View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Tomsk State University Repository

UDC 004.4

Asymptotic analysis of M/GI/1 retrial system with conflicts and afterservice

A. A. Nazarov, N. A. Pomortseva

Tomsk State University
Lenina ave. 36, Tomsk, 634050, Russia

Abstract. M/GI/1 retrial queueing system with conflicts and afterservice is investigated. The distribution of the request's number in the blocks of the orbit is studied. Throughput value and characteristic function of the number of requests in the blocks of the orbit are obtained for this system.

Keywords: retrial queueing system, conflict, afterservice, asymptotic analysis, long delay.

1. Introduction

A large number of important practical problems arising in connection with the rapid development of information, computer, telecommunication systems can be solved by queuing theory.

In this paper we consider a single-server retrial queueing system with conflicts. A conflict situation suggests that a request, which comes in the system and finds the server busy, and a request under service enter into a conflict, and the both are sent to the orbit. Such models are widely used in real life systems, for example, in computer networks managed by random multiple access protocols.

Falin and Sukharev [1] have analyzed the retrial queueing system with collision,called the queue with double connections. Choi et al. [2] have considered retrial queues with constant retrial rate and collision arising from unslotted CSMA/CD protocol. Krishna Kumar B., Vijayalakshmi G., Krishnamoorthy A., Sadiq Basha S.[3] have analyzed a Markovian single server feedback retrial queue with linear retrial rate and collisions of customers. For this system the joint steady-state probability generating functions of the server state and the orbit length are obtained and some important performance measures of this system are calculated.

In this paper we consider the M/GI/1 retrial system with conflicts of requests and afterservice. For a probability distribution of the system states, we obtain Kolmogorov system of equations, we get the throughput of this queueing system and the characteristic function of the number of requests in the blocks of the orbit.

2. System description and problem statement

Let us consider a single-server retrial queueing system (Figure 1) with a stationary Poisson arrival process of requests with parameter λ. The service time of the request is a random variable, which has a two-phase distribution with parameters μ_1 for the first phase and μ_2 for the second one. If an arriving request finds the server free, the request occupies the server and gets a service. If the server is busy, the arriving request and the request under service enter into a conflict, and the both are sent to the orbit, which consists of two blocks. In the first block of the orbit, a random delay is performed by the requests, service of which has not been completed at the first phase. In the second block of the orbit, a random delay is performed by the requests, service of which was interrupted by the conflict at the second phase. After a random delay, which has an exponential distribution with parameters σ_1 for the first block and σ_2 for the second block, each request from the k-th block of the orbit turns to the k-th phase of the service, that represents a procedure of requests afterservice.

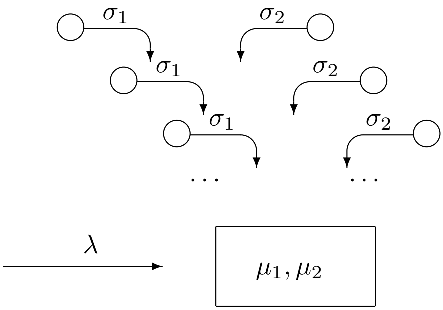

Figure 1. A single-server retrial system

Let $i_1(t)$ be a requests number in the first block of the orbit, $i_2(t)$ be a requests number in the second block of the orbit, and process $k(t)$ defines the state of the server. Our goal is to investigate a three-dimensional random process $\{k(t), i_1(t), i_2(t)\}$.

3. The system of Kolmogorov equations

Denoting

$$P_k(i_1, i_2, t) = P\{k(t) = k, i_1(t) = i_1, i_2(t) = i_2\},$$

$$H_k(u_1, u_2) = \sum_{i_1, i_2=0}^{\infty} exp\{ju_1 i_1 + ju_2 i_2\}\, P_k(i_1, i_2),$$

it is possible to write down the system of Kolmogorov equations for the partial characteristic functions in the steady state regime:

$$\begin{cases} -\lambda H_0(u_1,u_2)+\lambda e^{2ju_1}H_1(u_1,u_2)+H_2(u_1,u_2)[\mu_2+\lambda e^{ju_1}e^{ju_2}]+ \\ \quad +j\sigma_1\dfrac{\partial H_0(u_1,u_2)}{\partial u_1}+j\sigma_2\dfrac{\partial H_0(u_1,u_2)}{\partial u_2}- \\ \quad -j\sigma_1 e^{ju_1}\dfrac{\partial H_1(u_1,u_2)}{\partial u_1}-j\sigma_2 e^{ju_1}\dfrac{\partial H_1(u_1,u_2)}{\partial u_2}- \\ \quad -j\sigma_2 e^{ju_2}\dfrac{\partial H_2(u_1,u_2)}{\partial u_2}-j\sigma_1 e^{ju_2}\dfrac{\partial H_2(u_1,u_2)}{\partial u_1}=0, \\ \lambda H_0(u_1,u_2)-(\lambda+\mu_1)H_1(u_1,u_2)-j\sigma_1 e^{-ju_1}\dfrac{\partial H_0(u_1,u_2)}{\partial u_1}+ \\ \quad +j\sigma_1\dfrac{\partial H_1(u_1,u_2)}{\partial u_1}+j\sigma_2\dfrac{\partial H_1(u_1,u_2)}{\partial u_2}=0, \\ -(\lambda+\mu_2)H_2(u_1,u_2)-j\sigma_2 e^{-ju_2}\dfrac{\partial H_0(u_1,u_2)}{\partial u_2}+ \\ \quad +j\sigma_2\dfrac{\partial H_2(u_1,u_2)}{\partial u_2}+j\sigma_1\dfrac{\partial H_2(u_1,u_2)}{\partial u_1}+\mu_1 H_1(u_1,u_2)=0. \end{cases} \tag{1}$$

4. Asymptotic analysis under the long delay condition

Denote

$$\sigma_k=\gamma S_k.$$

System (1) will be solved by applying the method of asymptotic analysis under the long delay condition $\gamma\to 0$.

4.1. Asymptotic of the first order

Substituting

$$\sigma_k=\varepsilon S_k, u_k=\varepsilon w_k, H_k(u_1,u_2)=F_k(w_1,w_2,\varepsilon)$$

into the system (1) we obtain:

$$\begin{cases} -\lambda F_0(w_1,w_2,\varepsilon)+\lambda e^{2j\varepsilon w_1}F_1(w_1,w_2,\varepsilon)+jS_1\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_1}+ \\ \quad +(\mu_2+\lambda e^{j\varepsilon w_1}e^{j\varepsilon w_2})F_2(w_1,w_2,\varepsilon)+jS_2\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_2}- \\ \quad -jS_1e^{j\varepsilon w_1}\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_1}-jS_2e^{j\varepsilon w_1}\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_2}- \\ \quad -jS_2e^{j\varepsilon w_2}\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_2}-jS_1e^{j\varepsilon w_2}\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1}=0, \\ \lambda F_0(w_1,w_2,\varepsilon)-(\lambda+\mu_1)F_1(w_1,w_2,\varepsilon)+jS_1\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_1}- \\ \quad -jS_1e^{-j\varepsilon w_1}\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_1}+jS_2\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_2}=0, \\ -(\lambda+\mu_2)F_2(w_1,w_2,\varepsilon)-jS_2e^{-j\varepsilon w_2}\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_2}+ \\ \quad +jS_2\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_2}+jS_1\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1}+ \\ \quad +\mu_1F_1(w_1,w_2,\varepsilon)=0. \end{cases} \tag{2}$$

Theorem 1 *The limit value $F_k(w_1,w_2)$ of the solution $F_k(w_1,w_2,\varepsilon)$ of the system (2) has the form:*

$$F_k(w_1,w_2)=R_k exp\left\{jw_1a_1+jw_2a_2\right\},$$

where the parameters a_1,a_2 and R_k are defined as follows:

$$R_1=\frac{\lambda}{\mu_1},R_2=\frac{\lambda}{\mu_2},R_0=1-\frac{\lambda}{\mu_1}-\frac{\lambda}{\mu_2},$$

$$a_1=\frac{\lambda(2R_1+R_2)(R_0-R_2)+R_2R_1}{S_1(R_0-R_1)(R_0-R_2)-R_2R_1},$$

$$a_2=\frac{\lambda R_2(2R_1+R_2)+(R_0-R_1)}{S_2(R_0-R_1)(R_0-R_2)-R_2R_1}.$$

Parameters R_1,R_2,R_0 represent the stationary probabilities of the states of the device, a_1,a_2 are the average numbers of requests in the first and the second blocks of the orbit correspondingly.

