

# Probing the astrophysics of cluster outskirts (Research Note)

A. Lapi<sup>1,2</sup>, R. Fusco-Femiano<sup>3</sup>, and A. Cavaliere<sup>1</sup><sup>1</sup> Dip. Fisica, Univ. “Tor Vergata”, via Ricerca Scientifica 1, 00133 Roma, Italy  
e-mail: [lapi@roma2.infn.it](mailto:lapi@roma2.infn.it)<sup>2</sup> SISSA, via Bonomea 265, 34136 Trieste, Italy<sup>3</sup> INAF-IASF, via Fosso del Cavaliere, 00133 Roma, Italy

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## ABSTRACT

In galaxy clusters the entropy distribution of the intracluster plasma (ICP) modulates the latter’s equilibrium within the dark matter gravitational wells, as rendered by our supermodel. We argue the entropy production at the boundary shocks to be reduced or terminated as the accretion rates of DM and intergalactic gas peter out; this behavior is enforced by the slowdown in the outskirts development at late times, when the dark energy dominates the cosmology while the outer wings of the initial perturbation drive the growth. For these conditions, we predict the ICP temperature profiles to steepen into the cluster outskirts. The detailed expectations from our simple formalism agree with the X-ray data concerning five clusters whose temperature profiles have been recently measured out to the virial radius. We predict steep temperature declines to prevail in clusters at low  $z$ , tempered only by rich environs including adjacent filamentary structures.

**Key words.** galaxies: clusters: general – X-rays: galaxies: clusters – methods: analytical

## 1. Introduction

Galaxy clusters constitute the largest bound structures in the Universe, with their masses up to  $M \sim 10^{15} M_{\odot}$  and outskirts extending out to sizes  $R \sim$  a few Mpcs. These set the *interface* between the intergalactic environment keyed to the cosmology at large, and the confined intracluster plasma (ICP). The latter pervades the clusters at temperatures of  $k_{\text{B}}T \propto GM/R \sim 5$  keV and number densities of  $n \sim 10^{-3} \text{ cm}^{-3}$ , and thus emits copious X-ray powers mainly by thermal Bremsstrahlung (see [Sarazin 1988](#)). The ICP coexists with the gravitationally dominant dark matter (DM) component in the baryonic fraction  $m/M$  close to the cosmic value 0.16, and the two build up together from accretion across the cluster boundary.

The build-up comprises an early collapse of the cluster body, tailing off into a secular development of the outskirts by smooth accretion and minor mergers (see [Zhao et al. 2003](#); [Diemann et al. 2007](#); [Vass et al. 2008](#); [Navarro et al. 2010](#)). In radius, the body ranges out to  $r \sim r_{-2}$  where the slope of the DM density run  $n(r)$  equals  $-2$ ; the adjoining outskirts extend out to the current virial radius  $R$  with steepening density.

In time, the transition is marked by the redshift  $z_t$ ; thereafter  $r_{-2}$  stays the same, while  $R$  grows larger in a quasi-static, self-gravitating DM equilibrium (described through the Jeans equation, see [Lapi & Cavaliere 2009a](#)), to imply for the standard concentration parameter  $c \equiv R/r_{-2}$  observed values of  $c \approx 3.5 H(z_t)/H(z_{\text{obs}})$  in terms of the Hubble parameter  $H(z)$ . Below we adopt the standard flat cosmology (see [Dunkley et al. 2009](#)). This means that values of  $c$  ranging from 3 to 10 correspond for  $z_{\text{obs}} \approx 0-0.2$  to young or to old dynamical cluster ages  $z_t \sim 0.2-3$ . The concentration can be directly if laboriously probed with gravitational lensing (see [Broadhurst et al. 2008](#); [Lapi & Cavaliere 2009b](#)).

Secular accretion of DM goes along with an inflow of intergalactic gas. The ensuing ICP equilibrium is amenable to the powerful yet simple description provided by the supermodel<sup>1</sup> (SM; see [Cavaliere et al. 2009](#), hereafter CLFF09).

Clearly, inflows into the cluster outskirts are exposed to the cosmological grip. This is the focus of the present paper.

