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Supersymmetry Breaking, Gauge Mediation and Holography

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Acknowledgements

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I have had the good fortune to spend my years in SISSA in a positive environment, and with many of my mates I shared not only the training but also rewarding human relationships, Marco and Alberto in particular. I am indebted to all my friends which made my life in Trieste so enjoyable, Stefano, Martino, Roberto, Gemma, Federico to mention a few. Without them, it would have been difficult to persevere in my activities.

Finally, my gratitude goes to the members of my family, for their invaluable affection and support.

Foreword

List of PhD Publications

This thesis summarizes part of my research during the PhD, that is contained in the following three publications

- M. Bertolini, L. Di Pietro, F. Porri, "Dynamical completions of generalized O'Raifeartaigh models," *JHEP*, vol. 1201, p. 158, 2012, arXiv:1111.2307.

- R. Argurio, M. Bertolini, L. Di Pietro, F. Porri, D. Redigolo, "Holographic Correlators for General Gauge Mediation," *JHEP*, vol. 1208, p. 086, 2012, arXiv:1205.4709.

- R. Argurio, M. Bertolini, L. Di Pietro, F. Porri, D. Redigolo, "Exploring Holographic General Gauge Mediation," *JHEP*, vol. 1210, p. 179, 2012, arXiv:1208.3615.

My activity has also focused on other topics in supersymmetric quantum field theory and holography, resulting in the following two publications

- L. Di Pietro, S. Giacomelli, "Confining vacua in SQCD, the Konishi anomaly and the Dijkgraaf-Vafa superpotential", *JHEP*, vol. 1202, p. 087, 2012, arXiv:1108.6049.

- M. Bertolini, L. Di Pietro, F. Porri, "Holographic R-symmetric Flows and the τ_U conjecture", to appear in *JHEP*, pre-print arXiv:1304.1481.

Contents

Chapter 1

Introduction and outline

Supersymmetry is one of the best motivated options for physics beyond the Standard Model. The possibility that it is realized in a natural way and is responsible for the stabilization of the electro-weak scale, thus solving the Hierarchy Problem, is still open, even if this requires to go beyond the simplest realizations. Additional well-known motivations include gauge-coupling unification and the existence of Dark Matter candidates. Supersymmetry is also a prediction of top-down constructions coming from string theory, and is a compelling idea from a theoretical viewpoint in that it realizes the maximal possible space-time symmetry in a field theory.

In realistic models, supersymmetry must be broken spontaneously, and this is most easily obtained by relegating the dynamics responsible for supersymmetry breaking in a so-called hidden sector. The interactions with the supersymmetric extension of the Standard Model give rise to an effective theory with the desired supersymmetry breaking soft terms.

In particular, gravitational interactions between the hidden and the visible sector are unavoidable, because of their universality, and it is conceivable that they constitute the main effect responsible for the communication of supersymmetry breaking. In the resulting models, going under the name of Gravity Mediation models, supersymmetry must be broken at very high energies, due to the weakness of the interactions. The soft terms are generated by higher-dimensional operators suppressed by the Planck scale. Despite its conceptual elegance, this setting suffers from both a theoretical and a phenomenological drawback. The theoretical one is that the precise coefficients of the higher dimensional operators depend on the underlying fundamental theory at the Planck scale, and are not under control. The related phenomenological problem is that such coefficients, in general, would give rise to flavor and CP violating processes which have severe experimental constraints, avoided only via additional ad-hoc assumptions.

Alternatively, one can consider setups in which the hidden sector communicates with the visible one also via the SM gauge interactions, and this effect dominates over gravity. This class of models, which are called Gauge Mediation models and will be the main topic of this thesis, solve both the issues we have just mentioned. On one hand, since gauge interactions are flavor blind, the resulting soft terms will not introduce additional sources of flavor violation (as for CP, one has to assume the reality of some parameters to reach the same conclusion). On the other hand, gauge interactions are treatable by standard perturbative techniques, so that, once the hidden sector is known, the precise form of the soft terms can be actually calculated in this case. Moreover, one can allow a much lower scale of supersymmetry breaking, which, at least in principle, can be nicely accommodated within models of Dynamical Supersymmetry Breaking (DSB). In this context, therefore, one foresees the possibility of constructing a model which is satisfactory from a phenomenological point of view, and in which both the dynamics of the hidden sector and of the mediation mechanism are under theoretical control. This motivates the interest in studying Gauge Mediation.

