Synthesis of control actions with aggregate model

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***Abstract* — An approach to the formation of a control system for the combined synthesis while using mathematical reduced order model is observed. The algorithm of aggregation is given that allow to restore the original state system to the condition of the aggregate system optimally in the sense of quadratic meaning. Synthesis of control actions carried out on the evaluation of aggregate model based on the minimization of a quadratic criterion.**

***Key words*— algorithm of aggregation, control with reduced order model, the quadratic criterion**

INTRODUCTION

At present, computers are widely used in the management of dynamic objects included in the control loop. It is explained due to the increasing complexity of the objects itselves, as well as the development of computer technology, allowing to create flexible and compact digital controllers. In this case in order to achieve good functioning quality of the controlled object is necessary the following conditions that the formation of the control actions carried out during the time, small to negligible in the comparison with the speed of the external environment change and the object itself.

The use in real-time of synthesis of control actions algorithms for the objects, the mathematical models of which that are described with linear systems of large dimension, it is often feasible because of the high demands on calculating capacity of the control computers. In this regard, urgent is the task of developing such algorithms and methods of synthesis that would provide the required quality of the object functioning while reducing the computational cost that allows to create the control actions during some reasonable time for practice.

In this paper authors propose to aggregate a mathematical model of the object, for this purpose the generalization of the approach is used that was proposed in [1]. Algorithms of aggregation of linear discrete systems with constant input impact and additive disturbance in the form of white Gaussian noise with certain characteristics are proposed that allow to restore the original state system to the condition of the aggregate system optimally in the sense of quadratic meaning

Synthesis of control actions is implemented on the basis of mathematical minimizing expectation of an integral quadratic criterion according to state estimates, that are built at each time point by measuring the state of reduced order model.

1. MATHEMATICAL MODEL DESCRIPTION OF AN OBJECT

Let the mathematical model of the control object is defined by a system of linear stochastic differential equations:

$$\dot{x}(t) = \overline{A}(t)x(t) + \overline{B}(t)u(t) + \overline{F}(t)q(t),$$
$$x(t_0) = x_0,\ t \in [t_0, T], \qquad (1)$$

where $x(t) \in R^n$ – state vector, $u(t) \in R^m$ – vector of control, $q(t) \in R^{l_1}$ – vector of external disturbances, which are described by Gaussian random variables with the following characteristics:

$$M\{q(t)\} = \overline{q}(t),$$
$$M\{(q(t) - \overline{q}(t))(q(\tau) - \overline{q}(\tau))^T\} = Q(t)\delta(t-\tau).$$

In (1) matrices $\overline{A}(t), \overline{B}(t), \overline{F}(t)$ are matrices of dynamic properties, the effect of control actions and external disturbances.

Including the computer in the control loop makes it necessary to construct a discrete model of the object, which will be presented in the form:

$$x(k+1) = A(k)x(k) + B(k)u(k) + F(k)q(k),$$
$$x(0) = x_0,\ k = \overline{0, N-1}, \qquad (2)$$

where external forcing processes are described by sequences of Gaussian variables with the characteristics:

$$M\{q(k)\} = \overline{q}(k),$$
$$M\{(q(k) - \overline{q}(k))(q(j) - \overline{q}(j))^T\} = Q(k)\delta_{kj}.$$

Matrices of discrete system will be defined as follows:

$$A(k) = I_n + \Delta t \overline{A}(t_k),$$
$$B(k) = \Delta t \overline{B}(t_k),$$
$$F(k) = \sqrt{\Delta t} \overline{F}(t_k),$$

where *Δt* – sampling spacing, coincide with the period of control signal quantization, I_n – identity matrix of order *n*, $N=(T-t_0)/\Delta t$.

