Servo-System Simulation

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***Abstract* — The objective of this work is an adaptive control system modeling in servo-system in case of component control of a state vector. The integral quadratic functional is used to form the control actions under the conditions as an object's behavior and its description on a sliding range of stationary optimization of a predictive model. The results of the simulation process control system and modeling are given in this paper.**

Key words—predictive model, servo-system, adaptive control system

1. Introduction

Nowadays the phase of information is experienced and is characterized by the information expansion and computer systems, and more by creation and development of automatic control methods in engineering, economics, medicine, biology and other fields. Thus, more and more the servo- automatic control systems for the combined synthesis are being applied, as the most suitable to use in the control computers. The most promising methods that allow to solve problems of combined synthesis are those methods that are based on quadratic functional optimization, and use predictive models and a sliding time interval can deepen the synthesis ability of adaptive systems significantly.

The simulation of adaptive control of servo system is being discussed in this work in the case when modification of algorithms was given in [1, 2]. This system helps to control some parts of a state vector. The purpose of the modification is to reduce the number of components of the monitored state. In contrast to the proposed in [3], where predictive model reduction is carried out, proposed a fairly simple way to reduce the number of components of the monitored state, while providing the required quality of the managed object functioning. The control system allows to have incomplete measurement and provides the status evaluation and parameters of Kalman's filters. The results of the simulation of process control system are given.

2. Mathematical Model and Control Algorithm

In the case when the object control in the process of its operation is carried out with help of a computer, then some special conditions for the implementation of the control actions are formed, the main among them is discreteness of a forming control by time. Let us assume that the digital system for object control is used which forms a piecewise constant vector control point and a sampling period, coinciding with the moment and the receipt of the informational period about the object. At the same time the non-stop activity of the control signal (with influence on an object) is achieved by keeping its constant value between moments of formation. Thus, control belongs to the class of a piecewise constant continuous functions from the left with the fixed sampling period Δt.

A mathematical model of the object is described by a system of the ordinary differential stochastic equations of the following form:

$$\begin{aligned} &\dot{x}(t) = \overline{A}(t)x(t) + \overline{B}(t)u(t) + \overline{F}(t)q(t), \;\; x(t_0) = x_0, \\ &\dot{u}(t) = v(t), \;\; u(t_0) = u_0, \end{aligned} \tag{1}$$

where $x(t)\in R^n$ – is a state vector, $u(t),v(t)\in R^m$ – is a position vector of control body and a control vector.

Let us suppose that external disturbances are additive and are described by the vector of Gaussian noises $q(t)\in R^{l1}$ with specified data:

$$M\{q(t)\} = \overline{q}(t), \;\; M\{(q(t)-\overline{q}(t))(q(\tau)-\overline{q}(\tau))^T\} = Q(t)\delta(t-\tau),$$

where $\delta(t-\tau)$ – is a delta-Dirac's function.

In (1) matrices $\overline{A}(t), \overline{B}(t), \overline{F}(t)$ - are matrices of the dynamic properties, influence, control impacts and external disturbances respectively.

Modeling of the control system will be implemented in the assumption that the sampling step Δt of an object model coincides with the quantization control signal, but the sampling moment k – with the instant application of the control actions. Moreover, control and the external perturbation described by piecewise constant functions on the left-continuous range of the control actions.

Under a small value Δt for modeling Euler's method is often used. In this case, the system (1) is represented as:

$$\begin{aligned} &x(k+1) = A(k)x(k) + B(k)u(k) + F(k)q(k), \\ &x(0) = x_0, \\ &u(k+1) = u(k) + \Delta t v(k), \;\; u(0) = u_0, \end{aligned} \tag{2}$$

where $A(k) = I_n + \Delta t\overline{A}(t_k),\ B(k) = \Delta t\overline{B}(t_k),\ F(k) = \sqrt{\Delta t}\,\overline{F}(t_k)$, k corresponds to the time moment $t_k=t_0+k\Delta t$.

