ISSN 2070-0482, Mathematical Models and Computer Simulations, 2017, Vol. 9, No. 2, pp. 133–141. © Pleiades Publishing, Ltd., 2017.
Original Russian Text © A.V. Astanin, A.D. Dashkevich, I.B. Petrov, M.N. Petrov, S.V. Utyuzhnikov, N.I. Khokhlov, 2016, published in Matematicheskoe Modelirovanie, 2016, Vol. 28, No. 8, pp. 33–45.

Modeling the Influence of the Chelyabinsk Meteorite's Bow Shock Wave on the Earth's Surface

A. V. Astanin[a], A. D. Dashkevich[b], I. B. Petrov[b], M. N. Petrov[b], S. V. Utyuzhnikov[b, c], and N. I. Khokhlov[b]

[a] *National Research Tomsk State University, Tomsk, Russia*

[b] *Moscow Institute of Physics and Technology (State University), Dolgoprudny, Russia*

[c] *The University of Manchester, Manchester, United Kingdom*

e-mail: astanin@ftf.tsu.ru, anton.dashkevich.1@gmail.com, petrov@mipt.ru, mikhail.petrov@phystech.edu, s.utyuzhnikov@manchester.ac.uk, k_h@inbox.ru

Received May 25, 2015

Abstract—The influence of the Chelyabinsk meteorite's bow shock wave on the Earth's surface is modeled. For solving the problem, a 3D statement is considered for its gas-dynamic and seismic parts. The determining equations of a gas-dynamic model and a seismic model are Euler equations with respect to the radiation heat exchange and the equations of linear elasticity theory, respectively. Two parts of the problem are matched by the direct mechanism of the disturbance transmission. A model seismic signal is compared with the real data. Modeling a seismic effect makes it possible to specify the estimate of the typical parameters of the phenomenon (height of the explosion, and the density and characteristic sizes of the meteorite).

Keywords: Chelyabinsk meteorite, mathematical modeling, Euler equations, equations of linear elasticity theory, bow shock wave, seismic wave, surface wave, Rayleigh wave

DOI: 10.1134/S2070048217020028

1. INTRODUCTION

Quite a rare seismic event occurred on February 15, 2013 as a result of the atmospheric explosion of a space object that entered the atmosphere. The fall of the Chelyabinsk meteorite on February 15, 2013 aroused strong interest in the scientific community [1–9]. This is explained by the fact that such a rare natural phenomenon for such big meteorites happened in the immediate vicinity of the city of Chelyabinsk, was recorded by different scientific equipment (infrasound and seismic sensors), and was videotaped. The researchers had a lot of data that helped them study this phenomenon and similar events better. According to the NASA estimates, this was the biggest known celestial body that has fallen on the Earth since the Tunguska meteorite in 1908. Two big comprehensive studies of this phenomenon were carried out [10, 11]. We consider these works to be the base for our own study. Despite the fact that this event is studied in much detail in those works, a seismic effect that had been caused by a bow shock wave of the Chelyabinsk meteorite has remained least studied. Our work presents the study based on the computational experiment, thus completing the construction of the whole picture of the phenomenon. The modeling results are compared with the real seismic signal, which was caused by a bow shock wave of the Chelyabinsk meteorite and was processed at the seismic station in the city of Obninsk, Moscow oblast. In the model, we carried out a computational experiment on studying the formation and propagation of a seismic trace in the Earth's crust produced by the explosion of the Chelyabinsk meteorite. The software implementation of the 3D mathematical models of the gas dynamics and seismicity was used to estimate the evolution of the wave field in the Earth's crust initiated by the bow shock wave from the object that exploded in the atmosphere. The analysis and comparison of the modeling results with the real seismic data confirm the estimated parameters of the entry of the Chelyabinsk meteorite in [10, 11], making it possible to specify the average values and narrow the range of estimates.

