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# V Международной молодежной научной конференции <br> «МАТЕМАТИЧЕСКОЕ И ПРОГРАММНОЕ ОБЕСПЕЧЕНИЕ ИНФОРМАЦИОННЫХ, ТЕХНИЧЕСКИХ И ЭКОНОМИЧЕСКИХ СИСТЕМ» 

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ориентированного языка программирования GPSS, получена аналитическая модель, рассмотрен иллюстративный пример, который подтверждает работоспособность всех построенных моделей.

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# REGRESSION ANALYSIS SIMULATION OF RADIATION TRANSFER IN SYSTEMS VISION THROUGH THE ATMOSPHERE 

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## Introduction

Such method of studies as the system approach has been intensively developed in different fields of science, industry, and social life in recent decades. System approach is defined to be that which considers any system (object) to be a set of interrelated elements (components), and to have output (purpose), input (resources), connection with environment, and feedback. System approach represents a form of applications in the theory of knowledge and dialectics to studying the processes that occur in nature, community, and thinking. It essentially consists of fulfillment of the requirements of the general system theory; in accordance to this theory, each object in the process of its study should be considered as a large and complex system and, simultaneously, as an element of a more general system. One of the applications of the system approach is the optics [3] and, in particular, the atmospheric optics [4]. The main system characteristic in these fields is the point spread function (PSF); it is defined as the response $L$ of linear system to the input signal, representing a point mass $\delta\left(x-x_{1}\right) \delta\left(y-y_{1}\right)$, located at a certain point

$$
\left(x_{1}, y_{1}\right): L\left[\delta\left(x-x_{1}\right) \delta\left(y-y_{1}\right)\right]=h\left(x, y ; x_{1}, y_{1}\right)
$$

An arbitrary object (function) $f(x, y)$ can be considered as a set of point masses. For instance:

$$
f(x, y)=\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f\left(x_{1}, y_{1}\right) \delta\left(x-x_{1}, y-y_{1}\right) d x_{1} d y_{1} .
$$

Then, a result of the system impact (image) can be represented in the form:

$$
g(x, y)=L[f(x, y)]=\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f\left(x_{1}, y_{1}\right) h\left(x, y ; x_{1}, y_{1}\right) d x_{1} d y_{1} .
$$

Obviously, regularities of the image distortion due to impact of any system can be studied by analyzing the effect of this system on the point spread function.

We will consider a system "underlying system - atmosphere - receiving device" which can be regarded as a linear system [3]. These systems are conventionally called the vision systems. Image distortion in the vision systems may be caused by the properties of scattering medium and underlying surface, and by characteristics of receiving optical device. The available images of any objects should be analyzed, and possible distortions of object images should be predicted, by studying the point spread function of this system. One of the methods
for the PSF determination in this case is to calculate the angular distribution of brightness of surface-based point source, measured with receiving device at the top of the atmosphere (TOA).

The purpose of this work is study dependence the angular distribution of brightness to radiation at TOA on the geometrical and optical observation conditions, as well as clarify the applicability conditions of plane and spherical models.

## 1. Problem statement

Change brightness of radiation at point source in stationary case is described by the following integro-differential transfer equation:

$$
\begin{equation*}
(\vec{\omega}, \operatorname{grad} I(\vec{r}, \vec{\omega}))=-\sigma(\lambda, \vec{r}) I(\vec{r}, \vec{\omega})+\sigma_{s}(\lambda, \vec{r}) \int_{\Omega} I\left(\vec{r}, \vec{\omega}^{\prime}\right) g\left(\vec{r}, \vec{\omega}^{\prime}, \vec{\omega}\right) d \vec{\omega}^{\prime}+\Phi_{0}(\vec{r}, \vec{\omega}) . \tag{1}
\end{equation*}
$$

Here, $\vec{x}=(\vec{r}, \vec{\omega})$ is the point of phase space $\mathrm{X}=R \times \Omega$ of coordinates $\vec{r} \in R$ and directions $\vec{\omega} \in \Omega . \Phi_{0}(\vec{r}, \vec{\omega})$ is the distribution density of sources. $I(\vec{r}, \vec{\omega})$ is the intensity (brightness) at point $\vec{x}=(\vec{r}, \vec{\omega})$.

