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Supplier and Retailer Coordination under Stochastic Price-Dependent Demand and Fast Moving Items*

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***Abstract*—We consider a centralized supply chain system consisting of a one supplier and one retailer. The customers' demand is a compound Poisson process with price-dependent intensity and continuous batch size distribution. The intensity of the customers' arrivals is assumed to be sufficiently high to use a diffusion approximation of the demand process. We assume that the supplier has complete information about the rational retailer's behavior in the framework of the newsvendor problem. The objective is to find a joint pricing and ordering policy so as to maximize the retailer's expected profit and supplier's profit. The equations for the optimal prices main parts are obtained and the example of the price-intensity dependence is considered.**

Keywords—newsvendor; compound Poisson demand; supply chain coordination; pricing

I. Introduction and Statement of the Problem

Originally the retail and wholesale prices are exogenous variables in the newsvendor problem. Whitin [1] was the first who analyzes the price-dependent demand and determines the order size and selling price simultaneously. The topic has been the theme of many papers; see reviews by Petruzzi and Dada [2]; Yano and Gilbert [3]; Chen and Simchi-Levi [4].

In this paper the stochastic demand is modelled as a compound Poisson process. The model is rich enough to describe the uncertainty of demand and seems to be suitable for many inventory control processes; see, for example, [5 and 6] for early papers, [7 and 8] for more recent ones. The only drawback of the distribution is its complexity, which leads to expressions that are too complex to deal with analytically, so different approximation methods for representation of a compound Poisson distribution have been proposed; see [9, 10, and 11].

In this paper we approximate the demand by a diffusion process, which leads to a normal cumulative demand with the specific form of the dependence of the mean and variance parameters of the intensity. Such representation allows us to solve the problem analytically.

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We consider a supply chain consisting of a supplier, a retailer, and customers.

The product's lifetime is T. Let the demand be a compound Poisson process with price-dependent intensity $\lambda(c)$, where c is a selling (retail) price per unit of the product. The values of orders (batch sizes) are i.i.d. continuous random variables with finite first and second moments equal respectively a_1 and a_2.

At the beginning of a time period T the retailer purchases a quantity Q at a fixed price per unit d (wholesale price), $d < c$, and replenishment during the period is not allowed. So the retailer faces a newsvendor problem and needs to make a decision about the order quantity and retail price.

We assume that there are no capacity restrictions, and we do not consider the cost of production, holding, leftovers utilization, and lost sales. The objective is to maximize the expected profits of the retailer and supplier simultaneously.

Denote $X(t)$ the cumulative customer demand at $[0,t]$, $p(\cdot)$ the probability density function of $X(T) = X$.

Expected profit for the retailer at the end of the period

$$S = -Qd + cQ\int_{Q}^{\infty} p(x)dx + c\int_{0}^{Q} xp(x)dx.$$

The retailer is interested in determining an optimal value of lot size Q_0 and then corresponding value of c by maximizing the expected profit. The supplier is interested in maximizing the value of $P = Q_0 d$ with respect to wholesale price d. We assume that he correctly anticipates the retailer's order and selling price for any value of d.

This paper continues the research presented in [12]. Here we consider centralized system when the supplier and retailer make joint decisions.

The paper is organized as follows. In the next section, we consider the retailer's problem and present the numerical results for the profit increase due to price control for the following price-intensity dependence

$$\lambda(c) \sim \left(1+\left(\frac{c}{d}\right)^2\right)^{-1}.$$

In the third section, we solve the joint prices optimization problem. We conclude the paper with Section IV.

II. Retailer's Optimization

Let us consider the retailer's optimization task that consists of two steps: first, determining the optimal lot size and second, determining the corresponding selling price. Obviously the first task has unique solution Q_0 determined by the equation

$$\int_{Q_0}^{\infty} p(x)dx = \frac{d}{c}, \qquad (1)$$

and the retailer's corresponding profit

$$S_0 = c\int_{0}^{Q_0} xp(x)dx. \qquad (2)$$

The result is well known; see, for example, [13].

The problem is that in general the distribution $p(\cdot)$ is very complicated and it is impossible to solve (2) in closed form.

For example, for exponential batch size distribution with expectation a_1

$$p(x) = \delta(x)e^{-\lambda(c)T} + e^{-x/a_1-\lambda(c)T}\sqrt{\frac{\lambda(c)T}{a_1 x}}I_1\left(2\sqrt{\frac{\lambda(c)Tx}{a_1}}\right),$$

where $I_1(\cdot)$ is the modified Bessel function of the first kind and first order, $\delta(\cdot)$ is the Dirac delta function.