2. MODELING THE ENTRY OF THE METEORITE INTO THE EARTH'S ATMOSPHERE

There are many works on modeling the entry of a meteorite into the Earth's atmosphere. Nevertheless, the works in the past decade took a qualitatively new look at this process. For example, the estimate of the power of the explosion of the Tunguska meteorite was reduced from 18 Mt [12] to no more than 5 Mt [13]. In [13], the authors considered in detail the main errors in modeling that appear in the works on this subject. A model of a point source was proposed as an estimate in [13], but the subsequent analysis indicated that such a model is not appropriate for modeling a meteorite's explosion at low heights, since a meteorite's explosion is more destructive at low heights and has a greater impact on the surface than the influence of an explosion of a precise source. One of the reasons why the earlier predictions could be wrong is the occurrence of an unexpected phenomenon such as the formation and propagation of large-scale downward-directed vortex rings that are generated by the fast descent of air masses in some cases. The gas cloud that is formed after a meteorite's explosion moves towards the Earth's surface, carrying over a considerable energy and dissipating a shock wave before its front. As a result, large-scale vortex rings that decrease a decelerating force are formed in several simulations. Thus, a bow shock wave that reached the Earth's surface has a high kinetic energy and can cause much more extensive destruction. The authors suggest that a standard pancake model should be expanded by including a postexplosion phase and more correct consideration of the resistance forces to a quickly expanding and quickly descending gas cloud. The gas dynamic model used in the work is based on solving the Euler equations and on the assumption that the state of the meteorite in the postexplosion phase is determined by the model of progressive disintegration [14]. After the destructive pressure is reached on the front of the meteorite's surface, a destruction front starts moving over it. Under the impact of the front, the meteorite's matter will spread to the sides. This matter has properties that are close to a fluid's properties. The process of mechanical loading of the disintegrated mass will occur quasi-statically. This means that a typical time of changing the external load is long compared to the travel time of an elastic wave over the disintegrated mass. Under these conditions, the compressibility of the disintegrated mass can be neglected. The problem on the side flow of an aerodynamically failed meteorite is solved for two consecutive stages. The initial stage is described under the conditions of small strains and the stage of the developed flow is determined by the method of abridged equations. The estimates that were obtained in solving the equations for these stages show that at the onset of a significant deformation of the disintegrated mass due to the flow, the destruction front a fortiori passes through the entire body and the entire body gets disintegrated [15]. This simplifies the consideration of the subsequent motion, since the whole mass of the meteorite is involved in the flow. The statement proposed in [16, 17] is used within the gas-dynamic model. A homogeneous gas mixture is placed in the stratified atmosphere at the height of the explosion. It has the following parameters: the gas mass is assumed to be equal to the mass of the cosmic body; the gas velocity is assumed to be the velocity of entry into the atmosphere; the gas density corresponds to the density of the cosmic body matter; and the static pressure corresponds to the pressure of deceleration at this height. The ratio between the inner energy of the matter and the kinetic energy of the cosmic body within this statement is unambiguously determined by the ratio of the atmospheric density at the explosion height to the density of the body matter. This model is suitable for a 3D problem of shock-wave propagation from the meteorite's explosion to the interaction with the Earth's surface, which is confirmed by the calculations in [18].

3. MODELING A SEISMIC EFFECT CAUSED BY THE INFLUENCE OF A BOW SHOCK WAVE ON THE EARTH'S SURFACE

The physical space of the model of the Earth's surface represents a parallelepiped. The physics of seismic wave propagation in this space is determined by the equations of the linear dynamic theory of elasticity (the determining equations will be presented in the following section). At the upper boundary of the physical space corresponding to the Earth's surface, we defined a "free surface" boundary condition, and at the other boundaries, we assigned absorbing boundary conditions. Since the seismic station that recorded a real signal and provided the data used for comparison is located at almost 1600 km from Chelyabinsk, we selected the following dimensions of the physical space: 2000 km in length, 800 km wide, and 150 km deep. The dimensions have such a significant reserve because the numerical implementation of the absorbing boundary conditions is not ideal and a few reflected waves continue to return to the computational region, but this choice does not allow the reflected waves to influence the main disturbance. The seismic source (a disturbance from the bow shock wave) was arranged on the surface at an equal distance of 400 km from the three lateral boundaries of the region and at 1600 km from the fourth lateral boundary. The geological properties of the medium were assigned based on the following assumptions. The average value of the wave length amounted to approximately 100 km according to the real data and the estimate of the average velocity of the Raleigh surface wave [11] excited by the bow shock wave of the