The fundamental model-building input, in any theory of this kind, is a choice for the hidden sector responsible for supersymmetry breaking. Ideally, one would like to scan the space of possible hidden sectors, so to explore the most general predictions of this setting. To make this program more precise, one can rely on a characterization of Gauge Mediation models, called General Gauge Mediation (GGM), which has the virtue of being general and conceptually economic, and which we will review in chapter 2 of the thesis. Basically, in GGM one parametrizes the information about the hidden sector in terms of few form factors appearing in the two-point functions of operators in the supermultiplet of a conserved current. Then, the scanning of hidden sectors can be rephrased in terms of a study of the possible behaviors of these form factors.

An important requirement on the hidden sector is that supersymmetry is broken dynamically, which implies that the relevant dynamics occurs at strong coupling. This fact, in principle, poses a problem of calculability, which could spoil the good theoretical control that we referred to. There are mainly two handles on strongly coupled theories which we will discuss, corresponding to different types of theories in the hidden sector, and accordingly our dissertation is organized in two parts.

The fist possibility is in fact supersymmetry itself: one can use its powerful constraints to derive the form of the low energy effective theory in the supersymmetry breaking vacuum. Often, such effective theories turn out to be weakly coupled theories of chiral superfields, motivating the general study of hidden sectors of this sort, which we will be the topic of chapter 3. This setting allows a good degree of calculability and flexibility, which indeed constitutes an additional motivation, and allows a classification of the models. An important theme will be the role played by the R-symmetry, and the related issue of generating a large enough Majorana mass of the gauginos. We will see that, in this respect, a particular sub-class of the weakly coupled models is more promising as candidate hidden sector. Interestingly, most known examples of DSB theories do not reduce to this sub-class at low energies. In chapter 4 we will explicitly construct examples of asymptotically-free gauge theories which reduces at low energies to models falling in all possible classes, thus giving a map between dynamical theories and weakly coupled models. This will allow, in particular, to get a dynamical embedding of the more promising class of weakly coupled hidden sectors.

The second approach, which will be the topic of the second part of the thesis, consists in considering hidden sectors which admit a holographic dual description. In this case, one can replace the strongly coupled theory with a weakly-curved, asymptotically-AdS $5D$ geometry, where the relevant two-point functions can be calculated by exploiting the holographic prescription, which we review in chapter 5. The details of the hidden sector, and the supersymmetry breaking dynamics, are encoded in the gravitational background, which is a supersymmetry breaking solution in a certain supergravity theory. The introduction given in chapter 5 also includes a review of the relevant supergravity theories in $5D$, and an overview on the topic of holographic renormalization, which is a fundamental technical ingredient to perform the calculation of the form factors. In particular, it will be shown how to apply the holographic renormalization procedure to the whole supersymmetric multiplet of a massless vector, which is the field one has to consider in order to calculate the GGM two-point functions.

In the remaining part of the thesis we will construct concrete models of holographic hidden sectors. To achieve this goal, one can either follow a top-down approach, deriving the theory from consistent truncations of type IIB supergravity in $10D$ compactified on some internal manifold, or a complementary bottom-up approach, by constructing by hand the simplest possible background which captures the desired dynamics. The first method is pursued in chapter 6, where we consider a $5D$ supergravity theory with the minimal field content necessary to describe, holographically, a strongly coupled hidden sector, and such that, at least for some choices of parameters, it can in fact be derived as a truncation from $10D$. As we will see, this theory contains supersymmetry breaking solutions, both with and without an R-symmetry. Somewhat surprisingly, it will turn out that gauginos can get a mass even when the background does not break R-symmetry, thanks to massless fermionic excitations in the holographic hidden sector which can mix with gauginos, and provide a Dirac-like mass. Even if the holographic correspondence is on a firmer ground in such top-down constructions, they are often too constrained to allow a significant scan of the parameter space, and few concrete solutions are known which can be used for our scopes. Therefore, in chapter 7, we will turn to the alternative, bottom-up approach, and consider a hard wall model. This is defined by a simple AdS_5 throat truncated at some point in the extra dimension, where the geometry has an additional boundary, the so-called IR wall. In this case the background does not solve the supergravity equations of motion, and therefore is not dynamical. The properties of such hidden sectors are determined by the parameters entering the boundary conditions for the fields at the IR wall. Varying these parameters, one can explore which portion of the GGM parameter space can be covered. The outcome of this analysis is that one can indeed cover the whole GGM parameter space at strong coupling, using holography. However, we will see that some regions of the parameter space are actually favored, like higgsed mediation scenarios, while some others require a certain amount of tuning between the parameters to be met. Another virtue of this simple model is that many results can be derived in an analytic fashion, contrarily to the top-down examples mentioned before, which require to resort to numerics.