In (2) $q(k)$ – vector of array random variable with the data

$$M\{q(k)\} = \overline{q}(k) = \overline{q}(t_k),\ \ M\{(q(k)-\overline{q}(k))(q(j)-\overline{q}(j))^T\} = Q(k)\delta_{k,j},$$

where $\delta_{k,j}$ – is Kronecker's symbol.

Synthesis of the control actions in the process of object operation imposes the certain restrictions on its implementation. The main limitation is that the control formation in moment t_k, some information is known about the object behavior in the previous moments of time and is not known in the following. In addition, there is a wide range of tasks, where extends the object requirement to achieve a given state $x_z(t)$, followed by holding the state of the object in a small neighborhood $x_z(t)$.

Formation of the control actions will be carried out by using a modification of the algorithms proposed in [4]. In order to reduce the number of the vector components of the monitored state, let us introduce an S matrix of degree $p\times n$ of rank p, composed of zeros and identities, zero columns that define untracked components of a given state [5]. For the model of the object (1) let us represent the optimized functional in the following form:

$$J^{(S)}(t_k) = \frac{1}{2}M\Bigg\{\int_{t_k}^{t_k+l_p\Delta t}[(x^{(S)}(t)-x_z^{(S)}(t))^T C_S(x^{(S)}(t)-x_z^{(S)}(t)) + u^T(t)D_2u(t) + v^T(t)D_1v(t) + v_{on}^T(t)D_1v_{on}(t)]dt\Bigg\}, \quad (3)$$

where $x^{(S)}(t) = Sx(t)$, $x_z^{(S)}(t) = Sx_z(t)$; C_S – is a positive semi-definite matrix of order p; D_1, D_2 – is a positive definite and are semi-definite matrices of order m; $v_{on}(t)$ – is an optimal control.

The system order p, that describes the object's behavior on sliding range optimization $[t_k,\ t_k + l_p\Delta t]$, that is called as a predictive model, and it can be written like this:

$$\begin{aligned} &x_M^{(S)}(j+1) = A_S(k)x_M^{(S)}(j) + B_S(k)u(k) + F_S(k)\overline{q}(k),\\ &x_M^{(S)}(j=k) = Sx(k) \end{aligned} \quad (4)$$

where «M» indicates the belonging to the predictive model,

$$As(k)=SA(k)S^T,\ Bs(k)=SB(k),\ Fs(k)=SF(k)$$

The control influences under functional optimization (3) and use of the predictive model (4) is defined by the following:

$$v_{on}(k) = v(k) = -D^{-1}W_2^{(S)}(k), \quad (5)$$

where $W_2^{(S)}(k)$ is found with solution of reverse time difference equations

$$\begin{aligned} &g_M^{(S)}(j-1) = 2g_M^{(S)}(j) - A_S(k)g_M^{(S)}(j) - B_S(k)u(k) - F_S(k)\overline{q}(k),\\ &g_M^{(S)}(k+l_p) = x_M^{(S)}(k+l_p),\\ &W_1^{(S)}(j-1) = A_S^T(k)W_1^{(S)}(j) + \Delta tC_S(g_M^{(S)}(j) - x_z^{(S)}(t_k)),\\ &W_1^{(S)}(k+l_p) = 0_p,\\ &W_2^{(S)}(j-1) = W_2^{(S)}(j) + B_S^T(k)W_1^{(S)}(j) + \Delta tD_2u(k),\\ &W_2^{(S)}(k+l_p) = 0_m,\\ &j = k+l_p, k+l_p-1,..., k+1. \end{aligned} \quad (6)$$

In (6) $W_1^{(S)}(\cdot)$, $W_2^{(S)}(\cdot)$ – are columns-vector of vectors-columns orders p and m; O_p, O_m – are zero vectors-columns orders p and m respectively.

Usually in the case of control synthesis on the amount of control are applied restrictions. It is happen quite often, these restrictions are imposed in the form of inequalities:

$$U_{1i}(k) \le u_i(k) \le U_{2i}(k),\ \ i = \overline{1,m} \quad (7)$$

3. State value and model parameters

Some information about the object condition comes from the measuring complex in the formation of control influences in the real conditions. This complex often contains incomplete information about the object, which is distorted by measurement errors.

