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# Determination of the Parameters of Plasticity Models of Geological Media on the Base of Computer Simulation

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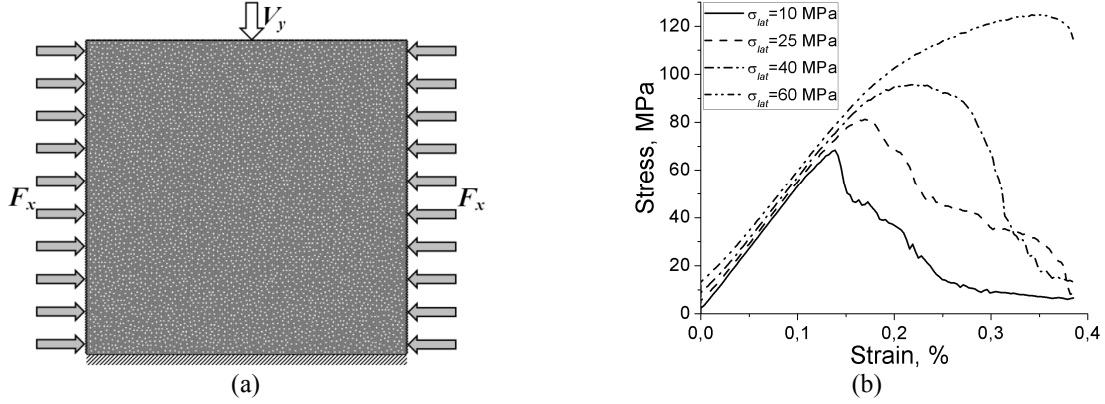
**Abstract.** The paper is devoted to theoretical investigation of peculiarities of inelastic deformation of porous brittle materials in constrained conditions. The study was based on computer-aided simulation by movable cellular automaton method. Analysis of the simulation results allowed to estimate values of some rheological parameters of porous brittle materials and to determine the limits of applicability of Mises–Schleicher equation for description of the inelastic deformation of such materials under axial compression in constrained conditions.

## INTRODUCTION

It is well known that rock massifs are characterized by complex and non-uniform stress state, an important feature of which is the presence of the constrained conditions. A substantial portion of the brittle geological materials is characterized by a developed pore structure. The presence of the pore structure has a significant, and in many cases a determining influence on the meso and macroscopic features of behavior and mechanical properties of these materials [1]. In particular, volume content of pores (porosity) and character of their spatial distribution are important factors which determine the possibility of brittle-ductile transition of mechanical response of material under constrained loading conditions. So at a certain degree of constraint brittle porous materials can exhibit inelastic response [1–3]. Plastic behavior of geological materials is usually described by the flow laws taking into account the dependence of the inelastic response on pressure [3–5]. Experimental determination of mechanical and rheological properties of porous brittle geological materials is rather difficult problem. Therefore, a promising way to solve it is to use computer-aided simulation. In the paper peculiarities of mechanical response of the porous model samples of sandstone under axial compression in the conditions of action of constant lateral pressure were investigated. Movable cellular automaton method [6–8] was used as a method of numerical simulation. This method is the representative of the class of discrete element methods. The aim of the investigation was determination of some rheological parameters of the porous brittle material under constrained loading conditions (coefficient of internal friction, coefficient of dilatancy, etc.).

## MODEL DESCRIPTION AND DISCUSSION OF RESULTS

Simulation of axial compression of constrained 2D-samples was performed for investigation of peculiarities of inelastic response of elastic-brittle geological materials. A two-step loading scheme was used for this purposes (Fig. 1a). In the first step the sample was subjected to lateral compression with a constant force  $F_x$  (position of the top and bottom surfaces of the sample was fixed). After specifying of the initial stress state axial compression of the sample with a low constant speed  $V_y$  was simulated.