2. Aggregation of linear discrete stochastic system

Classical aggregation task, according to [2], is in the following that for the completely controllable system of order *n*

$$\dot{x}(t) = \overline{A}x(t) + \overline{B}u(t),\ x(t_0) = x_0 \tag{3}$$

the parameters of an aggregated system are required to find

$$\dot{x}^{(p)}(t) = \overline{A}^{(p)}x^{(p)}(t) + \overline{B}^{(p)}u(t), \quad x^{(p)}(t_0) = x_0^{(p)} \tag{4}$$

order $p<n$, for which mapping is performed:

$$x^{(p)}(t) = H_p x(t). \tag{5}$$

Here $H_p -$ aggregation matrix of dimension $p \times n$ rank *p*. This last relation is done, if

$$\overline{A}^{(p)}H_p = H_p\overline{A}(t),$$
$$\overline{B}^{(p)} = H_p\overline{B},$$
$$x_0^{(p)} = H_p x_0.$$

For aggregation of the system (2) in the moment *k* we will use the discrete system of the type:

$$x(j+1) = A(k)x(j) + B(k)u(k) + F(k)q(j), \quad x(j=k) = x(k),\ j = \overline{k, N-1}, \tag{6}$$

Where *u(k)* – constant dimension, *q(j)* – Gaussian sequence of random variables with the characteristics:

$$M\{q(j)\} = \overline{q}(k),\ M\{(q(j)-\overline{q}(k))(q(i)-\overline{q}(k))^T\} = = Q(k)\delta_{ji}.$$

Let us consume, that

$$M\{x(j)\} = \overline{x}(j),$$

where $\overline{x}(\cdot)$ satisfies the system of equations

$$\overline{x}(j+1) = A(k)\overline{x}(j) + B(k)u(k) + F(k)\overline{q}(k), \quad \overline{x}(j=k) = x(k),\ j = \overline{k, N-1}. \tag{7}$$

Reduced order system $p < n$ with state vector $x^{(p)}(\cdot)$ for (6) it will be constructed in the form:

$$x^{(p)}(j+1) = A_k^{(p)}(j)x^{(p)}(j) + H_pB(k)u(k) + + H_pF(k)q(j), \quad x^{(p)}(j=k) = H_px(k),\ j = \overline{k, N-1}, \tag{8}$$

where $x^{(p)}(\cdot) \in R^p -$ state vector,

$$A_k^{(p)}(j) = H_pA(k)V(j) \tag{9}$$

dynamics matrix of reduced order system. In (9) *V(j)* – the matrix of the inverse transformer $V: R^p \to R^n$ dimension $n \times p$, which will be constructed in such form in order to restore the original state system as aggregate state

$$\widetilde{x}(\cdot) = V\{x^{(p)}(\cdot)\} \tag{10}$$

will be possible to obtain a state close to the initial state, i.e. $\widetilde{x}(\cdot) \approx x(\cdot)$.

Then matrix *V(j)* one can determine the ratio:

$$V(j) = H_p^+ + \overline{H}_pS(j), \tag{11}$$

where $H_p^+ -$ pseudoinverse matrix, $\overline{H}_p = I_n - H_p^+H_p -$ projection matrix, *S(j)* – arbitrary matrix of shape $n \times p$.

Because matrix *V(j)* of the type (11) is a calculation of equation

$$H_pV(j) = I_p,$$

which can be verified directly by substituting, then

$$H_pV(j)x^{(p)}(j) = x^{(p)}(j). \tag{12}$$

Matrix *S(j)* will be determined from the conditions of minimum functional:

$$J(k) = M\{(x(j)-\widetilde{x}(j))^T(x(j)-\widetilde{x}(j))\} = = trM\{((x(j)-V(j)x^{(p)}(j))(x(j)-V(j)x^{(p)}(j))^T\}. \tag{13}$$

Let's indicate *e(j)* – error approximation of the system (6) with system (8), and $P_e(j)$ – covariance matrix of error. Assuming that the vectors *x(j)* and *q(j)* are uncorrelated, and also uncorrelated vectors *e(j)* and *q(j)*, after transformations we will obtain the recursive formula for the matrix calculation после $A_k^{(p)}(\cdot)$ and matrices of inverse transformer $V(\cdot)$ [3]