We conclude in chapter 8 with an overview of the possible future directions.

Chapter 2

A general formulation of Gauge Mediation models

In the Gauge Mediation scenario, the hidden sector responsible for the breaking of supersymmetry is coupled only via gauge interactions to the visible sector, thus solving the Flavor Problem and permitting a perturbative analysis of the mediation mechanism. This idea can be made concrete in a variety of models, which differ by the choice of the hidden sector and can give rise to distinct phenomenological outcomes. In this chapter we will start by discussing some realizations, then we will show how to formulate in a general and compact way all possible models of Gauge Mediation. The result will be instrumental to discuss the model-independent consequences of Gauge Mediation.

2.1 An effective approach: Minimal Gauge Mediation

As a first simple and calculable example, one can consider a toy model dubbed Minimal Gauge Mediation (MGM) in the literature, in which the supersymmetry breaking is effectively encoded by the existence of a singlet chiral field with a non-zero F-component[1]

$$\langle X \rangle = M + \theta^2 F, \tag{2.1}$$

which is generated by some yet unspecified dynamics of the hidden sector [2–5]. Since X is taken to be a singlet, we need additional *messenger fields* charged under the Standard Model gauge group. These fields get a non-supersymmetric mass splitting due to direct interactions with the spurion X, which is then communicated radiatively to the visible sector. Denoting the messenger as the couple $(\phi, \tilde{\phi})$ of chiral superfields in a vector-like representation $\rho \oplus \bar{\rho}$ of the gauge group[2], the interaction

[1] Here and in what follows we will use the conventions of [1] for spinors and supersymmetry.

[2] In order to preserve gauge coupling unification, ρ must fill a complete representation of the unified gauge group.

