Locally Optimal Inventory Control with Time Delay in Deliveries and Incomplete Information on Demand

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***Abstract*—Algorithm for inventory control with incomplete information about the model of demand is proposed. Algorithm is synthesized taking into account time delay. Inventory control algorithm based on local criterion with using Kalman filtering for systems with unknown input is constructed. Examples are given to illustrate the usefulness of the proposed approach.**

***Keywords*—inventory control; time delay; local criterion; Kalman filter; unknown input**

I. Introduction

Locally optimal discrete control systems are a special case of the discrete model predictive control [1, 2] (MPC) with one step forecast. The main advantage of the method of locally optimal control is a significant simplification of the synthesis procedure. Last years, the field of the MPC application and, accordingly, the method of locally optimal control have been applied to technical systems, chemical processes, inventory control, production-inventory system, and portfolio optimization [3–9].

In this paper, we consider a control algorithm for the discrete model of the warehouse with time delay. It is assumed that the demand model contains unknown supplements.

II. Inventory Control with Exact Information on Demand

We consider the model of a warehouse, which is described by the discrete equation

$$x(k+1) = Ax(k) + Bu(k-h) - s(k)\,,$$

$$x(0) = x_0,\; u(j) = \psi(j),\; j = -h, -h+1, \dots, -1, \tag{1}$$

where $x(k) \in R^n$ is a vector of product volumes, $x_i(k)$ is a product volume for the i-th nomenclature, $u(k-h) \in R^m$ is a vector of deliveries, u_i is a volume of deliveries of the i-th nomenclature, h is a value of time delay, $s(k) \in R^n$ is a vector of demand at the k-th step, $s_i(k)$ is a demand for the product of the i-th nomenclature, x_0 and $\psi(j)$ ($j = -h, -h+1, \dots, -1,$) are the known vectors. Matrices A and B define of characteristics and the structure of a warehouse.

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The local criterion has the form

$$I(k) = (x(k+1) - z)^{\mathrm{T}} C (x(k+1) - z) + u^{\mathrm{T}}(k-h) D u(k-h), \tag{2}$$

where $C > 0, D \geq 0$ are weight matrices, z is a vector which is selected by additional criterion (see Section V). In this section, we assume that all the components of the vector $x(k)$ and $s(k)$ are measured exactly.

Transform criterion (2):

$$I(k) = u^{\mathrm{T}}(k-h)(B^{\mathrm{T}}CB + D)u(k-h) + u^{\mathrm{T}}(k-h)B^{\mathrm{T}}C(Ax(k)$$
$$-s(k) - z) + (Ax(k) - s(k) - z)^{\mathrm{T}} CBu(k-h).$$

Now, obtain the optimal control from the equation

$$\frac{dI(k)}{du(k-h)} = 0\,. \tag{3}$$

From (3), we have

$$(B^{\mathrm{T}}CB + D)u(k-h) + B^{\mathrm{T}}C(Ax(k) - s(k) - z) = 0\,. \tag{4}$$

Then, from (4)

$$u(k-h) = -(B^{\mathrm{T}}CB + D)^{-1} B^{\mathrm{T}} C (Ax(k) - s(k) - z), \tag{5}$$

according to (1), we get the following equalities:

$$x(k) = Ax(k-1) + Bu(k-h-1) - s(k-1)\,,$$
$$x(k-1) = Ax(k-2) + Bu(k-h-2) - s(k-2)\,,$$
$$\vdots$$
$$x(k-h+1) = Ax(k-h) + Bu(k-2h) - s(k-h)\,. \tag{6}$$

Now, using (6), the locally optimal control (5) is represented as follows:

$$u(k-h) = -(B^{\mathrm{T}}CB + D)^{-1} B^{\mathrm{T}} C (A^{h+1} x(k-h)$$
$$+ \sum_{i=1}^{h} A^i Bu(k-h-i) - \sum_{i=0}^{h} A^i s(k-i) - z)\,. \tag{7}$$

Note that the control (7), formed at the time $(k-h)$, demands the knowledge $x(k-h)$, $s(k-h)$, past values of

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controls $u(k-h-i)$, and forecasts for a vector of demand at the moments $k,\ k-1$,..., $k-h+1$.

