int dt \int_{\vec k} a^3\left[\frac{k^2/3}{1+k^2/3\epsilon H^2a^2}|\dot\pi_L + \epsilon H\pi_L|^2 - \epsilon H^2 c_L^2 k^2|\pi_L|^2\right] . \tag{5.10}$$

We can use the above actions to compute correlation functions. Late time observables must be expressed in terms of the gauge invariant ζ. In SFSG, it is given by

$$\zeta = \frac{\partial\pi}{3} + \mathcal{O}(\pi^2) , \tag{5.11}$$

where $\partial\pi \equiv \partial_i\pi^i$.[1]

5.1.2 Adiabaticity and Consistency Relations

The violation of the CRs in Solid Inflation has already been shown by directly computing the squeezed limit of the non-gaussian correlators $\langle\zeta\zeta\zeta\rangle$ and $\langle\gamma\zeta\zeta\rangle$ in [23, 77]. In this chapter, instead of re-proposing the calculations of the 3-point fuctions, we show that both scalar and tensor perturbations are non-adiabatic following the spirit of Section 3.1. This is equivalent to show the violation of the CRs.

Let us consider the effect of a rescaling of the coordinates $\xi^i = (\lambda\delta^i{}_j + \omega^i{}_j)x^j$, with $\omega^i_i = 0$ on the phonons. The first piece in the trasformation ($\propto\lambda$) is a dilation and it can be used to remove ζ, while the second piece ($\propto \omega^i{}_j$) consists in an anisotropic rescaling and mimics γ_{ij}. The rescaling excites the π^i fields,

$$\delta_\xi\pi^i = (\lambda\,\delta^i{}_j + \omega^i_j)x^j , \tag{5.12}$$

and $\partial_i\pi_j$ is a locally observable quantity. It leads to a long lasting super-horizon anisotropic stress. This allows us to relate the violation of the CRs to the squeezed limit of scalar correlation functions. Let us write the correlation of two short modes in the presence of long scalar π^i and tensor γ_{ij} fluctuations as [126]

$$\langle\zeta(\vec x_1)\zeta(\vec x_2)\rangle_{\gamma,\pi} = \xi(r) + \partial_i\pi_j\xi^{ij}_\pi(\vec r) + \gamma_{ij}\xi^{ij}_\gamma(\vec r) + \cdots \tag{5.13}$$

where $\vec r = \vec x_2 - \vec x_1$. Under a rescaling that removes either ζ or γ_{ij}, the phonons change respectively as $\partial_i\pi_j \to \partial_i\pi_j + \frac{\partial\pi}{6}\delta_{ij}$ and $\partial_i\pi_j \to \partial_i\pi_j + \frac{1}{2}\gamma_{ij}$. Thus we get the

[1] Notice that when we expand around the background $\phi^I = x^I$ one has $\partial_i\phi^I = \delta^I_i$, so that there is no distinction between capital and lower-case spatial indeces.

following corrections to the CRs

$$\text{scalar CR:} \quad \lim_{q\to 0} \frac{1}{P_\zeta(q)} \langle \zeta^s_{\vec q}\, \zeta(\vec x_1)\zeta(\vec x_2)\rangle - \text{CR} = \tfrac{1}{2}\left(\hat q_i \hat q_j - \delta_{ij}/3\right) \xi^{ij}_\pi(\vec r)\,, \tag{5.14}$$

$$\text{tensor CR:} \quad \lim_{q\to 0} \frac{1}{P_\gamma(q)} \langle \gamma^s_{\vec q}\, \zeta(\vec x_1)\zeta(\vec x_2)\rangle - \text{CR} = \frac{1}{2}\, \epsilon^{(s)}_{ij}\, \xi^{ij}_\gamma(\vec r)\,. \tag{5.15}$$

This confirm the explicit calculations of the squeezed limit of $\langle\zeta\zeta\zeta\rangle$ and $\langle\gamma\zeta\zeta\rangle$ of [77].

5.2 Long Isotropic Perturbation

The violation of the scalar CR, might be surprising at first glance. One might be tempted, indeed, to attribute this to the presence of isocurvature modes in addition to adiabatic ones, however, in Solid inflation there is only one scalar perturbation. Therefore the reason why a long perturbation is non-adiabatic must be different. From eq. (5.12) we see that the rescaling that gets rid of ζ generates also a term $\sim \partial_i \pi_j$ with $i \neq j$, that is responsible for the breaking of the scalar CR. Then a long ζ mode generates some anisotropy that does not decay, even on super-horizon scales. As we see from eq. (5.14), getting rid of the anisotropy, the RHS of the equation vanishes, therefore a spherically symmetric superposition of scalar modes should be adiabatic. Let us verify this statement.

5.2.1 Proof of adiabaticity

We have to prove that an isotropic superposition of scalar Fourier modes, can be brought back to the unperturbed solution via a suitable diffeomorphism.

We start from the SFSG. For the rest of this Section we will only be interested in scalar modes:

$$g_{ij} = a^2 \delta_{ij}\,, \qquad \phi^I = x^I + \pi^I\,. \tag{5.16}$$

The triplet π^I consists of a scalar, π_L, plus a transverse vector, π^I_T, which we neglect in the following. The constraint equations give the lapse $N = 1 + \delta N$ and the longitudinal part of the shift N_L [23]:

$$\delta N = -\frac{a^2 \dot H}{kH} \frac{\dot\pi_L - \dot H \pi_L/H}{1 - 3\dot H a^2/k^2}\,, \qquad N_L = \frac{-3a^2 \dot H \dot\pi_L/k^2 + \dot H \pi_L/H}{1 - 3\dot H a^2/k^2}\,. \tag{5.17}$$

Since in SFSG ζ is given by eq. (5.11), by performing the following time diff,

$$x^0 \to x^0 + \xi^0(t,\vec x)\,, \quad \text{with} \quad \xi^0(t,\vec x) = \frac{1}{3H}\partial\pi\,, \tag{5.18}$$

we go from SFSG to the $\zeta-gauge$, defined by the condition $\delta\rho = 0$, where ρ is the energy density. The spatial part of the metric now reads $g_{ij} = a(t)^2\,(1+2\,\zeta(t,\vec x))\,\delta_{ij}$, while one can write π_L as a function of ζ. Using eq.s (5.17) and (5.18), one can verify that the expression of δN in $\zeta-gauge$ is

$$\delta N = -\frac{k}{3}\frac{d}{dt}\left(\frac{\pi_L}{H}\right) \frac{1}{1 - 3\dot H \frac{a^2}{k^2}}\,. \tag{5.19}$$

In the limit $k/aH \to 0$ the time-dependence of π_L is slow-roll suppressed, see eq. (5.27), therefore $\delta N \to 0$ on super-horizon scales.

To reproduce the unperturbed FRW solution one has to eliminate the perturbation in the scalar fields ϕ^i. This can be done by a redefinition of the spatial coordinates

$$x^i \to x^i + \xi^i(t,\vec{x})\,, \quad \text{with} \quad \xi^i(t,\vec{x}) = -\pi^i(t,\vec{x})\,. \tag{5.20}$$

Since now the scalars are unperturbed, it is natural to call this Unitary Gauge (UG). In UG the shift vanishes on super-horizon scales

$$N_L = -\frac{d}{dt}\left(\frac{\pi_L}{H}\right)\,\frac{H}{1-3\dot{H}\frac{a^2}{k^2}}\,. \tag{5.21}$$

In this gauge, for long wavelength, $\delta N = N_L = \pi^i = 0$. However the spatial part of the metric is still perturbed

$$g_{ij} = a(t)^2\,\left(1 + 2\zeta(t,\vec{x})\delta_{ij} + \partial_i\partial_j\chi(t,\vec{x})\right), \quad \text{with} \quad \chi(t,\vec{x}) = -6\,\partial^{-2}\zeta(t,\vec{x})\,. \tag{5.22}$$

The perturbation is purely anisotropic, i.e. the volume is not perturbed because of the gauge condition $\delta\rho = 0$. Therefore if one considers a spherically symmetric superposition of scalar modes, the metric perturbations in eq. (5.22) average to zero

$$\int \frac{d^2\hat{k}}{4\pi}(2\zeta_{\vec{k}}\delta_{ij} - k_i k_j \chi_{\vec{k}}) = \int \frac{d^2\hat{k}}{4\pi}(2\zeta_{\vec{k}}\delta_{ij} - 6\hat{k}_i\hat{k}_j\zeta_{\vec{k}}) = 0\,. \tag{5.23}$$

This shows that a spherically symmetric superposition of scalar modes is adiabatic.

Notice that, if one works in $\zeta-gauge$, the transformation that eliminates a long-wavelength mode and goes back to FRW is a rescaling of the spatial coordinates: this is quite similar to the standard case of single-field inflation. However here the rescaling is time-dependent and adiabaticity requires an average over directions. In the next Section we will see that the adiabaticity gives rise to CRs: the only difference with the standard case is that they hold only after the spherical average. (The time dependence of the rescaling is immaterial because one is usually interested in correlation functions at equal time.)

Instead of using time-slices with $\delta\rho = 0$, one could use slices that are orthogonal to the 4-velocity of the solid. The perturbation of the spatial part of the metric is called ζ in the first case and $\mathcal{R}$ in the second. Contrary to the usual case, in Solid Inflation the variables ζ and $\mathcal{R}$ differ even on super-horizon scales. At linear level

$$\mathcal{R} = \frac{1}{\epsilon H}\frac{\dot{\zeta} + \epsilon\,H\,\zeta}{1 + k^2/3a^2H^2\epsilon}\,. \tag{5.24}$$

Since the two slicings do not coincide, one needs a time-diff to go from one to the other. This is the difference of the two time diff.s to go from SFSG to $\zeta-gauge$ and to $\mathcal{R}-gauge$ respectively:

$$\delta t_{\mathcal{R}\to\zeta} = \delta t_\zeta - \delta t_{\mathcal{R}} = \frac{\zeta}{H} - \frac{\mathcal{R}}{H} \simeq -\frac{\dot{\zeta}}{\epsilon H^2}\,, \tag{5.25}$$

where the last equation holds on super-horizon scales. The property that δN vanishes in $\zeta-gauge$ on large scales, eq. (5.19), will not hold in $\mathcal{R}-gauge$. This means that to go from $\mathcal{R}-gauge$ to the unperturbed FRW one has to supplement the rescaling of spatial coordinates with the time diff eq. (5.25). As we will discuss in the next Section this implies that the CR for $\mathcal{R}$ will contain an extra piece: the time diff induces a piece involving the time-derivative of the short modes.

5.2.2 Squeezed vs Super-Squeezed regimes

In taking the long-wavelength limit $k \to 0$ one ends up in the regime $k \ll aH\epsilon^{1/2}$. However one expects that the adiabaticity arguments above hold whenever k is comfortably outside the Hubble radius and in particular also in the intermediate regime $aH \gg k \gg aH\epsilon^{1/2}$. This is indeed the case. For example in this regime the expression for the lapse in $\zeta-gauge$ is

$$|\delta N_{\vec{k}}| = \left| \frac{d}{dt}\left(\frac{\zeta_{\vec{k}}}{H}\right) \frac{1}{1-3\dot{H}\frac{a^2}{k^2}} \right| < \left| \frac{d}{dt}\left(\frac{\zeta_{\vec{k}}}{H}\right) \frac{k^2}{a^2H^2}\frac{1}{3\dot{H}} \right| = \mathcal{O}\left(\frac{k^2}{a^2H^2}\right) . \tag{5.26}$$

(Notice that in the inequality above we are *not* assuming that the term proportional to $\dot{H}$ in the denominator dominates.) The same argument works for the shift: also in the intermediate regime the physical difference with the unperturbed solution are suppressed when the mode is superhorizon.

