Theoretical study of strength of elastic-plastic water-saturated interface under constrained shear

Andrey V. Dimaki, Evgeny V. Shilko, and Sergey G. Psakhie

Citation: AIP Conference Proceedings **1783**, 020042 (2016); doi: 10.1063/1.4966335
View online: http://dx.doi.org/10.1063/1.4966335
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1783?ver=pdfcov
Published by the AIP Publishing

Theoretical Study of Strength of Elastic-Plastic Water-Saturated Interface under Constrained Shear

Andrey V. Dimaki[1,a)], Evgeny V. Shilko[1,2], and Sergey G. Psakhie[1,3]

[1] *Institute of Strength Physics and Materials Science SB RAS, Tomsk, 634055 Russia*
[2] *National Research Tomsk State University, Tomsk, 634050 Russia*
[3] *National Research Tomsk Polytechnic University, Tomsk, 634050 Russia*

[a)] Corresponding author: dav@ispms.tsc.ru

Abstract. This paper presents a theoretical study of shear strength of an elastic-plastic water-filled interface between elastic permeable blocks under compression. The medium is described within the discrete element method. The relationship between the stress-strain state of the solid skeleton and pore pressure of a liquid is described in the framework of the Biot's model of poroelasticity. The simulation demonstrates that shear strength of an elastic-plastic interface depends non-linearly on the values of permeability and loading to a great extent. We have proposed an empirical relation that approximates the obtained results of the numerical simulation in assumption of the interplay of dilation of the material and mass transfer of the liquid.

Liquid in a pore volume influences the strength of porous permeable media, both brittle and elastic-plastic [1–3]. Despite the dilation of the latter, pore pressure of liquid can remain nonzero under constrained shear loading. As a result, liquid pressure contributes to the stress-strain state of the medium, and, consequently, to its strength. On the other hand, liquid pressure depends on transportation properties of the medium namely on porosity and permeability, which is determined by a characteristic diameter of filtration channels in the solid skeleton of the medium. Interplay of fluid filtration and its compression in pores generates dynamical distribution of pore pressure in the bulk of material. Non-linearity and interconnectedness of material dilation, fluid transfer and mechanical loading clearly demonstrate the necessity to apply numerical methods to study strength properties of such media.

In the paper we have carried out a numerical simulation of shearing of a water-filled elastic-plastic interface under constrained conditions. The elastic-plastic interface was located between purely elastic permeable blocks that have been loaded with a constant velocity in the lateral direction. Periodic boundary conditions were used in the lateral direction. Pre-loading was performed before shearing to create the initial hydrostatic compression in the volume. The elastic blocks and the sample had the same values of porosity, permeability, elastic moduli, etc. We varied the shear rate, the value of hydrostatic compression, the width of the interface and the permeability of the material. As the result, values of shear strength of the sample have been obtained for a wide range of input parameters.

We used the hybrid cellular automaton method, which is a variation of the discrete element method with an explicit account of liquid in pores. The method of hybrid cellular automaton is based on the decomposition of the considered problem into two ones: 1) description of the "porous solid skeleton – liquid" system and 2) simulation of liquid exchange between the pore volume and the environment. The method of movable cellular automaton (MCA) which represents an implementation of the discrete element method [4] was used to solve the first problem. The problem of fluid mass transfer in the filtration volume was solved within the MCA layer. Following the ideas of Biot [5, 6], we assume that the stress-strain state of a discrete element is directly interconnected with the variation of pore volume and pore pressure of fluid in the "micropores". The second problem of calculating the fluid mass transfer between the solid skeleton and external macroscopic voids was solved on a finer finite-difference mesh that was "frozen" into a laboratory coordinate system. The finite-difference mesh was also used to calculate macropore volumes by means of integrating upper nodes in the macropore network.

Advanced Materials with Hierarchical Structure for New Technologies and Reliable Structures 2016
AIP Conf. Proc. 1783, 020042-1–020042-4; doi: 10.1063/1.4966335
Published by AIP Publishing. 978-0-7354-1445-7/$30.00

The model of rock plasticity with a non-associated flow law and von Mises yield criterion (the so-called Nikolaevsky model [7, 8]) was designed to simulate mechanical response of fractured porous brittle materials. This model adequately describes response of a wide range of brittle materials (geological materials, ceramics etc.) taking into account the influence of lower-scale structures. The Nikolaevsky model postulates a linear relationship between the volume and shear deformation rates of the plastic deformation with coefficient Λ named the coefficient of dilatancy. We have adapted the Nikolaevsky model to the MCA method with the use of the so called Wilkins algorithm [9]. In the framework of this algorithm, solution of the elastic-plastic problem is reduced to solution of an elastic problem in increments and the following correction of specific forces between particles (discrete elements) following the rules of the Nikolaevsky model to calculate local pressure and the stress deviator [9]. A detailed description of the MCA and the model of inter-particle interaction can be found in [10, 11].

