The Electroweak Phase Transition in Models with Gauge-Higgs Unification

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The dynamics of five dimensional Wilson line phases at finite temperature is studied in the one-loop approximation. We show that at temperatures of order $T \sim 1/L$, where L is the length of the compact space, the gauge symmetry is always restored and the electroweak phase transition appears to be of first order.

We focus on a specific model where the Wilson line phase is identified with the Higgs field (gauge-Higgs unification). The transition is of first order even for values of the Higgs mass above the current experimental limit. If large localized gauge kinetic terms are present, the transition might be strong enough to give baryogenesis at the electroweak transition.

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1. Introduction

Theories with compact extra dimensions at the TeV scale [1] offer the possibility of identifying the Higgs field with the internal component of a gauge field.[1] The Electroweak Symmetry Breaking (EWSB) is equivalent to a Wilson line symmetry breaking, since the vacuum expectation value (VEV) of the Higgs field is proportional to a Wilson line phase along the compact extra dimensions. Such theories with gauge-Higgs unification (GHU) can provide a solution to the gauge hierarchy problem. In the minimal five-dimensional (5D) case, which seems the most interesting framework, interesting potentially realistic models of GHU have recently appeared in [3].

In this work we study if and how in such models an electroweak phase transition (EWPT) arises at some critical temperature T_c, above which the symmetry is restored. One of the main motivations to perform this study is related to the possibility of having a successful baryogenesis if a strong first-order phase transition occurs. We focus on models in flat space, analyzing first the dynamics of 5D Wilson line phases at finite temperature in general and then considering in detail the study of the Higgs potential in a specific class of $S^1/\mathbf{Z}_2$ orbifold models [4].

At one-loop level, we find that the gauge symmetry broken by the Wilson lines is restored at temperatures of order $T \sim 1/L$, where L is the length of the covering compact space [5]. The transition is typically of first-order, due to the presence of a term, cubic in the Wilson line phase, given by massless 5D bosons [6]. An analysis of the EWPT in the model of [4] shows that for low values of the Higgs mass, $M_H \lesssim 20\,\text{GeV}$, the transition is strongly of first-order, with a strength that is inversely proportional to the Higgs mass, similarly to what happens in the SM (see fig. 1). In the model of [4], realistic values of the Higgs mass can be obtained only by considering generalizations of the minimal model, introducing 5D bulk fermions in large representations of the gauge group or large localized gauge kinetic terms.[2] In the former case, the first order phase transition becomes very weak, whereas in the latter the strength of the transition is considerably larger.

2. The Phase Transition in the SM

The total tree-level and one-loop SM Higgs potential at high temperatures, for $M_H < M_W$, can be written as [7]

$$V_{tot}(T,H) \simeq D(T^2 - T_0^2)H^2 - ETH^3 + \frac{1}{4}\lambda H^4 , \qquad (2.1)$$

where T_0, D and E are three constants which encode the contributions of the gauge bosons and the top quark and λ is the tree-level Higgs quartic coupling. The coupling E is induced only by the Matsubara zero mode, which is present only for bosons, and determines the nature of the phase transition. If $E = 0$, one gets a second-order phase transition, whereas for $E > 0$ one finds a first-order phase transition with a strength proportional to $|H(T_c)|/T_c \simeq E/\lambda$. Since $E/\lambda \sim 1/M_H^2$, the transition is weaker and weaker for increasing values of M_H. The parameter $|H(T_C)|/T_C$ is the crucial parameter to look at if one wants to get baryogenesis at the EWPT. For the SM, the requirement is $|H(T_C)|/T_C > 1$. It turns out that only for M_H significantly lower than M_W the above

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[1]See [2] for a brief overview and earlier references.

[2]Both possibilities do not actually give rise to a phenomenologically acceptable model. The nature of the EWPT, however, do not depend much on the various issues that rule out the above models.

one-loop computation can be trusted. Around the critical temperature, as M_H approaches M_W, perturbation theory is less and less reliable, and for $M_H \gtrsim M_W$, perturbation theory breaks down. Lattice computations seem to indicate that for $M_H \gtrsim M_W$ the SM has a crossover, ruling then out baryogenesis at the SM EWPT.

3. Wilson Lines at Finite Temperature

Consider, for simplicity, a single Wilson line phase α and one massless 5D gauge boson and fermion, with charges q_B and q_F ($q_F > q_B$) with respect to α. The effective potential V at $T = 0$ has a minimum at $\alpha_{min} \simeq 1/(2q_F)$, where the "Higgs mass", neglecting the bosonic contribution, is approximately given by[3] $M_H^2 = (g_4 R/2)^2 V''(\alpha_{min}) \sim 24 g_4^2 q_F^2/(16\pi^2 L^2)$. At high temperatures $TL > 1$, V can roughly be written, by expanding in α, as [6]

$$\frac{L^4}{\pi^2} V(T, \alpha) \simeq a(x)\,\alpha^2 - b(x)\,\alpha^3 + c(x)\,\alpha^4, \qquad \begin{cases} a(x) = q_B^2 x - 8 q_F^2 \sqrt{2} x^{5/2} e^{-\pi x} \\ b(x) = 2x q_B^3 \\ c(x) = q_B^4 x + \frac{8}{3} q_F^4 \sqrt{2}\pi^2 x^{5/2} e^{-\pi x} \end{cases} \tag{3.1}$$