Chelyabinsk meteorite. For quite a long wavelength such as this, the main influence is exerted by the deeper, basaltic and granitoidal layers despite the difference in the structure of the surface layer, taking into account the structure of the Earth's crust in the Russian Plain and the Ural Mountains [19]. Taking this into consideration, the medium is considered to be homogeneous with the most appropriate weighted-average values of the longitudinal and lateral velocities of wave propagation $Vp = 7.18$ km/s and $Vs = 3.4$ km/s over the entire distance of the propagation from the source to the moment of recording [20]. We transfer the data from the gas-dynamic to the seismic part. The data were transferred by a direct mechanism; i.e., the excessive pressure that resulted from the distribution across the Earth's surface was transferred directly in the gas-dynamic calculation as a function that determines the elastic strains in the seismic model and formed a seismic source [21]. During the data transfer in the regions over 40 km × 40 km, the more detailed gas-dynamic grid (a uniform square grid on the surface with a step of 400 m) is changed to a less detailed seismic one (a uniform square grid on the surface with a step of 4 km). To evaluate the quality of data transfer, we performed additional calculations using a more detailed seismic grid (with a step of 400 m as for the gas-dynamic grid) to transfer the data node-to-node. We compared the more approximate transfer and the node-to-node data transfer and did not find any distinct losses in accuracy.

4. DETERMINING EQUATIONS AND APPROACHES TO NUMERICAL SOLUTIONS

In estimating the gas-dynamic part with the calculation of the meteorite explosion and the subsequent propagation of shock waves in the model, the system of Euler equations is solved with respect to the radiation heat exchange. In the Cartesian coordinate system, this system is written in a strongly conservative form, as

$$U_t + F_x + G_y + H_z = Q,$$

where

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho uw \\ (E+P)u + q_x \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho vw \\ (E+P)v + q_y \end{pmatrix}, \quad H = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + P \\ (E+P)v + q_y \end{pmatrix}, \quad Q = \begin{pmatrix} 0 \\ \rho g_x \\ \rho g_y \\ \rho g_z \\ \rho gu \end{pmatrix}.$$

Here, ρ is the density, $\mathbf{u} = (u, v, w)^T$ is the gas velocity in the Cartesian coordinate system, P is the pressure, $\mathbf{g}$ is the gravity acceleration, $\mathbf{q}$ is the total heat flow, and e is the specific internal energy of the gas volume unit. The total energy is obtained as follows:

$$E = \rho e + \frac{1}{2}(u^2 + v^2 + w^2).$$

The equation of state is

$$\frac{e}{K} = \frac{2}{\gamma - 1}\frac{\rho(h)}{\rho_0},$$

where K is the kinetic energy.

The Rodionov UNO-scheme was used as the numerical method for solving the system of equations [24]. The scheme has a second order of approximation in spatial variables. The second order of approximation in time is reached by using the "predictor-corrector" technique. The equations are approximated at the static Cartesian grid with a step refined in the high gradient region. In modeling the propagation of waves excited by the bow shock wave of the Chelyabinsk meteorite, the equations of the linear dynamic elasticity theory are solved numerically [23]. We represent the corresponding system of equations in the following way:

$$\frac{\partial u}{\partial t} = \sum_{j=1}^{3} A_j \frac{\partial u}{\partial x_j}, \tag{1}$$

where u is the nine-component vector for the required functions, $u = (v_1, v_2, v_3, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{22}, \sigma_{23}, \sigma_{33})^T$ (three components of the velocity and six components of the symmetric stress tensor). In the Cartesian coordinate system, the matrices A_j take on a simple form

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & -1/\rho & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 & 0 \\ -(\lambda + 2\mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda + 2\mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/\rho & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\rho & 0 \\ 0 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\lambda + 2\mu) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Here, λ and μ are Lame elastic constants. The system of equations (1) is a system of a hyperbolic type, each matrix has a full set of eigenvectors with real eigenvalues; therefore, the system can be represented in the characteristic form. After the method of disintegration is applied to the system and the variables are replaced with the Riemann invariants, the system is split into nine independent scalar transfer equations. In turn, the 1D equations are solved by the TVD-scheme with the monotonized central (MC) limiter [24]. The equations are solved on the uniform Cartesian grid with a 4-km step in all directions.

5. POSSIBLE SCENARIOS FOR THE CHELYABINSK METEORITE HIT

Before the main study, we carried out a preliminary study of the dependence of the parameters of the seismic signal (the amplitude and the spectrum) obtained in the model based on the entry parameters. The height of the explosion, entry velocity, entry angle, density of meteorite matter, and the typical diameter are the main entry parameters that determine the phenomenon's configuration. We note that entry parameters, such as velocity and angle, are known quite exactly from the observation data; and slight changes in them do not lead to a sensitive change in the signal parameters. Therefore, for the sake of variety in the study, we used the entry parameters with a greater uncertainty of the value: the height of the explosion, and the meteorite's density and diameter. We obtained that at fixed values of the entry velocity and angle, a decrease in the density, a decrease in the diameter, and an increase in the explosion height