III. INVENTORY CONTROL WITH INDIRECT OBSERVATIONS

In this case, we introduce the model of demand

$$s(k+1) = Rs(k) + f + q(k)\ ,\ \ s(0) = s_0, \quad (8)$$

where R is ($n\times n$)-matrix, f is a vector, $q(k)$ is a random vector. There is an indirect observation m_1-vector of demand

$$w(k) = Hs(k) + \tau(k), \quad (9)$$

where H is ($m_1\times n$)-matrix, $\tau(k)$ is a random vector of errors, $q(k), \tau(k)$ are sequences of the Gaussian random vectors with such characteristics:

$$M\{q(k)\} = 0,\ M\{\tau(k)\} = 0,$$

$$M\{q(k)q^T(j)\} = Q\delta_{kj},\ M\{\tau(k)\tau^T(j)\} = T\delta_{kj},$$

$$M\{q(k)\tau^T(j)\} = 0, \quad (10)$$

where $M\{\}$ is the mathematical expectation, δ_{kj} is Kronecker symbol.

In this case, the control will be calculated according to (7) with making use of the filtering estimates $\hat{s}_f(k-h)$ and the forecasts for demand $\hat{s}_p(k-i)$, $i=1,\ldots,h-1$

$$u^*(k-h) = -(B^{\mathrm{T}}CB+D)^{-1}B^{\mathrm{T}}C(A^{h+1}x(k-h)$$

$$+\sum_{i=1}^{h}A^iBu(k-h-i) - A^h\hat{s}_f(k-h) - \sum_{i=0}^{h-1}A^i\hat{s}_p(k-i) - z), \quad (11)$$

where $\hat{s}_f(k-h)$ is determined with using the algorithm of the optimal Kalman filtering:

$$\hat{s}_f(k-h) = R\hat{s}_f(k-h-1) + f + K_f(k-h)[w(k-h)\ ,$$

$$-H(R\hat{s}_f(k-h-1)+f\,],\ \hat{s}_f(0) = \overline{s}_0, \quad (12)$$

$$K_f(k-h) = P(k-h/k-h-1)H^T$$

$$\times(HP(k-h/k-h-1)H^T+T)^{-1}, \quad (13)$$

$$P(k-h/k-h-1) = RP(k-h-1)R^T+Q\ , \quad (14)$$

$$P(k-h) = (E-K_f(k-h)H)P(k-h/k-h-1),\ P(0)=P_0\ . \quad (15)$$

In (15), E is the identity matrix of the appropriate dimension. The filter (12) uses the information at the $(k-h)$-th step. In (11), it is required to estimate the forecast demand in the greater steps, than $(k-h)$. So, here it is necessary to use extrapolator, which will calculate the estimate of the forecast demand for on the 1-st step:

$$\hat{s}_p(k-h+1) = R\hat{s}_p(k-h) + f + K_p(k-h)(w(k-h)$$

$$-H\hat{s}_p(k-h)),\hat{s}_p(0) = \overline{s}_0\ , \quad (16)$$

$$K_p(k-h) = RP_p(k-h)H^T(HP_p(k-h)H^T+T)^{-1}, \quad (17)$$

$$P_p(k-h+1) = (R-K_p(k-h)H)P_p(k-h)(R-K_p(k-h)H)^T$$

$$+Q+K_p(k-h)TK_p^T(k-h),\ P_p(0)=P_0\ . \quad (18)$$

The forecasts for next steps $j=2,\ldots,h-1$ are determined by the formula

$$\hat{s}_p(k-h+j) = R\hat{s}_p(k-h+j-1)+f. \quad (19)$$

IV. INVENTORY CONTROL FOR MODEL OF DEMAND WITH INCOMPLETE INFORMATION

Here, we assume that the demand model with incomplete information contains unknown supplements:

$$s(k+1) = (R+\Delta R)s(k) + f + \Delta f + q(k)\ ,\ s(0) = s_0\ , \quad (20)$$

where R is the known matrix, f is the known vector, ΔR and Δf are some addition unknown matrix and vector, which can be interpreted as errors in the model parameters (8). Model (8) may be interpreted as a dynamic model with an unknown input

$$s(k+1) = Rs(k) + f + r(k) + q(k)\ ,\ s(0) = s_0\ , \quad (21)$$

where $r(k) = \Delta Rs(k) + \Delta f$ is an unknown input.

