Servo systems with incomplete information

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***Abstract* — The problem of control design formation is considered with a modification use of the local quadratic criterion, which allows to keep a part of vector component of the given state. Kalman's filter is used in order to assess the state of the object model. The results of control system simulation for non-stationary model of the ship are given while changing course.**

***Key words* — servo system, incomplete information, modification of the local criterion, non-stationary model**

INTRODUCTION

The concept of the combined synthesis of servo systems is dominated in the modern control theory. The most promising methods for solving such problems for objects are mathematical models which are described by the systems of linear non-stationary differential equations in state-space of non-stationary, these methods based on quadratic optimization criteria. Generally the solution is obtained in the standard feedback form under this condition. In [1, 2] the control algorithms are given, i.e. algorithms of control for the technical and economic models of the objects by the quadratic criteria.

Synthesis of servo control systems - one of the major technical tasks. The purpose of the synthesis is to create a system that would meet the required quality parameters. The situation becomes more complicated when the control is carried out at incomplete information, which may occur if a task of some of the components of the monitored condition is associated with a significant financial or technical efforts.

One of the ways that afford to control form in this case is to lower the order of models with help of aggregation algorithms. Hence, in [1] proposed the control algorithms when classical quadratic criterion with the reduced order models are used, in [3] a predictive model of decreased order is proposed to use for the integral quadratic criterion of generalized work, in [4] to control a local quadratic criterion with decreasing order of the model is used.

The proposed aggregation algorithms are iterative and occupy a certain time during the synthesis, which affects the quality of functioning of the managed object. Moreover difficulties arise in the synthesis of adaptive systems [5]. A significant simplification of the formation of control actions with incomplete information about the monitored state can be achieved by modification of quadratic functional. So, in [6] such method is proposed for the classical quadratic functional.

In this paper we propose to form some control actions when a modification of the local quadratic criterion is used. This allows to track out only a part of the vector components of a given state during control, and, in addition, reduce the computational delay in a control circuit significantly.

1. THE DESCRIPTION OF MATHEMATICAL MODELS OF THE OBJECT AND MEASURING COMPLEX

Let the plant model be described by a system of linear stochastic differential equations of the following form:

$$\dot{x}(t) = \overline{A}(t)x(t) + \overline{B}(t)u(t) + \overline{F}(t)q(t),\quad x(t_0) = x_0, \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $q(t) \in R^{l_1}$ - vectors defining the condition, control and external disturbances. It is assumed that the external perturbations are additive and set by Gaussian noise vector with specified characteristics. Matrices $\overline{A}(t), \overline{B}(t), \overline{F}(t)$ describe the dynamic properties, the impact of control actions and external disturbances.

The sampling model (1) is carried out on the assumption that $u(t)$ and $q(t)$ are both the piecewise constant left continuous functions in the formation of the control actions

$$u(t) = u(t_k),\ q(t) = q(t_k),\ t_k \le t < t_{k+1},\ t_{k+1} = t_0 + k\Delta t,\ k = \overline{0,N}. \tag{2}$$

Then the differential equation corresponds to (1) and has the form:

$$x(k+1) = A(k)x(k) + B(k)u(k) + F(k)q(k),\ x(0) = x_0, \tag{3}$$

where

$$A(k) = I_n + \sum_{i=1}^{L} \frac{\Delta t^i \overline{A}^i(t_k)}{i!},$$
$$B(k) = \sum_{i=1}^{L} \frac{\Delta t^i \overline{A}^{i-1}(t_k)\overline{B}(t_k)}{(i-1)!}, \tag{4}$$
$$F(k) = \sqrt{\Delta t}\sum_{i=1}^{L} \frac{\Delta t^{i-1} \overline{A}^{i-1}(t_k)\overline{F}(t_k)}{(i-1)!}.$$

In (4) I_n - the identity matrix of n order, Δt - sampling step, L gives the sampling accuracy. So, if $L = 1$, then for the construction of model (3) Euler's method is used, and if $L = 4$ Runge-Kutta's method is used [7].