Variants for the Meteorite's Entry Parameters

Variant number	Explosion height, h km	Density of cosmic matter, g/cm^3	Typical diameter, m
1	30	3.6	18
2	24	3.6	18
3	30	3.0	16

lead separately to a decrease in amplitude. We note also that varying these three parameters does not change the dominant signal frequency. Based on the estimates of density, diameter, and height of the explosion, we performed a series of calculations with the different values of these parameters. As a basic result, we present and analyze three variants (table). We clarify our choice. Variant 1 models the real signal most accurately among the whole series of the test calculations. Variant 2 represents the seismic response that is provided when we use the values of the parameters taken as the most probable in conformity with [10, 11]. Variant 3 gives the minimum seismic response when we chose the explosion height, density, and velocity from the acceptable ranges (the lower estimate). The upper estimate is not presented, since even Variant 1 slightly overestimates the signal's amplitude as compared to the real signal. For all signals, the velocity of atmospheric entry $V = 18.6$ km/s and the entry angle is 18 deg. The entry velocity and angle correspond to the values in [10, 11]. These works also have estimates of the explosion height at 23–30 km; the density (for chondrite) can be estimated at 3–3.7 g/cm^3 (with respect to the composition), and the typical diameter is selected equal to 16–19 m. Based on the studies carried out in [10, 11] (the analysis of the glow and composition of the meteor matter), Variant 2 may be considered the main one (with the most likely values of the parameters).

6. ANALYSIS OF THE RESULTS AND COMPARISON WITH THE REAL DATA

We compare the modeling results for each variant with the real data that were obtained at the seismic station in the city of Obninsk, Moscow oblast. Figure 1 shows the filtered signal recorded by the seismic sensors (on the left). We focus on the signal structure at $t < 490$ s and $t > 560$ s. For these moments, the disturbance is determined by a low-intensity earthquake, rather than by the impact of a meteorite's bow shock wave; therefore, the comparison of the signals and the spectral analysis should not take these components of the signal structure into account. Figure 1 presents a real signal's spectrum (on the right). The peak in the spectral pattern is recorded for a frequency of approximately 0.03 Hz.

To begin with, we compare the signals obtained for the three model variants with the real signal in shape and amplitude. Figure 2 shows that the signal's shape is the same in all cases; i.e., we may assume that the principles forming the basis for meteorite explosion and the subsequent propagation of a shock wave and its influence on the Earth's surface illustrate the real nature of the phenomenon. Variant 3 represents the least possible impact that could be made by a bow shock wave. In this variant, we selected the least values from the range of estimates for the meteorite's density and diameters, as well as the maximum

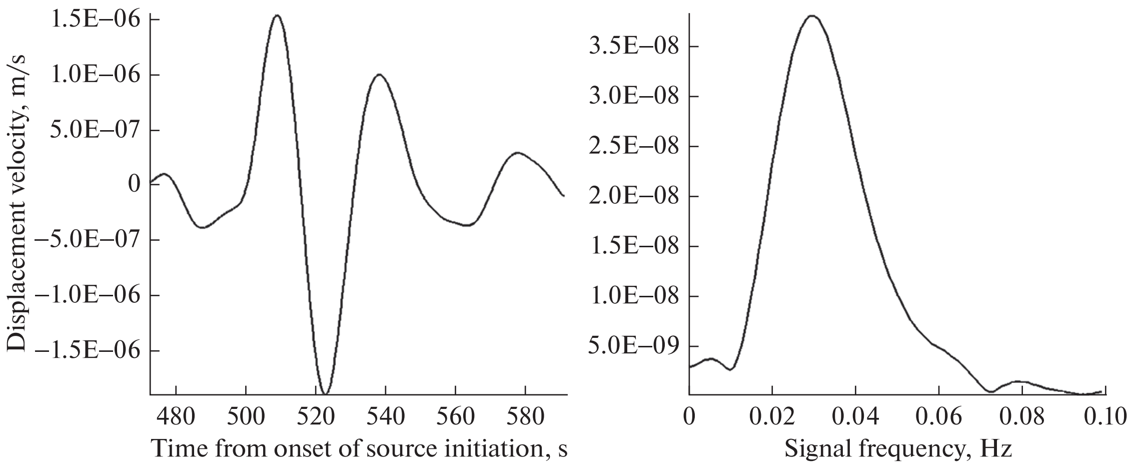

Fig. 1. Real signal recorded at the seismic station (on the left) and its spectrum (on the right).