**FIGURE 1.** Structure and loading scheme (a) and loading diagrams of the model sample for different values of  $\sigma_{lat}$  (b)

Elastic and strength characteristics of the material skeleton were equal to corresponding characteristics of sandstone [9]. Volume content of pores was equal to 10%. Inelastic behavior of brittle porous materials in constrained conditions is greatly associated with processes of generation and accumulation of damages in the volume of the deformed sample. In computer-aided simulation by discrete element methods, these processes are determined by the chosen criterion of breaking inter-element links (fracture criterion). In the framework of this paper to criteria Mises criterion (1) and criterion of Drucker–Prager (2) were used:

$$\sigma_{int} = \sigma_{crit}, \quad (1)$$

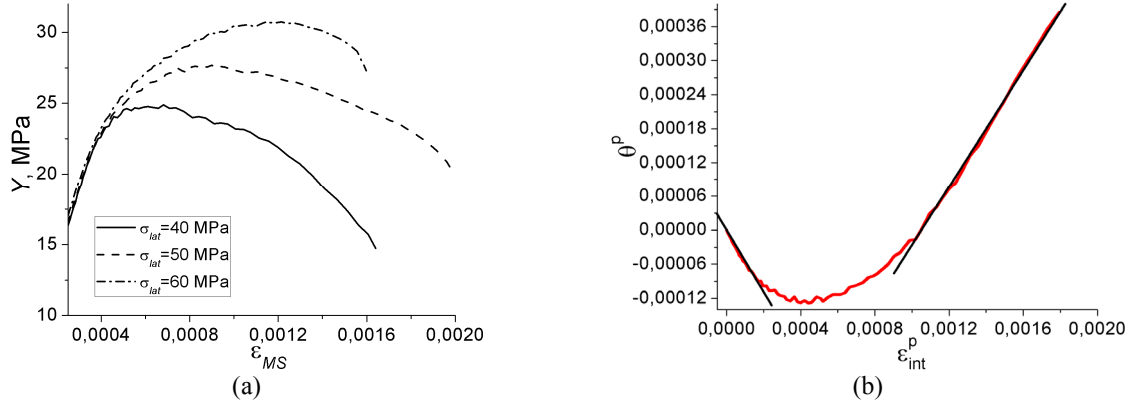
$$\sigma_{int} 0.5(a + 1) + \sigma_{mean} 1.5(a - 1) = \sigma_{crit}. \quad (2)$$

Here  $\sigma_{crit}$ —corresponding threshold value of strength for pair of cellular automata,  $\sigma_{int}$  and  $\sigma_{mean}$ —local values of von Mises stress and mean stress in the pair,  $a = \sigma_c / \sigma_t$ —ratio of compressive strength of the pair ( $\sigma_c$ ) to tensile strength ( $\sigma_t$ ). Fig. 1b shows examples of diagrams of axial compression of the porous sample for various values of the lateral pressure ( $\sigma_{lat}$ ). It is seen that from a certain value of lateral pressure (more than 25 MPa) constrained samples exhibit inelastic response, and increase in  $\sigma_{lat}$  leads to increase of yield stress and ultimate strain.

Equation of Mises–Schleicher is frequently used as a yield surface in plasticity models of geological media [4, 5]:

$$\sigma_{MS} = \beta \Sigma_{mean} + \frac{\Sigma_{int}}{\sqrt{3}} = Y. \quad (3)$$