Theorem 2 *The throughput value of M/GI/1 retrial system with conflicts and afterservice is defined by*

$$S=\frac{1}{2}.$$

4.2. Asymptotic of the second order

In the system (1) let us denote

$$H_k(u_1,u_2) = H_k^{(2)}(u_1,u_2)\exp\left\{ju_1\frac{a_1}{\gamma} + ju_2\frac{a_2}{\gamma}\right\}.$$

Substituting

$$\gamma = \varepsilon^2, \sigma_k = \varepsilon^2 S_k, u_k = \varepsilon w_k, H_k^{(2)}(u_1,u_2) = F_k(w_1,w_2,\varepsilon)$$

into the system (1) we get:

$$\begin{cases}
-\lambda F_0(w_1,w_2,\varepsilon) + \lambda e^{2j\varepsilon w_1}F_1(w_1,w_2,\varepsilon) + j\varepsilon S_1\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_1} + \\
\quad + (\mu_2 + \lambda e^{j\varepsilon w_1}e^{j\varepsilon w_2})F_2(w_1,w_2,\varepsilon) + j\varepsilon S_2\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_2} - \\
\quad - j\varepsilon S_1 e^{j\varepsilon w_1}\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_1} - j\varepsilon S_2 e^{j\varepsilon w_1}\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_2} - \\
\quad - j\varepsilon S_2 e^{j\varepsilon w_2}\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_2} - j\varepsilon S_1 e^{j\varepsilon w_2}\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1} - \\
\quad - S_1a_1F_0(w_1,w_2,\varepsilon) - S_2a_2F_0(w_1,w_2,\varepsilon) + \\
\quad + S_1a_1e^{j\varepsilon w_1}F_1(w_1,w_2,\varepsilon) + S_2a_2e^{j\varepsilon w_1}F_1(w_1,w_2,\varepsilon) + \\
\quad + S_2a_2e^{j\varepsilon w_2}F_2(w_1,w_2,\varepsilon) + S_1a_1e^{j\varepsilon w_2}F_2(w_1,w_2,\varepsilon) = 0, \\
\lambda F_0(w_1,w_2,\varepsilon) - (\lambda+\mu_1)F_1(w_1,w_2,\varepsilon) + j\varepsilon S_1\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_1} - \\
\quad - j\varepsilon S_1 e^{-j\varepsilon w_1}\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_1} + S_1a_1e^{-j\varepsilon w_1}F_0(w_1,w_2,\varepsilon) + \\
\quad + j\varepsilon S_2\dfrac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_2} - S_1a_1F_1(w_1,w_2,\varepsilon) - \\
\quad - S_2a_2F_1(w_1,w_2,\varepsilon) = 0, \\
-(\lambda+\mu_2)F_2(w_1,w_2,\varepsilon) - j\varepsilon S_2 e^{-j\varepsilon w_2}\dfrac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_2} + \\
\quad + j\varepsilon S_2\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_2} + j\varepsilon S_1\dfrac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1} + \\
\quad + \mu_1F_1(w_1,w_2,\varepsilon) + S_2a_2e^{-j\varepsilon w_2}F_0(w_1,w_2,\varepsilon) - \\
\quad - S_1a_1F_2(w_1,w_2,\varepsilon) - S_2a_2F_2(w_1,w_2,\varepsilon) = 0.
\end{cases}$$