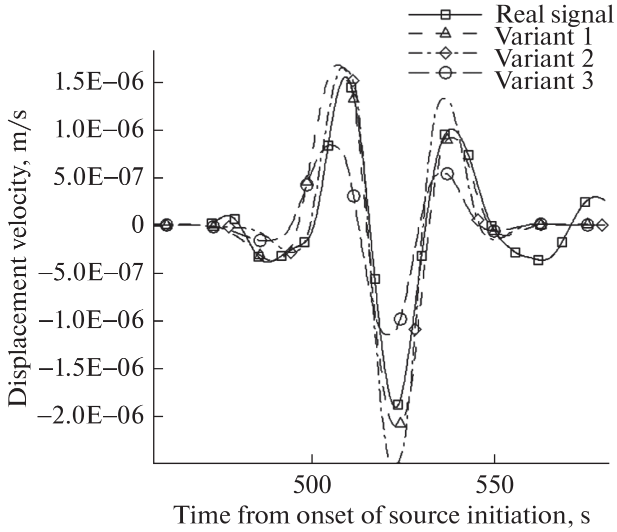

Fig. 2. Real signal and three variants of model signals.

values of the height of the meteorite's explosion. Variants 1 and 2 are the closest to the real signal in amplitude; Variant 2 displays the entry parameters that are usually considered as the most acceptable estimated values [11]. In Variant 1, the value of the explosion height is increased to the upper boundary of the height estimate at 30 km.

For illustrative purposes, we compare Variants 1 and 2 with the real signal separately. Figure 3 shows that Variant 2 represents best the first peak of the signal, but has a noticeably distinguished second peak (minimum). Variant 1 better represents the second and the third peaks. To find the preferable variant, we carry out a spectral analysis of the model signals based on fast Fourier transform and compare them with the real signal (Fig. 4). The plot shows that all three variants have the same dominant frequency, which almost coincides with the dominant frequency of a real signal. The analysis of the amplitude of the spectral patterns indicates that Variant 1 models the real signal more accurately, since the spectral amplitude of Variant 1 is almost similar to the spectral amplitude of the real signal. The spectral amplitude of Variant 2 considerably exceeds the spectral amplitude of the real signal and the spectral amplitude in Variant 1. Nevertheless, since the spectral amplitude in Variant 1 somewhat exceeds the spectral amplitude of the real signal, we need to specify some of the three entry parameters (density, diameter, and height of the explosion) in order to adjust the entry parameters. Since it is necessary to increase the explosion height in

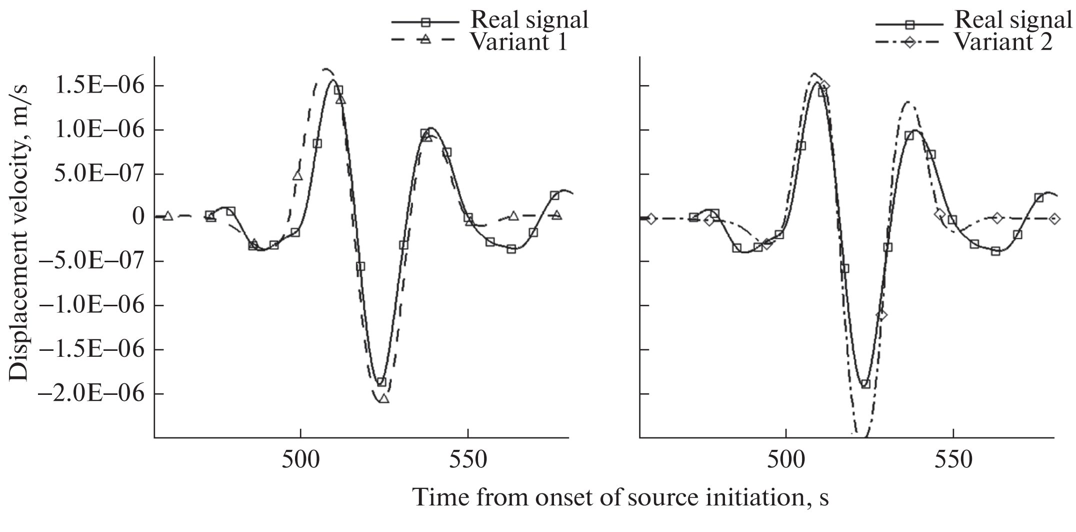

Fig. 3. Real signal compared to the signal in Variant 1 (on the left) and to the signal in Variant 2 (on the right).