In this paper, we consider two models of the atmosphere. The first model of planetary atmosphere represents as plane-parallel layered-homogeneous medium, i.e., all quantities in (1) depends on just one coordinate, namely, the depth $z$, while the intensity of the scattered radiation will be a function of the coordinate $z$ and direction of radiation, characterized by the zenith angle $\theta$ and azimuth $\varphi$ in the horizontal plane. Values $z$ are varies from $z=0$ at the bottom of the atmosphere to $z=H$ at TOA, where $H$ is the thickness of the medium. The source is on the Earth's surface, and it will be assumed to be the origin of the coordinates. The receiver is on the upper boundary of the medium and has the coordinates $(0,0, H)$. The system has been the circular symmetry; therefore, all movements of photons are proceed in the YOZ [1].

Layered-homogeneous plane-parallel model was chosen as the model of the medium; it consists on $n$ homogeneous layers, the geometrical thickness, which is characterized by coordinate $z$ (fig. 1).


Fig. 1. Shematic representation plane-parallel model of the atmosphere

The second model represents the layered-homogeneous spherical atmosphere. In this case, all quantities depend on the distance from Earth's surface $h$, while the intensity of scattering radiation will be the function of $h$ and direction of radiation $\vec{\omega}=(\theta, \varphi)$.

The Earth's center to be the origin of coordinates is assumed. The source has the following coordinates: $\left(0,0, R_{0}\right)$; while the receiver located at TOA and has the coordinates ( $0,0, R$ ), where $R_{0}$ is the radius of Earth, and $R$ is the outer radius of the atmosphere. This model of the medium atmosphere with the thickness $H=R-R_{0}$ is divided into $n$ spherical layers with radii $R_{i}, i=\overline{0, n}, R_{n}=R$ [2] (fig. 2).


Fig. 2. Shematic representation spherical model of the atmosphere
Optical model of the atmosphere include the following parameters.

1. The coefficients of scattering $\sigma_{s}(h, \lambda)$ and absorption $\sigma_{a}(h, \lambda)$. Here, $h$ is the height above the Earth's surface, and $\lambda$ is the wavelength. The coefficients $\sigma_{s}$ and $\sigma_{a}$ piecewise constant are assumed, since the atmosphere into homogeneous layers is divided.
2. The scattering phase function $g(h, \mu, \lambda)$. Here, $\mu=\left(\bar{\omega}^{\prime}, \vec{\omega}\right)$ is the cosine of the scattering angle.

The scattering phase function is specified by dividing the atmosphere into layers, in which, the scattering phase function constant is assumed. One of the most universal methods for solving equation (1) is the method of imitation simulation, or the Monte Carlo method. This method on the integral transfer equation of the second kind with generalized kernel for the particle collision density is based:

$$
f(\vec{x})=\int_{X} k\left(\vec{x}^{\prime}, \vec{x}\right) f\left(\vec{x}^{\prime}\right) d \vec{x}^{\prime}+\psi(\vec{x}) .
$$

Monte Carlo method is usually used to estimate linear functional of the form (2):

$$
\begin{equation*}
I_{\varphi}=(f, \varphi)=\int_{X} f(\vec{x}) \varphi(\vec{x}) d \vec{x} . \tag{2}
\end{equation*}
$$

If $\left\{\vec{x}_{n}\right\}$ is a "physical" chain of collisions, then $I_{\varphi}=M \xi$ where $\xi=\sum_{n=0}^{N} Q_{n} \cdot \varphi\left(x_{n}\right)$.
We single out two main algorithms of the Monte Carlo method.

1. The algorithm of direct simulation. This method is based on simulation of random trajectories of photon passage through the scattering medium. It is noteworthy that the characteristics of radiation process, necessary for analysis, are estimated in accordance with their physical meaning. A disadvantage of the direct simulation is that such characteristics as intensity, illumination, and others cannot be calculated with a sufficient accuracy. However, the direct simulation can be used to construct some other methods which make it possible to perform the required calculations.

One of these methods is the algorithm of local estimate.
2. Algorithm of local estimate.