We assume that the mathematical model of measurement system is as follows:

$$y(k) = Hx(k) + r(k), \quad (8)$$

where $y(k)\in R^l$ – is a vector of measurements, H – is a matrix of measurements channel, that consists from zeros and identity, where zero columns correspond to vector's behavior components inaccessible dimension, $r(k)$ – discrete Gaussian noise with characteristics:

$$M\{r(k)\} = 0,\ \ M\{r(k)r^T(j)\} = R\delta_{k,j}$$

Moreover, it is necessary to consider the presence of the time-varying parameters of the unknown parameters in the object model under the conditions when the synthesis of control is in real operating conditions of the object.

Let us use the mathematical model of the object in the form as follows in order to form the control actions:

$$\begin{aligned} &x(k+1) = A(k,\theta(k))x(k) + B(k,\theta(k))u(k) + F(k)q(k),\\ &x(0) = x_0, \end{aligned} \quad (9)$$

where $\theta(k)$ – N- is a dimensional vector of variables in time of the unknown parameters in the model of the object. It is assumed that the elements of the matrices $A(\cdot)$ and $B(\cdot)$ linearly dependent on the vector components $\theta(\cdot)$, and a priori distribution of the vectors of the initial conditions $\theta(0)$ and x_0 are Gaussian:

$$M\{\theta(0)\} = \bar{\theta}_0, \quad M\{(\theta(0) - \bar{\theta}_0)(\theta(0) - \bar{\theta}_0)^T\} = P_{\theta_0},$$
$$M\{x_0\} = \bar{x}_0, \quad M\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0) = P_{x_0}.$$

Here $P_{\theta 0}$ and P_{x0} – are covariance matrices of errors of the initial conditions of vector parameters and the state of the object model.

We will assess the state of the object model and parameters by using the discrete Kalman's filters [6]. A recurrent algorithm for assessing the state has the following form:

$$\begin{aligned}
&\hat{x}(k+1) = \hat{x}(k+1/k) + K(k)[y(k+1) - H\hat{x}(k+1/k)],\\
&\hat{x}(k+1/k) = A(k,\hat{\theta}(k))\hat{x}(k) + B(k,\hat{\theta}(k))u(k) + F(k)\bar{q}(k),\\
&\hat{x}(0) = \bar{x}_0,\\
&K(k) = P_x(k+1/k)H^T[HP_x(k+1/k)H^T + R]^{-1},\\
&P_x(k+1/k) = A(k,\hat{\theta}(k))P_x(k)A^T(k,\hat{\theta}(k)) + F(k)QF^T(k),\\
&P_x(k+1) = [I_n - K(k)H]P_x(k+1/k),\\
&P_x(0) = P_{x_0}.
\end{aligned} \tag{10}$$

Estimation of the parameters is being done by using the following algorithm:

$$\begin{aligned}
&\hat{\theta}(k+1) = \hat{\theta}(k) + L(k)[y(k+1) - H\Phi(\hat{x}(k),u(k))\hat{\theta}(k) -\\
&\qquad - Hf(\hat{x}(k),u(k))],\\
&\hat{\theta}(0) = \bar{\theta}_0,\\
&L(k) = P_\theta(k)\Phi^T(\hat{x}(k),u(k))M^{-1},\\
&M(k) = H\Phi(\hat{x}(k),u(k))P_\theta(k)\Phi^T(\hat{x}(k),u(k))H^T +\\
&\qquad + HF(K)QF^T(k)H^T + R,\\
&P_\theta(k+1) = [I_N - L(k)H\Phi(\hat{x}(k),u(k))]P_\theta(k),\\
&P_\theta(0) = P_{\theta_0}.
\end{aligned} \tag{11}$$