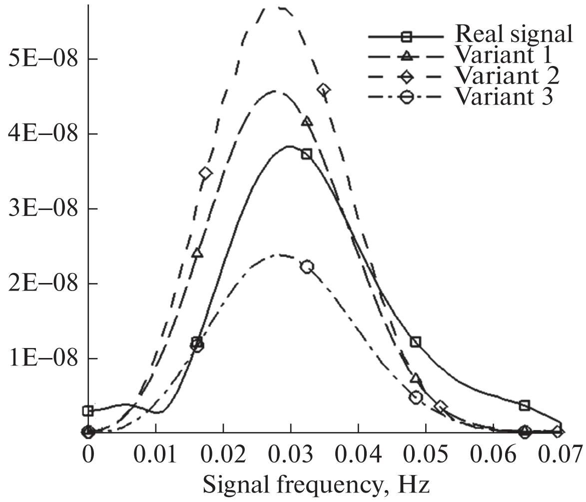

Fig. 4. Spectra of real model signals.

order to decrease the amplitude, and the height in Variant 1 is selected as the upper boundary of the respective estimate, this parameter remains unchanged. Most of the researchers, including the authors of [10, 11], believe that the meteorite consisted of homogeneous matter having a density that was determined accurately based on the study of the meteorite's fragments [11]. Therefore, the only parameter that can be adjusted in Variant 1 is the meteorite's diameter, which we decrease, selecting a value less than 18 m. Nevertheless, such a decrease should be insignificant, i.e., less than 1 meter. This conclusion can be made based on the comparison with the spectrum in Variant 3.

Some researchers believe that a considerable portion of the meteorite consisted of ice, because only a few fragments of the meteorite were found. Variant 3 shows that a decrease in the average density at a slight decrease in the diameter leads to a considerable decrease in the seismic signal's amplitude. However, for an ice-chondrite structure, the density value will be much less than in Variant 3, which certainly depends on what portion of the meteorite is considered icy; thus, the signal's amplitude will be considerably less than in the real signal. Therefore, according to the seismic analysis, this structure is impossible.

We perform an additional analysis of the seismic signal obtained in Variant 1. It is known that the velocity of the surface Rayleigh wave initiated by the Chelyabinsk meteorite's bow shock wave was slightly more than 3 km/s on the average [10]. Figure 5 shows a calculated seismogram for Variant 1 (on the left). The velocity of the Rayleigh wave of the model signal is easily determined from the slope of the seismic trace of the wave on the seismogram. In this case, it is approximately 3.1 km/s, which is consistent with

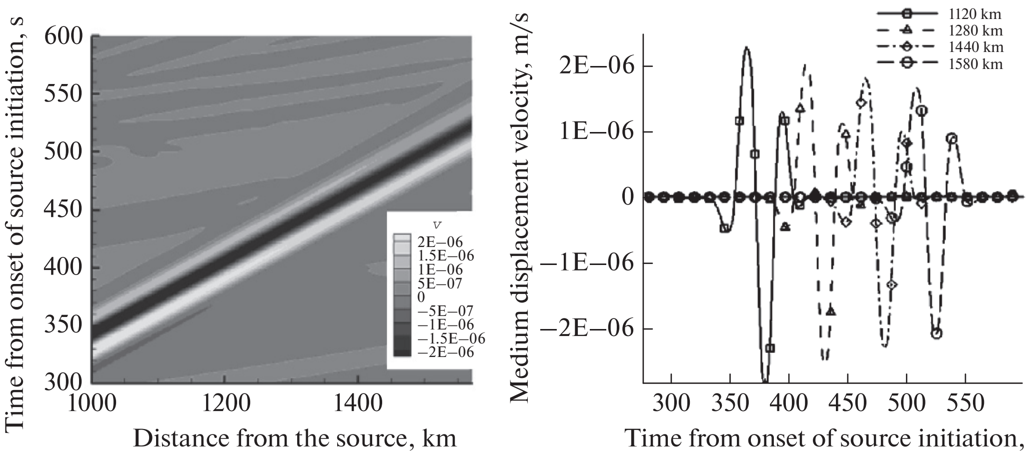

Fig. 5. Seismogram for Variant 1 (on the left) and signals obtained at the sensors at different distances from the source for Variant 1 (on the right).

