AIP Conference Proceedings

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Citation: AIP Conference Proceedings **1698**, 060005 (2016); doi: 10.1063/1.4937860
View online: http://dx.doi.org/10.1063/1.4937860
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1698?ver=pdfcov
Published by the AIP Publishing

The Mathematical Model of the Chevron-Arch Gearing Transmitter

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Abstract. The teeth of herringbone transmission wheels are obtained by docking two helical wheels with an opposite arrangement of teeth, which can solve the problem of the axial force. The mathematical model of coupling chevron teeth of the driving wheel in the area of their docking using the arch tooth fragment is developed. The conjugacy area surface of the driven wheel chevron teeth is obtained as the envelope of the surfaces family formed by the arched tooth during the process of the parts motion.

INTRODUCTION

The basis of most modern machines and mechanisms is cylindrical gears which, to a large extent, determine technical conditions and reliability of machines. The most common in practice of engineering are cylindrical gears with straight and helical teeth, the theoretical engagement contact of which takes place along a line. The eccentrically cycloidal gearing (EC-gearing) [1, 2] of the transmitter parts, developed at the Closed Joint Stock Company "Technology market" (Tomsk, Russia) in 2007, is a helical gearing having a number of advantages (it is less noisy, transmits more torque and has a greater efficiency). Unlike the classical involute gearing, the teeth profiles in the EC-gearing are a cycloidal curve and a circle. A significant disadvantage of any helical gearing is presence of the axial force. Herringbone gearing is used to compensate for the axial force, with a 2-times increase in axial dimensions of the parts. When manufacturing a chevron gearing, the chevron teeth docking area is observed in the centre of the part, which does not participate in torque transmission. To address this disadvantage in the case of an involute gearing the so-called arch tooth is placed in the docking area. Arched cylindrical gear wheels were studied by a great number of Russian and foreign scientists. V. N. Sevryuk [3] built an arch engagement on the basis of Novikov's gears. M. I. Yevstigneev [4] dealt with designing arch transmissions based on an involute engagement as applied to aircraft engines. M. N. Bobkov [5] investigated an arched transmission scheme with circular teeth of the drive shaft. R.A. Matsey [6] studied an arch-helical gear constructed in the framework of involute gearing. M. R. Varshavskiy [7] researched the stress-strained state of arch teeth of spur gears. A. I. Belyaev [8] analyzed features of cutting and calculating theoretically accurate arch gears. V. N. Syzrantsev [9] dealt with profiling cylindrical wheels with arched teeth. Arched engagement allows stabilizing the state of an operating gear by means of minimizing axial forces.

In this paper the mathematical (geometric) model is developed which can be applied for the construction, the chevron teeth of which are in the EC-gearing, while the arched teeth are located in the centre of the input part (the gear). These teeth are in contact with the output part (the wheel) teeth, the surfaces of which are obtained as the envelopes of the surfaces families of the arch teeth.

Advanced Materials in Technology and Construction (AMTC-2015)
AIP Conf. Proc. 1698, 060005-1–060005-6; doi: 10.1063/1.4937860

PARAMETRIC EQUATIONS OF THE ARCH GEAR TOOTH SURFACE

The Parametric equations of the curves and surfaces considered

$$x=f_0(u),\qquad x=g_0(u,v),\ y=f_1(u),\qquad y=g_1(u,v),\ z=f_2(u),\qquad z=g_2(u,v)$$

will be written in the form of vector-valued functions of one and two arguments, i.e. as :

$$\vec{\boldsymbol{r}}(u)=\{f_0(u),f_1(u),f_2(u)\},$$
$$\vec{\boldsymbol{R}}(u,v)=\{g_0(u,v),\ \mathrm{g}_1(u,v),g_2(u,v)\}.$$

The work surface of the arch tooth is the surface described by the curve (the tooth profile) in the plane perpendicular to the rotation axis of the part. The profile moves along this axis and rotates about it. The line *Lc* is the line of the arch tooth profile (the arch) displacement. It is placed on the cylinder and in the unfolded pattern of the cylinder it is observed as the circle *L* with the radius *R* (Fig. 1).