## 2. Entropy run vs. cluster build-up

The SM fully expresses the hydrostatic equilibrium (HE) of the ICP in terms of DM gravity and of the “entropy”  $k \equiv k_{\text{B}}T/n^{2/3}$ . In its basic form, the latter’s physical run may be represented as

$$k(r) = k_c + (k_R - k_c) (r/R)^a, \quad (1)$$

consistent out to  $r \approx R/2$  with recent analyses of wide cluster samples ([Cavagnolo et al. 2009](#); [Pratt et al. 2010](#)). This embodies two *specific* ICP parameters: the central level  $k_c$  and the outer powerlaw slope  $a$ .

The former is set at a basal level  $k_c \sim 10 \text{ keV cm}^2$  by intermittent entropy injections by central AGN feedback (e.g., [Cavaliere et al. 2002](#); [Valageas & Silk 1999](#); [Wu et al. 2000](#); [McNamara & Nulsen 2007](#)); the ensuing quasi-stable condition corresponds to cool-core morphologies (CC, see [Molendi & Pizzolato 2001](#)), featuring a limited central temperature dip and generally large concentrations of  $c \approx 6-10$ , as discussed by CLFF09.

On the other hand,  $k_c$  may be enhanced up to several  $10^2 \text{ keV cm}^2$  by deep mergers (e.g., [McCarthy et al. 2007](#); [Markevitch & Vikhlinin 2007](#)), which are frequent during the

<sup>1</sup> *IDL* and *FORTTRAN* algorithms which implement the supermodel and run in a fraction of a second on a standard laptop, can be found at <http://people.sissa.it/~lapi/Supermodel/>

cluster youth; these events give rise to non-cool core (NCC) clusters, featuring generally low concentrations of  $c \approx 3\text{--}5$  and a central temperature plateau, scarred in some instances by imprints from recently stalled blastwaves (see discussions by Fusco-Femiano et al. 2009; Rossetti & Molendi 2010).

### 2.1. The outer regions

The second term in Eq. (1) describes the powerlaw outward rise expected from the scale-free stratification of the entropy continuously produced by the boundary accretion shock, while the cluster grows larger by slow accretion.

The slope  $a_R$  at  $r \approx R$  with standard values around 1 has been derived by CLFF09 from the shock jumps and the adjoining HE maintained by thermal pressure, to read

$$a_R = 2.37 - 0.47 b_R. \quad (2)$$

Here  $b_R \equiv \mu m_p v_R^2 / k_B T_R$  marks the ratio of the potential to the thermal energy of the ICP (see Lapi et al. 2005; Voit 2005). This reads  $b_R \approx 3 v_R^2 / 2 \Delta\Phi$  when a strong shock efficiently thermalizes the infall energy into three degrees of freedom; the latter is expressed in terms of the potential drop  $\Delta\Phi = -\int_{R_{\text{ta}}}^R dr G \delta M / r^2$ , experienced by successive shells of DM and gas that expand; owing to the excess mass  $\delta M$  they turn around at the radius  $R_{\text{ta}} \approx 2R$  to start their infall toward the shock at  $R$ .

At any given cosmic time  $t$ , Eq. (2) holds at the current virial radius  $R(t)$ . On the other hand, the entropy deposited there is conserved during subsequent compressions of the accreted plasma into the DM gravitational well, while no other major sources or sinks of entropy occur down to the central  $10^2$  kpc. Thus while the cluster outskirts develop out to the current radius  $R(t)$ , the specific entropy stratifies with a running slope  $a(r) = a_{R(t)}$  that retains the sequence of original values set at the times of deposition (see Tozzi & Norman 2001).

Values  $a \approx 1$  are obtained adopting the standard ratio  $R/R_{\text{ta}} \approx 0.5$ , and the simple potential drop  $\Delta\Phi/v_R^2 \approx 1 - (R/R_{\text{ta}}) \approx 0.5$  associated to a flat initial mass perturbation  $\delta M/M \propto M^{-\epsilon}$  with  $\epsilon \sim 1$  that describes the collapse of the cluster *body* as a whole. This implies  $b_R \approx 3$  and  $a \approx 1$  from Eq. (2). Indeed,  $\Delta\Phi/v_R^2 \approx 0.57$  is obtained from the full DM  $\alpha$ -profile (see CLFF09), implying  $b_R \approx 2.7$  and the standard value  $a \approx 1.1$ .

However, considerable variations of  $a(r)$  are to be expected in the cluster outskirts, as discussed below.