The algorithm of local estimate consists of the calculation of the following functional:

$$
\begin{gathered}
J\left(\theta_{i}\right)=\int_{\theta_{i}} \Phi\left(\vec{r}^{*}, \vec{\omega}^{*}\right) d \vec{\sigma}^{*}=\int_{X} l_{i}\left(\vec{x}^{\prime}, \vec{x}^{*}\right) f\left(\vec{x}^{\prime}\right) d \vec{x}^{\prime}=M \sum_{n=0}^{N} Q_{n} l_{i}\left(\vec{x}_{n}, \vec{x}^{*}\right), \\
l_{i}\left(\vec{x}, \vec{x}^{*}\right)=\frac{\exp \left[-\tau\left(\vec{r}, \vec{r}^{*}\right)\right] g\left(\mu^{*}\right)}{2 \pi\left|\vec{r}-\vec{r}^{*}\right|^{2}} \Delta_{i}\left(\vec{s}^{*}\right) .
\end{gathered}
$$

Here, $\vec{s}^{*}=\frac{\vec{r}^{*}-\vec{r}}{\left|\vec{r}^{*}-\vec{r}\right|}, \mu^{*}=\left(\vec{\omega}, \vec{s}^{*}\right), \Delta_{i}(\vec{s})$ is the indicator of the region $\theta_{i}, \Phi\left(\vec{r}^{*}, \vec{\omega}^{*}\right)$ is the particle flux at a preset point of the phase space $\vec{x}^{*}=\left(\vec{r}^{*}, \vec{\omega}^{*}\right), Q_{n}$ is the particle "weight".

We do not simulate the absorption, but multiply the "weight" by the scattering probability, namely, by the single scattering albedo. It is noteworthy that the variance decreases, and the average time of trajectory simulation on computer increases.

This algorithm is used to calculate the angular distribution of brightness, which represents the following quantity: $I(\Theta)=\frac{J(\Theta)}{\Theta}$, where $\Theta$ is the value of the corresponding to $\theta$ solid angle.

## 2. Initial data

We will consider the process of radiative transfer through aerosol-molecular atmosphere, which comprises layer overcast, by neglecting the reflection from underlying surface. We used the following data:

1. Wavelength (mkm) in transparent windows: $0.347 ; 0.530 ; 0.694 ; 0.860 ; 1,060 ; 3,390$; 10.60 .
2. Lower boundary of atmosphere above Earth's surface, upper limit of the atmosphere H $=30 \mathrm{~km}$ above the Earth's surface.
3. Optical thickness for a cloudless atmosphere shown in Table 1 presented.

Table 1
Optical thickness of the cloudless atmosphere

| Wavelength, mkm | Optical thickness |
| ---: | ---: |
| 0,347 | 0,228 |
| 0,530 | 0,158 |
| 0,694 | 0,124 |
| 0,860 | 0,098 |
| 1,060 | 0,092 |
| 3,390 | 0,067 |
| 10,600 | 0,041 |

4. Lower boundary of the cloud layer - 1 km above the Earth's surface, Upper limit 2 km above the Earth's surface. The optical model of the cloud layer - haze $H$.
5. In this work, we considered two models of sources of radiation: Lambertian, isotropic, and sources. In case of isotropic source, density of the initial areas looks like $p(\stackrel{\rightharpoonup}{\varpi})=\frac{1}{2 \pi}$. For Lambertian this value is defined as $p(\stackrel{\rightharpoonup}{\varpi})=\frac{\mu}{\pi}$.
For the statistical experiments was used a local assessment method in the scheme of constructing conjugate trajectories. Calculations were carried out based software package, which designed for a cloudless atmosphere. This complex was modified in order to be able to con-
duct simulations under the presence of clouds. Numerical simulation results are shown in Table 2 .