Therefore we expect the CR to hold both in the intermediate and in the super-squeezed, $k \ll aH\epsilon^{1/2}$, regime. However, to get analytical results one is forced to expand the solution of the constraints in different ways in the two regimes and therefore one has to assume one of the two regimes. This also applies to the expression of the wavefunction, which can be expanded in the two limits to give [23]

$$\pi_L(\tau,\vec{k}) \simeq \begin{cases} \mathcal{B}_k \left(1 + ic_Lk\tau + \frac{1}{3}c_L^2k^2\tau^2\right) e^{ic_Lk\tau} , & \text{if } |c_Lk\tau| \geq \epsilon , \\ \mathcal{B}_k \left[1 + \epsilon(1+c_L^2)\log(-c_Lk\tau)\right] (-c_Lk\tau_c)^{-5s/2-\eta/2-\epsilon} , & \text{if } |c_Lk\tau| \leq \epsilon , \end{cases} \tag{5.27}$$

with

$$\mathcal{B}_k = -\frac{3}{2}\frac{H}{M_{Pl}\, c_L^{5/2}\, \epsilon^{1/2}}\frac{1}{k^{5/2}} . \tag{5.28}$$

5.2.3 Adiabatic modes and the Weinberg theorem

Solid Inflation is an interesting exception to many general theorems on cosmological perturbations. Weinberg [50, 127] showed that, under quite general assumptions, one can always find an adiabatic mode which features *identical* and *time-independent* ζ and $\mathcal{R}$ on super-horizon scales. In the Solid case, ζ and $\mathcal{R}$ are neither equal (see eq. (5.24)) nor time-independent (both ζ and $\mathcal{R}$ have a slow-roll suppressed time-dependence on super-horizon scales). Of course by linearity these properties are not changed by the spherical average. In the original paper on Solid Inflation [23] (see also [128]) the authors addressed the issue of why a scalar Fourier mode does not comply with Weinberg analysis. The point is that the solid supports a large anisotropic stress, so that even in the long-wavelength limit the stress energy tensor remains anisotropic and thus locally distinguishable from the unperturbed solution: the mode is not adiabatic. The theorem [50, 127] assumes the decay of the anisotropic stress for $k \to 0$.

Here we are considering a spherical average of Fourier modes and the problem takes a somewhat different form. Indeed, as we discussed, the perturbation is now adiabatic, since the anisotropic stress averages to zero. However, this adiabatic mode is still different from the one of Weinberg. One can still check that the assumptions of the theorem do not hold: the $0i$ component of Einstein equations is not regular for $k \to 0$, since δu diverges in that limit. This does not allow to continue a homogeneous perturbation to a physical one at finite momentum.

This however looks rather technical. What is the physical reason why the adiabatic mode we are considering is different from the standard case? Why doesn't adiabaticity ensure that ζ is constant? In the standard case, the conservation of ζ and the relation $\zeta = \mathcal{R}$ can be understood from a linearized version of a spatially curved FRW. Neglecting short-wavelength perturbations and working at linear order, the curvature of a constant ρ slice is given by

$$ {}^{(3)}R = -\frac{4}{a^2}\partial^2\zeta. \tag{5.29} $$

Since for a curved FRW the spatial curvature $\kappa = a^2\ {}^{(3)}R/6$ is constant, super-horizon ζ fluctuations better be time-independent. Moreover for a curved FRW the surfaces of constant density are perpendicular to the 4-velocity so we need $\zeta = \mathcal{R}$. The adiabatic mode we are discussing in Solid Inflation does not have these properties and this is related to the fact that one does not have curved FRW solution in this model. [2] This can be understood in terms of symmetries: the internal symmetries of the ϕ^I is isomorphic to the symmetries of flat Euclidean space. This allows to write flat FRW solutions, but not spatially curved solutions, which have a different group of isometries.[3]

5.3 Angle-averaged Consistency Relations

In $\zeta-gauge$ a long mode averaged over the direction can be removed by a rescaling of the spatial coordinates. The derivation of the CR is very similar to the standard case, apart from the required angular average. In the case of the scalar 3-point function we get

$$ \int \frac{d^2\hat{q}}{4\pi}\langle\zeta_{\vec{q}}\zeta_{\vec{k}}\zeta_{-\vec{k}-\vec{q}}\rangle'_{q\ll k} = -\frac{d\log k^3 P_\zeta(k)}{d\log k} P_\zeta(q)P_\zeta(k)\,. \tag{5.30} $$

We stress that this result, in the limit of exact scale-invariance when the RHS of eq. (5.30) vanishes, was already discussed in [77].

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[2]Curved FRW solutions are allowed if we change the internal metric in the Lagrangian, see [129]. However, our argument for constancy of ζ in the standard case is based on the fact that curvature is a free parameter of the background solution. In the models discussed in [129] curvature is uniquely fixed in terms of energy density, so we don't expect ζ to be conserved.

[3]This also implies the usual curvature problem takes a somewhat different flavour in this class of models.

5.3.1 Check of the consistency relation for $\langle\zeta\zeta\zeta\rangle$

Let us check the CR eq. (5.30). In Solid Inflation the quadratic action is $\mathcal{O}(\epsilon)$ while the cubic action is $\mathcal{O}(\epsilon^0)$. Thus the 3-point function is slow-roll *enhanced*, $f_{NL} \sim \langle\zeta^3\rangle/\langle\zeta^2\rangle^2 = \mathcal{O}(\epsilon^{-1})$ [23]. Since the tilt of the 2-point function outside the horizon is $\mathcal{O}(\epsilon)$, a non-trivial (in the sense of non-zero) verification of eq. (5.30) in this regime would require taking into account corrections to the leading bispectrum at second-order in slow-roll. This is quite challenging. We content ourselves with the first order correction: at this order the two sides of eq. (5.30) should vanish when all the modes are outside the horizon. When the short modes are inside the horizon the scale-dependence of the spectrum is not slow-roll suppressed and the LHS of eq. (5.30) should thus be non-zero.[4] The check will be done in the regime $k \gg aH\epsilon^{1/2}$ for all the modes.

To do this we compute the cubic Lagrangian in SFSG up to $\mathcal{O}(\epsilon)$, calculate the bispectrum and then transform to ζ*−gauge*. The $\mathcal{O}(\epsilon)$ corrections to the bispectrum of three super-horizon modes was studied in detail in [88]. Thus, we skip most of the technical steps. However, we identify a missing term in [88] that is important for the CR to work. The *in-in* calculation up to this order consists of the sum of three pieces. Schematically,

$$\langle\zeta\zeta\zeta\rangle \sim \mathcal{L}^{(3)}_{\mathcal{O}(1)} \times \pi(\tau,\vec{k})_{\mathcal{O}(1)} + \mathcal{L}^{(3)}_{\mathcal{O}(1)} \times \pi(\tau,\vec{k})_{\mathcal{O}(\epsilon)} + \mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)} \times \pi(\tau,\vec{k})_{\mathcal{O}(1)} \,, \tag{5.31}$$

where $\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon^n)}$ and $\pi(\tau,\vec{k})_{\mathcal{O}(\epsilon^n)}$ are respectively the cubic Lagrangian and the wave-functions evaluated at n^{th} order in slow-roll. The leading cubic Lagrangian, $\mathcal{L}^{(3)}_{\mathcal{O}(1)}$, was calculated in [23] (eq. (D.2)). In Fourier space and in the squeezed limit it reduces to

$$\mathcal{L}^{(3)}_{\mathcal{O}(1)}\Big|_{\text{squeezed}} = -\frac{8}{81} F_Y \left(1 - 3\cos^2(\theta)\right) \pi_{L,\vec{q}}\, \pi_{L,\vec{k}}\, \pi_{L,-\vec{q}-\vec{k}} \,, \tag{5.32}$$

where θ is the relative angle between the long and the short modes. The above expression gives zero when one takes the angular average. This means there is no contribution $\mathcal{O}(\epsilon^{-1})$ to the LHS of eq. (5.30). Notice that the cancellation after angular average holds independently of the explicit form of the wavefunctions. Therefore, the second term of eq. (5.31) vanishes and only the last term is relevant for checking the CR.

Cubic scalar Lagrangian at $\mathcal{O}(\epsilon)$. At first look, expanding eq. (D.1) of [23] up to first order in slow-roll seems like a formidable task. Since Y and Z in (5.1) are defined in such a way that they start from second order in perturbations, one has to expand

$$\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)} = F_X \delta X^{(3)} + F_{XX}\delta X^{(1)}\delta X^{(2)} + \frac{1}{6} F_{XXX} (\delta X^{(1)})^3 + F_{XY}\delta X^{(1)}\delta Y^{(2)} + (F_Y \delta Y^{(3)})_{\mathcal{O}(\epsilon)} + (Y \leftrightarrow Z), \tag{5.33}$$

[4] For a discussion of CRs in standard inflation when the short modes are inside the horizon see [130].

where subscripts on F denote partial derivatives. However, there are several simplifications [88]. As we will see, one only needs the SFSG deformation matrix B^{IJ} at zeroth order in slow-roll parameters. This allows neglecting N^i and δN (which are slow-roll suppressed in the regime we are considering):

$$B^{IJ} = \frac{1}{a^2}\left(\delta^{IJ} + \partial^I\pi^J + \partial^J\pi^I + \partial_k\pi^I\partial_k\pi^J\right) - \dot{\pi}^I\dot{\pi}^J + \mathcal{O}(\epsilon) \tag{5.34}$$

Therefore, δX terminates at quadratic order, apart from slow-roll suppressed corrections

$$\delta X = \delta[B] = \frac{3}{a^2}\left(\frac{2}{3}\partial\pi + \frac{1}{3}\partial_i\pi_j\partial_i\pi_j - \frac{a^2}{3}\dot{\pi}_i^2\right) + \mathcal{O}(\epsilon). \tag{5.35}$$

Given that derivatives of F with respect to X are slow-roll suppressed, there is no contribution from $F_X\delta X$ to $\mathcal{O}(\epsilon)$ cubic Lagrangian. Another simplification is that if at some order in perturbations the corrections to $[B^n]$ involve at most $m \leq n$ of the B^{IJ} factors, then

$$\delta\left(\frac{[B^n]}{[B]^n}\right) = \delta\left(\frac{[B^m]}{[B]^m}\right), \quad \text{for all } n \geq m\ . \tag{5.36}$$

This implies that

$$\delta Y^{(2)} = \delta Z^{(2)}, \tag{5.37}$$

and since slow-roll corrections to B^{IJ} start at $\mathcal{O}(\pi^2)$

$$\delta Y^{(3)}_{\mathcal{O}(\epsilon)} = \delta Z^{(3)}_{\mathcal{O}(\epsilon)}. \tag{5.38}$$

Therefore, the following combinations appear in (5.33)

$$(F_Y + F_Z)\delta Y^{(3)}_{\mathcal{O}(\epsilon)}, \quad (F_{XY} + F_{XZ})\delta X^{(1)}\delta Y^{(2)}. \tag{5.39}$$

However $F_Y + F_Z = \mathcal{O}(\epsilon)$ and $F_{XY} + F_{XZ} = \mathcal{O}(\epsilon^2)$, so these terms are negligible. Using

$$F_{XX} = -\frac{a^4}{9}\epsilon F, \quad F_{XXX} = \frac{2a^6}{27}\epsilon F, \quad F = -3M_{\rm Pl}^2H^2, \tag{5.40}$$

one gets

$$\begin{aligned}\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)} \quad = \quad & \epsilon M_{Pl}^2H^2a^3\left[\frac{2}{3}(\partial\pi)\partial_j\pi^k\partial_j\pi^k - \frac{8}{27}(\partial\pi)^3\right] - \frac{2}{3}\epsilon M_{Pl}^2H^2a^2(\partial\pi)\dot{\pi}_i^2 \\ & + \frac{4}{27}(F_Y + F_Z)a^2\left[(\partial\pi)\dot{\pi}_i^2 - 3\partial_i\pi^j\dot{\pi}^i\dot{\pi}^j\right]\ . \end{aligned} \tag{5.41}$$

The terms with time derivatives in this equation are absent from eq. (35) of [88]. Note that the appearance of the combination $F_Y + F_Z$ on the second line is a consequence of (5.36) since time-derivatives appear in B^{IJ} starting from quadratic order. The angular average of this term is zero, hence it does not contribute to CR while the last term on the first line does contribute.