It follows that (1) can be written as

$$\sqrt{\lambda(c)T}\int_{Q_0/a_1}^{\infty}\frac{1}{\sqrt{u}}I_1\left(2\sqrt{\lambda(c)Tu}\right)e^{-u-\lambda(c)T}du = \frac{d}{c}. \qquad (3)$$

Only numerical solution of (3) with respect to Q_0 is possible and the following task of price optimization given Q_0 becomes extremely difficult, not to mention the analytical solution of the problem.

So below we consider an approximate solution for fast moving items, i.e. for $\lambda(c)T >> 1$. In this case, the optimal order size also needs to be large enough.

Consider diffusion approximation of demand process $X(t)$

$$dX(t) = a_1\lambda dt + \sqrt{a_2\lambda}dw(t), \qquad (4)$$

where $w(\cdot)$ is the Wiener process.

Diffusion methods have been applied to inventory models in a variety of domains to begin with the papers by Bather [14] and Puterman [15], and nowadays the Brownian motion process is one of the most commonly used demand processes in the inventory literature; see, e.g. [16, 17, and 18]. In [19] the diffusion approximation of stock level process has been used to find the steady-state distribution and to solve the problem of on/off control minimizing the variance of the process. In [20] a theoretical justification of a diffusion approximation with some numerical results is given.

In [12] approximation (4) is used to obtain an approximate distribution of the selling time of a large order, and the demand parameters estimation procedures based on two censored samples – the observed selling durations and truncated cumulative demands – are proposed. As the results of simulation show, the accuracy of the estimators is good enough (see, e.g., [21]), that can be considered as a practical justification of the approximation used. Here we only need the cumulative demand distribution that is defined by (4).

From (4) it follows that we can consider X as the normal random variable with the mean $\mu_X = a_1\lambda(c)T$ and the variance $\sigma_X{}^2 = a_2\lambda(c)T$. It is well known that the normal distribution provides a good empirical fit to observed demand data for fast moving items.

From (1) we receive the well-known result for optimal value of Q_0

$$Q_0 = a_1\lambda(c)T + \sqrt{a_2\lambda(c)T}\Psi\left(1-\frac{d}{c}\right),$$

where $\Psi(\cdot) = \Phi^{-1}(\cdot)$ is a standard normal quantile function and $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(\frac{-t^2}{2}\right)dt$ is the standard normal cumulative distribution function.

From (2) we get

$$S_0 = c\left[a_1\lambda(c)T\left(1-\frac{d}{c}\right) - \sqrt{\frac{a_2\lambda(c)T}{2\pi}}\exp\left(-\frac{1}{2}\Psi^2\left(1-\frac{d}{c}\right)\right)\right].$$

Let $\lambda(c) = \lambda_0 F(c)$, where $F(\cdot)\downarrow$ is a twice differentiable function. Then

$$S_0 = a_1\lambda_0 T\left[F(c)(c-d) - c\sqrt{\frac{a_2 F(c)}{2\pi a_1^2\lambda_0 T}}\exp\left(-\frac{1}{2}\Psi^2\left(1-\frac{d}{c}\right)\right)\right]. \qquad (5)$$

Taking derivative $\frac{dS_0}{dc}$ we receive the equation for the optimal value of retail price

$$F'(c)(c-d)+F(c)-\sqrt{\frac{a_2}{2\pi a_1^2\lambda_0 T}}\times$$

$$\times\left[\left(\sqrt{F(c)}+\frac{cF'(c)}{2\sqrt{F(c)}}\right)\exp\left(-\frac{1}{2}\Psi^2\left(1-\frac{d}{c}\right)\right)+\right.$$

$$\left.+\sqrt{2\pi F(c)}\frac{d}{c}\Psi\left(1-\frac{d}{c}\right)\right]=0. \qquad (6)$$

If $\lambda(c)T>>1$ then considering only the first summand in (5) we get a main part of the optimal value of a retail price as $\lambda_0 T\to\infty$

$$c_0=\arg\max_c\left(F(c)(c-d)\right).$$

The main part is defined by equation

$$F'(c_0)\left(c_0-d\right)+F(c_0)=0. \qquad (7)$$

Price c_0 corresponds to the case when the number of orders and, consequently, the lot size are very large.

In [12] the correction term $\Delta c=O\left(1/\sqrt{\lambda_0 T}\right)$ as $\lambda_0 T\to\infty$ is also considered and the profits $S_0(c)$ for $c\neq c_0$ and $S_0(c_0)$ for linear price-intensity dependence function $F(c)=1-a\frac{c-c_0}{d}$, $a>0$, are compared.

In this paper we are going to consider more complicated price-intensity dependence form

$$F(c)=\left(1+\left(\frac{c}{d}\right)^{\gamma}\right)^{-1},$$

where parameter $\gamma>1$.