Table 2
The brightness of the scattered radiation to the atmosphere from the cloud layer type "H Haze"

| Angle of <br> reception, <br> grad | Wavelengths, mkm |  |  |  |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\lambda=0,374$ | $\lambda=0,530$ | $\lambda=0,694$ | $\lambda=0,860$ | $\lambda=1,060$ | $\lambda=3,390$ | $\lambda=10,60$ |  |  |
| 4,5 | $2,24 \mathrm{E}-05$ | $1,67 \mathrm{E}-05$ | $1,38 \mathrm{E}-05$ | $1,13 \mathrm{E}-05$ | $1,19 \mathrm{E}-05$ | $5,55 \mathrm{E}-06$ | $3,51 \mathrm{E}-06$ |  |  |
| 13,5 | $1,09 \mathrm{E}-06$ | $7,65 \mathrm{E}-07$ | $6,60 \mathrm{E}-07$ | $5,30 \mathrm{E}-07$ | $4,56 \mathrm{E}-07$ | $2,09 \mathrm{E}-07$ | $1,43 \mathrm{E}-07$ |  |  |
| 22,5 | $2,23 \mathrm{E}-07$ | $1,83 \mathrm{E}-07$ | $1,55 \mathrm{E}-07$ | $1,39 \mathrm{E}-07$ | $1,12 \mathrm{E}-07$ | $4,84 \mathrm{E}-08$ | $3,51 \mathrm{E}-08$ |  |  |
| 31,5 | $8,44 \mathrm{E}-08$ | $5,63 \mathrm{E}-08$ | $5,54 \mathrm{E}-08$ | $4,73 \mathrm{E}-08$ | $3,85 \mathrm{E}-08$ | $1,95 \mathrm{E}-08$ | $1,32 \mathrm{E}-08$ |  |  |
| 40,5 | $3,49 \mathrm{E}-08$ | $2,73 \mathrm{E}-08$ | $2,81 \mathrm{E}-08$ | $6,65 \mathrm{E}-08$ | $4,33 \mathrm{E}-08$ | $8,70 \mathrm{E}-09$ | $5,38 \mathrm{E}-09$ |  |  |
| 49,5 | $1,86 \mathrm{E}-08$ | $5,15 \mathrm{E}-08$ | $1,54 \mathrm{E}-08$ | $1,18 \mathrm{E}-08$ | $9,30 \mathrm{E}-09$ | $4,23 \mathrm{E}-09$ | $4,59 \mathrm{E}-09$ |  |  |
| 58,5 | $1,30 \mathrm{E}-08$ | $1,10 \mathrm{E}-08$ | $1,71 \mathrm{E}-08$ | $7,75 \mathrm{E}-09$ | $6,64 \mathrm{E}-09$ | $3,58 \mathrm{E}-09$ | $3,57 \mathrm{E}-09$ |  |  |
| 67,5 | $1,94 \mathrm{E}-08$ | $1,12 \mathrm{E}-08$ | $7,95 \mathrm{E}-09$ | $2,11 \mathrm{E}-08$ | $1,73 \mathrm{E}-08$ | $8,16 \mathrm{E}-08$ | $6,37 \mathrm{E}-09$ |  |  |
| 76,5 | $1,58 \mathrm{E}-07$ | $2,13 \mathrm{E}-08$ | $4,96 \mathrm{E}-09$ | $3,68 \mathrm{E}-09$ | $2,90 \mathrm{E}-09$ | $1,99 \mathrm{E}-09$ | $1,18 \mathrm{E}-09$ |  |  |
| 85,5 | $1,20 \mathrm{E}-08$ | $6,37 \mathrm{E}-09$ | $1,23 \mathrm{E}-09$ | $1,03 \mathrm{E}-09$ | $1,00 \mathrm{E}-09$ | $5,13 \mathrm{E}-11$ | $1,16 \mathrm{E}-11$ |  |  |

## 3. Statistical analysis of simulation result

Regression analysis was used to establish a functional link between the angular distribution of brightness and optical parameters. This analysis is widely used to restore the characteristics of aerosol and clouds, as well as to assess their impact on the climate.

For the angular distribution of brightness in the aerosol and molecular cloud and the atmosphere relative to the wavelength, the regression equation was built. This regression equation was obtained in the form of $y=e^{b_{0}} x^{h_{1}}$ for all wavelengths. The regression coefficients are shown in Tables 4-5. coefficients of determination were also given for each equation.

Similarly, the regression equation was built for the angular distribution of brightness in the cloud spherical atmosphere at fixed angles reception. Regression coefficients and the coefficient of determination are shown in Table 3.