Theorem 3 *The limit value $F_k(w_1, w_2)$ of the solution $F_k(w_1, w_2, \varepsilon)$ of the system (12) has the form:*

$$F_k(w_1, w_2) = R_k exp\left\{\frac{(jw_1)^2}{2}\kappa_{11} + jw_1 jw_2 \kappa_{12} + \frac{(jw_2)^2}{2}\kappa_{22}\right\},$$

where parameters R_k are defined by expression 7 and parameters κ_{ii} are the solution of the following system:

$$\begin{cases} \frac{B}{\mu_2}\left\{S_1a_1 - \lambda - \frac{(2\lambda + S_2a_2)(\lambda + S_1a_1)}{\mu_1 + S_2a_2}\right\} + A\frac{2\lambda + S_2a_2}{\mu_1 + S_2a_2} - \\ \quad - R_1(2\lambda + \frac{1}{2})(S_1a_1 + S_2a_2) + S_1\kappa_{11}(R_0 - R_1) - \\ \quad - \frac{1}{2}\lambda R_2 - S_2R_1\kappa_{12} = 0, \\ \frac{C}{\mu_2}\left\{S_2a_2\frac{\lambda + S_1a_1}{\mu_1 + S_2a_2} - (\lambda + S_1a_1)\right\} - D\frac{S_2a_2}{\mu_1 + S_2a_2} - \frac{1}{2}S_2a_2R_0 - \\ \quad - \frac{1}{2}R_2(\lambda + S_1a_1 + S_2a_2) + S_2(R_0 - R_2)\kappa_{22} - S_1R_2\kappa_{12} = 0, \\ \frac{B}{\mu_2}\left\{-S_1a_1 - \lambda + \frac{S_2a_2(\lambda + S_1a_1)}{\mu_1 + S_2a_2}\right\} - A\frac{(\lambda + S_1a_1)S_2a_2}{\mu_1 + S_2a_2} + \\ \quad + \frac{C}{\mu_2}\left\{S_1a_1 - (2\lambda + S_2a_2)\frac{\lambda + S_1a_1}{\mu_1 + S_2a_2} - \lambda\right\} - \lambda R_2 + \\ \quad + D\frac{(\lambda + S_1a_1)(2\lambda + S_2a_2)}{\mu_1 + S_2a_2} + [(S_1 + S_2)R_0 - S_1R_1 - S_2R_2]\kappa_{12} - \\ \quad - S_2R_1\kappa_{22} - S_1R_2\kappa_{11} = 0. \end{cases}$$

where A, B, C, D are defined as follows:

$$A = S_1a_1R_0 + S_1(R_1 - R_0)\kappa_{11} + S_2R_1\kappa_{12},$$

$$B = -R_1(2\lambda + S_1a_1 + S_2a_2) - \lambda R_2 + S_1(R_0 - R_1 - R_2)\kappa_{11} + S_2(R_0 - R_1 - R_2)\kappa_{12},$$

$$C = -R_2(\lambda + S_1a_1 + S_2a_2) + S_1(R_0 - R_1 - R_2)\kappa_{12} + S_2(R_0 - R_1 - R_2)\kappa_{22},$$

$$D = S_1(R_1 - R_0)\kappa_{12} + S_2R_1\kappa_{22}.$$

5. Conclusions

Thus, in this paper we considered M/GI/1 retrial system with conflicts and afterservice. We found the throughput value, the stationary probabilities of the states of the device, the average number of requests in the blocks of the orbit and the characteristic function of the number of requests in the blocks of the orbit for this system.

Acknowledgments

The reported study was funded by RFBR according to the research project No. 16-31-00292 mol-a.

References

1. *Falin G. I., Sukharev Yu. I.* On single-line queue with double connection –All-Union Institute for scientific and Technical Information. Moscow, 1985.
2. *Choi B. D., Shin Y. W., Ahn W. C.* Retrial Queues with collision arising from unslotted CSMA/CD protocol – Queueing Systems. – 1992 – no. 11. – P. 335–356.
3. *Krishna Kumar B., Vijayalakshmi G., Krishnamoorthy A., Sadiq Basha S.* A single server feedback retrial queue with collisions. – Computer and Operations Research. – 2010. – no. 37. – P. 1247–1255.
4. *Artalejo J. R., Gomez-Corral A.* Retrial Queueing Systems. – Springer. – 2008.
5. *Jeongsim Kim.* Retrial queueing systems with collision and impatience. –Commun. Korean Math. Soc. –25(2010). –no. 4. –P. 647-653