Field redefinition. Since we are interested in the bispectrum of ζ at $f_{NL} = \mathcal{O}(1)$, we need to find the relation between ζ and π at quadratic order and to zeroth order in ϵ. We start from B^{IJ} in SFSG given in (5.34). The last term can be neglected because in the squeezed limit at least one of the two π's will be out of the horizon with a slow-roll suppressed time evolution. The assumption of spherical symmetry simplifies the expression,

$$B^{IJ} = \delta^{IJ} X(t) = \frac{\delta^{IJ}}{a^2}\left(1 + \frac{2}{3}(\partial\pi) + \frac{1}{9}(\partial\pi)^2\right). \tag{5.42}$$

Now we perform the time diffeomorphism that leads to the $\zeta-gauge$ (the analogue of eq. (5.18) but now at second order), where $X(t)$ takes its unperturbed value

$$X(t + \xi^0(t, \boldsymbol{x}); \boldsymbol{x}) = \bar{X}(t) = a^{-2}. \tag{5.43}$$

At non-linear order the spatial part of the $\zeta - gauge$ metric is defined as $g_{ij} = a^2 e^{2\zeta}\delta_{ij}$. Therefore up to quadratic order in π, we obtain

$$\zeta = H\xi^0 = \frac{1}{3}\partial\pi - \frac{1}{18}(\partial\pi)^2 + \frac{1}{9H}(\partial\dot{\pi})(\partial\pi) + \mathcal{O}((\partial\pi)^3). \tag{5.44}$$

One can now put together the *in-in* computation, using the Lagrangian in the first line of eq. (5.41), with the definition of ζ, eq. (5.44), to get

$$\int \frac{d^2\hat{q}}{4\pi}\langle\zeta_{\vec{q}}\zeta_{\vec{k}}\zeta_{-\vec{q}-\vec{k}}\rangle'_{q\ll k} = \frac{1}{27}\int\frac{d^2\hat{q}}{4\pi}\langle(\partial\pi)_{\vec{q}}(\partial\pi)_{\vec{k}}(\partial\pi)_{-\vec{k}-\vec{q}}\rangle'_{q\ll k} - 2P_\zeta(q)P_\zeta(k) + \frac{1}{H}P_\zeta(q)\dot{P}_\zeta(k). \tag{5.45}$$

When all the modes are outside the horizon the last term on the RHS of eq. (5.45) is slow-roll suppressed and can be neglected. A straightforward calculation shows that the other two terms cancel each other confirming that

$$\int \frac{d^2\hat{q}}{4\pi}\langle\zeta_{\vec{q}}\zeta_{\vec{k}}\zeta_{-\vec{k}-\vec{q}}\rangle'_{q\ll k} = \mathcal{O}(\epsilon), \tag{5.46}$$

as implied by eq. (5.30).[5] When the short modes are inside the horizon the cancellation between the first two terms on the RHS of eq. (5.45) still holds, but now the last term is non-negligible since the time dependence is not slow-roll suppressed in this regime. One has

$$\int \frac{d^2\hat{q}}{4\pi}\langle\zeta_{\vec{q}}\zeta_{\vec{k}}\zeta_{-\vec{k}-\vec{q}}\rangle'_{q\ll k} = \frac{1}{H}P_\zeta(q)\dot{P}_\zeta(k) = -\frac{d\log k^3 P_\zeta(k,\tau)}{d\log k}P_\zeta(q)P_\zeta(k,\tau), \tag{5.47}$$

where in the last passage we used that the power spectrum is of the form $P_\zeta = k^{-3}f(k\tau)$ as dictated by scale invariance, up to corrections of order slow-roll. The CR eq. (5.30) is verified.

[5] It is challenging to perform the calculation at next order in slow-roll, to test the CR. The cancellation after angular average in eq. (5.45) can be seen only after the explicit *in-in* integral (in contrast to what happens at leading order, eq. (5.32)). This means that, at higher order, we should compute time integrals involving the wavefunctions at $\mathcal{O}(\epsilon)$.

5.3.2 Check of the consistency relation for $\langle\zeta\gamma\gamma\rangle$

One novel feature of our analysis is that the (spherically averaged) adiabatic modes of solid inflation feature a time-dependent ζ outside the horizon. Since this time dependence arises at $\mathcal{O}(\epsilon)$ it would be nice to check the CR at this order. As discussed this is quite challenging for $\langle\zeta\zeta\zeta\rangle$, but it is doable for $\langle\zeta\gamma\gamma\rangle$, where the scalar mode is taken to be long. The leading term in $\langle\zeta\gamma\gamma\rangle$ is $\mathcal{O}(1)$ [77], so one just needs to do the calculation including the first-order slow-roll corrections. The leading Lagrangian has the form (see eq. (A.7) of [77])

$$\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon^0)} \propto -\frac{1}{3}(\partial\pi)\gamma_{ij}\gamma_{ij} + \gamma_{ij}\gamma_{jk}\partial^k\pi^i. \tag{5.48}$$

It averages to zero in the squeezed limit $\pi_{L\,\vec{q}\to 0}$ independently of the wavefunctions. Thus we do not need to consider the slow-roll corrections to the wavefunctions. We go directly to the computation of $\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)}$.

Cubic Lagrangian at $\mathcal{O}(\epsilon)$. There are two terms which contribute to $\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)}$. The first arises from the expansion of the function $F(X, Y, Z)$, while the second is from the Einstein-Hilbert action. The expansion of F gives

$$\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)} \supset F_X\delta X + F_Z\delta Z + \frac{F_{XX}}{2}\delta X^2 + F_{XY}\delta X\delta Y + F_{XZ}\delta X\delta Z \tag{5.49}$$

where,

$$\delta X = a^{-2}\left[2(\partial\pi) - 2\gamma_{ij}\partial_i\pi^j + \frac{1}{2}{\gamma_{ij}}^2 + \gamma_{ij}\gamma_{jk}\partial_k\pi^i\right], \tag{5.50}$$

$$\delta Y = \left[-\frac{4}{9}\gamma_{ij}\partial_i\pi^j + \frac{1}{9}{\gamma_{ij}}^2 + \frac{2}{3}\gamma_{ij}\gamma_{jk}\partial_k\pi^i - \frac{2}{9}{\gamma_{ij}}^2(\partial\pi)\right], \tag{5.51}$$

$$\delta Z = \left[-\frac{4}{9}\gamma_{ij}\partial_i\pi^j + \frac{1}{9}{\gamma_{ij}}^2 + \frac{8}{9}\gamma_{ij}\gamma_{jk}\partial_k\pi^i - \frac{8}{27}{\gamma_{ij}}^2(\partial\pi)\right]. \tag{5.52}$$

One gets

$$\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)} = -3\epsilon a^2 M_{Pl}^2 H^2\left[\gamma_{ij}\gamma_{jk}\partial_k\pi^j - \frac{1}{3}\gamma_{ij}^2(\partial\pi)\right]. \tag{5.53}$$

This gives zero after the angular average. The contribution which arises from the Einstein-Hilbert action (plus the appropriate boundary terms) is

$$\begin{aligned}\mathcal{L}^{(3)}_{\mathcal{O}(\epsilon)} &\supset a^3\left[N\,{}^{(3)}R + N^{-1}(E_{ij}E^{ij} - E^2)\right]_{\mathcal{O}(\epsilon)\pi\gamma\gamma} \\ &= -\frac{a^2}{8}\delta N\left[{\gamma'_{ij}}^2 + (\partial_l\gamma_{ij})^2\right] - \frac{a^3}{4}\gamma'_{ij}\partial_k\gamma_{ij}N^k.\end{aligned} \tag{5.54}$$

The *in-in* calculation can be done separately in the intermediate regime $aH \gg q \gg aH\epsilon^{1/2}$ and in the super-squeezed regime $q \ll aH\epsilon^{1/2}$. [6] In both regimes the *in-in*

[6] Notice that eq. (5.54), evaluated in the super-squeezed limit is $\mathcal{O}(1)$. But δN, $N_L \propto \dot{\pi}_L$ which has a slow-roll suppressed time dependence and so the final expression of the bispectrum is $\mathcal{O}(\epsilon)$. Notice also that, naively, the term proportional to the shift seems to cancel when one performs the angular average. However in the super-squeezed limit this term would give a bispectrum $\propto 1/q^4$ for $q \to 0$ (see the expression for N^i in eq. (5.17)). This leading behaviour cancels, even before taking the angular average. The subleading contribution has the correct $1/q^3$ dependence and does contribute to the angular average.

computation of the 3-point function gives

$$\frac{1}{3}\int \frac{d^2\hat{q}}{4\pi}\langle(\partial\pi)_{\vec{q}}\gamma^s_{\vec{k}}\gamma^s_{-\vec{k}}\rangle_{q\ll k} = 2\epsilon\, P_\zeta(q)\, P_\gamma(k)\,. \tag{5.55}$$

Tensor modes at quadratic order in perturbations. The final expression of the bispectrum is given once one considers the contribution coming from the time diff that has to be performed to go from SFSG to $\zeta-gauge$. This changes the tensor perturbations at quadratic level (see eq. (A.8) of [20]). The interesting part (for us) is

$$\gamma_\zeta = \gamma_\pi + \frac{1}{H}\dot{\gamma}_\pi\frac{(\partial\pi)}{3}\,, \tag{5.56}$$

where γ_ζ and γ_π denote tensor perturbations respectively in $\zeta-gauge$ and SFSG. This adds a contribution to the bispectrum: $\langle\zeta\gamma_\zeta\gamma_\zeta\rangle = \langle\zeta\gamma_\pi\gamma_\pi\rangle + \frac{2}{3H}\langle\zeta\partial\pi\gamma_\pi\dot{\gamma}_\pi\rangle$, which is given to leading order by

$$\frac{1}{H}\,P_\zeta(q)\,\frac{d}{dt}P_\gamma(k) = -2\epsilon(1+c_L^2)P_\zeta(q)P_\gamma(k)\,. \tag{5.57}$$

Eqs. (5.55) and (5.57) give

$$\int\frac{d^2\hat{q}}{4\pi}\langle\zeta_{\vec{q}}\gamma^s_{\vec{k}}\gamma^s_{-\vec{k}}\rangle_{q\ll k} = -2c_L^2\epsilon P_\zeta(q)P_\gamma(k) = -\frac{d\log k^3P_\gamma(k)}{d\log k}P_\zeta(q)P_\gamma(k)\,. \tag{5.58}$$

This is exactly what is predicted by the CR. In conclusion, this computation confirms that CRs after spherical average hold even at slow roll order, i.e. when the time dependence of the long mode cannot be neglected.

5.3.3 Consistency relation for $\mathcal{R}$?

There are two differences if one wants to use the variable $\mathcal{R}$ instead of ζ. First, as discussed above, starting from $\mathcal{R}-gauge$ one needs an extra time diff to map a long mode into an unperturbed FRW. This changes the CR and introduces a time derivative of the short mode 2-point function. Moreover on super-horizon scale, in both squeezed and super-squeezed regimes, the relation between $\mathcal{R}$ and ζ depends on c_L and therefore is non-universal,

$$\mathcal{R}\simeq -c_L^2\zeta. \tag{5.59}$$

Hence, the spatial rescaling (5.20), which is determined in terms of ζ, will be model-dependent when written in terms of $\mathcal{R}$. This means that in this gauge we are not going to be able to write an explicit CR, i.e. a model independent relation among correlation functions.

For instance, consider the squeezed limit of $\langle\mathcal{R}\gamma\gamma\rangle$. One needs to go from $\zeta-gauge$ to $\mathcal{R}-gauge$ with the time diff $\delta_{\mathcal{R}\to\zeta}$, see eq. (5.25). This implies

$$\begin{aligned}\int\frac{d^2\hat{q}}{4\pi}\langle\mathcal{R}_{\vec{q}}\gamma^s_{\vec{k}}\gamma^s_{-\vec{k}}\rangle'_{q\ll k} &= n_t\langle\mathcal{R}_{\vec{q}}\zeta_{-\vec{q}}\rangle'\langle\gamma_{\vec{k}}\gamma_{-\vec{k}}\rangle' + \frac{1}{\epsilon\,H^2}\langle\mathcal{R}_{\vec{q}}\dot{\zeta}_{-\vec{q}}\rangle'\frac{d}{dt}\langle\gamma_{\vec{k}}\gamma_{-\vec{k}}\rangle' = \\ &= 2\epsilon\left(1-\frac{(c_L^2+1)^2}{c_L^2}\right)P_{\mathcal{R}}(q)P_\gamma(k)\,;\end{aligned} \tag{5.60}$$

where we used the fact that, at first order in slow roll, $\dot{\zeta}_{\vec{q}} = -H(1+c_L^2)\,\epsilon\,\zeta_{\vec{q}}$. This form of CR is not very useful since it contains parameters that make the relation model-dependent. However, after reheating $\zeta = \mathcal{R}$ and they are both time-independent. Therefore for observational purposes the CRs take the form of eqs. (5.30) and (5.58).

5.4 Solid Inflation as a spin-2 multiplet

In Solid Inflation, tensor modes have a non-vanishing mass. This is a consequence of the fact that gravitational waves are propagating inside a medium. Being massive, we expect that all the five helicities of the tensor field propagate. This is exactly what happens, we know indeed that, in Solid Inflation, perturbations (tensor, vector and scalars) on super-horizon scales are physical. This implies that in the limit $k \to 0$ they must recombine to form a spin-2 multiplet [7]. Taking a closer look at the quadratic action eq. (5.7) evaluated on super-horizon scales we see that this is the case. Notice for instance, that all the modes have the same mass. In fact, evaluating eqs. (5.8) - (5.10) in the ultra-squeezed limit regime, on can check that the (squared) mass of tensors and vectors is $4\epsilon H^2 c_T^2$. Taking into account the relation between c_T and c_S, it easy to verify that the same holds for scalars at any order in slow-roll [8]. The non-vanishing mass makes ζ and γ evolve on super-horizon scales. However, being different helicities of the same spin-2 multiplet, they evolve in the same way. This must be true even during re-heating, when the solid "melts". This implies that *the tensor-to-scalar ratio cannot be changed by reheating.* Notice however that this does not imply anything on the relation tensor/scalar tilt: they depend not only on the time-dependence (which is the same) but on horizon-crossing quantities. The same holds for the normalisation.