The developed model of fluid transfer has been based on the following assumptions: 1) fluid can occupy a pore volume completely or partially; 2) fluid is weakly compressible; 3) adsorption of fluid on internal surfaces of pores, capillary effects and the effect of adsorption reduction of strength are not taken into account; 4) the influence of gravity is neglected, and 5) all micropores are assumed to be of the same size. In the framework of the latter assumption, a pore volume is completely described by the following parameters: a value of open "microscopic" porosity and a characteristic diameter of filtration channel d_{ch}. Note that the value of d_{ch} is determined by the size of the smallest channels that control the rate of filtration of fluid through the porous skeleton. An adequate choice of d_{ch} allows correct description of fluid mass transfer despite the simplicity of the assumptions given above [12].

A state of compressible liquid (both in microscopic pores and macroscopic voids) is described by a linear equation for weakly-compressible liquid [13]. A transfer of liquid due to pore pressure gradient is described by a variation of the Leibenzon's equation [14]. The developed algorithm finds nodes of finite-difference mesh belonging to a boundary between a solid skeleton and macropores in order to simulate mass transfer between the solid skeleton and macroscopic pores. Calculation of mass transfer of fluid between a discrete element and a macropore(s) is performed through these boundary nodes. Redistribution of fluid in the volume of micropores in a discrete element or in macroscopic pores is described within the approximation of equal pressure. Following this assumption, fluid density and pressure are distributed uniformly in any closed volume at each time step. This simplification remains adequate for relatively slow processes under consideration, where inertia effects can be neglected.

We have considered a shear loading of an infinitely long fragment of material under constrained conditions (see Fig. 1a). The mentioned loading regime is typical for boundaries in geological media [15]. Pore volumes of permeable elastic blocks and the elastic-plastic interface have been saturated with water under initial atmospheric pressure. The diagrams of uni-axial loading of materials of the elastic blocks and the elastic-plastic interface are given in Fig. 1b. The considered fragment of material has been mounted between thin impermeable layers of the material to which an external loading has been applied. Periodic boundary conditions were applied in the lateral direction.

The loading was performed in two stages. In the first stage an initial pre-loading with compression normal stress σ_N was performed. After that, we fixed the loading until fading of elastic waves in the sample. In the second stage a shear loading in the lateral direction with the constant velocity V_x was applied till fracture of the sample. At that, top and bottom layers were fixed in the vertical direction during the shear loading.

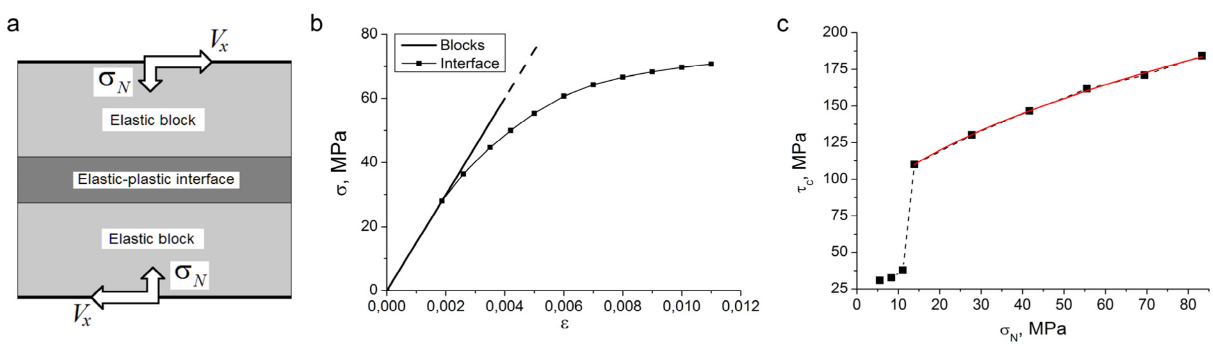

FIGURE 1. (a) Scheme of loading; (b) diagrams of uni-axial loading of materials of the blocks and the interface; (c) typical dependence of shear strength on normal load. $d_{ch} = 0$