$$\begin{aligned} &G_x(j+1) = A(k)G_x(j)A^T(k) + \\ &+ A(k)\bar{x}(j)b(k)^T + b(k)\bar{x}^T(j)A^T(k) + C(k), \\ &L(j+1) = G_x(j+1) - A(k)P_e(j)A^T(k), \\ &V(j+1) = L(j+1)H_p^T[H_pL(j+1)H_p^T]^{-1}, \\ &A_k^{(p)}(j+1) = H_pA(k)V(j+1), \\ &P_e(j+1) = G_x(j+1) - V(j+1)H_pL(j+1), \\ &\bar{x}(j+1) = A(k)\bar{x}(j) + B(k)u(k) + F(k)\bar{q}(k), \\ &j = \overline{k, N-1}, \end{aligned} \tag{14}$$

with the following initial and permanent values:

$$\begin{aligned} &G_x(j=k) = G_{x_k}, \\ &b(k) = B(k)u(k) + F(k)\bar{q}(k) \\ &V(j=k) = G_x(j=k)H_p^T[H_pG_x(j=k)H_p^T]^{-1}, \\ &P_e(j=k) = [I_n - V(j=k)H_p]G_x(j=k)[I_n - \\ &-V(j=k)H_p]^T, \\ &\bar{x}(j=k) = x(k), \\ &C(k) = F(k)Q(k)F^T(k) + b(k)b(k)^T, \end{aligned} \tag{15}$$

where G_{x_k} – matrix of the initial moments of the second order.

The convergence of the algorithm (14), (15) is estimated

$$\alpha(j) = \frac{\left| \left\| A_k^{(p)}(j) \right\| - \left\| A_k^{(p)}(j-1) \right\| \right|}{\left\| A_k^{(p)}(j-1) \right\|}.$$

Steady value $A_k^{(p)}(\cdot)$, referred to $A_p(k)$, determined under the conditions:

$$A_p(k) = \begin{cases} A_k^{(p)}(j), \ \alpha(j) \le \varepsilon, \ j < N, \\ A_k^{(p)}(N), \ j = N, \end{cases} \tag{16}$$

where $\varepsilon -$ the given accuracy of aggregation, $V_k^{(p)}$ – the matrix of the inverse transformer, determined with the dynamics matrix of low-order system simultaneously $A_p(k)$.

Thus, the reduced order system in accordance with (8) will have the following form:

$$\begin{aligned} &x^{(p)}(j+1) = A_p(k)x^{(p)}(j) + B^{(p)}(k)u(k) + \\ &\qquad + F^{(p)}(k)q(j), \\ &x^{(p)}(j=k) = H_px(k), \ j = \overline{k, N-1}. \end{aligned} \tag{17}$$

Note that the algorithm of the dynamics matrix of low-order system constructing and matrix of the inverse conversion in accordance with (14)-(15) is recurrent, which is convenient for implementation and for calculation accuracy control.

It is essential the question about the choice of aggregation matrix for the given task H_p. For example, in [4], it is proposed to select it on the base of the major components analysis of the system. However, in general case, as specified in [5], it is not clear how this matrix corresponds to the required quality management of the functioning system. In general, aggregation matrix can be set on the basis of some considerations or selection of preliminary design with the further refinement methods of simulation modeling.

3. Synthesis of Control Actions with Use of the Reduced-Order Model

In the synthesis of control actions in the real conditions the information about the state of the object arrives at discrete points of time from the measuring complex, which often contain measurement errors.

We assume that the mathematical model of measurement system is as follows:

$$y(k) = x(k) + r(k), \tag{18}$$

where *y(k)* – measurement vector, and *r(k)* – vector of measurement errors, which will be assumed Gaussian random sequences with the characteristics:

$$M\{r(k)\} = 0, \ M\{r(k)r(j)^T\} = R\delta_{kj}, \\ M\{q(k)r(j)^T\} = 0.$$

In addition, we will assume that the vector vector distribution x_0 is Gaussian

$$M\{x_0\} = \bar{x}_0, \ M\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\} = P_{x_0},$$

where P_{x_0} – variance matrix of initial state estimation errors.