Such a form of the dependence is quite natural: when the retail price decreases (or increases) the intensity of the customers' arrival increases (or correspondently decreases), and the parameter regulates the sensitivity of the demand intensity to the retail price.

In this case (7) takes the following form:

$$(1-\gamma)\left(\frac{c_0}{d}\right)^{\gamma}+\gamma\left(\frac{c_0}{d}\right)^{\gamma-1}+1=0\,. \qquad (8)$$

It is easy to see that (8) has the unique solution for $c_0>d$.

Let us consider $\gamma=2$. Then $\frac{c_0}{d}=1+\sqrt{2}$ and $F(c_0)=\frac{2-\sqrt{2}}{4}$.

To illustrate the increase in profit due to price optimization let us compare the profits $S_0(c)$ for $c\neq c_0$ and $S_0(c_0)$ for the price-intensity dependence defined by $F(c)$ for $\gamma=2$.

Let us denote the coefficient of variation of the demand corresponding to the case $F(c)=1$, i.e. when the demand does not depend on the price, as

$$r=\frac{\sigma_X}{\mu_X}=\sqrt{\frac{a_2}{a_1^2\lambda_0 T}}.$$

We denote

$$\bar{c}=c/d=\left(c_0+\Delta c\right)/d=1+\sqrt{2}+\Delta>1,$$

$\Delta=\Delta c/d\neq 0$, i.e. the price c should be less or greater than c_0; and $F_c=F(\bar{c})$ is the corresponding price-dependent multiplier in the intensity of customers' arrivals.

Let us consider the relative profit

$$R_1=\frac{S_0(c_0)}{S_0(c)}=$$

$$=\frac{2\sqrt{\pi}F(c_0)-r\left(1+\sqrt{2}\right)\sqrt{F(c_0)}\exp\left\{-\frac{1}{2}\psi^2\left(2-\sqrt{2}\right)\right\}}{\sqrt{2\pi}\left(\sqrt{2}+\Delta\right)F_c-r\bar{c}\sqrt{F_c}\exp\left\{-\frac{1}{2}\psi^2\left(1-\frac{1}{\bar{c}}\right)\right\}}.$$

Table I presents the values of R_1 for different values of parameter *r* and the price deviations $\Delta=\pm 0.5,\pm 0.7,\pm 0.9$.

TABLE I. RELATIVE PROFIT.

r \ Δ	0.5	− 0.5	0.7	− 0.7	0.9	− 0.9
1/4	1.021	1.087	1.042	1.206	1.068	1.468
1/6	1.024	1.073	1.045	1.175	1.070	1.398
1/8	1.025	1.067	1.046	1.165	1.071	1.374

Note that the approximation should not work well for large r values, so here we do not present the numeric results for $r > 1/4$.

We can see from Table I that the relative profit is more sensitive to the negative deviations from the optimal price; in this case, the greater r, the greater the loss of profit. The positive deviations comparatively have no such a strong effect to the profit; and the influence of the coefficient of variation on the relative profit is reversed.

III. Supplier's Optimization

We suppose that the supplier has complete information about the rational retailer's decisions in the framework of the newsvendor problem. Note that the supplier's profit in such a setting is deterministic and depends on the retailer's decision about the optimal lot size Q_0, which is a function of the whole sale price.

Let us find the main part of the optimal wholesale price as $\lambda_0 T \to \infty$. Let in (6) $c = c_0$ and c_0 satisfies (7).

Thus, we need to find

$$d_0 = \arg\max_d P =$$

$$= \arg\max_d d\left(a_1\lambda(c_0)T + \sqrt{a_2\lambda(c_0)T}\,\Psi\left(1-\frac{d}{c_0}\right)\right),$$

where c_0 is defined by (7).

Let us denote $\varphi(d)$ the solution of (7) with respect to c_0. Differentiating P with respect to d we get

$$a_1F(c_0) - \frac{a_1F(c_0)\varphi'(d)d}{(c_0-d)} + \frac{G(d)}{\sqrt{\lambda_0 T}} = 0, \qquad (9)$$

where

$$G(d) = \sqrt{\frac{a_2}{F(c_0)}}\left[\Psi\left(1-\frac{d}{c_0}\right)\left(1+\frac{\varphi'(d)d}{2(c_0-d)}\right) - \right.$$

$$\left. -\sqrt{2\pi}\left(1+\frac{d}{c_0}\varphi'(d)\right)\exp\left(\frac{1}{2}\Psi^2\left(1-\frac{d}{c_0}\right)\right)\right].$$

From (7) we get

$$\varphi'(d) = \frac{F'(c_0)}{2F'(c_0)+(c_0-d)F''(c_0)} =$$

$$= \frac{F(c_0)}{2F(c_0)+(c_0-d)^2F''(c_0)}. \qquad (10)$$

Thus, d_0 is defined by equation

$$\frac{F(\varphi(d_0))}{2F(\varphi(d_0))+(\varphi(d_0)-d_0)^2F''(\varphi(d_0))} = \frac{\varphi(d_0)}{d_0} - 1.$$

Let us consider

$$F(c) = \left(1+\left(\frac{c}{b}\right)^{\gamma}\right)^{-1},$$

where $b > 0$ and $\gamma > 1$ are some parameters.