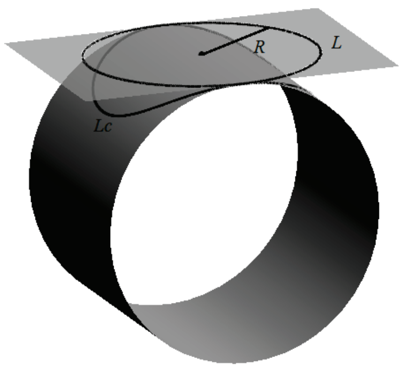

FIGURE 1. The line *Lc* on the cylinder – the line of the arch tooth profile (the arch) displacement. The circle *L* with the radius *R* – the image of the line *Lc* in the unfolded pattern of the cylinder

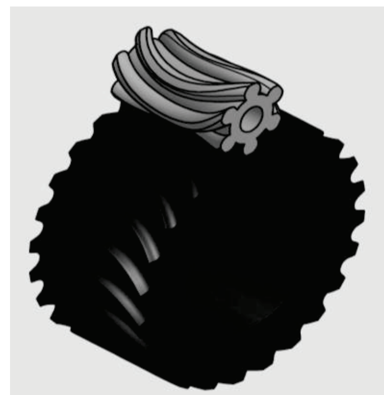

FIGURE 2. Arch teeth of the input and output parts of the gearing

The tooth profile of the input part is the circle with the radius ρ. A family of such circles with centres on the line *Lc,* located in the planes perpendicular to the rotation axis of the part, forms the arch tooth surface of the input part (Fig. 2).

Let the rotation axis of the wheel coincide with the axis *OZ* and the rotation axis of the gear parallel to it be displaced along the axis *OX* by the value of centre-to-centre spacing *Aw.* This displacement is determined by the vector $\overrightarrow{\boldsymbol{Aw}}=(Aw,0,0)$.

Then the equations of the cylinder with the radius ε, the line of the gear arch tooth profile displacement lies on, can be written as:

$$\overrightarrow{\boldsymbol{CL}}(u,v)=\left\{-\varepsilon\cos\left(\frac{u}{\varepsilon}\right),\ -\varepsilon\sin\left(\frac{u}{\varepsilon}\right),v\right\}+\overrightarrow{\boldsymbol{Aw}}\,, \tag{1}$$

where $u=0,..,2\pi\varepsilon, v=\frac{-lr}{2},..,\frac{lr}{2}$ and *lr* is the axial dimension of the part.

With this parameterization of the cylinder its first differential quadratic form coincides with the first differential quadratic form of the parameters (u, v) plane. This reduces the problem of the space curves touch (the lines on the cylinder) to the touch of the plane curves (the images of these lines when displaying the cylinder onto the plane). We will need two intersecting helices on the cylinder – the displacement lines of the chevron EC-gear teeth profiles and the *Lc*-arch touching these helices. The equations of the helices on the cylinder intersecting at the point $\overrightarrow{\boldsymbol{CL}}(0,0)$ have the form:

$$\overrightarrow{\boldsymbol{Wnt}_1}(u)=\left\{-\varepsilon\cos\left(\frac{u}{\varepsilon}\right),-\varepsilon\sin\left(\frac{u}{\varepsilon}\right),\frac{lr\,z\,u}{2\pi\varepsilon}\right\}+\overrightarrow{\boldsymbol{Aw}},\quad \overrightarrow{\boldsymbol{Wnt}_2}(u)=\left\{-\varepsilon\cos\left(\frac{u}{\varepsilon}\right),\ \varepsilon\sin\left(\frac{u}{\varepsilon}\right),\frac{lr\,z\,u}{2\pi\varepsilon}\right\}+\overrightarrow{\boldsymbol{Aw}}, \tag{2}$$

where $u=0,..,\frac{\pi\varepsilon}{z}$ and z is the number of the gear teeth. Since the arch in the parameter plane is the circle *L* with the radius *R*, then the parametric equations of the circle is written as:

$$ua(\alpha)=R\cos\alpha+h,\quad va(\alpha)=R\sin\alpha\,. \tag{3}$$

We will define the parameter *h* from the condition of the circle tangency (3) with the images of the helices (2), i.e. with two symmetric lines about the axis *OX* passing through the origin of coordinates of the parameters plane.

The line touching the circle *L* is defined by two points: *O* (0,0) and *M0* – the image of one of the helix points $\overrightarrow{\boldsymbol{Wnt}_1}(u)$, for example, $\overrightarrow{\boldsymbol{Wnt}_1}\left(\frac{-\pi\varepsilon}{z}\right)$, i.e. $\overrightarrow{\boldsymbol{M0}}\left(\frac{-\pi\varepsilon}{z},\frac{-lr}{2}\right)$. Denoting the coordinates of the point $\overrightarrow{\boldsymbol{M0}}$ through $M0_0$, $M0_1$, let us write the equation of the line in normal form:

$$\frac{M0_1\,x-M0_0\,y}{\left|\overrightarrow{\boldsymbol{M0}}\right|}=0\,. \tag{4}$$