In (11) matrix $\Phi(\cdot)$ and vector $f(\cdot)$ are obtained as a result of the system representation (9) in the form:

$$x(k+1) = \Phi(x(k),u(k))\theta(k) + f(x(k),u(k)) + F(k)q(k),$$
$$x(0) = x_0$$

In the synthesis of adaptive control in (4), (6) the following vectors and matrices are used:

$$x^{(S)}(t_k) = S\hat{x}(t_k), \quad A_S(k) = SA(k,\hat{\theta}(k))S^T, \quad B_S(k) = SB(k,\hat{\theta}(k))$$

4. The Simulation Results of Adaptive Process Control System

As a process, let's consider a mixing tank, the scheme is shown in figure 1. The tank has two input streams of fluids with a constant concentration. The task control is in automatic maintenance of the flow and liquid concentration of output stream in accordance with the specified values.

The linearized mathematical model of the mixing tank can be written as (1) [7], where vector components $x(t)$ and $u(t)$ have the following meaning: $x_1(t)$ – is a flow rate deviation (m^3/s), $x_2(t)$ – is an output concentration deviation ($Kmol/m^3$), $x_3(t)$, $x_4(t)$ – are variables, simulating fluctuations of linearized model (Gaussian correlated perturbations $u_1(t)$ – the deviation of the flow of the first input (m^3/s), $u_2(t)$ – the deviation of the flow of the second input (m^3/s).

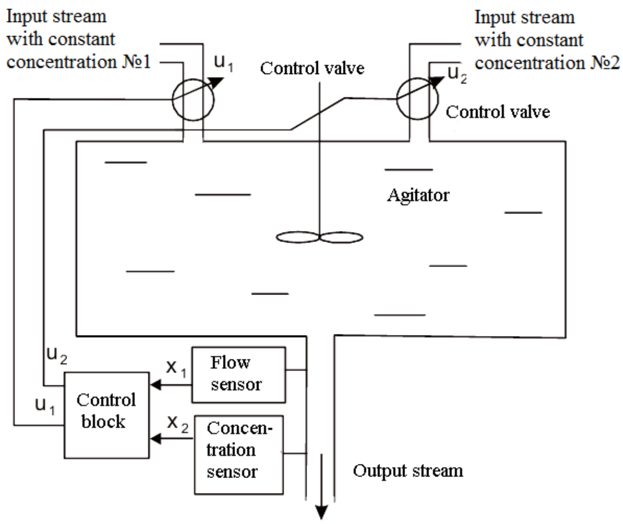

Fig.1. Mixing tank scheme

External perturbations in the model of the object is described by vector $q(t)$, which components are independent normal Gaussian variables with matrix influence $\bar{F}(t)$.

Matrices $\bar{A}, \bar{B}, \bar{F}$ are equal respectively:

$$\bar{A} = \begin{pmatrix} -0.01 & 0 & 0 & 0 \\ 0 & -0.02 & 0.01 & 0.005 \\ 0 & 0 & -0.025 & 0 \\ 0 & 0 & 0 & -0.02 \end{pmatrix},$$

$$\bar{B} = \begin{pmatrix} 1 & 1 \\ -0.25 & 0.75 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{F} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.35\cdot 10^{-2} & 0 \\ 0 & 0.55\cdot 10^{-2} \end{pmatrix}$$

vector of initial state $x_0 = (0.025;0;0;0)^T$, $t_0 = 0$.

In order to simulate Euler's method with step $\Delta t = 0{,}1s$ is used. Time of modeling $T = 40s$. Synthesis of control actions is carried out by measuring the deviation of the output concentration of the solution. This mathematical model of measurement system has the form (8), where

$$H = (0 \quad 1 \quad 0 \quad 0),\ R = 10^{-5}.$$

The unknown parameters are the elements of the matrix $\bar{B}$: $b_{2,1}$; $b_{2,2}$. Control actions are formed on the estimated state vectors $\hat{x}(k)$ and parameters $\hat{\theta}(k)$, the following initial values:

$$\hat{x}(0) = (0.03;0;0;0)^T, \quad P_{x_0} = I_4$$

$$\hat{\theta}(0) = (-0.02;0.07)^T, \ P_{\theta_0} = I_2,$$

where I_4, I_2 – are identity matrices of corresponding orders.