The fact that the physical (not gauge artifact) curvature perturbation ζ is nothing but the helicity-zero component of a spin-2 representation of $SO(3)$, can be used to understand why the scalar CRs hold after spherical average: from the discussion of Section 4.4.1 follows that the squeezed $\langle\zeta\zeta\zeta\rangle$ can only be in the form of eq. (4.46), implying that the physical bispectrum vanishes after angular average at any order in slow-roll.

5.5 Discussion and Outlook

It is remarkable that in Solid Inflation one has evolution outside the horizon, even when the mode is spherically averaged and therefore adiabatic. The evolution is slow-roll suppressed during inflation, but it will become significant during reheating unless some extra assumption about this phase is made. (For instance the limit of instantaneous reheating was taken in [23].) Therefore one cannot even relate the normalization of the spectrum to the parameters during inflation, since the change during reheating may be significant. From this point of view, it is quite surprising that one can derive model-independent (and reheating independent) relations like eq. (5.30).

[7] Notice that this is different from standard single-filed inflation. Being super-horizon modes non-physical, there cannot be any recombination for $k \to 0$.

[8] To all order in ϵ and η, the relation among c_T and c_L is $c_T^2 = \frac{3}{4}\left(1 + c_L^2 - \frac{2}{3}\epsilon + \frac{1}{3}\eta\right)$, [23].

Physically the angular-averaged consistency relations we found imply, within the symmetry pattern of solid inflation, that local (angle-independent) non-Gaussianity cannot be generated. (As in the usual case, the tiny non-Gaussianity in the squeezed limit, which is implied by the CR should be considered in some sense “unobservable", see for example [79].) This however does not prevent large non-Gaussianity in the squeezed limit, as long as it vanishes after angular average.

The consistency relation we discussed here is the analogue of the original Maldacena's CR. A natural question would be to look for other CRs, analogue of the conformal ones in standard inflation [115, 63]. Another natural question is to try and apply these methods to other symmetry breaking patterns for instance Gauge-flation [109] or Supersolid Inflation [61, 125]. In all these cases however there are multiple scalar excitations: similarly to standard multifield inflation, one does not expect any CR, unless further conditions are imposed.

Finally, even though our explicit checks were limited to the lowest derivative solid action, the CRs are just based on symmetry considerations, and therefore they are robust when one considers higher derivative operators.

Chapter 6

Conclusions

Inflation is arguably the most promising observational window onto very high-energy physics. By amplifying quantum fluctuations of fields lighter than the Hubble scale, inflation acts like an extremely powerful particle collider. The spectrum of light fields produced by the inflationary background and their interactions are encoded in the statistics of the primordial fluctuations. Understanding how additional fields affects the primordial correlation functions and how to extract informations about these changes in the initial conditions of the universe is one of the main goals of modern cosmology.

The minimal set of light degrees of freedom during inflation is given by a scalar mode, the Goldstone of broken time-translations, and the graviton. The addition of extra light scalars has been studied for many years. They modify the predictions for the scalar spectrum and their presence gives unmistakeable signatures: isocurvature perturbations and non-Gaussianity of the local form [21].

On the other hand, the study of particles with spin is much more recent [14, 101]. Nonetheless, the phenomenology they give rise could be as rich as the scalar one. In this thesis, we made progress in characterising the signatures on the inflationary correlation functions which involve also tensor perturbations, besides scalar ones. Very peculiar signatures are generated in the primordial correlators if the action for the graviton deviates from General Relativity or if there are additional particles with spin which are active during inflation (like a light spin-2 field).

To look for hints of new physics one firstly has to work out the predictions for the tensor non-gaussinaity in the simplest models. This is the reason why this thesis starts with a discussion about the *effective field theory of inflation*: its construction has been reviewed in Chapter 1. We have seen that in this framework the tensor sector is pretty standard. In fact, it has been shown that, at lowest order in a derivative expansion, the quadratic Lagrangian for the tensor sector is given by General Relativity, [28]. This implies for instance, that tensor modes always have a trivial speed of propagation and the tensor bispectrum that involves three gravitons, is fixed by the power spectrum. In Chapter 2 we have further investigated this robustness, focussing on the graviton cubic interactions. In particular, after having identified a class of field redefinitions (generalised disformal transformations) that leave observables invariant, we used them to simplify the cubic action that involves gravitons simplifying some redundant operators. The conclusion is that, despite the appearances, there is only one shape for the scalar-tensor-tensor correlation function and two shapes for the tensor-scalar-scalar bispectrum. Following the same spirit we have briefly studied some imprints that could arise from higher derivatives operators: we highlighted a new shape of the 3-point function that involves 2 tensor and one scalar perturbations, that, arising from the coupling with the foliation is not slow-roll suppressed and therefore could be sizeable.

In Chapter 3 we have studied in detail the squuezed limit of the tensor-scalar-scalar 3-point function. We have shown that its leading squeezed behaviour is set by the so called *tensor consistency relation*, that holds not only in the framework of the EFTI but even in presence of additional scalar fileds. The tensor bispectrum can be enhanced either if the underlying inflationary model has a symmetry pattern different from the standard one, or if additional particles with spin were active. These states could arise if the energy scale of inflation is not too far from the energy scale of String Theory [14]. Then, the so-called tensor CR can be seen as a smoking gun for modify gravity or String Theory.

There is however a sort of no-go theorem that severely constrains the presence of light degrees of freedom with spin: the Higuchi bound states that the mass m of a particle with spin s must satisfy $m^2 > s(s-1)H^2$. Fields heavier than the Hubble rate decay exponentially fast outside the horizon and cannot lead to a violation of the tensor CR. The Higuchi bound relies on the symmetries of the de Sitter spacetime and therefore it implies that the tensor CR is quite robust and hard to be violated. There are, however, a few ways to evade it that we investigated:

i. In Chapter 4 we have shown that in the framework of the EFTI a light spin-s field can be made healthy by coupling it with the foliation given by the inflaton. We have then spelled out a recipe that allows to systematically incorporate higher spin excitations in the EFTI. Being, non-Lorentz invariant, the theory for these new states posseses several peculiar features. For instance, even in the massless limit, a spin-s field propagates $2s+1$ degrees of freedom[1] and different helicities, in general, travel with different speeds. All these peculiarities show up in the primordial correlators. As an illustrative example, we studied in detail the theory for a light spin-2 field. We have then computed the primordial squeezed bispectra, generated by the exchange of this extra light field, explicitly showing that both the scalar and tensor CRs are broken. This breaking generate some anisotropy in the sky. As we have claimed in Chapter 3, one could probe it by looking at the quadrupolar modulation of the scalar power specturm or at the counter-collapsed limit of the scalar trispectrum.

ii. Inflationary theories that break spatial diffs invariance can make the graviton massive. In Chapter 5 we have described one of these models: Solid Inflation [23]. After having verified that all the "usual" CRs are broken, we have argued that in Solid Inflation only one shape for the scalar non-gaussianity can be generated. It peaks in the squeezed limit (and in fact the scalar CR is violated) but it is orthogonal to the local template. We thus concluded that no local f_{NL} can be generated in Solid Inflation and hence the Maldacena CR holds after angular average. We have proved this, firstly, by a "brute force" calculation of the bispectra that involve a long scalar mode up to first order in the slow-roll parameters; later we have also provided a more intuitive explanation for

[1]Of course the situation is different in Lorentz invariant theories. As an integer-spin particle approaches the limit $m \to 0$ the gauge symmetry arises, making all helicties but two disappear.

this. It is based on the fact that perturbations in Solid Inflation form a spin-2 multiplet on super-horizon scales and therefore they cannot generate any local non-Gaussianity.

Still a lot of work remains to be done: there are many possible possible extensions for this thesis. From the theoretical point of view there are many problems that are still open and certainly deserve further investigation. For instance it is not clear how to incorporate auxiliary fields in the EFT for higher-spin particles provided in Chapter 4, we know however that they are necessary to reproduce the Proca action. Alternatively the study of the phenomenology generated by particles with spin greater than 2 could reveal some surprise. At the same time it would be nice understand how to distinguish between models with a different symmetry pattern. The discovery of the CRs that hold after spherical average is just a first step in this direction.

On the other hand, more urgent questions need to be answered from the experimental point of view. In the foreseeable future, many forthcoming surveys will provide new data, especially regarding the LSS, therefore we need to connect primordial predictions with the late time observables. The presence of light fields during inflation generates unique features in several cosmological observables that could be probed by forthcoming experiments:

i. The $\langle BTT \rangle$ correlation function in the CMB gets enhanced by the presence of light tensor fields. This could be detected in the foreseeable future by a Stage IV experiment [131].

ii. From the LSS side, the violation of the tensor CR leaves its imprint on the matter power spectrum that acquires a quadrupolar modulation. Moreover particles with spin induce a scale-dependent tidal alignment of galaxies. Both these effects could be observed in future surveys [132].

iii. Because of the exchange of a long vector or tensor mode, the trispectrum of scalar perturbations gets enhanced. The shape induced by the presence of spinning particles is orthogonal to the standard parameterization of the trispectrum. No analysis has been performed on this shape, so far. A dedicated analysis could reveal the presence of light fields during inflation already with the Planck data.

Appendix A

Field redefintions: explicit calculations

A.1 Transformation of the operators $\mathcal{O}_I^{(2)}$ and $\mathcal{O}_I^{(3)}$

In this appendix we compute how the couplings a_I and b_I change under the transformations f_i. The transformation of the metric that we are considering is of the form

$$g_{\mu\nu} \to \widetilde{g}_{\mu\nu} = C(t,N)g_{\mu\nu} + D(t,N)n_\mu n_\nu \ . \tag{A.1}$$

This amounts to a rescaling of the three-metric and the normal one-form: more precisely, we have $\widetilde{h}_{\mu\nu} = C(t,N)h_{\mu\nu}$, $\widetilde{n}_\mu = \sqrt{C(t,N) - D(t,N)}\, n_\mu$. Recalling that $n_\mu = -N dt_\mu$, this amounts to a rescaling of the lapse function $\widetilde{N} = \sqrt{C(t,N) - D(t,N)}\, N$. In order to see how the coefficients a_I and b_I transform, we need to invert these transformations, that is

$$h_{\mu\nu} = \frac{\widetilde{h}_{\mu\nu}}{C(t,N)} \ , \tag{A.2a}$$

$$n^\mu = \sqrt{C(t,N) - D(t,N)}\, \widetilde{n}^\mu \ . \tag{A.2b}$$

We also need to do a time rescaling to bring the background value of $\widetilde{g}_{00}$ to one for the unperturbed transformations f_1 and f_2. That is, we want $N = 1$ for $\widetilde{N} = 1$. To find the expression for the rescaling, we use the following facts:

- Standard results of the ADM decomposition (see, e.g., [36]) tell us that

$$N = (dt_\mu n^\mu)^{-1} = \frac{(dt_\mu \widetilde{n}^\mu)^{-1}}{\sqrt{C(t,N) - D(t,N)}} \ . \tag{A.3}$$

- The normal $\widetilde{n}^\mu$ does not change under a time rescaling.