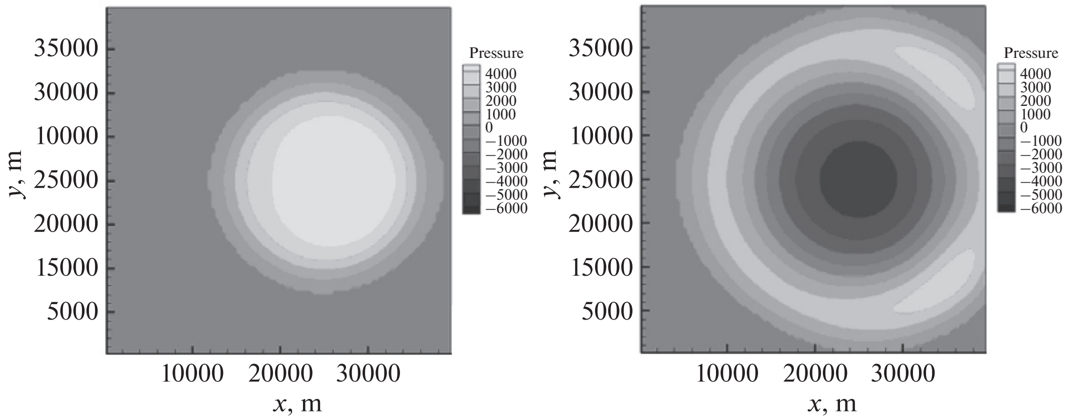

Fig. 6. Distribution of dynamic pressure (Pa) as a result of the bow shock wave at $t = 13$ s (on the left, achievement of the maximum overload) and the moment $t = 28$ s (on the right).

the observations. The same value can be obtained from the dynamics of the seismic signal at four consecutive sensors (Fig. 5 (on the right)). Another method for the analysis of the model data is to compare the estimate of dynamic overload caused by the Chelyabinsk meteorite's bow shock wave to the data obtained by the gas-dynamic calculation. The values in the range of 3–5 kPa were presented as an estimate of the dynamic overload in [10]. Such conclusions are based on the fact that if the excessive pressure is over 7 kPa, the walls and roofs are destroyed partially. However, due to the absence of any serous failures in the city of Chelyabinsk, the excessive pressure should be estimated at less than 7 kPa. Figure 6 presents two different moments of time from the onset of the initiation of the seismic source of the bow shock wave. The plots show that the maximum value of the dynamic overload is 4 kPa, which is the average value of the dynamic overload estimate; i.e., it is consistent with the real phenomenon.

7. CONCLUSIONS

This work presents the study of the Chelyabinsk event based on modeling the seismic effect initiated by the Chelyabinsk meteorite's bow shock wave. The modeling reproduced the seismic pattern caused by the influence of the bow shock wave on the Earth's surface. The most probable scenario of the real seismic effect was represented. The analysis of the seismic pattern showed that the values of density, diameter, and height of the Chelyabinsk meteorite's explosion selected in [11] as the most appropriate overestimate of the signal amplitude, although it is close to the real values. Therefore, the adjustment of some parameters is required: the diameter should be decreased and the real height of the explosion should be increased. Since both an increase in the height and a decrease in the radius lead to a decrease in the signal amplitude, a certain uncertainty still remains when these parameters vary. Variant 1 selected as the main scenario has the highest possible height of the explosion and therefore has only one clarifying parameter, the radius of the meteorite. It also somewhat overestimates the signal amplitude. Therefore, we may unambiguously state that the radius should be decreased. Our next work will specify the initial mass of the meteorite in order to obtain more accurate consistence with the observation data. The explosion height, as the parameter having the greatest scatter in the real value, makes it possible to adjust the model more finely. The analysis of the seismic signal, the propagation velocity of the surface wave, and the distribution of the dynamic pressure from the bow shock wave confirms the correctness of the model.

ACKNOWLEDGMENTS

This work was supported by the grant of the Government of the Russian Federation, Resolution no. 220 "On Measures to Attract Leading Scientists to the Russian Educational Institutions of Higher Professional Education" under contract no. 11.G34.31.0072 between the Ministry of Education and Science of the Russian Federation, leading scientists, and Moscow Institute of Physics and Technology (State University).

The authors are grateful to A.S. Kholodov for the general formulation of the problem.

REFERENCES

1. V. E. Fortov, V. G. Sultanov, and A. G. Shutov, "Chelyabinsk superbolide explosion in the Earth's atmosphere: a common phenomenon or unique coincidence?," Geochem. Int. **51**, 549 (2013).

2. V. A. Andrushchenko and Yu. D. Shevelev, The Earth attack from space - the chronicle of centure," Komp. Issled. Model. **5**, 907–916 (2013).

3. M. M. Golomazov, "Simulation of asteroid braking in the Earth atmosphere," Komp. Issled. Model. **5**, 917–926 (2013).