### 2.2. Development of the outskirts

The cluster *outskirts* for  $r > r_{-2}$  originate from the wings of a realistically bell-shaped perturbation, which may be described by  $\delta M/M \propto M^{-\epsilon}$  for  $\epsilon$  exceeding 1 (e.g., Lu et al. 2006). Then the outer potential drop

$$\frac{\Delta\Phi}{v_R^2} = \frac{1 - (R/R_{\text{ta}})^{3\epsilon-2}}{3\epsilon-2} \quad (3)$$

is shallower relative to the body value, so leading to higher values of  $b_R$  and lower values of  $a$ . Thus as  $r$  increases outwards we expect  $k(r)$  to deviate downward from a simple powerlaw.

The argument may be phrased in terms of the accretion rate  $\dot{M}$ . A shell  $\delta M$  enclosing the mass  $M$  will collapse when  $\delta M/M$  attains the critical threshold  $1.686 D^{-1}(t)$  in terms of the linear growth factor  $D(t)$  (e.g., Weinberg 2008). Accordingly, the shape parameter  $\epsilon$  also governs the mass buildup after  $\dot{M} \propto D^{1/\epsilon} \propto t^{d/\epsilon}$ ; here we represent the growth factor as  $D(t) \propto t^d$  with  $d$

ranging from  $2/3$  for  $z \gtrsim 1$  to approach  $1/2$  as  $z \rightarrow 0$ . So the outskirts develop from the inside-out, at accretion rates  $\dot{M}/M \approx d/\epsilon t$  that lower for  $\epsilon$  exceeding 1, and for  $d$  decreasing toward  $1/2$  at late cosmic times<sup>2</sup>. We add that at small accretion rates the shock position outgrows  $R$  (see Voit et al. 2003), while the shock strength may weaken; both these effects will decrease  $a$  relative to Eqs. (2) and (3), and will be discussed in detail elsewhere.

We see that flatter slopes  $a$  of the entropy are to prevail for decreasing accretion rates  $\dot{M}$  of DM and gas; these have a twofold origin. First, the cosmological structure growth slows down at later cosmic times (low  $z_{\text{obs}}$ ), as expressed by  $d < 2/3$ . Second, perturbation wings marked by  $\epsilon > 1$  imply shallow gravitational wells and little available mass to accrete in average environs; the effect may be locally offset (and represented with a smaller effective  $\epsilon$ ) in specifically rich environments, including adjacent filamentary large-scale structures.

The decline of  $a$  from the body value and the entropy bending set in at a radius  $r_b \approx r_{-2}$  where matter began to stratify onto the outskirts just after  $z_t$ . Such a radius is evaluated in terms of the observed concentration in the form  $r_b/R \approx r_{-2}/R \approx 1/c$ , to take on values around 0.2–0.3 for typical concentrations  $c \approx 6\text{--}8$  of CC clusters. Hints of this trend are discerned in the data by Pratt et al. (2010) and Hoshino et al. (2010).

To summarize, with the lower accretion rates prevailing at later times in average environs, we expect the entropy run to flatten or even decline into the cluster outskirts; then the temperature will decline as  $k_B T(r) = k n^{2/3} \propto r^{-2}$  or steeper, after Eq. (7) of CLFF09. Do such behaviors show up in real clusters?

## 3. A case study on current data

To answer this question, we use the SM to provide profiles of density and temperature from expressing the expected entropy run in a simple form: the initial slope  $a$  still applies in the cluster body for  $r \leq r_b$ ; but for  $r > r_b$  it goes over to a decline toward the current boundary value  $a_R < a$  following  $a - a' (r/r_b - 1)$  with a constant gradient  $a'$ . The entropy profile then reads

$$k(r) = \begin{cases} k_c + (k_b - k_c) (r/r_b)^a & r \leq r_b \\ k_R (r/R)^{a+a'} e^{a'(R-r)/r_b} & r > r_b. \end{cases} \quad (4)$$

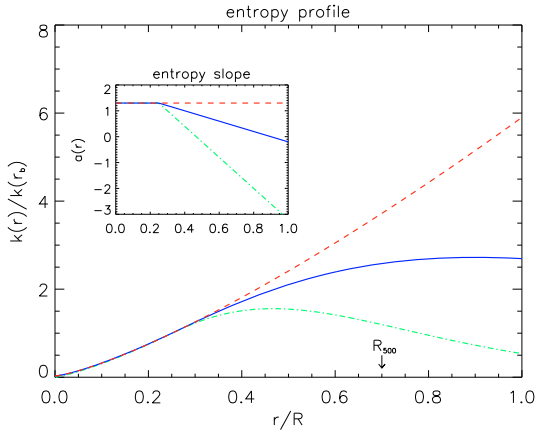
The outer branch describes a linear decline of the slope with the gradient  $a' \equiv (a - a_R)/(R/r_b - 1)$ ; normalizations have been set to obtain a continuous function and derivative for  $k(r)$ . We show in Fig. 1 examples of entropy profiles according to Eqs. (1) and (4), with parameters indicated by the analyses below.