where $x = LT$. Eq.(3.1) is valid for $0 \leq \alpha \leq 1/(2q_F)$, which is the relevant range in α for the study of the phase transition. The latter is of first-order and occurs with a critical temperature $T_c \sim 1/L$. At $T = T_C$, $\alpha_{min}(x_C) = b(x_C)/(2c(x_C)) \simeq 6q_B/(\pi^2 q_F^2)$. In terms of $|H(T_C)|/T_C = 2\alpha_{min}(T_C)/(g_4 R T_C)$, we get $|H(T_C)|/T_C \sim q_B/q_F^2$. The strength of the first-order phase transition is inversely proportional to q_F^2 and hence to the value of the squared Higgs mass.

4. The Phase Transition in a Model with Gauge-Higgs Unification

We consider here the EWPT in the model of [4], where we refer for a detailed description. All the results have been obtained by a numerical computation of the one-loop Higgs potential (see also [8]). In the minimal model of [4], the top mass is too low and it has been fixed to $M_{top} = 45$ GeV. In agreement with the previous considerations, the EWPT is of first order, with a critical temperature of order $1/L$ (see fig. 1 (right)). For comparison, in fig. 1 (left) we plot $|H(T_C)|/T_C$ as a function of the Higgs mass for both the 5D model and the one-loop SM potential, with $M_{top} = 45$ GeV. The phase transition is strongly first order, as expected for such low values of M_H.

The problem of a too low value for the Higgs mass in the minimal model of [4] can be solved by the introduction of additional bulk fermions, in high rank representations of the underlying $SU(3)_w$ gauge group. By adding a massive bulk fermion in the symmetric rank 8 representation of $SU(3)_w$ and still fixing the top mass to $M_{top} = 45$ GeV, the first-order phase transition becomes much weaker, with $|H(T_C)|/T_C \sim 0.13$ for $110 \leq M_H \leq 150$ GeV. This can be understood by noting that V is now dominated by the high rank fermion and that the strength of the phase transition decreases with the rank, as explained in the last section. No comparison with the SM is given, since for such values of the Higgs mass perturbation theory breaks down in the SM close to T_C. The introduction of localized gauge kinetic terms represents another way to get realistic values for

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[3]The precise coefficient relating α to the Higgs VEV H is model dependent. For definiteness, we have taken here and in the following the one appearing in [4].
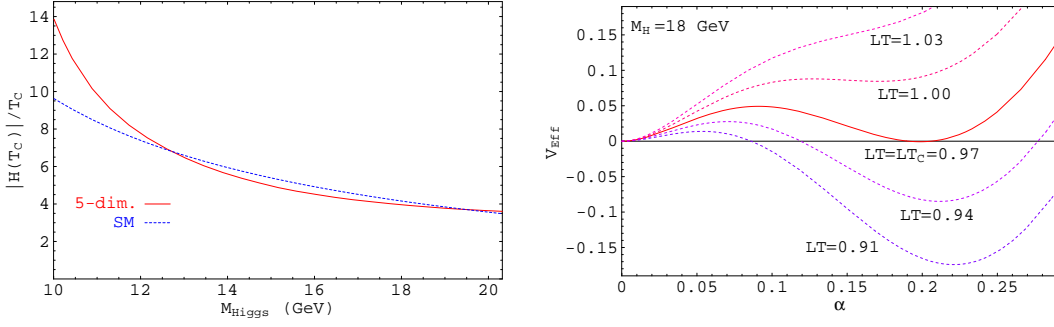
Figure 1: (Left) Phase transition strength and (right) detail of the potential near the phase transition (minimal model).

the Higgs mass. In the notation of [4], we take $c_1 \equiv c = 6$, $c_2 = 0$ and we fix $M_{top} = 110$ GeV. The phase transition is moderately strong of first order, with $|H(T_C)|/T_C \sim 0.7$ for $110 \leq M_H \leq 170$. The behaviour of the phase transition for large values of c is analytically hard to be studied. At the critical temperature, the ratio between $b(x)$ and $c(x)$ (see eq.(3.1)) depends on the bulk fermion charges like $1/q_F^2$, but it has a milder dependence on c, like $1/\sqrt{c}$. This explains why in the present case, in which the bulk fermion charges are small ($q_F^{max} = 2$), the first order phase transition is considerably stronger than in the case with high rank bulk fermions (in which $q_F^{max} = 8$).

All these results are based on one-loop perturbative studies, for any value of $M_H < M_W$ or $M_H \geq M_W$. In light of the breakdown of perturbation theory for $M_H \gtrsim M_W$ in the SM, it is natural to ask if and to what extent one can trust our results. It is possible to give an estimate of the relevance of the leading higher loop corrections in a 5D Wilson line based toy model [6]. Due to the good UV properties of the potential of Wilson line phases, it has been shown that higher order diagrams give a negligible contribution for $T \sim 1/L$. We believe that these results provide a strong evidence that perturbation theory in 5D models with GHU is valid around the critical temperature.

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