4. V. A. Andrushchenko, N. G. Syzranova, and Yu. D. Shevelev, "Modeling of Chelyabinsk meteorite fall," Komp. Issled. Model. **5**, 927–940 (2013).

5. A. A. Ivankov and V. S. Finchenko, "Numerical study of thermal destruction of the 'Chelyabinsk' meteorite-when entering the Earth's atmosphere," Komp. Issled. Model. **5**, 941–956 (2013).

6. I. B. Petrov, V. A. Miryakha, A. V. Sannikov, and A. V. Shevtsov, "Computational modeling of a meteor entering atmosphere dense layers using elastoplastic approximation," Komp. Issled. Model. **5**, 957–968 (2013).

7. F. A. Maksimov, "Supersonic flow of system of bodies," Komp. Issled. Model. **5**, 969–980 (2013).

8. I. S. Nikitin, A. V. Filimonov, and V. L. Yakushev, "Propagation of Rayleigh waves at oblique impact of the meteorite aboutthe Earth's surface and their effects on buildings and structures," Komp. Issled. Model. **5**, 981–992 (2013).

9. A. S. Kholodov, "About the evolution of perturbations caused by the movement of meteoroids in the Earth's atmosphere," Komp. Issled. Model. **5**, 993–1030 (2013).

10. P. G. Brown, J. D. Assink, L. Astiz, R. Blaauw, M. B. Boslough, et al., "A 500-kiloton airburst over Chelyabinsk and an enhanced hazard from small impactors," Nature **503**, 238–241 (2013).

11. V. V. Emel'yanenko et al., "Astronomical and physical aspects of the Chelyabinsk event (February 15, 2013)," Solar Syst. Res. **47**, 240–254 (2013).

12. V. P. Korobeinikov, L. V. Shurshalov, V. I. Vlasov, and I. V. Semenov, "Complex modeling of the Tunguska catastrophe," Planet. Space Sci. **46**, 231–244 (1998).

13. M. B. E. Boslough and D. A. Crawford, "Low-altitude airbursts and the impact threat," Int. J. Impact Eng. **5**, 1441–1448 (2008).

14. S. S. Grigoryan, "Motion and destruction of meteorites in planetary atmospheres," Kosm. Issled. **17**, 875–893 (1979).

15. S. S. Grigoryan, F. S. Ibodov, and S. I. Ibadov, "Physical mechanism of Chelyabinsk superbolide explosion," Solar Syst. Res. **47**, 268 (2013).

16. V. I. Kondaurov, A. V. Konyukhov, V. V. Polukhin, and S. V. Utyuzhnikov, "Mathematical simulation of gas cloud motion after meteoroid explosion in atmosphere," Izv. Akad. Nauk, Mekh. Zhidk. Gaza, No. 1, 29-37 (1998).

17. B. A. Klumov, V. I. Kondaurov, A. V. Konyukhov, Yu. D. Medvedev, A. G. Sokol'skii, S. V. Utyuzhnikov, and V. E. Fortov, "Collision of comet Shoemaker-Levi 9 with Jupiter: what shall we see?," Phys. Usp. **37**, 577–588 (1994).

18. D. V. Rudenko and S. V. Utyuzhnikov, "Use of dynamically adaptive grids for modeling three-dimensional unsteady gas flows with high gradients," Comput. Math. Math. Phys. **42**, 377–390 (2002).

19. G. L. Hebeler and G. J. Rix, "Site characterization in Shelby county, Tennessee using advanced surface wave methods," Report (Georgia Inst. Technol., 2001).

20. L. M. Brekhovskikh, *Waves in Layered Media* (Nauka, Moscow, 1973; Academic, New York, 1980).

21. W. N. Edwards, D. W. Eaton, and P. G. Brown, "Seismic observation of meteors: coupling theory and observations," Rev. Geophys. **46** (4), 96–110 (2008).

22. A. V. Rodionov, "Monotonic scheme of the second order of approximation for the continuous calculation of nonequilibrium flows," USSS Comput. Math. Math. Phys. **27**, 175–180 (1987).

23. I. B. Petrov and N. I. Khokhlov, "Modeling 3D seismic problems using high-performance computing systems," Math. Models Comput. Simul. **6**, 342–350 (2014).

24. B. van Leer, "Towards the ultimate conservative difference scheme. III. Upstream-centered finite-difference schemes for ideal compressible flow. IV. A new approach to numerical convection," J. Comput. Phys. **23**, 263–299 (1977).

Translated by L. Mukhortova

SPELL: 1. OK