In addition to the several CCs and NCCs previously analyzed out to  $R/2$  in terms of the standard entropy run (see CLFF09 and Fusco-Femiano et al. 2009), we focus here on five CCs with data now available out to  $r \approx R$ , and analyze them in terms of the entropy run of Eq. (4). We report in Table 1 the resulting parameters with 68%-level uncertainties. For these old CC clusters we take  $k_c \approx 10$  keV cm<sup>2</sup> as anticipated in Sect. 2.

## 4. Discussion and conclusions

These data clearly bear out our expectations of steep runs for  $T(r)$ , which arise because the outer entropy is reduced as the accretion rates  $\dot{M}$  of DM and intergalactic gas peter out (see

<sup>2</sup> Note that  $d \rightarrow 0$  would occur if the dark energy density increased with time to cause an ultimate cosmic Doomsday (see Caldwell et al. 2003); this would imply truly vanishing accretion rates, and result in a cutoff of the temperature profiles.



**Fig. 1.** Examples of entropy profiles (normalized at  $r = r_b$ ) and slopes (inset). Dashed line: after Eq. (1) with  $k_c = 0$  and  $a = 1.3$ . Solid line: Eq. (4) with  $k_c = 0$ ,  $a = 1.3$ ,  $r_b = 0.25 R$ , and  $a' = 0.5$ . Dot-dashed line: Eq. (4) with  $k_c = 0$ ,  $a = 1.3$ ,  $r_b = 0.25 R$ , and  $a' = 1.5$ .

**Table 1.** Fitting parameters from ICP temperature profiles.

| Cluster            | $c^*$                | $a$                 | $r_b/R$                | $a'$                  | $\chi^2_{**}$ |
|--------------------|----------------------|---------------------|------------------------|-----------------------|---------------|
| A1795              | $8.5^{+1.9}_{-1.9}$  | $1.2^{+0.3}_{-0.3}$ | $0.28^{+0.02}_{-0.02}$ | $1.8^{+1.3}_{-1.3}$   | 0.3 (2.6)     |
| PKS0745-191        | $7.6^{+1.7}_{-1.7}$  | $1.9^{+1.3}_{-1.3}$ | $0.23^{+0.03}_{-0.03}$ | $1.1^{+0.7}_{-0.7}$   | 1.4 (4.4)     |
| A2204              | $5.5^{+1.1}_{-1.1}$  | $1.5^{+1.1}_{-1.1}$ | $0.31^{+0.07}_{-0.07}$ | $0.6^{+0.4}_{-0.4}$   | 1.1 (2.1)     |
| A1413              | $8.3^{+1.7}_{-1.7}$  | $0.9^{+0.3}_{-0.3}$ | $0.27^{+0.07}_{-0.07}$ | $0.2^{+0.03}_{-0.03}$ | 1.2 (1.9)     |
| A1689 <sup>†</sup> | $12.4^{+5.3}_{-5.3}$ | $0.7^{+0.3}_{-0.3}$ | $0.5^{+0.1}_{-0.1}$    | $1.6^{+1.2}_{-1.2}$   | 1.5 (1.7)     |

**Notes.** (\*) DM concentration estimated from X rays. (\*\*) Reduced  $\chi^2$  values for fits with the entropy run in Eq. (4); in parenthesis the values with  $a' = 0$ , corresponding to Eq. (1). (†) CC classification is controversial (see [Riemer-Sørensen et al. 2009](#)).