To generate the control actions we will use a mathematical model of the object of reduced order type

$$x^{(p)}(k+1) = A_p(k)x^{(p)}(k) + B^{(p)}(k)u(k) + \\ + F^{(p)}(k)q(k), \quad (19)$$
$$x^{(p)}(0) = H_p x(0),$$

where $A_p(k)$ – matrix of dynamics system (19). It is determined according to (14)-(15),

$$B^{(p)}(k) = H_p B(k),\ F^{(p)}(k) = H_p F(k).$$

The mathematical model of measuring complex for the system (19) represented in the form:

$$y^{(p)}(k) = H_p x(k) + r^{(p)}(k), \quad (20)$$

where $r^{(p)}(k)$ – measurement errors, which are defined by Gaussian variables with characteristics:

$$M\left\{r^{(p)}(k)\right\} = 0,\ M\left\{r^{(p)}(k)(r^{(p)}(j))^T\right\} = R^{(p)}\delta_{kj},$$

where $R^{(p)} = H_p R H_p^T$.

Control actions are formed on the evaluation of reduced-order model based on minimizing of the mathematical expectation of an integral quadratic criterion, which we will present with a reduced state vector:

$$J = 0,5M\left\{\int_{t_0}^{T}[\widetilde{x}(t)^T C\widetilde{x}(t) + u^T(t)Du(t)]dt\right\}, \quad (21)$$

where *C, D* – weight matrix of correspond orders.

Because,

$$\widetilde{x}(\cdot) = V_k^{(p)} x^{(p)}(\cdot), \quad (22)$$

then (21) it can be written as the following form:

$$J = 0,5M\left\{\int_{t_0}^{T}[x^{(p)}(t)^T C^{(p)}(x^{(p)} + u^T(t)Du(t)]dt\right\}, \quad (23)$$

where $C^{(p)} = (V_k^{(p)})^T C V_k^{(p)}$ – weight matrix for (23) of the order *p*.

Then, according to [6],

$$u(k) = -D_d^{-1}(B^{(p)}(k))^T S_p \hat{x}^{(p)}(k), \quad (24)$$

where S_p – calculation of Riccati equation for the model (19), $D_d = \Delta t D$. Evaluation of reduced-order model $\hat{x}^{(p)}(k)$ determined by the relations, which are listed in [6,7] if:

$$\hat{x}^{(p)}(0) = H_p \bar{x}(0),\ P_{x_0}^{(p)} = H_p P_{x_0} H_p^T.$$

4. Numerical simulation

Numerical simulation was carried out for the linearized model of lateral movement of the plane relatively of the steady horizontal level flight [8], which can be written as (1), where the state vectors components $x(t) = (\omega_x, \omega_y, \beta, \gamma)^T$ and the control circuit $u(t) = (\delta_a, \delta_d)^T$ have the following meanings: $\omega_x -$ the deviation of the angular roll velocity (*rad/s*), $\omega_y -$ the deviation of the yaw angular velocity (*rad/s*), $\beta -$ sideslip excursion (*rad*), $\gamma -$ the deviation of the roll angle (*rad*), $\delta_a -$ perturbation angle of ailerons deviation (*rad*), δ_d - perturbation angle of rudder deviation (*rad*).

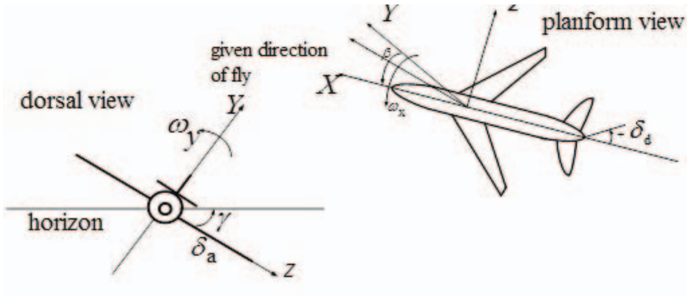


Fig.1 Spatial orientation of a plane

External perturbations are described with the vector *q(t)* whose components are normal Gaussian variables with the matrix of influence $\overline{F}$.

