Current status of the muon g-2

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Abstract. The current status of the muon g-2 problem is briefly discussed. We briefly discuss the latest results on the muon g-2 measured in experiment and obtained theoretically within the standard model. Special attention is for the hadronic corrections and in particular the corrections due to the light by light scattering mechanism. For latter we present the results found in the leading in $1/N_c$ approximation with the nonlocal chiral quark model.

1. Introduction

The anomalous magnetic moment (AMM) of charged leptons ($l = e, \mu, \tau$) is defined by

$$a_l = \frac{g_l - 2}{2}, \tag{1}$$

with the gyromagnetic ratio g_l of the lepton magnetic moment to its spin, in Bohr magneton units. For a free pointlike fermion one has $g = 2$ in accordance with the Dirac equation. However, deviations appear when taking into account the interactions leading to fermion substructure and thus to nonzero a_l.

During the first years of the lepton AMM studies the fundamental task was to test the foundations of quantum field theory in general and quantum electrodynamics (QED) in particular. At present, the measurements of the lepton AMM are one of the major low-energy tests of the standard model (SM) and play an important role in the search for new interactions beyond the SM.

2. Muon anomalous magnetic moment in the standard model

The nonzero lepton AMMs are induced by radiative corrections due to the coupling of the lepton spin to virtual fields, which in the SM are induced by QED, weak and strong (hadronic) interactions[1]

$$a^{\rm SM} = a^{\rm QED} + a^{\rm weak} + a^{\rm hadr}. \tag{2}$$

The electron and muon AMMs are among the most accurately measured quantities in elementary particle physics. Today, the electron AMM serves as the best quantity to determine the fine structure constant with the highest accuracy. At the same time, for a_μ, there is a deviation at the level of 3-4 σ of the SM prediction from the measured value. Even if this does not give a clear indication for the existence of New Physics, it allows us to provide stringent constraints on the parameters of hypothetical models.

In 2006, there were published the results on a_μ measurements by the E821 collaboration at Brookhaven National Laboratory [5]. The combined result, based on nearly equal samples of positive and negative muons, is

$$a_\mu^{\rm BNL} = 116\ 592\ 08.0\ (6.3) \times 10^{-10} \quad [0.54\ {\rm ppm}]. \tag{3}$$

This exiting result is still limited by the statistical errors and proposals to measure a_μ with a fourthfold improvement in accuracy have been proposed at Fermilab (USA) [6] and J-PARC (Japan) [7]. A future experiments plan to reduce the present experimental error to a precision of 0.14 ppm.

In SM the dominant contribution to the lepton AMM comes from QED. The complete tenth-order QED contribution to a_μ was reported in [8]

$$a_\mu^{\rm QED} = 11\ 658\ 471.8951\ (0.0080) \times 10^{-10}. \tag{4}$$

The accuracy of these calculations is enough for any planed experiments in new future.

In general, the weak contributions are small due to suppressing factor $\alpha/\pi \cdot m_\mu^2/M_w^2 \sim 10^{-9}$, where M_w is a typical mass of heavy $W^\pm, Z$ and H bosons. The one- and two-loop evaluations indicate that they are known with a sufficiently high accuracy [9, 10]

$$a_\mu^{\rm weak} = 15.36\ \ (0.10) \times 10^{-10}, \tag{5}$$

where the remaining theory error comes from unknown three-loop contributions and dominantly from light hadronic uncertainties in the second-order electroweak diagrams with quark triangle loops. The most important feature of these new estimates, that significantly increase the theoretical precision, is to use the LHC result on the Higgs-boson mass measured by ATLAS and CMS Collaborations.

3. Hadronic contributions to the muon anomalous magnetic moment

Strong (hadronic) interaction produces relatively small contributions to a_μ, however they are known with an accuracy comparable to the experimental uncertainty in (3). In leading in α orders, these contributions can be separated into three terms

$$a_\mu^{\rm hadr} = a_\mu^{\rm HVP} + a_\mu^{\rm ho} + a_\mu^{\rm HLbL}. \tag{6}$$

In (6), $a_\mu^{\rm HVP}$ is the leading in α contribution due to the hadron vacuum polarization (HVP) effect in the internal photon propagator of the one-loop diagram , $a_\mu^{\rm ho}$ is the next-to-leading and next-to-next-to-leading order contributions related to iteration of HVP. The last term is

[1] For comprehensive reviews see [1, 2, 3, 4].

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not reduced to HVP iteration and it is due to the hadronic light-by-light (HLbL) scattering mechanism.

Hadronic contributions in (6) are determined by effects dominated by long distance dynamics, the region where the methods of perturbation theory of Quantum Chromodynamics (QCD) do not applicable and one must use less reliable nonperturbative approaches. However, in case of HVP, using analyticity and unitarity (the optical theorem) $a_\mu^{\rm HVP}$ can be expressed as the spectral representation integral

$$a_\mu^{\rm HVP} = \frac{\alpha}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt}{t}K(t)\rho_{\rm V}^{\rm (H)}(t)\,, \tag{7}$$

which is a convolution of the hadronic spectral function $\rho_{\rm V}^{\rm (H)}$ with the known from QED kinematical factor $K(t)$. The QED factor is sharply peaked at low invariant masses t and decreases monotonically with increasing t. Thus, the integral defining $a_\mu^{\rm HVP}$ is sensitive to the details of the spectral function $\rho_V^{\rm (H)}(t)$ at low t, which is related to the total $e^+e^- \to \gamma^* \to$ hadrons cross-section $\sigma(t)$ at center-of-mass energy squared t by

$$\sigma^{e^+e^-\to{\rm hadrons}}(t) = \frac{4\pi\alpha}{t}\rho_{\rm V}^{\rm (H)}(t)\,. \tag{8}$$