Obtain the filtering estimate on the base of the algorithm of the optimal Kalman filtering with unknown input:

$$\hat{s}_f(k-h) = R\hat{s}_f(k-h-1) + f + \hat{r}(k-h-1)$$

$$+K_f(k-h)[w(k-h) - H(R\hat{s}_f(k-h-1) + f + \hat{r}(k-h-1))]\ ,$$

$$\hat{s}_f(0) = \overline{s}_0\ , \quad (22)$$

$$K_f(k-h) = P(k-h/k-h-1)H^{\mathrm{T}}$$

$$\times(HP(k-h/k-h-1)H^{\mathrm{T}}+\mathrm{T})^{-1}, \quad (23)$$

$$P(k-h/k-h-1) = RP(k-h-1)R^{\mathrm{T}}+Q\ , \quad (24)$$

$$P(k-h) = (E_{n_1}-K_f(k-h)H)P(k-h/k-h-1)\ ,$$

$$P(0) = P_0, \quad (25)$$

where $\hat{r}(\cdot)$ is given bellow in (31).

The extrapolator, which will estimate the forecast for the 1-st step $\hat{s}_p(k-h+1)$, is defined as follows:

$$\hat{s}_p(k-h+1) = R\hat{s}_p(k-h) + f + \hat{r}(k-h)$$

$$-K_p(k-h)(w(k-h)-H\hat{s}_p(k-h)),\ \hat{s}_p(0)=\bar{s}_0, \quad (26)$$

$$K_p(k-h)=RP_{pr}(k-h)H^{\mathrm{T}}(HP_{pr}(k-h)\Phi^{\mathrm{T}}+T)^{-1}, \quad (27)$$

$$P_{pr}(k-h+1)=(R-K_p(k-h)H)P_{pr}(k-h)(R$$

$$-K_p(k-h)H)^{\mathrm{T}}+Q+K_p(k-h)TK_p^{\mathrm{T}}(k-h),\ P_{pr}(0)=P_0. \quad (28)$$

According to (8), the estimates of forecasts $\hat{s}_p(k-h+j)$ for $j\geq 2$ are given as

$$\hat{s}_p(k-h+j)=R\hat{s}_p(k-h+j-1)+f+\hat{r}(k-h+j-1). \quad (29)$$

Note that in (29) $\hat{r}(k-h+j-1)$ for $j\geq 2$ can be calculated using the analysis methods of time series [10–12].

The estimate $\hat{r}$ is calculated by the least mean square method by criterion [13]

$$J=\sum_{i=1}^{k}\left\{\|\chi(i)\|_V^2+\|r(i-1)\|_W^2\right\}, \quad (30)$$

where $\chi(i)=w(i)-H\tilde{s}(i)$ ($\tilde{s}(i)=R\hat{s}(i-1)+f$); $V>0$, $W\geq 0$ are weight matrices of the appropriate dimensions, $\|\chi(i)\|_V^2=\chi^{\mathrm{T}}(i)V\chi(i)$. Then, minimizing (30), we obtain

$$\hat{r}(k)=[H^{\mathrm{T}}VH+W]^{-1}H^{\mathrm{T}}V\{w(k+1)-H[R\hat{s}(k)+f]\}. \quad (31)$$

The estimates (22), (26), and (31) are used by synthesis of control (11) for system with incomplete information (1), (20).

Also, the filtering problems for systems with unknown input can be solved with making use of the compensation approach [14] or non-parametric technique [15–17].

V. Cost Minimization of Products Storage

Let the cost of storage of products in the sliding time interval [*k, k + T*] is defined as the additional criterion

$$J_1(k,z)=\sum_{i=1}^{n}\sum_{t=k}^{k+T}c_i x_i(t,z), \quad (32)$$

and the following restrictions hold:

$$x_i(k)\geq X_i,\ \ \forall k\in[k,k+T],\ \ i=1,...,n, \quad (33)$$

where c_i is a storage cost of product unit for the *i*-th nomenclature and in the unit time interval, X_i is a safety stock for the *i*-th nomenclature. In (32), the dependence $x_i(t,z)$ on z is due control (11) for system with incomplete information (1), (20).