The tangency condition for the circle *L* with the radius *R* and the line (4) can be replaced by the requirement that the centre of the circle, the point $\vec{\boldsymbol{C}}(h,0)$, is disposed from the line *R*, which leads to the relation:

$$\left|\frac{M0_1\cdot h-M0_0\cdot 0}{\left|\overrightarrow{\boldsymbol{M0}}\right|}\right|=R,$$

from which we obtain:

$$h=\frac{R\left|\overrightarrow{\boldsymbol{M0}}\right|}{M0_1}, \tag{5}$$

after introducing (5) into (3), and (3) into (1) we obtain the equation of the arch on the cylinder:

$$\overrightarrow{\boldsymbol{Ar}}(\alpha)=\left\{-\varepsilon\cos\left(\frac{R\cos\alpha+h}{\varepsilon}\right),-\varepsilon\sin\left(\frac{R\cos\alpha+h}{\varepsilon}\right),R\sin\alpha\right\}+\overrightarrow{\boldsymbol{Aw}}\,.$$

To simplify further calculations when defining the envelope let us turn the arch round the rotation axis of the cylinder at the angle θ up to the extremum point of the arch $\overrightarrow{\boldsymbol{Ar}}(0)$ hitting the line *OX*, i.e. the point (Aw-ε, 0, 0). Then the equation of the arch on the cylinder becomes:

$$\overrightarrow{\boldsymbol{Ark}}(\alpha)=\left\{-\varepsilon\cos\left(\frac{R\cos\alpha+h}{\varepsilon}\right),-\varepsilon\sin\left(\frac{R\cos\alpha+h}{\varepsilon}\right),R\sin\alpha\right\}Q(\theta)+\overrightarrow{\boldsymbol{Aw}}, \tag{6}$$

where $Q(\theta)$ denotes the matrix of the turn which the vector function has round the axis *OZ* at the angle θ:

$$Q(\theta)=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix},$$

and the angle θ is determined from the equation which is obtained by equating the first coordinate of the vector function $\overrightarrow{\boldsymbol{Ark}}(0)$ from (6) to Aw-ε:

$$\theta = \frac{-R+h}{\varepsilon}.$$

The helices (2) are to be rotated at the same angle, after which their equations take the form:

$$\overrightarrow{\boldsymbol{Wint}_1}(u) = \left\{-\varepsilon\cos\left(\frac{u}{\varepsilon}\right), -\varepsilon\sin\left(\frac{u}{\varepsilon}\right), \frac{lr\,z\,u}{2\pi\varepsilon}\right\} Q(\theta) + \overrightarrow{\boldsymbol{Aw}},\quad \overrightarrow{\boldsymbol{Wint}_2}(u) = \left\{-\varepsilon\cos\left(\frac{u}{\varepsilon}\right), \varepsilon\sin\left(\frac{u}{\varepsilon}\right), \frac{lr\,z\,u}{2\pi\varepsilon}\right\} Q(\theta) + \overrightarrow{\boldsymbol{Aw}}. \quad (7)$$

To find the tangency point of the arch and the helix images let us write this point as:

$$\overrightarrow{\boldsymbol{Mk}} = \Lambda \cdot \overrightarrow{\boldsymbol{M0}},$$

where the value Λ is found from the vectors perpendicularity condition ($(\vec{\boldsymbol{C}} - \Lambda \cdot \overrightarrow{\boldsymbol{M0}})$ and $\overrightarrow{\boldsymbol{M0}}$):

$$\Lambda = \frac{h\,M0_0}{\left|\overrightarrow{\boldsymbol{M0}}\right|^2}.$$

The coincidence of the first differential quadratic forms of the cylinder (1) and the parameters space plane preserves the fact of touch between the plane curves for their images on the cylinder – the arch and the helix. Therefore, the spatial coordinates of the point $\overrightarrow{\boldsymbol{Mkp}}$, the tangency point of the arch and the helix, are obtained if we substitute u for the first coordinate Mk_0 of the point $\overrightarrow{\boldsymbol{Mk}}$ in the equation $\overrightarrow{\boldsymbol{Wint}_1}(u)$ from (7), or if we substitute α for the parameter value of the circle L providing the point $\overrightarrow{\boldsymbol{Mk}}$ in the equation of the arch (6):

$$\overrightarrow{\boldsymbol{Mkp}} = \overrightarrow{\boldsymbol{Wint}_1}(Mk_0) = \overrightarrow{\boldsymbol{Ark}}(\alpha_0),$$

where

$$\alpha_0 = \operatorname{arctg}\left(\frac{-Mk_1}{h - Mk_0}\right).$$

The gear arch tooth surface is formed by the circles with the radius ρ, the centres lying on the line $\overrightarrow{\boldsymbol{Ark}}(\alpha)$ in the planes perpendicular to the axis OZ. The equation of this surface can be written as:

$$\overrightarrow{\boldsymbol{Sark}}(\alpha,\tau) = \overrightarrow{\boldsymbol{Ark}}(\alpha) + \rho(\cos\tau, \sin\tau, 0), \quad (8)$$

where $\alpha = -\alpha_0,.., \alpha_0$, $\tau = 0,..2\pi$.