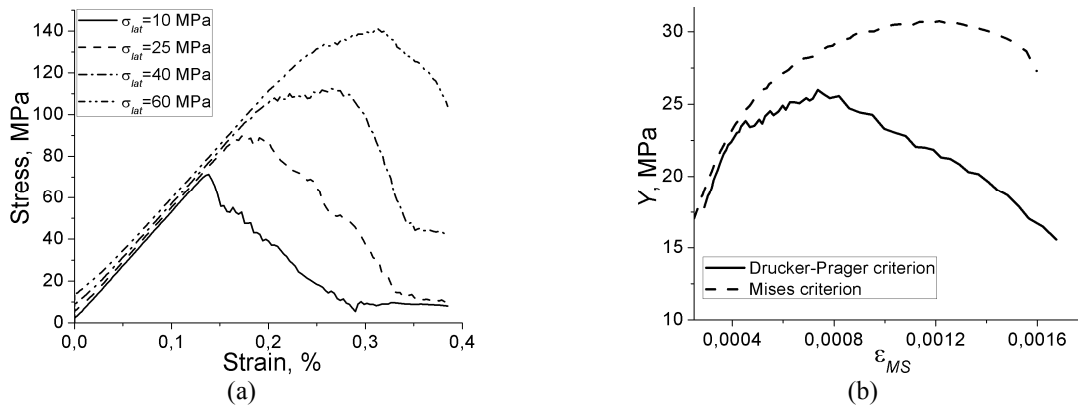
Here  $\beta$ —coefficient of internal friction,  $Y$ —stress at which yield state occurs in conditions of pure shear,  $\Sigma_{mean}$  and  $\Sigma_{int}$ —mean stress and von Mises stress (they are determined by loading and boundary conditions and current strain of the sample). So, to determine the yield surface it is necessary to define two unknowns ( $\beta$  and  $Y$ ) in equation (3). In first approximation, this can be done based on two assumptions. Firstly, the value of  $\beta$  does not depend on strain value ( $\beta = \text{const}$ ). And, secondly, value of  $Y$  at the beginning of stage of plastic deformation is characteristic of the material and does not depend on conditions of constraint ( $\sigma_{MS}(\sigma_{lat}) = \text{const}$ ). It means that knowing the value of yield stress of the material at different values of lateral pressure ( $\sigma_{lat}$ ), we can determine such value of  $\beta$  at which the dependence  $\sigma_{MS}(\sigma_{lat}) \rightarrow \text{const}$ . Analysis of the results of computer simulation showed that for the considered model system this value of  $\beta$  exists (for the considered porous material  $\beta \approx 0.514$ ). This confirms the adequacy of using of Mises–Schleicher equation for the determination of the yield surface for brittle porous materials. Substituting the obtained value of  $\beta$  in (3), plastic flow rule  $Y(\epsilon_{MS})$  could be determined law, where  $\epsilon_{MS} = \epsilon_{int} / \sqrt{3} + K\beta\epsilon_{mean} / 3G$  ( $K$  and  $G$ —bulk and shear moduli, respectively,  $\epsilon_{int}$  and  $\epsilon_{mean}$  are the total von Mises strain and mean strain, respectively). Figure 3a shows the obtained dependences  $Y(\epsilon_{MS})$  for different values of  $\sigma_{lat}$ . It can be seen that the  $Y(\epsilon_{MS})$  curves are characterized by two main stages. On the first stage (up to a certain value of  $\epsilon_{MS} \approx 0.00045$ ) they have almost linear character and close to each other.



**FIGURE 2.**  $Y(\varepsilon_{MS})$  curves for different values of lateral pressure  $\sigma_{lat}$  (a) and dependence  $\theta^p(\varepsilon_{int}^p)$  at  $\sigma_{lat} = 60$  MPa (b). Black lines in figure (b) denote linear parts of downward and upward stages of  $\theta^p(\varepsilon_{int}^p)$  curve

This indicates that the coefficient of internal friction in this area is a constant, and that the proposed form of the “single hardening curve” as dependence of  $Y(\varepsilon_{MS})$  is correct. At  $\varepsilon_{MS} > 0.00045$  curves  $Y(\varepsilon_{MS})$  become essentially nonlinear, and begin to diverge. This behavior of the  $Y(\varepsilon_{MS})$  curves is due to the fact that the beginning of the inelastic deformation stage is associated with formation of individual damages and their merging into small cracks (this occurs at the first stage of  $Y(\varepsilon_{MS})$  curves). At  $\varepsilon_{MS} \approx 0.00045$  this small cracks and damages are united in macro-cracks and the material loses its integrity. The inelastic behavior of the sample at  $\varepsilon_{MS} > 0.00045$  is associated with the localization of deformation on macro-cracks in the form of relative sliding of the formed blocks. Thus, the stage of divergence of  $Y(\varepsilon_{MS})$  curves corresponds to deformation of fragmented (block) system instead of consolidated material. The results indicate the need for caution in interpretation of the experimental loading curves obtained in constrained conditions. In particular, the Mises–Schleicher equation (3) can be correctly used to describe the initial stages of inelastic deformation of brittle material (until its fragmentation). For fragmented material dependence of plastic behavior on pressure has nonlinear character and must be described on the base of more complex flow laws.