This fact is used to get quite accurate estimate of $a_\mu^{\rm HVP}$. The most precise recent phenomenological evaluations of $a_\mu^{\rm HVP}$, using recent $e^+e^- \to$ hadrons data, provide the results

$$a_\mu^{HVP,LO} = \begin{cases} 692.3\ \ (4.2)\times 10^{-10}, & [11] \\ 694.91\ \ (4.27)\times 10^{-10}. & [12] \end{cases} \tag{9}$$

The next-to-leading order corrections are suppressed by the absolute value by extra degree of α. The hadron vacuum polarization contributions in the next-to-leading and next-next-to-leading order are

$$a_\mu^{\rm HVP,NLO} = -9.84(0.06)(0.04)\times 10^{-10} \qquad [12], \tag{10}$$
$$a_\mu^{\rm HVP,NNLO} = 1.24(0.01)\times 10^{-10} \qquad [13], \tag{11}$$

However, one kind of these contributions corresponding to the HLbL is amount to the range from 0.5 to 1.5 ppm and known with accuracy of order 50% It gives an error comparable in magnitude with the uncertainty induced by HVP (9). The problem is that the HLbL scattering contribution can not be calculated from first principles or (unlike to HVP) directly extracted from phenomenological considerations. Instead, it has to be evaluated using various QCD inspired hadronic models that correctly reproduce low- and high- energy properties of strong interaction.

Different approaches to the calculation of the contributions from the HLbL scattering process to a_μ were used. These approaches can be separated in several groups. The first one consists of various extended versions of the vector meson dominance model (VMD) supplemented by ideas of the chiral effective theory, such as the hidden local symmetry model (HLS) [14], the lowest meson dominance (LMD) [15, 16, 17], the resonance chiral theory (RχT) [18, 19]. The second group is based on consideration the effective models of QCD that use the dynamical quarks as effective degrees of freedom. The latter include different versions of the (extended) Nambu–Jona-Lasinio model (E)NJL [20, 21], the Constituent Quark Models with local interaction (CQM) [22, 23, 24, 25], the models based on nonperturbative quark-gluon dynamics, like the non-local chiral quark model (NχQM) [26, 27, 28, 3, 29], the Dyson-Schwinger model [30] (DS), or the holographic models (HM) [31, 32]. More recently, there have been attempts to estimate $a_\mu^{\rm HLbL}$ within the dispersive approach (DA) [33, 34] and the so-called rational approximation (RA) approach [35]. The lattice calculations of HLbL are still at an exploratory stage [36].

4. Results on hadronic light-by-light scattering contributions in the leading in $1/N_c$ approximation

For the numerical estimates, the $SU(2)$- and $SU(3)$- versions of the NχQM model are used by us. In order to check the model dependence of the final results, we also perform calculations for different sets of model parameters.

In the $SU(2)$ model, the scheme of fixing the model parameters was suggested in [27, 28]: fitting the parameters of the nonlocality Λ and the light current quark mass m_c by the physical values of the π^0 mass and the $\pi^0 \to \gamma\gamma$ decay width, and varying the dynamical quark mass m_D in the region $150 - 400$ MeV. For estimation of a_μ^{HLbL} and its error, we use the region for m_D from 200 to 350 MeV.

For the $SU(3)$ version of the model, it is necessary to fix two more parameters: the current and dynamical masses of the strange quark. We suggest to fix them by fitting the K^0 mass and obtaining reasonable values for the η meson mass and the $\eta \to \gamma\gamma$ decay width.

The estimates for the partial contributions to a_μ^{HLbL} (in 10^{-10}) are the π^0 contribution 5.01(0.37) [27], the sum of the contributions from π^0, η and η' mesons 5.85(0.87) [27], the scalar σ, $a_0(980)$ and $f_0(980)$ mesons contribution 0.34(0.48) [3, 28], and the quark loop contribution is 11.0(0.9) [3, 29]. In all cases we estimate the absolute value of the result and its error by calculating $a_\mu^{\mathrm{HLbL,N\chi QM}}$ for the space of model parameters fixed by above mentioned observables, except one, varying m_D. Because in all cases the behavior of the result is quite smooth, it gives to us a credit to point out rather small model errors ($\leq 10\%$) for the intermediate and final results. Thus our claim is that the total contribution obtained in the leading order in the $1/N_c$ expansion within the nonlocal chiral quark model is)

$$a_\mu^{\mathrm{HLbL,N\chi QM}} = 16.8(1.25) \cdot 10^{-10}. \tag{12}$$

This value accounts for the spread of the results depending on reasonable variation of the model parameters and sensitivity to the different choice of the nonlocality shapes.

Summarizing the results of our works [27, 28, 3, 29], we get the total hadronic contribution to a_μ^{HLbL} within the NχQM in the leading order in the $1/N_c$ expansion. The total result is given in Eq. (12). To estimate the uncertainty of this result, we vary some of the model parameters in physically reasonable interval and also study the sensitivity of the result with respect to different model parameterizations.

If we add the result (12) to all other known contributions of the standard model to a_μ, (4)-(11), we get that the difference between experiment (3) and theory is

$$a_\mu^{\mathrm{BNL}} - a_\mu^{\mathrm{SM}} = 18.73 \times 10^{-10}, \tag{13}$$

which corresponds to 2.43σ. The total result becomes more close to the experiment one, however it is still not enough to explain the muon anomaly.

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