From this, we see that a redefinition $dt_\mu = \left[\overline{C}(t(\tilde{t})) - \overline{D}(t(\tilde{t}))\right]^{-1/2} d\tilde{t}_\mu$, where a bar denotes a background quantity, does the job. We then arrive at

$$N = \frac{\sqrt{\overline{C}(t(\tilde{t})) - \overline{D}(t(\tilde{t}))}}{\sqrt{C(t(\tilde{t}), N(\widetilde{N})) - D(t(\tilde{t}), N(\widetilde{N}))}} \widetilde{N} \ . \tag{A.4}$$

This relation between N and $\widetilde{N}$ can be expanded around 1 in perturbations, and can be used to solve perturbatively for C and D in terms of $\widetilde{\delta N}$. With some abuse of notation, in the following we will use C and D to mean the conformal and disformal factors solved in terms of $\widetilde{\delta N}$. The final two things we need are how the volume element $d^4x\,\sqrt{-g}$ and the Hubble factor transform. The transformation of the volume element is straightforward to compute, since it is not affected by the time rescaling. The relation between H and $\widetilde{H}$ follows from the change in the scale factor due to $\overline{C}$, and the time rescaling. It is given by (we will also suppress the time arguments of C and D, in the following)

$$H = \sqrt{\overline{C}(t(\tilde{t})) - \overline{D}(t(\tilde{t}))}\left[\widetilde{H} - \frac{1}{2}\frac{d\log\overline{C}(t(\tilde{t}))}{d\tilde{t}}\right] . \tag{A.5}$$

These results allow us to compute how a generic operator transforms under (A.2):

- Let us start from the transformation of $\delta K_{\mu\nu}$. We have that

$$\delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu} = K_{\mu\nu} - \frac{H(\widetilde{H})}{C}\tilde{h}_{\mu\nu} , \tag{A.6}$$

  so we just need to see how $K_{\mu\nu}$ transforms. Recall that

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_{\vec{n}} h_{\mu\nu} , \tag{A.7}$$

  so it is straightforward to plug in this formula the relation of $h_{\mu\nu}$ and n^μ to $\widetilde{h}_{\mu\nu}$ and $\widetilde{n}^\mu$ to arrive at

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_{\sqrt{C-D}\,\widetilde{\vec{n}}}\left(C^{-1}\widetilde{h}_{\mu\nu}\right) = \frac{\sqrt{C-D}}{C}\left[\widetilde{K}_{\mu\nu} + \left(\mathcal{L}_{\widetilde{\vec{n}}}\log C^{-1/2}\right)\widetilde{h}_{\mu\nu}\right] . \tag{A.8}$$

  Expanding $\widetilde{K}_{\mu\nu}$ as $\widetilde{\delta K}_{\mu\nu} + \widetilde{H}\widetilde{h}_{\mu\nu}$, and plugging it in (A.6), we arrive at the transformation of $\delta K_{\mu\nu}$. Notice that the terms $\propto \mathcal{L}_{\widetilde{\vec{n}}}\log C$ will give rise to terms $\widetilde{\delta N}^n \times \widetilde{V}$, which must be integrated by parts to stay in Table 2.1 and Table 2.2.

- The transformation of ${}^{(3)}\!R\,\delta N^n$ includes a straightforward conformal transformation. We point out that terms $\widetilde{D}_\mu\widetilde{D}^\mu\widetilde{\delta N}$ will be generated, which must also be integrated by parts to yield $\widetilde{A}_\mu\widetilde{A}^\mu$.

- Finally, we list the transformation properties of V and $A_\mu = D_\mu \log N$:

$$V = \sqrt{C-D}\left[\widetilde{V} + \widetilde{N}\,\widetilde{n}^\mu\widetilde{\nabla}_\mu\frac{\sqrt{\overline{C}-\overline{D}}}{\sqrt{C-D}}\right] , \tag{A.9a}$$

$$A_\mu = \widetilde{A}_\mu - \widetilde{D}_\mu\log\sqrt{C-D} . \tag{A.9b}$$

These relations can be used to derive the transformations of the coefficients a_I and b_I. We start from the action eq. (2.7) where a_0 and a_1 have been set to zero by the use of f_1 and f_2, as discussed in Section 2.1 [28]. For simplicity, we assume that the coefficients a_I and b_I are time independent and we first consider the effect of f_3 and f_4 only on the operators that are not affected by f_5 and f_6. Moreover, for convenience we define

$$\mathcal{A} \equiv \frac{1}{1+\frac{f_3}{2}-\frac{f_4}{2}} \, . \tag{A.10}$$

Here we assume that f_3 and f_4 are time independent but not necessarily small. In particular, the transformations are non-linear in these two parameters. With these assumptions, the action $\widetilde{S}$ will be of the same form of eq. (2.7), with coefficients $\widetilde{a}_I$, $\tilde{b}_I$ for the actions $\widetilde{\mathcal{L}}^{(2)}$ and $\widetilde{\mathcal{L}}^{(3)}$ and a $\widetilde{S}_0$ given by the standard Einstein-Hilbert plus the minimal inflaton action with coefficient $\widetilde{M}_{\rm Pl}^2/2 = M_{\rm Pl}^2/2$. The operator coefficients, obtained by writing eq. (2.7) in terms of the metric on the r.h.s. of eq. (2.4), read

$$\widetilde{a}_2 = \mathcal{A}\left[a_2 + \frac{1}{2}\left(-f_3 + \frac{f_4}{2}\right)\right] \, , \tag{A.11a}$$

$$\widetilde{a}_3 = \mathcal{A}^2\left[a_3 - 2f_3a_2 - f_3\left(1-\frac{f_3}{4}\right)\right] \, , \tag{A.11b}$$

$$\begin{aligned}\widetilde{a}_4 = \mathcal{A}^2\Bigg\{ &\left[a_4 + \frac{3}{4}(f_3-f_4)\left((4f_3-f_4)(3a_6-1)-3a_8\right) + \frac{3}{4}(5f_3-2f_4)\,a_5\right] \\ &+ \frac{\dot H}{H^2}\left[-1+\frac{3}{4}(f_3-f_4)\left(f_3(3a_6-1)-a_8\right)+\frac{3}{4}f_3a_5\right]\Bigg\} + \frac{\dot H}{H^2} \, ,\end{aligned} \tag{A.11c}$$

$$\widetilde{a}_5 = \mathcal{A}\left[a_5 + (f_3-f_4)(3a_6-1)\right] \, , \tag{A.11d}$$

$$\widetilde{a}_7 = \mathcal{A}^2\left[a_7 + \frac{3f_3^2}{4}(3a_6-1) - \frac{3f_3}{2}a_8\right] \, , \tag{A.11e}$$

$$\widetilde{a}_8 = \mathcal{A}\left[a_8 - f_3(3a_6-1)\right] \, , \tag{A.11f}$$

$$\tilde{b}_1 = \mathcal{A}\left[b_1 - \frac{1}{2}\left(f_3 + \frac{f_4}{2}\right)\right] \, , \tag{A.11g}$$

$$\tilde{b}_6 = \mathcal{A}\left[b_6 - \frac{1}{2}\left(f_3 + \frac{f_4}{2}\right)(2a_6-1)\right] \, , \tag{A.11h}$$

while a_6 does not transform, i.e. $\widetilde{a}_6 = a_6$. As explained in the main text, one can choose f_3 and f_4 to set $\widetilde{a}_2$ and $\tilde{b}_1$ to zero, i.e.,

$$\widetilde{a}_2 = 0 \, , \qquad \tilde{b}_1 = 0 \qquad \Longleftrightarrow \qquad f_3 = a_2 + b_1 \, , \qquad f_4 = -2(a_2 - b_1) \, , \tag{A.12}$$

which changes the other coefficients according to the transformations above. We can then explicitly compute the effect of f_5 and f_6 on the other operator coefficients, assuming that they are time independent and that $f_3 = f_4 = 0$. In this case we have

$$\tilde{b}_2 = b_2 - \frac{1}{2}(f_5 - f_6)a_2 - \frac{1}{4}(2f_5 - f_6) \ , \tag{A.13a}$$

$$\tilde{b}_3 = b_3 - 4f_5a_2 - 2(f_5 - f_6)a_3 - 2f_5 \ , \tag{A.13b}$$

$$\tilde{b}_4 = b_4 - (f_5 - f_6)\left(a_4 + \frac{3}{2}a_8\right) + \frac{3}{2}(3f_5 - f_6)a_5 + \frac{\dot{H}}{H^2}\left[(f_5 - f_6)\left(1 - \frac{1}{2}a_8\right) + f_5a_5\right] \ , \tag{A.13c}$$

$$\tilde{b}_5 = b_5 - \frac{1}{2}(f_5 - f_6)(a_5 - 6a_6 + 2) \ , \tag{A.13d}$$

$$\tilde{b}_7 = b_7 - 2(f_5 - f_6)a_7 - 3f_5a_8 \ , \tag{A.13e}$$

$$\tilde{b}_8 = b_8 - 6f_5a_6 - (f_5 - f_6)a_8 + 2f_5 \ . \tag{A.13f}$$

This shows that f_5 can be used to set b_2, b_3, b_5 or b_8 to zero, while f_6 can be used only to set to zero b_2 or b_5.

A.2 $\langle\gamma\gamma\zeta\rangle$ FROM ${}^{(3)}R^2_{\mu\nu}$

In this section we compute the $\gamma\gamma\zeta$ cubic action, and the corresponding bispectrum, associated to the change of the tensor power spectrum discussed in the second paragraph of Section 2.2.2. As explained in the main text, we focus on ${}^{(3)}R^2_{\mu\nu}$. After integration by parts, we find that the cubic action is equal to

$$S_4^{(3)} \supseteq \frac{M_{\rm Pl}^2}{\Lambda^2}\int d\eta\, d^3x \left[\partial_i\partial_j\zeta\,\partial^2\gamma_{ik}\,\gamma_{kj} + \frac{1}{2}\partial^2\zeta\,\partial_i\gamma_{kl}\,\partial_i\gamma_{kl} + \frac{1}{2}\partial_i\partial_j\zeta\,\partial_i\gamma_{kl}\,\partial_j\gamma_{kl} + \partial_i\partial_j\zeta\,\partial_i\partial_l\gamma_{kj}\,\gamma_{kl} - \frac{1}{4}\zeta\,\partial^2\gamma_{ij}\,\partial^2\gamma_{ij} + \frac{1}{2}\partial_k\zeta\,\partial_k\gamma_{ij}\,\partial^2\gamma_{ij}\right] \ , \tag{A.14}$$

i.e. it is not slow-roll suppressed, as expected. Using the in-in formalism one can easily compute the associated three-point function. It is equal to

$$\langle\zeta_{\vec{k}_1}\gamma^{s_2}_{\vec{k}_2}\gamma^{s_3}_{\vec{k}_3}\rangle' = \frac{H^2}{\Lambda^2}\frac{1}{\epsilon}\left(\frac{H}{M_{\rm Pl}}\right)^4\left[\mathcal{I}(\vec{k}_1,\vec{k}_2,\vec{k}_3) + \mathcal{I}(\vec{k}_1,\vec{k}_3,\vec{k}_2)\right] \times \frac{1}{(k_1\,k_2\,k_3)^3}\,\frac{1}{K^4}\left(\sum_i k_i^3 + 4\sum_{i\neq j}k_ik_j^2 + 12\,k_1k_2k_3\right) \ , \tag{A.15}$$

where $K = k_1 + k_2 + k_3$. The function $\mathcal{I}$ is, instead, given by

$$\mathcal{I}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \Big\{k_2^2\,(\vec{k}_1\cdot\epsilon^2\cdot\epsilon^3\cdot\vec{k}_1) + \vec{k}_1\cdot\vec{k}_2\,(\vec{k}_1\cdot\epsilon^2\cdot\epsilon^3\cdot\vec{k}_2) + \frac{1}{2}\left(k_1^2\,(\vec{k}_2\cdot\vec{k}_3) + (\vec{k}_1\cdot\vec{k}_2)\,(\vec{k}_1\cdot\vec{k}_3) - \frac{1}{2}k_2^2\,k_3^2 + k_3^2\,(\vec{k}_1\cdot\vec{k}_2)\right)[\epsilon^2\cdot\epsilon^3]\Big\} \ , \tag{A.16}$$

where $\epsilon^i = \epsilon^{s_i}_{ij}(\vec{k}_i)$ and $[\,\cdot\,]$ denotes the trace.

Appendix B

Inflation as a Conformal Field Theory

B.1 Composite spin-1 and spin-2 fields

Spin-1 in Fourier space. Let us consider a primary spin 1 operator, $\mathcal{A}_i$ with conformal weight Δ. Its 2-point function is

$$\langle \epsilon.\mathcal{A}(\vec{x})\, \tilde{\epsilon}.A(\vec{0})\rangle \propto \frac{(\,\epsilon.\tilde{\epsilon}\,) - 2(\epsilon.\hat{x})(\tilde{\epsilon}.\hat{x})}{|x|^{2\Delta}}. \tag{B.1}$$

Fourier transforming the above expression we get a power spectrum with the same pathology of the spin-2 case: the helicity zero mode gives a negative contribution if $\Delta < 1$. Indeed,

$$\langle \epsilon.\mathcal{A}_{\vec{k}}\, \tilde{\epsilon}.\mathcal{A}_{-\vec{k}}\rangle' \propto k^{2\Delta-3}\left(e^{i\psi} + 2\frac{2-\Delta}{\Delta-1} + e^{-i\psi}\right). \tag{B.2}$$

However, if we compute the power spectrum of a composite vector directly in Fourier space, it will be always positive. To see this concretely consider the composite operator,

$$\mathcal{A}_i \equiv \Delta_1 \phi \partial_i \sigma - \Delta_2\, \sigma \partial_i \phi, \tag{B.3}$$

with ϕ and σ two scalar fields with conformal weights, respectively Δ_1 and Δ_2. After having verified that $\mathcal{A}_i$ satisfies Eq. (B.1) with weight $\Delta_v = \Delta_1 + \Delta_2 + 1$, we move to Fourier space.