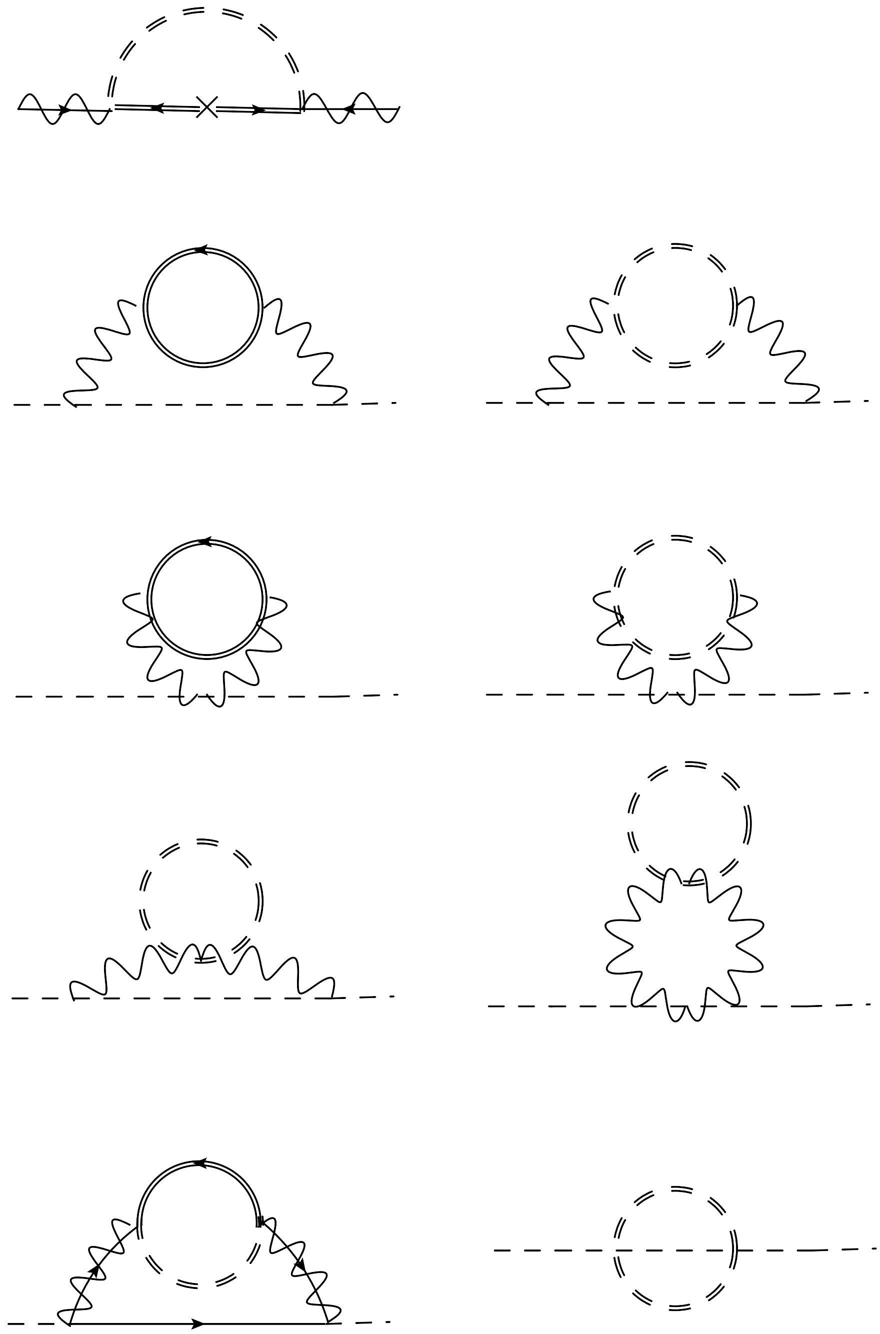

Figure 2.1: Diagrams generating the soft masses in a minimal gauge mediation model. Double lines represent messenger fields. Dashed lines are associated to scalars, while solid lines are fermionic. The gaugino mass is generated at 1 loop by the diagram at the top-left. The scalar masses arise at two loops by the diagrams displayed below.

is

$$\mathcal{W} = X\phi\tilde{\phi}, \tag{2.2}$$

so that the fermions get a supersymmetric mass M while the scalars are splitted by the F-term, with mass eigenvalues $M^2 \pm F$, where we are assuming M and F to be real (which is always the case up to a phase shift of the fields). Notice that we need $F < M^2$ to avoid tachyons, and in the following we will consider for simplicity the case $F \ll M^2$. One can easily compute the leading radiative contributions to soft masses for the visible gauginos and scalars (sleptons and squarks), which arise respectively at one loop and at two loops, via the diagrams in Fig 2.1. The calculation of the whole set of diagrams can be significantly simplified in the limit $F/M^2 \ll 1$ by the technique of analytic continuation into superspace [6, 7], which determines all the two-loops diagrams in terms of the 1-loop result for the wave-function renormalization of the quark/lepton superfields. At the leading order in the ratio F/M^2 the results for the gaugino soft masses $M_{\tilde{g},r}$ and for the scalar soft masses $m^2_{\tilde{f}}$ are

$$M_{\tilde{g},r} = \frac{g_r^2}{16\pi^2}\Lambda_G\,, \quad m^2_{\tilde{f}} = 2\sum_r C^r_{\tilde{f}}\left(\frac{g_r^2}{16\pi^2}\right)^2\Lambda_S^2\,, \quad \Lambda_G = \sqrt{N}\Lambda_S = N\frac{F}{M}, \tag{2.3}$$

where N is twice the Dynkin index of the representation ρ, the index r runs over $U(1)_Y$, $SU(2)$ and $SU(3)$, and $C^r_{\tilde{f}}$ is the quadratic Casimir of the representation which $\tilde{f}$ belongs to.

Notice that for $N \sim O(1)$ the resulting spectrum have soft masses of the same order for the gauginos and the squarks/sleptons. Therefore, in natural models MGM would predict all those masses to be close to the electro-weak scale.

2.2 Overview of Gauge Mediation models

In order to build a satisfactory model, one must go beyond the previous toy-example and provide an explanation of the supersymmetry-breaking expectation value in terms of a fully-fledged hidden sector. In addition, the scale of supersymmetry breaking is usually assumed to be dynamically generated [8], so that the hierarchy between the electro-weak scale and the UV scale (for instance M_{pl}, M_{GUT} or whatever is the high-energy cutoff of the effective theory) is not only stabilized, but also explained as an exponential suppression

$$M_{weak} \sim M_{SB} = e^{-\frac{8\pi^2}{g^2}} M_{UV}. \tag{2.4}$$

It turns out that the embedding of the minimal example in a Dynamical Supersymmetry Breaking (DSB) model runs into a number of difficulties, such as possible large tadpoles for the singlet X destabilizing the vacuum. Those problems can be overcome [9–11], but the solution necessitate a rather baroque structure for the hidden sector.