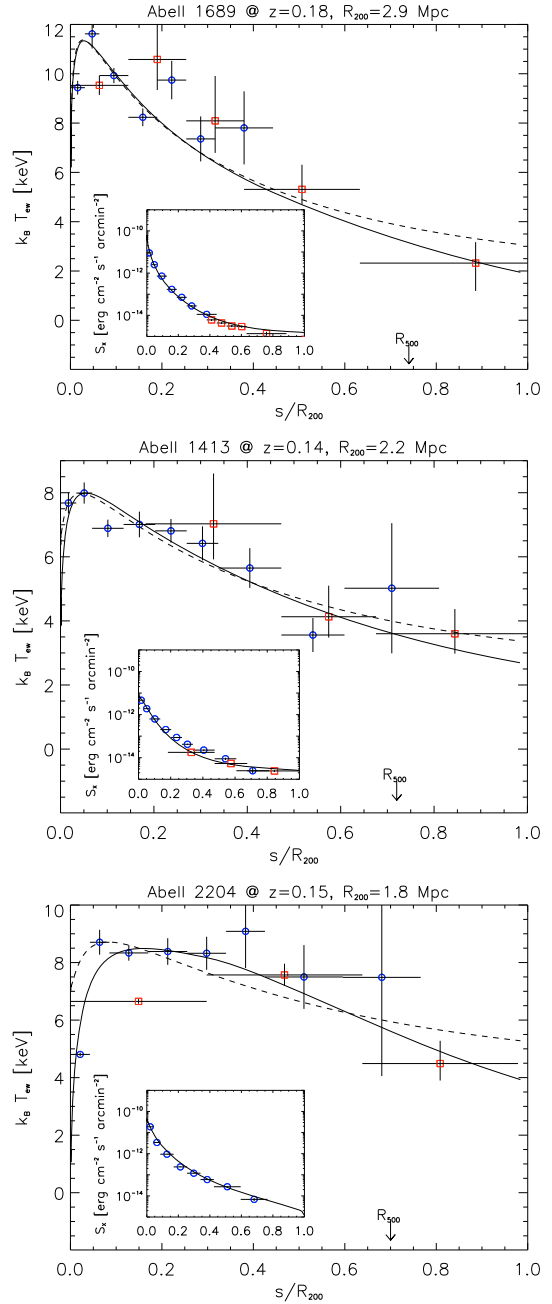
Sect. 2). We trace this behavior back to two concurring sources: (i) the cosmological slowdown in the growth of outskirts developing at late cosmic times; (ii) shallow perturbation wings scantily feeding the outskirts growth in average or poor environs.

Under these conditions, we show that on average the entropy profiles flatten out or even decline into the cluster outskirts as represented by Eq. (4). Then our supermodel yields *steep* outer profiles of projected temperatures (and flatter densities), in close agreement with current data out to  $R$ .

We also expect the cosmological decrease of the accretion rates to be locally offset in rich environs biased high, or in cluster sectors adjacent to filamentary large-scale structures; more standard runs of temperature are predicted there. These loom out in a sector of A1689 as observed by [Kawaharada et al. \(2010\)](#).

Compared with numerical simulations like those by [Nagai et al. \(2007\)](#) and [Roncarelli et al. \(2006\)](#), our temperature declines and entropy dearths are comparable for clusters like A2204 and A1413; but they are quite sharper for low  $z$  clusters like A1795 and PKS0745-191, well beyond possible contaminations due to *Suzaku* PSF smearing (see [Reiprich et al. 2009](#)). Low-noise, high-resolution simulations addressing the issue of entropy production related to the richness of the surrounding environment or adjoining filamentary structures will be most helpful.

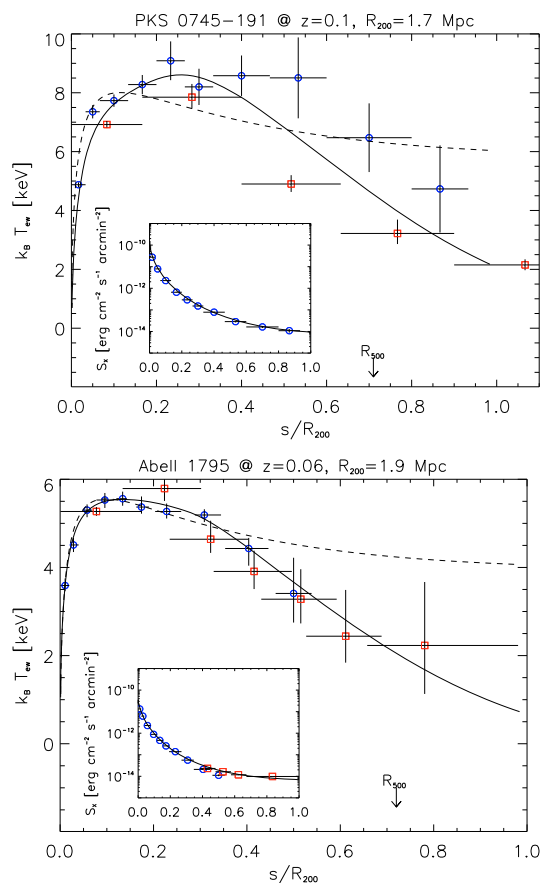
We have checked that with the SM the gas mass grows monotonically outwards, and actually faster than the DM's does, so