In Fourier space $\mathcal{A}_i(\vec{k})$ is a convolution of ϕ and σ:

$$\mathcal{A}_i(\vec{k}) = \int \frac{d^3p}{(2\pi)^3} i \left[\Delta_1\, \phi_{\vec{p}}\, (k-p)_i\, \sigma_{\vec{k}-\vec{p}} - \Delta_2\, p_i\, \phi_{\vec{p}}\, \sigma_{\vec{k}-\vec{p}}\right], \tag{B.4}$$

hence its power spectrum is given by

$$\begin{aligned}\langle \epsilon.O_{\vec{k}}\, \tilde{\epsilon}.O_{-\vec{k}}\rangle \quad \propto \quad \epsilon_i \tilde{\epsilon}_j \int \frac{d^3p}{(2\pi)^3} \Big[& \frac{-\Delta_1^2\, p_i\, p_j - \Delta_1 \Delta_2 [(p-k)_i\, p_j + p_i (p-k)_j]}{|\vec{p}-\vec{k}|^{3-2\Delta_1}\; p^{3-2\Delta_2}} \\ & + \frac{\Delta_2^2 (k-p)_i (k-p)_j}{|\vec{p}-\vec{k}|^{3-2\Delta_1}\; p^{3-2\Delta_2}}\Big]. \end{aligned} \tag{B.5}$$

The explicit expressions of ϵ and $\tilde{\epsilon}$ are given in Eq.(3.23). We can split the two helicities contributions in the two point function. Then we integrate over $\vec{p}$. We find that the helicity 1 contribution is IR divergent for $\Delta_1 < -1$ or $\Delta_2 < -1$ while it is

UV divergent for $\Delta_1 > 1/4$ or $\Delta_2 > 1/4$. Actually, the helicity 0 component has a smaller range of convergence: $0 < \Delta_1, \Delta_2 < 1/4$.

In the range of convergence, the ratio among the two helicities agrees with the ratio of the corresponding coefficients in Eq. (B.2). In terms of Δ_v the range of convergence is $1 < \Delta_v < \frac{3}{2}$. For $\Delta_v > \frac{3}{2}$, the two helicity components diverge in the same way keeping their ratio constant. This hints that this divergence is not physical, the 2-point function should be renormalized. For $0 < \Delta_v < 1$, the helicity zero component is still positive, however, it diverges.

Spin-2 case. Let us verify that the 2-point correlation function of S_{ij} defined in Eq. (3.25) is of the form (3.20). Since we are going to contract the operator with null polarization vectors $\vec{\epsilon}$ and $\vec{\tilde{\epsilon}}$, the terms proportional δ_{ij} can be dropped:

$$\begin{aligned}\langle S^{ij}(\vec{x})S^{kl}(0)\rangle \quad = \quad & \Delta^2\langle\phi\ \partial_i\partial_j\phi(\vec{x})\ \phi\ \partial_k\partial_l\phi(0)\rangle \\ & -\Delta(\Delta+1)\langle\phi\ \partial_i\partial_j\phi(\vec{x})\ \partial_k\phi\ \partial_l\phi(0)\rangle \\ & -\Delta(\Delta+1)\langle\partial_i\phi\ \partial_j\phi(\vec{x})\ \phi\ \partial_k\partial_l\phi(0)\rangle \\ & +(\Delta+1)^2\langle\partial_i\phi\ \partial_j\phi(\vec{x})\ \partial_k\phi\ \partial_l\phi(0)\rangle\,. \end{aligned} \tag{B.6}$$

In the Gaussian approximation, this can be evaluated by taking products of derivatives of

$$\langle\phi(\vec{x})\phi(0)\rangle = \frac{1}{|x|^{2\Delta}}. \tag{B.7}$$

For instance

$$\begin{aligned}\langle\phi\partial_i\partial_j\phi(\vec{x})\ \phi\partial_k\partial_l\phi(\vec{y})\rangle \quad = \quad & \langle\phi(\vec{x})\ \phi(\vec{y})\rangle\langle\partial_i\partial_j\phi(\vec{x})\ \partial_k\partial_l\phi(\vec{y})\rangle \\ & +\langle\phi(\vec{x})\ \partial_k\partial_l\phi(\vec{y})\rangle\langle\partial_i\partial_j\phi(\vec{x})\ \phi(\vec{y})\rangle. \end{aligned} \tag{B.8}$$

Neglecting terms proportional to δ_{ij} or δ_{kl} we get for this term

$$\begin{aligned}\langle\phi\ \partial_i\partial_j\phi(\vec{x})\ \phi\partial_k\partial_l\phi(0)\rangle \quad = \quad & \frac{1}{|x|^{2(2\Delta+2)}}\Big[4\Delta(\Delta+1)(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk}) \\ & -8\Delta(\Delta+1)(\Delta+2)(\delta^{ik}\hat{x}^j\hat{x}^l+3\ \text{Perm.}) \\ & +16\Delta(\Delta+1)[(\Delta+2)(\Delta+3)+\Delta(\Delta+1)]\hat{x}^i\hat{x}^j\hat{x}^k\hat{x}^l\Big]\,. \end{aligned} \tag{B.9}$$

The other two contributions are given by

$$\begin{aligned}\langle\phi\ \partial_i\partial_j\phi(\vec{x})\ \partial_k\phi\partial_l\phi(0)\rangle \quad = \quad & \frac{1}{|x|^{2(2\Delta+2)}}\Big[-8\Delta^2(\Delta+1)(\delta^{ik}\hat{x}^j\hat{x}^l+3\ \text{Perm.}) \\ & +32\Delta^2(\Delta+1)(\Delta+2)\hat{x}^i\hat{x}^j\hat{x}^k\hat{x}^l\Big]\,. \end{aligned} \tag{B.10}$$

$$\begin{aligned}\langle\partial_i\phi\ \partial_j\phi(\vec{x})\ \partial_k\phi\partial_l\phi(0) \quad = \quad & \frac{1}{|x|^{2(2\Delta+2)}}\Big[4\Delta^2(\delta^{ik}-2(\Delta+1)\hat{x}^i\hat{x}^k) \\ & \times(\delta^{jl}-2(\Delta+1)\hat{x}^j\hat{x}^l)+(k\leftrightarrow l)\Big]\,. \end{aligned} \tag{B.11}$$

Summing them up we get

$$\langle S^{ij}(\vec{x})S^{kl}(0)\rangle = \frac{4\Delta^2(2\Delta+1)}{|x|^{2(2\Delta+2)}}(\delta^{ik}-2\hat{x}^i\hat{x}^k)(\delta^{jl}-2\hat{x}^j\hat{x}^l)+(k\leftrightarrow l) \tag{B.12}$$

plus terms that vanish when contracted with null polarization vectors.

B.2 Wave function of the Universe

An alternative way to compute expectation values of the primordial perturbations consists in considering the wave function of the Universe. In this approach cosmological expectation values are casted in terms of the functional $\psi[\chi]$, with χ standing for a generic perturbation. The perturbations produced during inflation are known to be approximately Gaussian. This allows the wave function to be written as a power series expansion of the form,

$$\Psi[\chi(\vec{x})] = \exp\left[-\frac{1}{2}\int d^3x\, d^3y\ \chi(\vec{x})\chi(\vec{y})\,\langle O(\vec{x})O(\vec{y})\rangle + \dots\right], \tag{B.13}$$

where the coefficient $\langle O(x)O(y)\rangle$ determines the two-point correlators.

If dS isometries are respected by the fluctuations then the coefficient functions will transform under the $SO(4,1)$ symmetries like correlation functions of appropriate operators in a Euclidean CFT. This means that also the functions $\langle O(\vec{x})O(\vec{y})\rangle$ must satisfies Eq. (3.20). We focus just on the tensor perturbation S_{ij}, hence, in Fourier space,

$$\Psi[S_{ij}] \sim \exp\left[-\frac{1}{2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\, S_{ij}S_{kl}\langle\sigma_{ij}(\vec{k})\sigma_{kl}(-\vec{k})\rangle'\right]. \tag{B.14}$$

In the above expression, the function $\langle\epsilon^2.\sigma_{\vec{k}}\tilde{\epsilon}^2.\sigma_{-\vec{k}}\rangle'$ satisfies eq. (3.22) with $\Delta = \Delta_+$. Let us verify this by deriving the two point function for the spin-2 field S_{ij} assuming that $\langle\epsilon^2.\sigma_{\vec{k}}\tilde{\epsilon}^2.\sigma_{-\vec{k}}\rangle'$ satisfies eq. (3.22) with the larger dimension $\Delta \equiv \Delta_+$. First we need to derive $\langle\sigma_{ij}\sigma_{kl}\rangle$ from eq. (3.22) which can be parameterized as

$$\langle\epsilon^2.\sigma\,\tilde{\epsilon}^2.\sigma\rangle \propto \tilde{a}e^{2i\theta} + \tilde{b}e^{i\theta} + \tilde{c} + \tilde{b}e^{-i\theta} + \tilde{a}e^{-2i\theta} \tag{B.15}$$

with

$$\tilde{a} = 1, \quad \tilde{b} = 4\frac{3-\Delta}{\Delta}, \quad \tilde{c} = 6\frac{(3-\Delta)(2-\Delta)}{(\Delta-1)\Delta}. \tag{B.16}$$

Eq. (B.15) can be written as

$$a(\epsilon\cdot\tilde{\epsilon})^2 + b(\epsilon\cdot\tilde{\epsilon}) + c \tag{B.17}$$

with

$$a = 4\tilde{a}, \quad b = 2\tilde{b} - 8\tilde{a}, \quad c = \tilde{c} + 2\tilde{a} - 2\tilde{b}. \tag{B.18}$$

Therefore, we have

$$\begin{aligned}\langle\sigma_{ij}\sigma_{kl}\rangle \quad \propto \quad & \frac{1}{2}a(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \frac{1}{4}b(\delta_{ik}\hat{k}_j\hat{k}_l + \delta_{il}\hat{k}_j\hat{k}_k + \delta_{jk}\hat{k}_i\hat{k}_l + \delta_{jl}\hat{k}_i\hat{k}_k) \\ & + c\hat{k}_i\hat{k}_j\hat{k}_k\hat{k}_l + d\,\delta_{ij}\delta_{kl} + e(\delta_{ij}\hat{k}_l\hat{k}_k + \delta_{kl}\hat{k}_i\hat{k}_j).\end{aligned} \tag{B.19}$$

The coefficients d, e are determined in terms of the other three by the tracelessness condition:

$$e = -\frac{1}{3}(b+c), \qquad d = -\frac{1}{3}a + \frac{1}{9}(b+c). \tag{B.20}$$

To derive $\langle S_{ij}S_{kl}\rangle$ we should invert this matrix:

$$\begin{aligned}\langle S_{ij}S_{kl}\rangle \quad\propto\quad & A(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+B(\delta_{ik}\hat{k}_j\hat{k}_l+\delta_{il}\hat{k}_j\hat{k}_k+\delta_{jk}\hat{k}_i\hat{k}_l+\delta_{jl}\hat{k}_i\hat{k}_k)\\ & +C\hat{k}_i\hat{k}_j\hat{k}_k\hat{k}_l+D\delta_{ij}\delta_{kl}+e(\delta_{ij}\hat{k}_l\hat{k}_k+\delta_{kl}\hat{k}_i\hat{k}_j),\end{aligned} \tag{B.21}$$

where tracelessness condition fixes

$$E=-\frac{1}{3}(4B+C),\qquad D=-\frac{2}{3}A-\frac{1}{9}(4B+C), \tag{B.22}$$

and the other three coefficients are [1]

$$B=\frac{-b}{(2a+b)}A,\qquad C=\frac{-2cA-(4c+2b+4e)B}{a+b+c+e}. \tag{B.23}$$

The correlator Eq. (B.21) can be contracted with polarization vectors to be brought back to the form Eq. (B.15), with

$$\tilde{A}=\frac{1}{2}A,\quad \tilde{B}=4(\frac{B}{A}+1)\tilde{A},\quad \tilde{C}=C+3A+4B. \tag{B.24}$$

One obtains

$$\begin{aligned}\frac{\tilde{B}}{\tilde{A}} &= 4\frac{\Delta}{3-\Delta}=4\frac{3-\Delta_-}{\Delta_-},\\ \frac{\tilde{C}}{\tilde{A}} &= 6\frac{\Delta(\Delta-1)}{(3-\Delta)(2-\Delta)}=6\frac{(3-\Delta_-)(2-\Delta_-)}{\Delta_-(\Delta_--1)},\end{aligned} \tag{B.25}$$

which are the same as Eq. (B.15) with $\Delta\to\Delta_-$.

[1] A simplification arises by noting that Eq. (B.19) is traceless.