The simulation results also allow to study the dynamics of change of volume of model fragment of the medium and estimate the value of the dilatancy coefficient  $\Lambda$ . This can be done on the basis of analysis of dependencies  $\theta^p(\varepsilon_{int}^p)$ , where  $\varepsilon_{int}^p$  and  $\theta^p$  are von Mises plastic strain and volume of plastic strain, respectively. Figure 3b shows an example of this dependence at  $\sigma_{lat} = 60$  MPa. This curve has pronounced nonlinear and nonmonotonic character. Thus, at the initial stage of inelastic deformation, until the formation of macro-cracks ( $\varepsilon_{int}^p < 0.00045$ ) a compaction of the material takes place. When  $\varepsilon_{int}^p > 0.00045$  increase in the volume of the simulated fragment takes place. This character of dependence  $\theta^p(\varepsilon_{int}^p)$  is associated with fracture features of the constrained material. At the initial stages of plastic deformation there is an accumulation of damages by means of breaking of the “crosspieces” between the adjacent pores. This process is accompanied by local subsidence of the surrounding areas of the material into these pores (small integral compaction of the material takes place). Further deformation of the material results in unification of previously formed damages into small cracks, on which starts a local slippage. This process is manifested in the form of the transition region from the drop-down stage to a growing part of the  $\theta^p(\varepsilon_{int}^p)$  curve. Further deformation leads to formation of macro-crack and beginning of slippage of material fragments along the surfaces of this crack. This process is accompanied by a substantial increase in volume of the medium. Note that the downward and upward parts of the  $\theta^p(\varepsilon_{int}^p)$  curve have almost linear character. This allows to estimate a value of the dilatancy coefficient  $\Lambda$  by defining the inclination angle ( $\alpha$ ) of different parts of the  $\theta^p(\varepsilon_{int}^p)$  curve ( $\Lambda = tq(\alpha)/2$ ). For the considered sample  $\Lambda_{com} \approx -0.27$  at the compaction stage and  $\Lambda_{dil} \approx 0.25$  at dilatation one. It should be noted that the inclination angles of different stages of  $\theta^p(\varepsilon_{int}^p)$  curves are practically independent of the value of lateral pressure. So the dilatancy coefficients  $\Lambda_{com}$  and  $\Lambda_{dil}$  are material parameters.



**FIGURE 3.** Loading diagrams of the model samples for different values of  $\sigma_{lat}$  obtained using Drucker–Prager fracture criterion (a) and example of comparing of  $Y(\epsilon_{MS})$  curves for different fracture criteria at  $\sigma_{lat} = 60$  MPa (b)

The above results were obtained using Mises fracture criterion (1), which assumes absence of dependence shear strength of the material on pressure. Generally for brittle materials such dependence exists. Therefore, for brittle materials it is common to use more complex multi-parameter fracture criteria, for example, two-parameter criterion of Drucker–Prager (2). Analysis of the obtained results showed that loading diagrams for constrained porous samples are similar when using different fracture criteria (compare curves on Figs. 3a and 1b). The main difference consists in higher values of yield stress for Drucker–Prager fracture criterion (2) in comparison with curves obtained using Mises criterion (1). The consequence is an increase in the coefficient of internal friction  $\beta_{DP}$  ( $\beta_{DP} = 0.67$  vs  $\beta_M = 0.514$ ). Compare of  $Y(\epsilon_{MS})$  curves for different fracture criterion (Fig. 3b) shows that both curves have a similar character, and the first (almost linear) stage of the both curves have substantially the same slope. This indicates that until the fragmentation of the medium in constrained conditions plastic deformation of the material does not depend on the selected fracture criterion.

## CONCLUSION

On the base of computer-aided simulation the features of inelastic behavior of porous brittle materials in constrained conditions were investigated. It is shown that the Mises-Schleicher equation can be applied correctly to describe the initial stages of inelastic deformation of brittle materials. The value of the internal friction coefficient of the porous material is retained almost constant up to the fragmentation of the sample. At the same time the value of the dilatancy coefficient varies nonlinearly and nonmonotonically. At the initial stage of inelastic deformation it is negative, due to compaction of the porous material. After the formation of macro-cracks dilatancy coefficient becomes positive and its value is close to experimentally determined values for the considered material.

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