**Fig. 2.** Profiles of projected X-ray temperature (brightness in the insets) for the CC clusters A1689 (top), A1413 (middle) and A2204 (bottom). Data are from [Snowden et al. \(2008\)](#) with *XMM-Newton* (blue circles) and from [Kawaharada et al. \(2010\)](#), [Hoshino et al. \(2010\)](#), and [Reiprich et al. \(2009\)](#) with *Suzaku* (red squares). Our best-fits with the SM after Eq. (4) are shown by the solid lines, while dashed lines refer to the fits based on Eq. (1).

that it produces an increasing baryonic fraction by a factor 10 from  $R/20$  to  $R$  (cf. [Zhang et al. 2010](#)). So sharply declining  $T(r)$  and increasing masses are actually consistent with thermal HE. Note that as  $\dot{M}$  decreases to the point that the infall velocities decrease to transonic values, the shocks weaken, thermalization becomes inefficient (see CLFF09), the entropy production is terminated, and thermal pressure alone cannot support HE any longer.

An overall equilibrium in the outskirts can be helped by bulk or turbulent, merger-induced motions; these contribute up to 10–20% of the total support in relaxed clusters, as gauged



**Fig. 3.** Same as Fig. 2 for the low- $z$  clusters PKS0745-191 (*top*) and A1795 (*bottom*). Data are from Snowden et al. (2008) with *XMM-Newton* (blue circles) and from George et al. (2009) and Bautz et al. (2009) with *Suzaku* (red squares).

in terms of pressure and X-ray masses both observationally (Mahdavi et al. 2008; Zhang et al. 2010, adding to the references in Sect. 3) and numerically (Nagai et al. 2007; Piffaretti & Valdarnini 2008; Bode et al. 2009; Lau et al. 2010; Meneghetti et al. 2010). A similar accuracy is intended for our predictions based on thermal HE, while a finer approximation with non-thermal contributions included in the SM will be provided elsewhere.

On the other hand, even extended HE constitutes just a useful approximation, due to fail even in relaxed clusters: locally, when minor lumps of cold gas fall into the cluster from an adjacent filament, along with smooth accretion; globally, beyond  $R$  where the DM equilibrium is in jeopardy.

In summary, from the SM with thermal HE and optimal energy conversion in strong shocks, we find temperature profiles declining sharply outwards. These stem from progressive *exhaustion* of mass inflow, especially for clusters in average or poor environments at low  $z_{\text{obs}}$ ; this predicted trend is consistent with the current evidence for the handful of clusters in Figs. 2 and 3.

Our punchline is that cosmology – besides affecting the cluster statistics like mass or  $r_{200}$  temperature distributions (see

Vikhlinin et al. 2009) – concurs with the perturbation shape to set the outer structure of *individual* clusters and their development.

All these rich phenomena expected at the interface between the ICP and the intergalactic medium call for extensive probing even at  $z \gtrsim 0.2$  with the next generation of X-ray telescopes planned to detect at high resolution low-surface brightness plasma (see Giacconi et al. 2009), like *WFXT* (see <http://wfx.pha.jhu.edu/>) and eventually *IXO* (see <http://ixo.gsfc.nasa.gov/>).