PARAMETRIC EQUATIONS OF THE CHEVRON TEETH SURFACES

The chevron EC-gear teeth profiles are the circles with the radius ρ with the centres lying on the helices $\overrightarrow{\boldsymbol{Wint}_1}(u)$ and $\overrightarrow{\boldsymbol{Wint}_2}(u)$ located in the planes perpendicular to the rotation axis OZ of the part [2]. The equations of the surfaces for these teeth can be written as:

$$\overrightarrow{\boldsymbol{Ss}_1}(u,\tau) = \overrightarrow{\boldsymbol{Wint}_1}(u) + \rho(\cos\tau, \sin\tau, 0),\quad \overrightarrow{\boldsymbol{Ss}_2}(u,\tau) = \overrightarrow{\boldsymbol{Wint}_2}(u) + \rho(\cos\tau, \sin\tau, 0),$$

where $u = \pm M0_0, .., \pm Mk_0$ ("+" for $\overrightarrow{Ss_1}$ and "-" for $\overrightarrow{Ss_2}$).

The profile of the chevron EC - wheel tooth is the epitrochoid equidistant, i.e. the envelope of the circles family with the radius ρ, the centres of which are on the epitrochoid. Let us write the epitrochoid equation as the vector function of one argument:

$$\overrightarrow{\boldsymbol{Te}}(t) = \left\{ -\varepsilon\cos t + Aw\cos\left(\frac{t}{n+1}\right), -\varepsilon\sin t + Aw\sin\left(\frac{t}{n+1}\right) \right\},$$

where n is the gearing ratio, i.e. the ratio of the wheel teeth number to the gear teeth number (z). Denoting the epitrochoid normal vector directed toward the centre of the part as:

$$\overrightarrow{\boldsymbol{N}}(t) = \left\{ \varepsilon\cos t - \frac{Aw}{n+1}\cos\left(\frac{t}{n+1}\right), \varepsilon\sin t - \frac{Aw}{n+1}\sin\left(\frac{t}{n+1}\right) \right\},$$

we can write the epitrochoid equidistant equation as:

$$\vec{\boldsymbol{E}}(t) = \overrightarrow{\boldsymbol{Te}}(t) + \rho\frac{\overrightarrow{\boldsymbol{N}}(t)}{\left|\overrightarrow{\boldsymbol{N}}(t)\right|}.$$

As shown in [2], the surface of the wheel tooth is formed by these equidistants located in the planes perpendicular to the rotation axis OZ of the part, and rotating round this axis while shifting along it. The equations of the chevron wheel teeth surfaces are of the form:

$$\overrightarrow{\boldsymbol{Ev}_1}(\upsilon,\tau) = \left\{ E(\tau)_0, E(\tau)_1, \frac{lr\,z\,\upsilon}{2\pi} \right\} Q\left(\frac{-\upsilon-\theta}{n}\right), \ \overrightarrow{\boldsymbol{Ev}_2}(\upsilon,\tau) = \left\{ E(\tau)_0, E(\tau)_1, \frac{lr\,z\,\upsilon}{2\pi} \right\} Q\left(\frac{\upsilon-\theta}{n}\right),$$

where $\upsilon = \pm\frac{M0_0}{\varepsilon}, .., \pm\frac{Mk_0}{\varepsilon}$ ("+" for $\overrightarrow{\boldsymbol{Ev}_1}$ and "–" for $\overrightarrow{\boldsymbol{Ev}_2}$).

CONCLUSION

Mathematical modelling of the wheel gearing geometry is an essential part of the process of modern transmitters design. In this paper the mathematical model of the helical chevron teeth gearing, conjugated at the junction by arch teeth, is a set of exact analytical solutions to the problems of finding the equations of teeth surfaces of parts and their kinematically coherent motion equations, i.e. the equations of parts motion in the presence of constant contact (the surfaces touch). The surface of the conjugacy area of the driven wheel chevron teeth is obtained as the envelope of the surfaces family formed by the arch tooth during the movement of the parts. In the paper the teeth of the eccentrically cycloidal engagement are examined as the chevron teeth, but the method for constructing conjugacy of the chevron teeth can be applied to other gearing types (e.g. involute). On the basis of the EC-engagement the Closed Joint Stock Company "Technology market" (Tomsk, Russia) has developed a series of practical transmission systems movement. The site of the company: http://www.ec-gearing.com/ provides information on the technology of manufacturing engagement parts. "Technology market" has constructed a large number of gearboxes with different types of gearing, as well as created prototypes that have been successfully tested in Russia (BELAZ, KAMAZ and other) and Germany (SEW Eurodrive).

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