Information about the state of the object comes from the measuring complex, which is quite often contains incomplete information about the object, distorted by measurement errors in the synthesis of control actions. We assume that the mathematical model of the measuring complex is as follows:

$$y(k) = Hx(k) + r(k) \tag{5}$$

where $y(k)$ - l-dimensional vector of measurements $(l \le n)$, H - the matrix of channel dimension measurements $l \times n$, consisting of zeros and ones, zero columns which correspond to unmeasured components of the state vector, $r(k)$ - discrete Gaussian noise with specified characteristics.

2. Formation of the control actions with use of the local quadratic criterion modification

Because the object control is performed with the results of the measurements in real conditions, and all measurements are made with some errors and, moreover, it can be the components of the state vector in the object model, those measurements are inaccessible to measure, then it is necessary to use the state estimation to form control actions $\hat{x}(k)$.

The mathematical models of the object and measuring complex have the form (3) and (5), where $q(k)$ and $r(k)$ - Gaussian sequence with characteristics:

$$M\{q(k)\} = \bar{q}(k),\ \ M\{(q(k) - \bar{q}(k))(q(j) - \bar{q}(j))^T\} = Q(k)\delta_{k,j},$$
$$M\{r(k)\} = 0,\ \ M\{r(k)r^T(j)\} = R\delta_{k,j},$$
$$M\{q(k)r^T(j)\} = 0.$$

It is assumed that the prior vector distribution x_0 is also Gaussian:

$$M\{x_0\} = \bar{x}_0,\ \ M\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\} = P_{x_0}, \tag{6}$$

where P_{x_0} – variance matrix of estimation errors of initial state. We will use the assessment $\hat{x}(k)$ state vector $x(k)$, got with results of the given measurements $y(k)$, satisfies the requirement of unbiasedness:

$$M\{\hat{x}(k)\} = M\{x(k)\} \tag{7}$$

with minimum error variance estimation. Expressions for the assessment of the state $\hat{x}(k+1)$ have the following form [7, 8]:

$$\begin{aligned}
&\hat{x}(k+1) = \hat{x}(k+1/k) + K(k)[y(k+1) - H\hat{x}(k+1/k)],\\
&\hat{x}(k+1/k) = A(k)\hat{x}(k) + B(k)u(k) + F(k)\bar{q}(k), \hat{x}(0) = \bar{x}_0,\\
&K(k) = P(k+1/k)H^T[HP(k+1/k)H^T + R]^{-1},\\
&P(k+1/k) = A(k)P(k)A^T(k) + F(k)Q(k)F^T(k),\\
&P(k+1) = [I_n - K(k)H]P(k+1/k),\\
&P(0) = P_{x_0}.
\end{aligned} \tag{8}$$

Control actions $u(k)$ we will build while minimizing the expectation of local quadratic criterion

$$J(k) = M\{(x(k+1) - x_z(k))^T C(x(k+1) - x_z(k)) + u^T(k)Du(k)\} \tag{9}$$

where C – nonnegative definite form, but D – positive definite weight matrices, $x_z(k)$ – state, after which track is done at the moment $t_k = t_0 + k\Delta t$. Plug in (9) equation (3),and, consider (7) operation condition $tr(\cdot)$, we will have

$$\begin{aligned}
J(k) = {} & tr(CF(k)Q(k)F(k)^T) + u(k)^T[B(k)^T CB(k) + D]u(k) + \\
& + [A(k)\hat{x}(k) + F(k)\bar{q}(k) - x_z(k)]^T CB(k)u(k) + \\
& + u(k)^T B(k)^T C[A(k)\hat{x}(k) + F(k)\bar{q}(k) - x_z(k)].
\end{aligned} \tag{10}$$

From the optimality conditions

$$\begin{aligned}
\frac{dJ(k)}{du(k)} = {} & 2[B(k)^T CB(k) + D]u(k) + \\
& + 2B(k)^T C[A(k)\hat{x}(k) + F(k)\bar{q}(k) - x_z(k)] = 0
\end{aligned} \tag{11}$$

we will get:

$$u(k) = -[B^T(k)CB(k) + D]^{-1}B^T(k)C[A(k)\hat{x}(k) + F(k)\bar{q}(k) - x_z(t_k)] \tag{12}$$

To modify criterion (9) weight matrix C we will show in the form $S^T SCS^T S$, where S – matrix dimension $p \times n$ rank p, $(p \le n)$, consists of zeros and units, null columns are components of a given state and indicate that are not tracked in the formation of the control actions. For matrices S correct equalities $SS^+ = I_p$, $S^+ = S^T$, where S^+ – pseudoinverse matrix, I_p – unity matrix of p order.