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## References

- Bautz, M. W., Miller, E. D., Sanders, J. S., et al. 2009, *PASJ*, 61, 1117  
 Bode, P., Ostriker, J. P., & Vikhlinin, A. 2009, *ApJ*, 700, 989  
 Broadhurst, T., Umetsu, K., Medezinski, E., Oguri, M., & Rephaeli, Y. 2008, *ApJ*, 685, L9  
 Cavagnolo, K., Donahue, M., Voit, G. M., & Sun, M. 2009, *ApJS*, 182, 12  
 Caldwell, R. R., Kamionkowski, M., & Weinberg, N. N. 2003, *PhRvL*, 91g1301  
 Cavaliere, A., Lapi, A., & Fusco-Femiano, R. 2009, *ApJ*, 698, 580 (CLFF09)  
 Cavaliere, A., Lapi, A., & Menci, N. 2002, *ApJ*, 581, L1  
 Diemand, J., Kuhlen, M., & Madau, P. 2007, *ApJ*, 667, 859  
 Dunkley, J., Komatsu, E., Nolta, M. R., et al. 2009, *ApJS*, 180, 306  
 Fusco-Femiano, R., Cavaliere, A., & Lapi, A. 2009, *ApJ*, 705, 1019  
 George, M. R., Fabian, A. C., Sanders, J. S., Young, A. J., & Russell, H. R. 2009, *MNRAS*, 395, 657  
 Giacconi, R., et al. 2009, in *Astro2010: The Astronomy and Astrophysics Decadal Survey*, Science White Papers, No. 90  
 Hoshino, A., Patrick Henry, J., Sato, K., et al. 2010, *PASJ*, 62, 371  
 Kawaharada, M., Okabe, N., Umetsu, K., et al. 2010, *ApJ*, 714, 423  
 Lapi, A., & Cavaliere, A. 2009a, *ApJ*, 692, 174  
 Lapi, A., & Cavaliere, A. 2009b, *ApJ*, 695, L125  
 Lapi, A., Cavaliere, A., & Menci, N. 2005, *ApJ*, 619, 60  
 Lau, E. T., et al. 2010, *ApJ*, submitted [arXiv:1003.2270]  
 Lu, Y., Mo, H. J., Katz, N., & Weinberg, M. D. 2006, *MNRAS*, 368, 1931  
 Mahdavi, A., Hoekstra, H., Babul, A., & Henry, J. P. 2008, *MNRAS*, 384, 1567  
 Markevitch, M., & Vikhlinin, A. 2007, *Phys. Rep.*, 443, 1  
 McCarthy, I. G., Bower, R. G., Balogh, M. L., et al. 2007, *MNRAS*, 376, 497  
 McNamara, B. R., & Nulsen, P. E. J. 2007, *ARA&A*, 45, 117  
 Meneghetti, M., et al. 2010, *A&A*, 514, A93  
 Molendi, S., & Pizzolato, F. 2001, *ApJ*, 560, 194  
 Nagai, D., Kravtsov, A. V., & Vikhlinin, A. 2007, *ApJ*, 668, 1  
 Navarro, J. F., et al. 2009, *MNRAS*, 402, 21  
 Piffaretti, R., & Valdarnini, R. 2008, *A&A*, 491, 71  
 Pratt, G. W., Arnaud, M., Piffaretti, R., et al. 2010, *A&A*, 511, 85  
 Reiprich, T. H., Hudson, D. S., Zhang, Y.-Y., et al. 2009, *A&A*, 501, 899  
 Riemer-Sørensen, S., Paraficz, D., Ferreira, D. D. M., et al. 2009, *ApJ*, 693, 1570  
 Roncarelli, M., Ettori, S., Dolag, K., et al. 2006, *MNRAS*, 373, 1339  
 Rossetti, M., & Molendi, S. 2010, *A&A*, 510, 83  
 Sarazin, C. L. 1988, *X-ray Emission from Clusters of Galaxies* (Cambridge: Cambridge Univ. Press)  
 Snowden, S. L., Mushotzky, R. F., Kuntz, K. D., & Davis, D. S., et al. 2008, *A&A*, 478, 615  
 Tozzi, P., & Norman, C. 2001, *ApJ*, 546, 63  
 Valageas, P., & Silk, J. 1999, *A&A*, 350, 725  
 Vass, I., et al. 2008, *MNRAS*, 395, 1225  
 Vikhlinin, A., Kravtsov, A. V., Burenin, R. A., et al. 2009, *ApJ*, 692, 1060  
 Voit, G. M. 2005, *Rev. Mod. Phys.*, 77, 207  
 Voit, G. M., Balogh, M. L., Bower, R. G., Lacey, C. G., & Bryan, G. L. 2003, *ApJ*, 593, 272  
 Weinberg, S. 2008, *Cosmology* (Oxford: Oxford Univ. Press)  
 Wu, K. K. S., Fabian, A. C., & Nulsen, P. E. J. 2000, *MNRAS*, 318, 889  
 Zhang, Y.-Y., Okabe, N., Finoguenov, A., et al. 2010, *ApJ*, 711, 1033  
 Zhao, D. H., Mo, H. J., Jing, Y. P., & Börner, G. 2003, *MNRAS*, 339, 12