Table 3
The coefficients of the regression equation for cloudless atmosphere

| Angle of reception, grad | Coefficient $b_{0}$ | Coefficient $b_{1}$ | Coefficient of determination $R^{2}$ |
| ---: | ---: | ---: | ---: |
| 4,5 | $1,19595 \mathrm{E}-05$ | $3,25534 \mathrm{E}-05$ | 0,993636438 |
| 13,5 | $2,31401 \mathrm{E}-07$ | $5,05694 \mathrm{E}-07$ | 0,976885608 |
| 22,5 | $6,98539 \mathrm{E}-08$ | $1,19799 \mathrm{E}-07$ | 0,927503534 |
| 31,5 | $3,92659 \mathrm{E}-08$ | $4,04551 \mathrm{E}-08$ | 0,784773208 |
| 40,5 | $2,4403 \mathrm{E}-08$ | $1,51266 \mathrm{E}-08$ | 0,570060008 |
| 49,5 | $1,39106 \mathrm{E}-08$ | $6,51344 \mathrm{E}-09$ | 0,482543448 |
| 58,5 | $6,82884 \mathrm{E}-09$ | $3,64745-09$ | 0,609744683 |
| 67,5 | $2,74359 \mathrm{E}-09$ | $2,35355 \mathrm{E}-09$ | 0,771155104 |
| 76,5 | $5,21933 \mathrm{E}-10$ | $1,23544 \mathrm{E}-09$ | 0,907220265 |
| 85,5 | $3,00774 \mathrm{E}-11$ | $1,7624 \mathrm{E}-10$ | 0,929769833 |

Table 4
The coefficients of the regression equation for a cloudy atmosphere

| Angle of reception, grad | Coefficient $b_{0}$ | Coefficient $b_{1}$ | Coefficient of determination $R^{2}$ |
| ---: | ---: | ---: | ---: |
| 4,5 | $3,52884 \mathrm{E}-06$ | $7,12796 \mathrm{E}-06$ | 0,98542826 |
| 13,5 | $1,08652 \mathrm{E}-07$ | $3,63939 \mathrm{E}-07$ | 0,997272 |
| 22,5 | $3,63765-08$ | $7,53979 \mathrm{E}-08$ | 0,973075322 |
| 31,5 | $1,26022 \mathrm{E}-08$ | $2,66418 \mathrm{E}-08$ | 0,976384876 |
| 58,5 | $3,68558 \mathrm{E}-09^{*}$ | $4,34001 \mathrm{E}-09$ | 0,598507887 |
| 76,5 | $-3,34288 \mathrm{E}-08^{*}$ | $5,0401 \mathrm{E}-09$ | 0,606097561 |
| 85,5 | $-2,37483 \mathrm{E}-09^{*}$ | $4,50822 \mathrm{E}-09$ | 0,809969895 |

Statistical evaluation of the regression equations on the significance was conducted by Ftest and Student's t-test. With $90 \%$ confidence it can be argued that the considered dependence are statistically significant.

It should be noted that the obtained analytical data for the cloudless atmosphere are more accurate as compared with the data from the atmosphere cloud layer.

This once again confirms that the turbid layered homogeneous environment analysis and the preparation of some of the radiation brightness characteristics of a distorted unlike characteristics in the clear atmosphere.

The analysis shows that between the obtained angular distributions of intensity and wavelength for both cloudless and cloudy for the atmosphere, there is a link that can be with a good degree of accuracy to describe hyperbolic regression equation. This fact can be used for prediction, assessment and analysis of images of objects observed through the scattering medium.

## Conclusion

In the course of paper the following problem was posed and solved:

1. Simulation model for the propagation of radiation is constructed, which based on the dual transport equation using a local assessment method.
2. Algorithm of the Monte Carlo method for calculation angular distribution of the radiation intensity in a cloudless sky is implemented.
3. Dependence angular distribution of brightness on the wavelength, the thickness of the atmospheric model and radiation sources is investigated.
4. Applying regression analysis to establish functional relationship between the angular distribution of brightness and parameters of model the atmosphere.

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