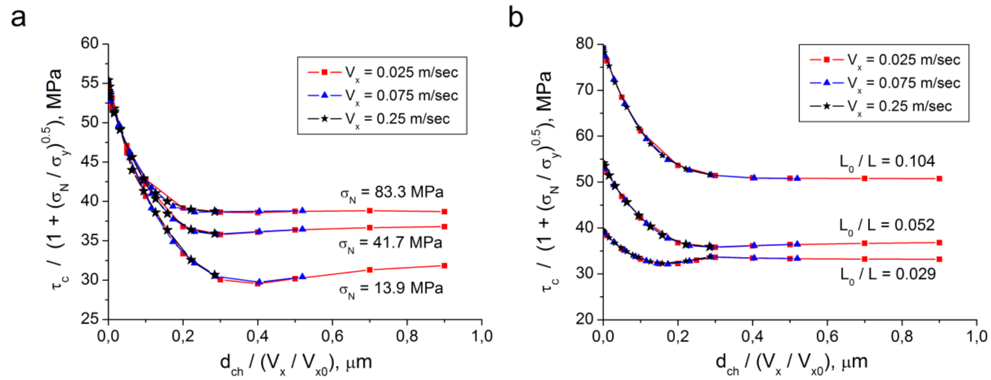

FIGURE 2. Dependencies of shear strength on the characteristic diameter of a filtration channel and a deformation rate: (a) under different values of normal load σ_N and $L/L_0 = 0.052$, (b) for different values of L/L_0 and $\sigma_N = 41.7$ MPa

We have found that fracture of the elastic-plastic interface occurs before the beginning of plastic deformation of the interface under relatively small values of normal pre-loading. At certain value of normal pre-loading, the fracture of the interface happens after the plastic deformation begins and takes place at relatively high values of plastic deformations (see Fig. 1c). The latter demonstrates a "brittle-to-ductile" transition, which takes place in real material, in particular, in geological media.

Further results have been obtained in the "ductile" zone of fracture, in other words, when fracture occurs significantly after reaching a yield point. At that, the dependence of shear strength on a value of normal confining pressure can be approximated with the following equation:

$$\tau_c = \sigma_0 (1 + \sqrt{\sigma_N / \sigma_y}), \tag{1}$$

where σ_0 is a scale factor that has the strength dimension and σ_y is a yield strength. Under these conditions, dependencies of shear strength on permeability of the material demonstrate an exponential decrease and further growth with the increase of the value of permeability (see Fig. 3a).

Dependencies of shear strength on permeability (that depends on a square of the characteristic diameter of a filtration channel in a solid skeleton) obtained for different values of a shear rate can be reduced to a single dependence of shear strength on an effective filtration channel diameter

$$d_{ch} \big/ \sqrt{V_x / V_{x0}}\,, \tag{2}$$

where V_{x0} is a scale factor that has a velocity dimension. The mentioned growth of shear strength $\tau_c(d_{ch})$ with an increase of permeability most strongly manifests for samples with the relation of $L_0/L = 0.026$ (see Fig. 3b), in other words, for samples with the least thickness of the elastic-plastic interface.

The mentioned nonmonotonicity of the dependencies of shear strength on permeability has the following explanation. At relatively small values of permeability, liquid pressure in an elastic-plastic interface rapidly decreases down to zero due to increase of pore volume under dilation of an elastic-plastic material. At that an effective stiffness of the elastic-plastic interface remains constant and depends only on elastic moduli of the solid skeleton. In turn, liquid pressure in elastic blocks decreases due to filtration of liquid from the elastic blocks to the elastic-plastic interface. As a result, an effective stiffness of the elastic blocks decreases, which leads to reduction of the mean stress in the interface and, correspondingly, to reduction of its strength in accordance with the Drucker-Prager criterion.

At relatively high values of permeability, liquid pressure in the interface remains non-zero due to a rapid inflow of liquid from the bulk of elastic blocks. At that an effective stiffness of the blocks decreases while an effective stiffness of the interface increases. As a result, mean pressure in the interface and, correspondingly, its strength grow. Competition of these processes results in the occurrence of a minimal value of shear strength, where the rate of liquid filtration is still not enough to provide non-zero liquid pressure in the whole cross-section of the elastic-plastic interface and, at the same time, is enough to significantly decrease liquid pressure in the bulk of elastic blocks. So, we can conclude that shear strength of the interface is determined by the cooperation of the following processes:

1) increase of the mean stress in material under shear; 2) dilation of the elastic-plastic interface after reaching the yield point; 3) mass transfer of liquid in the pore volume of a sample and elastic blocks and 4) redistribution of liquid pressure due to its mass transfer.