Then criterion (9) one can show in the following form

$$\begin{aligned}
J^{(S)}(k) = M\{ & (Sx(k+1) - x_z^{(S)}(k))^T C_S (Sx(k+1) - x_z^{(S)}(k)) + \\
& + u^T(k)Du(k)\},
\end{aligned} \tag{13}$$

where $C_S = SCS^T$ is a weight matrix of p order for criterion (13), but $x_z^{(S)}(k) = Sx_z(k)$ – vector of monitor the status of the dimension p, and

$$u(k) = -[B^{\mathrm{T}}(k)S^{\mathrm{T}}C_S SB(k) + D]^{-1} B^{\mathrm{T}}(k)S^{\mathrm{T}}C_S[SA(k)\hat{x}(k) + + SF(k)\overline{q}(k) - x_z^{(S)}(k)] \quad (14)$$

Note that the tracking part of a vector component of a given state is greatly simplified the process of determining the weight matrices by reducing its order.

Moreover, if the tracking is performed by a single component target of the state vector, then the weight matrix degenerates into a weight factor.

Some restrictions are imposed on the control magnitude that are usually take the form of inequalities:

$$U_1(k) \le u_i(k) \le U_2(k) \;\; i = \overline{1,m} \quad (15)$$

For the state vector of constraints define the maximum amount of deviation from the track status and determine the quality of the operation of a control object.

3. Ship control when change the course

Mathematical model of a ship movement is given in the form (1), where components of state vectors $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T = (\beta, \varpi, V, \psi)^T$ and control $u(t) = (u_1(t) = \delta, u_2(t))^T$ of the object model have the following meaning [9]:

β – drift angle deviation (rad),

ω – angular deviation of the drift velocity (rad/s),

V – speed deviation (m/s),

ψ – heading angle deviation (rad),

u_1 – deviation rudder angle (rad),

u_2 – deviation mode of the main engine.

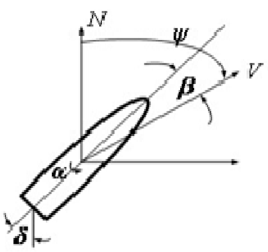

Fig.1. Spatial orientation of a plane

In the picture 1 variable N points the direction to the North.

External disturbances in the object model describes by the vector $q(t)$, whose components are independent and normal Gaussian variables with influence matrix $\overline{F}(t)$.

The following restrictions are applied to the control magnitude:

$|u_1| \le 0.61, \;\; |u_2| \le 0.01$

In this case, starting from a certain point in time, the state vector components must meet the following conditions:

$$|\beta| \le 0.46 rad; \;\; |\varpi| \le 1.74 \cdot 10^{-1} rad/s;$$
$$|V| \le 0.5 m/s; \;\; |\psi - \psi_z| \le 0.53 \cdot 10^{-1} rad.$$

Object model is a nonstationary model and according to the data for the three time moments $\tau_1 = 1s,\ \tau_2 = 40s,\ \tau_3 = 60s$ with help of polynomial of Lagrange of a second degree [7]

$$L_2(t,\tau,z) = \sum_{j=1}^{3} z_j \prod_{\substack{i=1 \\ i \ne j}}^{3} \frac{t-\tau_i}{\tau_j - \tau_i}, \quad (16)$$

calculated the matrices elements $\overline{A}(t), \overline{B}(t), \overline{F}(t)$, depend on time. In (16) t – time moment, which calculates the value of Lagrange polynomial $z = (z_1, z_2, z_3)$ – vector values specified in points τ_1, τ_2, τ_3.