Appendix C

Light spin-2: explicit calculations

C.1 Power Spectra

Let us study the quadratic action (4.40). In Fourier space, the equations of motion for σ read

$$\sigma_k^{(s)}(\eta)'' - \frac{2}{\eta}\sigma_k^{(s)}(\eta)' - \left(\frac{m^2}{H^2\eta^2} + c_s^2 k^2\right)\sigma_{\vec{k}}^{(s)}(\eta) = 0\,, \tag{C.1}$$

where $' \equiv \partial_\eta$ and the wavefunctions $\sigma_k^{(s)}(\eta)$ are defined by the following expression,

$$\sigma_{ij}(\vec{k},\eta) \equiv \sum_{s=-2}^{2} \sigma_k^{(s)}(\eta)\, \epsilon_{ij}^{(s)}(\hat{k})\,. \tag{C.2}$$

The polarisation tensors are defined such that $\sum_{i,j} \epsilon_{ij}^{(s)}(\hat{k})\,(\epsilon_{ij}^{(s)}(\hat{k}))^* = 2\delta^{ss'}$.

Let us write an explicit expression for the $\epsilon_{ij}^{(s)}(\hat{k})$. The helicity-zero polarisation tensor is just given in terms of the direction $\hat{k}$:

$$\epsilon_{ij}^{(0)}(\hat{k}) = \sqrt{3}\left(\hat{k}_i\hat{k}_j - \frac{\delta_{ij}}{3}\right)\,. \tag{C.3}$$

To define the higher-helicity tensors we introduce the vectors orthogonal to $\hat{k}$, i.e. $\hat{v}$ and $\hat{u} \equiv \hat{k} \times \hat{v}/|\hat{k} \times \hat{v}|$. Hence the helicity-one polarisation tensors are given by

$$\epsilon_{ij}^{(\pm1)}(\hat{k}) = \frac{\hat{k}_i\,(\hat{v} \pm \hat{u})_j + (\hat{v} \pm \hat{u})_i\,k_j}{\sqrt{2}}\,, \tag{C.4}$$

while the helicity-two tensors are

$$\epsilon_{ij}^{(\pm2)}(\hat{k}) = \frac{(\hat{u}_i\hat{u}_j - \hat{v}_i\hat{v}_j) \mp i\,(\hat{u}_i\hat{v}_j + \hat{v}_i\hat{u}_j)}{\sqrt{2}}\,. \tag{C.5}$$

The solution to the equations of motion are

$$\sigma_k^{(s)}(\eta) = \frac{\sqrt{\pi}}{2}\,H\,(-\eta)^{\frac{3}{2}} H_\nu^{(2)}(-c_s k\eta)\,, \tag{C.6}$$

with $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$. At late times, $\eta \ll 1$, the Hankel function can be expanded in a power series of $c_s k\eta$ and the wavefunction can be approximated with

$$\sigma_q^{(s)}(\eta \to 0) = \frac{i\,2^{\nu-1}\,\Gamma(\nu)}{\sqrt{\pi}}\,H\,\frac{(-\eta)^{\frac{3}{2}-\nu}}{(c_s\,q)^\nu}\,. \tag{C.7}$$

To compute the changes in the scalar and tensor power spectra we need to evaluate integrals of the form

$$\mathcal{I}_1(n,c) \equiv \int_0^\infty dx\; e^{-ix}\; x^n\; H_\nu^{(1)}(c\,x) \tag{C.8}$$

and

$$\mathcal{I}_2(n,c) \equiv \int_0^\infty dx\; e^{-ix}\; x^n\; H_\nu^{(1)}(c\,x) \int_x^\infty e^{-iy}\; y^n\; H_\nu^{(2)}(c\,y)\,, \tag{C.9}$$

with $c \ll 1$. To compute them one can slightly rotate the contour of integration in the complex plane to pick up the interaction vacuum (see for example [133, 134]). Then, because of the exponential suppression the integrals take support only if $x, y \lesssim 1$. Since $c \ll 1$, the Hankel functions can be Taylor expanded. Keeping the first order is enough. The integrals can now be computed easily with Mathematica, giving

$$\mathcal{I}_1(n,c) = -\frac{i\; 2^\nu\; e^{-i\pi(1+n-\nu)/2}\; \Gamma(1+n-\nu)\; \Gamma(\nu)}{\pi\; c^\nu}\,, \tag{C.10}$$

$$\mathcal{I}_2(n,c) = \frac{4^\nu\; e^{-i\pi(1-\nu)}\; \Gamma(2-2\nu)\; \Gamma(\nu)^2\; {}_2F_1(2-2\nu, 1+n-\nu, 2+n-\nu, -1)}{(1+n-\nu)\; \pi^2\; c^{2\nu}}\,. \tag{C.11}$$

Using these results, the contributions to the scalar and tensor power spectra follow straightforwardly. For the scalar power spectrum we get

$$C_{\zeta,1}(\nu) = \frac{2^{2\nu-3}\; (3-2\nu)^2\; \Gamma(\frac{1}{2}-\nu)^2\; \Gamma(\nu)^2}{3\;\pi}\,, \tag{C.12}$$

$$C_{\zeta,2}(\nu) = -\frac{2^{2\nu-3}\; (3-2\nu)^2\; \Gamma(\frac{1}{2}-\nu)^2\; \Gamma(\nu)^2\; \sin(\pi\,\nu)}{3\;\pi}\,. \tag{C.13}$$

The changes in the tensor power spectrum are instead given by

$$C_{\gamma,1}(\nu) = \frac{2^{2\nu-1}\; \Gamma(\frac{1}{2}-\nu)^2\; \Gamma(\nu)^2}{\pi}\,, \tag{C.14}$$

$$C_{\gamma,2}(\nu) = -\frac{2^{2\nu-1}\; \Gamma(\frac{1}{2}-\nu)^2\; \Gamma(\nu)^2\; \sin(\pi\,\nu)}{\pi}\,. \tag{C.15}$$

C.2 Non-perturbative treatment of the γ-σ mixing

In this Appendix we discuss the γ-σ mixing by studying the mode functions of the coupled system. A very similar analysis can be applied for the π-σ mixing in quasi-single field inflation, [16]. The perturbative calculation of the tensor power spectrum presented above leads to a spurious divergence as the mass of σ vanishes. The coefficient $\mathcal{C}_\gamma$ given by expression (4.64) blows up for small mass as $\mathcal{C}_\gamma \simeq \frac{9}{m^4}$. In the massless case the time integrals are divergent at late times. It is possible to regulate them by calculating the power spectrum at some finite time η_*. The result is given in equation (4.65) and grows as the square of the number of e-foldings passed since the horizon crossing for a mode with a given wavenumber k, $N_k = \log(-k\eta_*)$. This result motivates us to study the two-point function of the γ-σ system by finding the solutions to the coupled system. In fact, as we show shortly, all the modes in the system either remain constant or decay at late times and the power spectrum cannot grow unbounded with N_k. In this appendix we assume the sound speed of the helicity-2 mode to be small $c_2 \ll 1$ and, as usual, $\rho \ll c_0\sqrt{\epsilon}H \ll H$.

Given the quadratic action (4.40) and the mixing term from the action (4.42) we can write the system of coupled equations for the modes of given momentum k and polarisation $s = \pm 2$:

$$g'' + k^2\, g - \frac{2}{\eta^2}\, g - \frac{2\rho}{H\eta}\, s' + \frac{4\rho}{H\eta^2}\, s = 0\,, \tag{C.16}$$

$$s'' + c_2^2\, k^2\, s + \frac{m^2 - 2H^2}{H^2\eta^2}\, s + \frac{\rho}{H\eta}\, g' + \frac{\rho}{H\eta^2}\, g = 0\,, \tag{C.17}$$

where we have introduced rescaled fields $g \equiv a(\eta)\, \gamma_c^{(s)}$ and $s \equiv a(\eta)\, \sigma^{(s)}$. In order to obtain the power spectra of γ at the late times we have to find the late time behaviour of the mode functions. At late times $-k\eta \ll 1$ one can neglect the terms proportional to k^2 in the equation of motion. Since all the terms scale in the same way under rescaling of η, one should look for solutions of the form $g = g_\Delta(-k\eta)^{\Delta-1}$ and $s = s_\Delta(-k\eta)^{\Delta-1}$, where g_Δ and s_Δ are constant amplitudes and Δ is the scaling dimension for the original fields γ and σ. Plugging this ansatz in the field equations we obtain the following system:

$$\begin{pmatrix} \Delta(3-\Delta) & -\frac{2\rho}{H}(3-\Delta) \\ -\frac{\rho}{H}\Delta & \Delta(3-\Delta) - \frac{m^2}{H^2} \end{pmatrix} \begin{pmatrix} g_\Delta \\ s_\Delta \end{pmatrix} = 0\,. \tag{C.18}$$

This has non-trivial solution only if the matrix of the coefficients is singular. The solutions correspond to the eigenvectors with zero eigenvalue. The singularity condition fixes the scaling dimension to take one of the four values: $\Delta = 0$, 3, or $\Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m^2+2\rho^2}{H^2}}$. The former pair of scaling dimensions coincide with those of a massless scalar field and the latter pair corresponds to a scalar of mass $m^2 + 2\rho^2$. It means that for γ and σ there are two growing modes, one of which is constant and another decays as $(-k\eta)^{\Delta_-} \simeq (-k\eta)^{\frac{m^2+2\rho^2}{3H^2}}$ at late times. The general solution for the system at late times is specified by the four constant amplitudes of the corresponding modes and reads

$$\begin{pmatrix} g(\eta) \\ s(\eta) \end{pmatrix} = A_-(-k\eta)^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + A_+(-k\eta)^2\begin{pmatrix} \frac{m^2}{H^2} \\ -3\frac{\rho}{H} \end{pmatrix}$$
$$+ B_-(-k\eta)^{\Delta_- -1}\begin{pmatrix} \frac{2\rho}{H}\Delta_+ \\ \frac{m^2+2\rho^2}{H^2} \end{pmatrix} + B_+(-k\eta)^{\Delta_+ -1}\begin{pmatrix} \frac{2\rho}{H} \\ \Delta_+ \end{pmatrix}\,. \tag{C.19}$$

Here $A_\pm$ and $B_\pm$ are four integration constants. The composition of the modes crucially depends on the relation between the mass and the mixing. If the mass is larger than the mixing, $\rho^2 \ll m^2 \ll H^2$, then the massless modes are in the γ

direction and the massive modes are in the σ direction, just as one would expect:

$$\begin{pmatrix} g(\eta) \\ s(\eta) \end{pmatrix} \simeq A_-(-k\eta)^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + A_+(-k\eta)^2\begin{pmatrix} \frac{m^2}{H^2} \\ 0 \end{pmatrix} + B_-(-k\eta)^{\frac{m^2}{3H^2}-1}\begin{pmatrix} 0 \\ \frac{m^2}{H^2} \end{pmatrix} + B_+(-k\eta)^{2-\frac{m^2}{3H^2}}\begin{pmatrix} 0 \\ 3 \end{pmatrix} . \quad \text{(C.20)}$$

If the mixing dominates, $m^2 \ll \rho^2 \ll H^2$, then both growing modes appear to be in γ direction and both decaying modes are mostly in σ:

$$\begin{pmatrix} g(\eta) \\ s(\eta) \end{pmatrix} \simeq A_-(-k\eta)^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 3\frac{\rho}{H}A_+(-k\eta)^2\begin{pmatrix} 0 \\ 1 \end{pmatrix} + 6\frac{\rho}{H}B_-(-k\eta)^{\frac{2\rho^2}{3H^2}-1}\begin{pmatrix} 1 \\ \frac{\rho}{3H} \end{pmatrix} + 3B_+(-k\eta)^{2-\frac{2\rho^2}{3H^2}}\begin{pmatrix} \frac{2\rho}{3H} \\ 1 \end{pmatrix} . \quad \text{(C.21)}$$

Only the B_- growing mode has a component in the s direction and it is parametrically smaller than its component in g direction. In order to accommodate a growing solution only in the s direction at horizon crossing, $s\big|_{-k\eta=1} = s_-(-k\eta)^{\frac{2\rho^2}{3H^2}-1}$ and $g\big|_{-k\eta=1} = 0$, one has to take the coefficients $B_- = s_-\frac{H^2}{2\rho^2}$ and $A_- = -6\frac{\rho}{H}B_-$ to be parametrically large. The solution for g exhibits an exact cancellation between two large $\mathcal{O}(\frac{H}{\rho})$ terms at horizon crossing

$$g = -s_-\frac{3H}{\rho}\left(1-(-k\eta)^{\frac{2\rho^2}{3H^2}}\right)(-k\eta)^{-1} . \quad \text{(C.22)}$$