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The aim of control is to maintain the system in the state neighborhood $x_z = (0.04;0;0;0)^T$. The following restrictions are imposed at the maximum deflection of vector component control:

$$|u_1(k)| \le 0.0174(m^3/s); \quad |u_2(k)| \le 0.058(m^3/s)$$

The quality control system must meet the specified requirements, i.e. for the state vector components must satisfy the following restrictions:

$$|x_1 - x_{z_1}| \le 0.04(m^3/s); \quad |x_2 - x_{z_2}| \le 0.01(Kmol/s)$$

The tracking is carried for the first two components of the state vector, i.e. matrix in the formation of the control actions

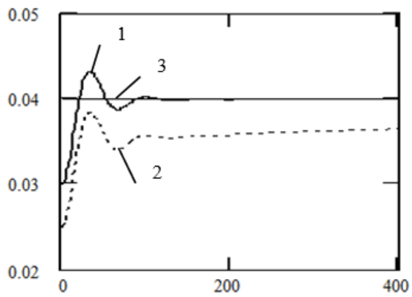
$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The length of the sliding range of optimization is supposed to be equal $l_p\,\Delta t = 0.4s$, value C_S, D_1, D_2 were given in the form:

$$C_S = \begin{pmatrix} 0.515 \cdot 10 & 0 \\ 0 & 0.6305 \cdot 10^{-2} \end{pmatrix},$$

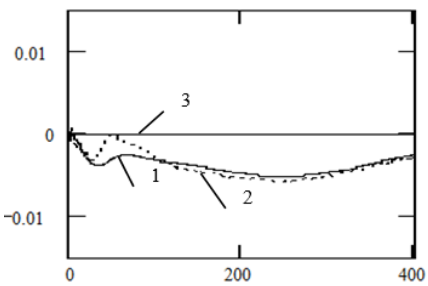
$$D_1 = D_2 = \begin{pmatrix} 0.35 & 0 \\ 0 & 3 \end{pmatrix}$$

The simulation results are shown in figure 2. In figures 2a and 2b a curve 1 shows a component of the state vector, curve 2 shows assessment of this component and the curve 3 shows a state, where control is done. In figure 2c the curve 1 shows the position of the first control body, and the curve 2 - the second.



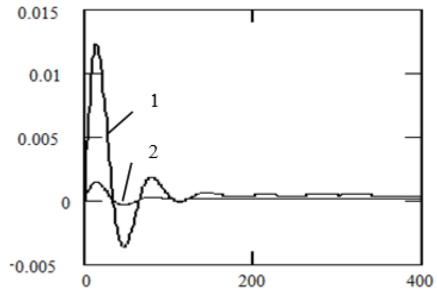
curve 1 – $x_1(k)$, curve 2 – $\hat{x}_1(k)$, curve 3 – $x_{z1}(k)$

a)



curve 1 – $x_2(k)$, curve 2 – $\hat{x}_2(k)$, curve 3 – $x_{z2}(k)$

b)



curve 1– $u_1(k)$, curve 2 – $u_2(k)$

c)

Fig.2. Simulation results

The given results are shown that all requirements to the quality of a control function process are being done. Besides, use of the functional form (5) greatly simplifies the task force of the weight matrix to reduce its order.

V. Conclusion

The algorithms of an adaptive control system designing are given in this paper, these algorithms monitor the components of the state vector. Object model is described by means of a linear system of ordinary differential equations with Gaussian a priori disturbances. The control system includes a model of the measuring complex, which allows incomplete measurement errors that are described by Gaussian variables, algorithms for the status and parameters evaluation of the object model, algorithms for constraint satisfaction of control.

The control mixing tank is being given as the example in order to illustrate the efficiency and quality of the proposed algorithms, which mathematical model is described by the fourth-order system. Simulation of the control system is performed only under the following conditions when output stream is measured, and the control is done in order to see fluid flow rate and output concentration. The simulation results shown in the graphs represent that all requirements to the operation quality of the controlled process are being carried out.

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