Construct vectors of matrix elements in the points for this ship model τ_1, τ_2, τ_3:

$$\overline{A}_{2,1} = (0.0142; 0.0185; 0.0244),$$
$$\overline{A}_{2,2} = (-0.167; -0.192; -0.220),$$
$$\overline{A}_{3,3} = (-0.0353; -0.040; -0.046),$$
$$\overline{B}_{1,1} = (0.0158; 0.0184; 0.021),$$
$$\overline{B}_{2,1} = (0.0084; 0.0108; 0.0146),$$
$$\overline{B}_{3,2} = (0.1; 0.087; 0.076),$$
$$\overline{F}_{2,2} = (0.17 \cdot 10^{-3}; 0.18 \cdot 10^{-3}; 0.20 \cdot 10^{-3}),$$
$$\overline{F}_{3,3} = (0.1 \cdot 10^{-1}; 0.101 \cdot 10^{-1}; 0.103 \cdot 10^{-1}).$$

Then in the moment t matrix, specifies the object model, are defined as follows:

$$\overline{A}(t) = \begin{pmatrix} -0.007 & 0.444 & 0 & 0 \\ L_2(t,\tau,\overline{A}_{2,1}) & L_2(t,\tau,\overline{A}_{2,2}) & 0 & 0 \\ 0 & 0 & L_2(t,\tau,\overline{A}_{3,3}) & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\overline{B}(t) = \begin{pmatrix} L_2(t,\tau,\overline{B}_{1,1}) & 0 \\ L_2(t,\tau,\overline{B}_{2,1}) & 0 \\ 0 & L_2(t,\tau,\overline{B}_{3,2}) \\ 0 & 0 \end{pmatrix},$$

$$\overline{F}(t) = \begin{pmatrix} 0.1 \cdot 10^{-2} & 0 & 0 \\ 0 & L_2(t,\tau,\overline{F}_{2,2}) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L_2(t,\tau,\overline{F}_{3,3}) \end{pmatrix}.$$

A discrete model of the form is used in the formation of control actions (3), obtained by Euler's method with step

$\Delta t = 0.1s$, where $k = \overline{0, N}$ corresponds to the time $t_k = t_0 + k\Delta t,\ t_0 = 0,\ T = 60s,\ N = (T - t_0)/\Delta t = 600.$

Mathematical model of measurement system is given in the form (5), where matrix $H = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$ which corresponds to the dimension of the course only. Measurement error $r(k)$ it describes a discrete Gaussian variable with zero mean and variance $R = 0.15 \cdot 10^{-2}$.

To assess the state the ratio is used (8) following a priori information: $\hat{x}(0) = (0;0;0;0.15)^T$, $P_x = diag\{1;1;1;1\}$.

Synthesis of controls carried out in accordance (14) modeling the interval $[0;60s]$ when following a deflection angle of the drift velocity $\varpi_z(t)$ and course $\psi_z(t)$, that given by the matrix:

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We solve the problem of transfer the ship to another course. Course $\psi_z(t)$ for which tracking is carried out, it is defined by the Lagrange polynomial (16) for the following values: $\psi_z(\tau_1) = 0.1\,rad$, $\psi_z(\tau_2) = 0.3\,rad$, $\psi_z(\tau_3) = -0.4\,rad$. To reject an angular drift tracking is the speed of zero.

Figure 2 shows the simulation results: curve 1 shows the estimated rate, curve 2 - implemented, curve 3 - deflection of rudder angle.

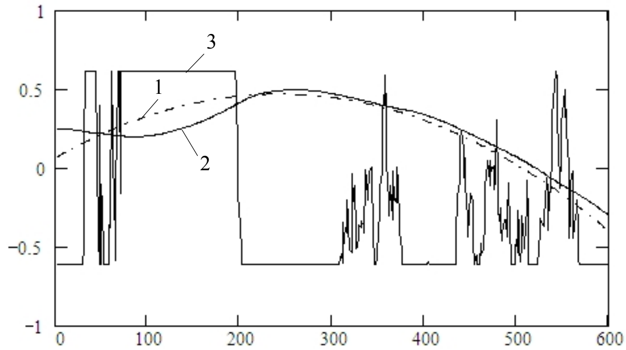

Fig. 2 .Given course– 1, implemented course – 2, deflection of rudder angle – 3

These given results illustrate the quality of tracking a given course in the operation of the controlled object, in this case clearly shows the steering control action to change the course. All other components of the state vector also satisfy the given restrictions.