Minimization of the criterion (32) under the restrictions (33) is carried over the vector z by numerical method, and at every step the control $u(k)$ is recalculated. The obtained value of the optimal vector z^* provides the minimum cost in the interval [*k, k + T*]. Vector z^* determines the volume of deliveries (control (11)), and then, by analogy, we solve the problem of minimizing the new criterion $J_1(k+1,z)$ under the constraints (33) ($\forall k\in[k+1,k+T+1]$). Such procedure is realized recursively.

VI. Simulation Results

Consider the application of control algorithm (11), (22), (26), (31) to model warehouse (1), (20), $m=n=m_1=2$, and observations (9) with

$$A=\begin{pmatrix}0.98 & 0\\ 0 & 0.96\end{pmatrix},\ R=\begin{pmatrix}0 & 1\\ 0.1 & 0.5\end{pmatrix},\ Q=\mathrm{diag}\{0.1\ \ 0.05\},$$
$$T=\mathrm{diag}\{0.05\ \ 0.04\},\ f=\begin{pmatrix}2.7 & 2.3\end{pmatrix}^{\mathrm{T}},$$
$$B=H=F=C=V=P_0=\mathrm{diag}\{1\ \ 1\},\ D=W=0.$$

The addition unknown matrix and vector ΔR and Δf we define as

$$\Delta R=\begin{pmatrix}0 & 0.02\\ 0.05 & 0.06\end{pmatrix},\ \Delta f_1(k)=\begin{cases}-0.1, & \text{if } 0\leq k<10,\\ -1.3, & \text{if } 10\leq k<20,\\ 1.2, & \text{if } 20\leq k\leq 30,\end{cases}$$

$$\Delta f_2(k)=\begin{cases}0.2, & \text{if } 0\leq k<10,\\ -0.8, & \text{if } 10\leq k<20,\\ 0.7, & \text{if } 20\leq k\leq 30.\end{cases}$$

Simulation is done for two vehicles under the following restrictions:

$$\bar{u}_i(k)=\begin{cases}0, \text{ if } u_i^*(k)\leq U\min_i,\\ u_i^*(k), \text{ if } U\min_i\leq u_i^*(k)\leq U\max_i,\\ U\max_i, \text{ if } u_i^*(k)\geq U\max_i,\end{cases} \quad (34)$$

where $U\max_i$ is a vehicle capacity ($i=\overline{1,2}$). The value of $U\min_i$ is usually defined by the condition

$$0.8U\max_i\leq U\min_i\leq U\max_i. \quad (35)$$

The condition (35) provides a high efficient using of the vehicles.

The results of simulation are presented in Figs. 1–8.

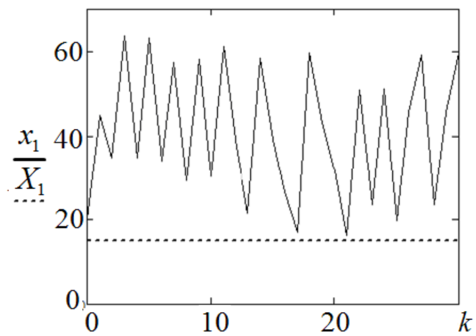

Figure 1. The volume of the stored products x_1 for the first nomenclature, X_1 is their safety stock

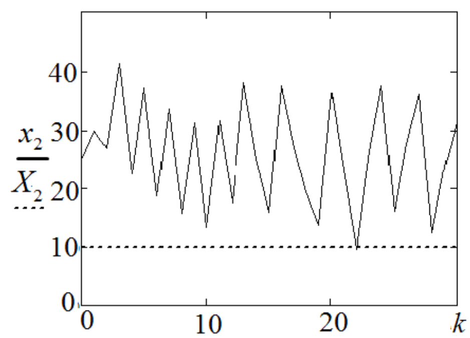

Figure 2. The volume of the stored products x_2 for the second nomenclature, X_2 is their safety stock