In this case we can write (7) in the following way:

$$(1-\gamma)\left(\frac{c_0}{b}\right)^{\gamma} + \gamma\frac{d}{b}\left(\frac{c_0}{b}\right)^{\gamma-1} + 1 = 0 .$$

From (10) it follows

$$\varphi'(d) = \frac{c_0}{(\gamma-1)(c_0-d)}. \qquad (11)$$

Substituting (11) into (9) and neglecting the last term as $\lambda_0 T >> 1$ we get

$$c_0 - d - \frac{c_0 d}{(\gamma-1)(c_0-d)} = 0 .$$

Solve the following system of equations

$$\begin{cases} (1-\gamma)\left(\frac{c_0}{b}\right)^{\gamma} + \gamma\frac{d_0}{b}\left(\frac{c_0}{b}\right)^{\gamma-1} + 1 = 0, \\ c_0 - d_0 - \frac{c_0 d_0}{(\gamma-1)(c_0-d_0)} = 0 \end{cases}$$

with respect to c_0 and d_0.

From the first equation we obtain

$$d_0 = c_0 - \frac{c_0}{\gamma} - \frac{b}{\gamma}\left(\frac{b}{c_0}\right)^{\gamma}. \qquad (12)$$

Substituting (12) into the second equation of the system we get

$$(\gamma-1)\left(\frac{b}{c_0}\right)^{2\gamma} + (3\gamma-2)\left(\frac{b}{c_0}\right)^{\gamma} - (1-\gamma)^2 = 0. \qquad (13)$$

Solution of (13) has the form

$$\tilde{c}_0 = \frac{b}{\sqrt[\gamma]{h}}, \qquad (14)$$

where

$$h = \frac{\sqrt{(3\alpha+1)^2 + 4\alpha^3} - 3\alpha - 1}{2\alpha}, \quad \alpha = \gamma - 1.$$

It is easy to see that $0 < h < \alpha$.

Substituting (14) into (12) we receive

$$d_0 = \frac{\tilde{c}_0}{\gamma}(\alpha - h) = \frac{b}{\gamma\sqrt[\gamma]{h}}(\alpha - h).$$

Thus, we find the pair $(d_0, \tilde{c}_0)$, $d_0 < \tilde{c}_0$ of the optimal retail and wholesale prices.

Let us consider the case $\gamma = 2$. Then the ratio

$$\frac{\tilde{c}_0}{d_0} = \frac{2}{3-\sqrt{5}} \approx 2.618 > \frac{c_0}{d} = 1 + \sqrt{2} \approx 2.414,$$

and

$$\frac{c_0}{\tilde{c}_0} = \frac{d}{b}\left(1+\sqrt{2}\right)\sqrt{\sqrt{5}-2} \approx \frac{d}{b}1.173.$$

The ratio of profits

$$\frac{S_0(c_0)}{S_0(\tilde{c}_0, d_0)} = \frac{d}{b}K.$$

It follows that if $\frac{d}{b} > K^{-1}$ than the retailer's profit increases, i.e. the joint decision making is profitable for the retailer.

In Table II values of K for different coefficients of variation r are presented.

TABLE II. K VALUES.

r	0.125	0.15	0.175	0.2	0.225	0.25
K	2.37	2.22	2.09	1.96	1.85	1.74

IV. Conclusion

Considering the approximation for fast moving items provides us with the opportunity to solve the joint retail and wholesale prices optimization problem for a compound Poisson demand with price-dependent intensity in the newsvendor problem framework analytically.

It should be also mentioned that the obtained results concerning the optimal prices are of interest regardless of the approximation to a compound Poisson distribution: we can use them for the normally distributed items with such a form of the price dependence of the distribution parameters.

This work is related to the topic of supply chain coordination. Here we consider centralized system when the supplier and retailer make joint decisions to increase their profits. Nowadays there are a lot of complex models devoted to supply chain coordination considering the subject from different points of view; see, for example, [22 and 23]. As to the interaction between the chain participants our model is very simple. The main contribution of our paper is the specific form of price-demand dependence as well as the approximations used to receive results in closed form.

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