CONCLUSION

Synthesis of servo control systems have to do enough often with incomplete information. Kalman's filter is used with incomplete measurements of an object condition, which is estimated by using a state vector and its recovered dimension. But there is a situation where unknown information about the part of the components of the monitored state. This occurs, for example, if such information involves considerable material and technical costs.

In this paper we proposed algorithms to generate the control actions with incomplete information about the monitored state when we use a modification of the local quadratic criterion. This significantly simplifies the process of determining the weight matrix criteria and reduced the time delay in the formation of control.

Illustration of efficiency and quality of the proposed algorithms is carried out on the example of the control for non-stationary stochastic model of the ship while changing course.

REFERENCES

[1] Reshetnikova G.N. Adaptive control of delivery and advertising. Algorithms. Models. Limitations. Monograph. Saarbrucken (Germany) LAP LAMBERT Academic Publishing, 2012. 152 p. ISBN: 978-3-659-29616-1

[2] Reshetnikova G.N., Krasnov I.U. Unsteady object optimum control algorithm (Conference Paper) [Electronic resources] / G. N. Reshetnikova [et al.] // Modern Techniques and Technologies, 2003. MTT 2003. Proceedings of the 9th International Scientific and Practical Conference of Students, Post-graduates and Young Scientists, 2003. [5 p.]. DOI: 10.1109/SPCMTT.2003.1438195 Available at: 10.1109/SPCMTT.2003.1438195 (Accessed 10 January 2017)

[3] Reshetnikova G. N., Khabibulina N. Yu., Kotcubinskiy V. P., Polonskaya M. S. Synthesis of control actions with aggregate model [Electronic resources] / G. N. Reshetnikova [et al.] // Control and Communications (SIBCON) : Proceedings of International Siberian Conference, Moscow, May 12-14, 2016. [S. l.]: IEEE, 2016. [5 p.]. DOI: 10.1109/SIBCON.2016.7491806 Available at: http://dx.doi.org/10.1109/SIBCON.2016.7491806 (Accessed 28 January 2017)

[4] Reshetnikova G. N. Tracking systems of adaptive control with the predictive model of reduced order. Tomsk State University Journal, 2006, no. 290, pp. 241 -243.

[5] Reshetnikova G. N. Adaptive Systems: work-book / G.N. Reshetnikova; Tomsk: Publishing house of Tomsk State University, 2016. 112 p.

[6] Reshetnikova G. N., Khabibulina N. Yu., Kotcubinskiy V. P., Polonskaya M. S. Servo-System simulation [Electronic resources] / G. N. Reshetnikova [et al.] // Mechanical Engineering, Automation and Control Systems (MEACS) : proceedings of the International Conference, Tomsk, 1-4 December, 2015 / National Research Tomsk Polytechnic University (TPU) ; Institute of Electrical and Electronics Engineers (IEEE). [S. l.]: IEEE, 2015. [4 p.]. DOI: 10.1109/MEACS.2015.7414890 Available at: http://dx.doi.org/10.1109/MEACS.2015.7414890 (Accessed 20 January 2017)

[7] Reshetnikova G.N. Modeling systems: work-book / G.N. Reshetnikova; Federal Agency for Education, Tomsk State University of control systems and radioelectronics. 2nd edition, revised and enlarged. Tomsk: Tomsk State University of control systems and radioelectronics, 2007. 441 p. ISBN 978-55-86889-364-3

[8] Brammer, Ziffling G. Kalman-Bucy filter. M.: Science Publ, 1972. 200 p.

[9] Sobolev G.V. Manageability and automation of ship navigation. L: Navigation, 1976. 477 p.