Matrices $\overline{A}, \overline{B}, \overline{F}$ have the following meanings:

$$\overline{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -1.89 & 0.39 & -5.55 \\ 0 & -0.034 & -2.99 & 2.43 \\ 0.035 & -0.0011 & 0.99 & -0.21 \end{pmatrix},$$

$$\overline{B} = \begin{pmatrix} 0 & 0 \\ 0.76 & -1.6 \\ -0.95 & -0.032 \\ 0.03 & 0 \end{pmatrix},$$

$$\overline{F} = \begin{pmatrix} 0.1\cdot10^{-3} & 0 \\ 0 & 0.35\cdot10^{-4} \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$t_0 = 0,$$

$$x_0 = (0.003;\ 0.005;\ 0;\ 0.002)^T,$$

$$R = diag\left(0.4\cdot10^{-5};\ 0.3\cdot10^{-5};\ 0.2\cdot10^{-5};\ 0.1\cdot10^{-5}\right),$$

$$C = diag\left(0.4\cdot10^{-1};\ 1.7;\ 2;\ 3.5\right),$$

$$D = diag(1;\ 1).$$

The following restrictions are imposed at the maximum deviation of the vector component of control:

$$|\delta_н| \le 0.22\ rad.,\ \ |\delta_э| \le 0.22\ rad.$$

Performance system of control shall meet the specified requirements, i.e. for the vector components of the state the following restrictions must be met:

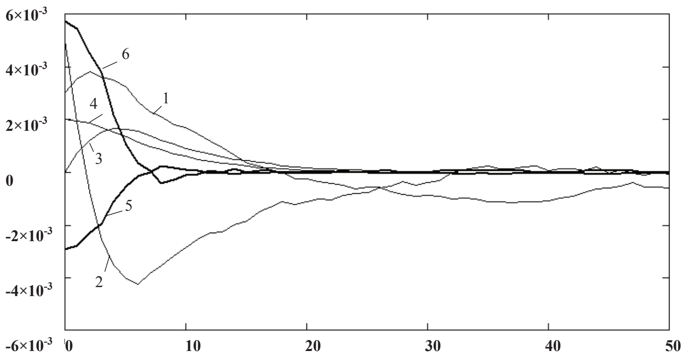
$$|\gamma| \le 0.53\cdot10^{-2}\,rad;\ \ |\omega_x| \le 0.61\cdot10^{-2}\,rad;$$

$$|\omega_y| \le 0.61\cdot10^{-2}\,rad;\ \ |\beta| \le 0.53\cdot10^{-2}\,rad.$$

Aggregation matrix assumed have the value of

$$Hp = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 1 \end{pmatrix}.$$

Figure 2 shows the simulation results: curve 1 shows the deviation of the angular roll velocity, curve 2 - the deviation of the yaw angular velocity, curve 3 - sideslip excursion, curve 4 - deviation of the roll angle, the curve 5 - perturbation angle of ailerons deviation, curve 6 - perturbation angle of rudder deviation.



curve 1 – $\omega_x(k)$, curve 2 – $\omega_y(k)$, curve 3 – $\beta(k)$, curve 4 – $\gamma(k)$, curve 5 – $\delta_a(k)$, curve 6 – $\delta_d(k)$

Fig. 2. The simulation results

These given results show that all requirements to the performance system of the controlled object are realized. Furthermore, due to the fact that the model of the object is stationary, the significant computational cost decrease is achieved by model aggregating only at the initial time. One can use this approach to reduce the computational cost for the synthesis of non-stationary models and with the adaptive control. In this case, the intervals should be allocated, in which the model of the object is considered to be stationary and implement the aggregation of the initial time of each interval.

CONCLUSION

In order to achieve a good functioning of the controlled object is necessary the following restriction - the formation of control actions carried out at the time, negligible compared to the rate of change of the external environment and the object itself.

In this regard are very important such tasks of algorithms development and methods of synthesis that would contribute the required quality of object performance in a reasonable time for practice.

The algorithms of synthesis of control actions by minimizing the mathematical expectation of an integral quadratic criterion while using mathematical reduced order model are proposed in this paper. In order to reduce the order of the model proposed algorithm of aggregation of linear discrete stochastic systems with a constant input stimulus which allow to restore the original state system optimally in a quadratic sense on condition of the aggregate system. This synthesis of control actions is carried out with estimated reduced-order model, which is measured with errors.

An example of control lateral movement of the aircraft is considered to illustrate the efficiency and quality of the proposed algorithms, which mathematical model described by the stochastic linear system of fourth-order. The simulation results shown in the graph, show that all requirements to the performance system of the controlled object are realized.

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