The observed effects together with the results of numerical simulations allowed us to suggest a binomial dependence of shear strength of an elastic-plastic interface on permeability and shear rate for a given normal load σ_N:

$$\tau_c(\sigma_N) = \sigma_0 + \sigma_1 \exp(-c_1 d_{ch} \big/ \sqrt{V_x/V_{x0}}) + \frac{\sigma_2}{1 + (c_2 d_{ch} \big/ \sqrt{V_x/V_{x0}})^{-p}}, \tag{3}$$

where the value $(\sigma_0 + \sigma_1)$ corresponds to the strength of impermeable water-filled sample (in so called "undrained conditions") and the value $(\sigma_0 + \sigma_2)$ represents the strength of a "dry" sample. Parameters c_1 and c_2 characterize the change rate of exponential and sigmoidal branches of the dependence (3) with the change of permeability. Note that values σ_0, σ_1 and σ_2 are not constant but depend on the thickness of the elastic-plastic interface, physical-mechanical parameters of the material and boundary conditions. The second term of the dependence (3) describes the exponential decrease of strength of the sample due to decrease of effective stiffness of the elastic blocks under outflow of liquid into an excess pore volume of the sample. Note that the latter leads to decrease in the degree of constraint. The third term characterizes the influence of filtration on increase of effective stiffness of the interface due to the growth of pore pressure of liquid. This, in turn, leads to increase of the degree of constraint of the interface. The parameters of the given dependence represent the combinations of the values of loading, width of the interface, physical-mechanical properties of the sample, including permeability, and physical mechanical properties of liquid. Obtaining relations for these parameters is the aim of a future study.

ACKNOWLEDGMENTS

The work was carried out in the framework of the Fundamental Research Program of the State Academies of Sciences for 2013–2020.

REFERENCES

1. M. S. Paterson and T.-F. Wong, *Experimental Rock Deformation—the Brittle Field* (Springer-Verlag, Berlin–Heidelberg, 2005).
2. A. V. Bishop and A. E. Skinner, Philos. Trans. A: Math. Phys. Eng. Sci. **284**, 91–130 (1975).
3. S. Zavšek, A. V. Dimaki, A. I. Dmitriev, E. V. Shilko, J. Pezdič, and S. G. Psakhie, Phys. Mesomech. **16**(1), 42–51 (2013).
4. L. Jing and O. Stephansson, *Fundamentals of Discrete Element Method for Rock Engineering: Theory and Applications* (Elsevier, London, 2007).
5. M. A. Biot, J. Appl. Phys. **12**, 155–164 (1941).
6. M. A. Biot, J. Appl. Mech. **24**, 594–601 (1957).
7. I. A. Garagash and V. N. Nikolaevskiy, Adv. Mech. **12**, 131–183 (1989).
8. Yu. P. Stefanov, Phys. Mesomech. **5**(5–6), 67–77 (2002).
9. M. L. Wilkins, *Computer Simulation of Dynamic Phenomena* (Springer-Verlag, Heidelberg, 1999).
10. S. G. Psakhie, E. V. Shilko, A. S. Grigoriev, S. V. Astafurov, A. V. Dimaki, and A. Y. Smolin, Eng. Fract. Mech. **130**, 96–115 (2014)
11. S. G. Psakhie, A. V. Dimaki, E. V. Shilko, and S. V. Astafurov, Int. J. Numerical Meth. Eng. **106**, 623–643 (2016).
12. C. Tien, *Principles of Filtration* (Elsevier, Amsterdam, 2012).
13. K. S. Basniev, N. M. Dmitriev, G. V. Chilingar, and M. Gorfunkle, *Mechanics of Fluid Flow* (John Wiley & Sons, Inc., Hoboken, 2012).
14. L. G. Loytsyanskii, *Mechanics of Liquids and Gases* (Pergamon-Press, Oxford, 1966).
15. G. G. Kocharyan and V. A. Novikov, Phys. Mesomech. **19**(2), 189–199 (2016).