At late times the second term decays and the amplitude of the growing g mode builds up to be large $g \propto \mathcal{O}(\frac{H}{\rho})$. Even if the growing g mode is present at horizon crossing with a comparable amplitude $g\big|_{-k\eta=1} \sim s_-$ it provides a contribution to $A_- \sim s_-$, which is parametrically smaller. At late times the solution for g is therefore dominated by the mode given in equation (C.22). Similar effects happen for the purely decaying solution in g direction, but we are interested only in the growing modes in g at the late times. For a generic mixing the solution (C.22) reads

$$\begin{aligned} g &\simeq -s_-\frac{6\rho H}{m^2+2\rho^2}\left(1-(-k\eta)^{\frac{m^2+2\rho^2}{3H^2}}\right)(-k\eta)^{-1} \\ &= -s_-\frac{6\rho H}{m^2+2\rho^2}\left(1-e^{-\frac{m^2+2\rho^2}{3H^2}N_k}\right)(-k\eta)^{-1} . \end{aligned} \quad \text{(C.23)}$$

The corresponding tensor mode $\gamma_c = (-H\eta)\,g$ grows in time till the massive mode decays and it saturates at a maximum value. The amplitude of the mode depends on the number of e-folds N_k between the horizon crossing and the time of the end of inflation. Since we require the mixing of σ with the scalar perturbations to be small, $\rho^2 \lesssim \epsilon H^2 \lesssim H^2/N_k$ for the observable modes, the ρ^2 term is never important in the exponent. Hence, independently of ρ there are two regimes. If the mass of σ is large enough that the Δ_- mode decays before we observe it, $\frac{m^2}{H^2}N_k \gtrsim 1$, then the amplitude of g is given by the constant term $g \simeq -s_-\frac{6\rho H}{m^2}(-k\eta)^{-1}$. In the opposite case, when the massive mode does not have enough time to decay, $\frac{m^2}{H^2}N_k \lesssim 1$, one can expand the exponent and obtain the amplitude to be $g \simeq -s_- \frac{2\rho N_k}{H}(-k\eta)^{-1}$. Both these regimes are captured by the perturbative calculations. The former one corresponds to the case of massive σ and asymptotically late times, and the latter to the massless σ and a finite time cutoff. The non-perturbative result (C.23) implies that the coefficient of the m^{-4} divergence of the massive σ power spectrum (4.60) is 3^2 times the coefficient of the N_k^2 enhancement of the massless σ result (4.65), which is indeed the case.

We have shown that the late time solution for γ is dominated for sufficiently large mixing by the growing mode of σ at horizon crossing. Let us now study the solutions at early times and find the amplitude s_- to be matched with the dominant g mode at late times (C.23). At early times, when both σ and γ modes are inside horizon, $-k\,\eta \gg c_2^{-1} \gg 1$, the mixing is not important and the solutions for g and s are just plane waves. The fields s and g have a canonical time kinetic term in conformal time η (up to a factor $\frac{1}{2}$ for g) and we can choose two independent positive frequency solutions to be

$$\begin{pmatrix} g \\ s \end{pmatrix}_{-c_2 k\eta \gg 1} \simeq \frac{e^{-ik\eta}}{\sqrt{k}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \qquad \text{and} \qquad \begin{pmatrix} g \\ s \end{pmatrix}_{-c_2 k\eta \gg 1} \simeq \frac{e^{-ic_2 k\eta}}{\sqrt{2\,c_2 k}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{C.24}$$

The choice of the Minkowski-like vacuum state for the modes deep inside the horizon corresponds to choosing these mode-functions to multiply annihilation operators for the γ and σ particles respectively. Plugging these early time solutions in the mixing terms of the equations (C.16) and (C.17) one can check that for $\rho \ll H$ the back reaction from the mixing does not become important until the γ horizon crossing time $-k\eta \sim 1$. It means that the second solution in equation (C.24), which correspond to the σ mode at early times, has much larger s-component $-k\eta \sim 1$, than the first solution, which correspond to the γ mode. In addition, its amplitude gets enhanced if $c_2 \ll 1$, and will dominate the γ power spectrum at late times. For this solution the σ mode freezes out at its horizon crossing at $-c_2 k\eta \sim 1$ and continues to behave like a free field of mass $m^2 + 2\rho^2$ till $-k\eta \sim 1$:

$$s\Big|_{1/c_2 \gtrsim -k\eta \gtrsim 1} \simeq \frac{1}{\sqrt{2\,c_2 k}} \frac{1}{(-c_2 k\eta)^{1-\Delta_-}} . \tag{C.25}$$

Because of the early freeze out, by the moment when γ crosses horizon $-k\eta \sim 1$ the derivatives of σ are suppressed by the factor c_2^2 with respect to its amplitude and field s behaves like a pure growing mode up to the $\mathcal{O}(c_2^2)$ corrections. Its amplitude is given by $s_- \simeq s(-k\eta = 1) \simeq 1/\sqrt{2kc_2^{\nu}}$, where we have used that at small mixing the mass of σ is dominated by m^2: $3/2 - \Delta_- = \nu + \mathcal{O}(\rho^2/H^2)$. The fact that the σ mode at $-k\eta \sim 1$ is a purely growing mode allows us to use the expression (C.23) together with the value of s_- in order to obtain the late time behaviour of g.

Recalling the definition of the canonical tensor mode $\gamma_c^{(s)} = M_{\rm Pl}\,\gamma^{(s)} = (-H\eta)\,g$ we can write the late times power spectrum of γ as

$$P_\gamma(k,\eta) \equiv \langle \gamma_{ij,\vec{k}}\,\gamma_{ij,-\vec{k}} \rangle' = 2 \sum_{s=\pm 2} \langle \gamma_{\vec{k}}^{(s)}\,\gamma_{-\vec{k}}^{(s)} \rangle' = 4 \frac{H^2}{M_{\rm Pl}^2}\,\eta^2\,|g(\eta)|^2 . \tag{C.26}$$

Using the solution (**??**) with the value of s_- inferred above we obtain the power spectrum of P_γ in the case when it is dominated by the mixing with σ:

$$P_\gamma = \frac{4H^2}{M_{\rm Pl}^2}\frac{1}{2\,c_2^{2\nu}\,k^3}\left(\frac{6\rho H}{m^2+2\rho^2}\right)^2\left(1-e^{-\frac{m^2+2\rho^2}{3H^2}N_k}\right)^2\,, \qquad c_2 \ll 1\,. \tag{C.27}$$

It is straightforward to check that this expression coincides with the perturbative results (**??**) and (**??**) in the regimes $m^2 \gg \rho^2,\ H^2/N_k$ and $m^2 \ll \rho^2 \ll H^2/N_k$ respectively.

C.3 Three-point correlation functions

Let us start computing the mixing $\langle\pi\sigma\rangle$, eq. (4.73). We can use the results of the Appendix C.1, more in detail eq. (C.10). For $c_0 \ll 1$, we thus get

$$\begin{aligned}\langle\pi_{-\vec q}\,\sigma^{(0)}_{\vec q}\rangle' &= \frac{4}{\sqrt3}\,\rho\,M_{\rm Pl}\,k^2\,{\rm Re}\left\{i\left(\sigma^{(0)}_q(\eta_*)\,\pi_q(\eta_*)\right)\int_{-\infty}^{\eta_*}d\eta\,a^2\,\sigma^{(0)\,*}_q(\eta)\,\pi_q{}^*(\eta)\right\}\\ &\simeq \frac{4}{\sqrt3}\,\rho\,\frac{\mathcal N_\sigma}{\sqrt\epsilon\,H}\,{\rm Re}\left\{\sigma^{(0)}_{\vec q}(\eta_*)\;\pi_k(\eta_*)\;\left[\mathcal I_1\left(-\frac12,c_0\right)+\mathcal I_1\left(\frac12,c_0\right)\right]\right\}\\ &= \frac{d_\pi(\nu)}{c_0^{2\nu}}\,M_{\rm Pl}\;\rho\;(-q\,\eta_*)^{3/2-\nu}\;P_\pi(k)\,.\end{aligned} \tag{C.28}$$

Here η_* is the conformal time at the end of inflation. The coefficient $d_\pi(\nu)$ is,

$$d_\pi(\nu) = \frac{2^{2\nu-3/2}(2\nu-3)\Gamma\left(\frac12-\nu\right)\Gamma(\nu)^2\left[\cos\left(\frac{\pi\nu}{2}\right)-\sin\left(\frac{\pi\nu}{2}\right)\right]}{\sqrt3\,\pi}\,. \tag{C.29}$$

Now we compute the squeezed $\langle\sigma^{(0)}\zeta\zeta\rangle$ correlation function. For generic ν, the integral inside eq. (4.72) cannot be computed analytically. However, since we are interested only in the contribution in which $\sigma^{(0)}$ is soft, we can use the late time expansion of the $\sigma^{(0)}$ wavefunction, eq. (C.7). We then obtain

$$\langle\sigma^{(0)}_{\vec q}\,\pi_{\vec k}\,\pi_{-\vec k}\rangle'_{q\to0} = \frac{\sqrt3\,c(\nu)}{\epsilon\,M_{\rm Pl}}\,\frac{\tilde\rho}{H}\,(-k\,\eta_*)^{-\frac32+\nu}\,P_{\sigma^{(0)}}(q)\,P_\pi(k)\left((\hat q\cdot\hat k)^2-\frac13\right)\,, \tag{C.30}$$

where the coefficient $c(\nu)$ is given by

$$c(\nu) \equiv 2^{-\frac72+\nu}\;(2\nu-9)\;\cos\left(\frac{\pi}{4}(1+2\nu)\right)\;\Gamma\left(\frac52-\nu\right)\,, \tag{C.31}$$

and η_* is the conformal time at the end of inflation. One can check that the above result agrees with the exact calculation that can be performed if σ is massless (i.e. $\nu = 3/2$). Notice, also, that $c(1/2) = 0$, then the leading term of $\langle\sigma^{(s)}\pi\pi\rangle$ vanishes if $\nu = 1/2$. In fact, using the expression of the wavefunctions for $\nu = 1/2$, one can easily check that the leading contribution to $\langle\sigma^{(s)}\pi\pi\rangle$ is of order η_*^2 instead of being proportional to η_*, as one might naively expect.

***

The computation of the squeezed tensor bispectrum $\langle\gamma\zeta\zeta\rangle$ follows very closely the one of $\langle\zeta\zeta\zeta\rangle$. The mixing $\langle\gamma\sigma\rangle$, eq. (4.73), reads

$$\begin{aligned}
\langle\gamma^{(s)}_{-\vec{q}}\,\sigma^{(s)}_{-\vec{q}}\rangle' &= 2\,M_{\rm Pl}\,\rho\;{\rm Re}\left\{i\left(\sigma^{(s)}_q(\eta_*)\,\gamma^{(s)}_q(\eta_*)\right)\int_{-\infty}^{\eta_*}d\eta\;a^3\,\left(\sigma^{(s)}_q(\eta)\right)^*\left(\partial_\eta\,\gamma_q{}^{(s)}(\eta)\right)^*\right\}\\
&\simeq 2\,\mathcal{N}_\sigma\,\frac{\rho}{H}\;{\rm Re}\left\{\sigma^{(s)}(\eta_*)\;\gamma^{(s)}(0)\;\mathcal{I}_1\left(-\frac{1}{2},c_2\right)\right\}\\
&= \frac{d_\gamma(\nu)}{c_2^{2\nu}}\,M_{\rm Pl}\,\frac{\rho}{H}\,(-q\,\eta_*)^{\frac{3}{2}-\nu}\;P_\gamma(q)\,. \qquad \text{(C.32)}
\end{aligned}$$

Again, the above expression assumes $c_2 \ll 1$. The coefficient $d_\gamma(\nu)$ is

$$d_\gamma(\nu) = -\frac{2^{2\nu-7/2}\;\Gamma\left(\frac{1}{2}-\nu\right)\Gamma(\nu)^2\left[\cos\left(\frac{\pi\nu}{2}\right)-\sin\left(\frac{\pi\nu}{2}\right)\right]}{\pi}\,. \qquad \text{(C.33)}$$

The 3-point function $\langle\sigma^{(\pm2)}\pi\pi\rangle$ in the squeezed limit is given by

$$\langle\sigma^{(\pm2)}_{\vec{q}}\;\pi_{\vec{k}}\,\pi_{-\vec{k}}\rangle' = \frac{c(\nu)}{\epsilon\;M_{\rm Pl}}\,\frac{\tilde{\rho}}{H}\,(-k\,\eta_*)^{-\frac{3}{2}+\nu}\;P_{\sigma^{(2)}}(q)\,P_\pi(k)\epsilon^{(\pm2)}_{ij}\hat{k}_i\hat{k}_j\,, \qquad \text{(C.34)}$$

with $c(\nu)$ given in eq. (C.31).

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