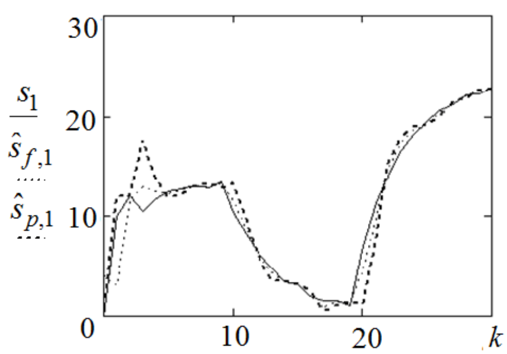

Figure 3. Estimates of filtering and prediction for the first component of demand

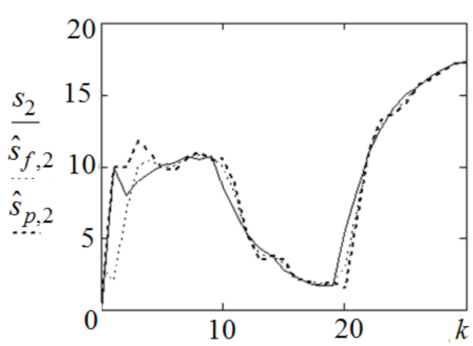

Figure 4. Estimates of filtering and prediction for the second component of demand

On Fig. 3 and Fig. 4 the components of demand (s_i is the demand of the *i*-th product, $\hat{s}_{f,i}$ is the estimate of filtration, $\hat{s}_{p,i}$ is the estimate of extrapolation, $i=\overline{1,2}$). On Fig. 5 and Fig. 6 the diagrams of product deliveries are given

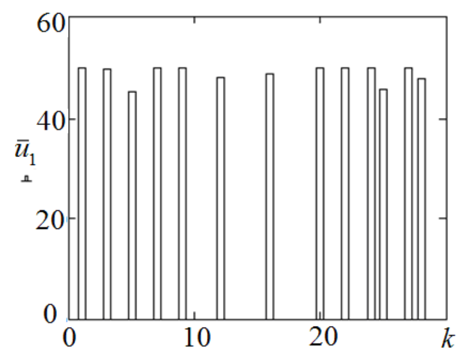

Figure 5. Diagram of product deliveries for the first nomenclature

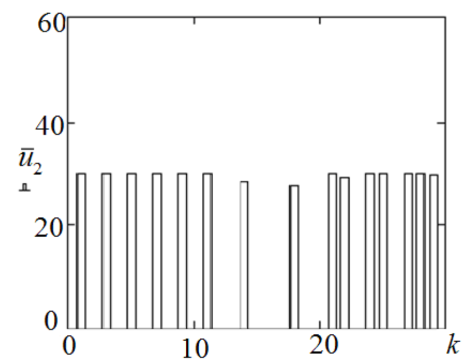

Figure 6. Diagram of product deliveries for the second nomenclature

The unknown inputs r_1 and their estimates are presented on Fig. 7 and Fig. 8

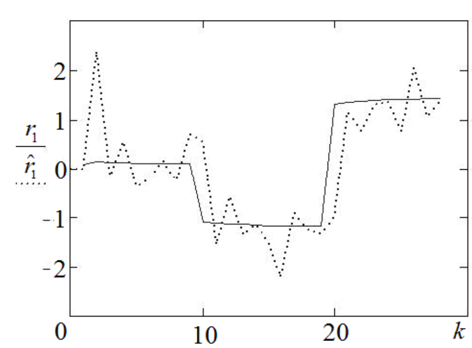

Figure 7. Unknown inputs r_1 and their estimates

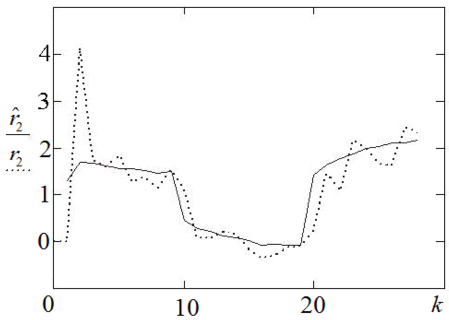

Figure 8. Unknown inputs r_2 and their estimates

VII. Conclusion

The algorithm of inventory control with delays and with incomplete information on model of demand is synthesized. The proposed method has been verified by simulations. The figures show that the algorithm inventory control can be used to calculate the supply of products in the conditions of incomplete information on the model of demand.

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