This observation led several authors to the exploration of dynamical models which avoid the somewhat artificial splitting of the hidden sector in messenger sector/supersymmetry-breaking sector. In these class of theories, going under the name of *Direct Gauge Mediation* models, the hidden sector is a gauge theory whose strong-coupling dynamics is responsible for supersymmetry breaking, and the gauge group of the Standard Model is embedded in the flavor symmetry group of the gauge theory [12, 13]. The role of the messengers is played by (part of) the same fields involved in the supersymmetry breaking dynamics. A typical problem of such models (which is indeed one of the reasons why a separate messenger sector was introduced in the first place) is that the rank of the hidden-sector gauge group may be required to be large, resulting in a large multiplicity for the flavor fields which are in turn charged under the SM gauge group, and possibly leading to Landau poles at unacceptably low energies. Viable realizations include some field which takes a large supersymmetric VEV, which results in a comparably large mass for the charged fields, in such a way that the drastic changes to the β functions kick in only at high energies.

A third possibility which lies somewhat in between the Minimal and Direct case is the approach of *Semi-Direct Gauge Mediation* [14]. In this setting one re-introduces messenger fields which are not necessary for the supersymmetry breaking dynamics as in MGM, but in contrast to MGM the only allowed interactions between the messengers and the source of supersymmetry breaking are (hidden sector) gauge interactions. Assigning a mass m_m to the messengers via a superpotential term, and calling Λ the dynamical scale of the (hidden sector) gauge theory, one can interpolate in a controllable way between the two regimes

- $m_m \gg \Lambda$: in this case the messengers are weakly coupled to the rest of the hidden sector, and they get the non-supersymmetric mass splitting in a calculable way which is then radiatively communicated to the visible sector, in a MGM-like fashion,
- $m_m \ll \Lambda$: in this case the messengers take part in the strong-coupling dynamics of the gauge theory, and the hidden sector can only be studied as a whole like in direct gauge mediation.

Even the more sophisticated constructions of Direct/Semi-Direct Gauge Mediation are not free of model-building issues. An interesting one, in light of what will follow, is that in many examples the soft gaugino masses turn out to be suppressed with respect to the squarks/sleptons ones [7,14–17]. This gap in the spectrum is in tension with the requirement that supersymmetry solves the naturalness problem, because fixing the squark masses to be close to the electro-weak scale would result in too light gauginos (but still it can be compatible with gauge-coupling unification and the existence of a Dark Matter candidate [18–20]). The problem is ultimately related to the fact that gaugino masses (of Majorana type) are protected by an R-symmetry. We will return on the problem of the suppression of gaugino masses and the related issue of R-symmetry breaking in the next chapter.

2.3 General Gauge Mediation

In this section, following [21] (see also subsequent work [22–26]), we focus on the model-independent property of Gauge Mediation, namely the existence of a supersymmetry breaking theory which is coupled to the visible sector via the SM gauge group. This will led to a formulation, called General Gauge Mediation (GGM), which does not depend on the particular theory which is used as hidden sector. The idea of GGM is to consider the limit in which the gauge couplings are turned off, and extract all the relevant data from the decoupled sector which breaks supersymmetry.

In this limit, the SM gauge group becomes a global symmetry of the hidden sector, and there always exists an associated conserved current operator j_μ, together with its supersymmetric partner operators[3], a fermionic operator j_α, where $\alpha = 1, 2$ is a Weyl index, and a real scalar operator J. They can be nicely organized in terms of a real superfield with a linear constraint, often called *linear superfield*

$$\mathcal{J}(x, \theta, \bar{\theta}) = J(x) + i\theta\, j(x) - i\bar{\theta}\,\bar{j}(x) - \theta\sigma^\mu\bar{\theta}\, j_\mu(x) + \ldots, \tag{2.5}$$

where the dots indicate higher components which can be expressed in terms of the lower ones, and the linear constraint, which implies current conservation, is

$$D^2\mathcal{J} = 0 = \bar{D}^2\mathcal{J} \;\Rightarrow \partial_\mu j^\mu = 0. \tag{2.6}$$

All the data necessary to compute the soft masses can be extracted from correlators of these operators, which can be parametrized in (Euclidean) momentum space as

$$\langle j_\mu(k) j_\nu(-k)\rangle = -(k^2\eta_{\mu\nu} - k_\mu k_\nu) C_1\left(k^2/M^2\right) \tag{2.7}$$

$$\langle j_\alpha(k) \bar{j}_{\dot{\alpha}}\rangle = -\sigma^\mu_{\alpha\dot{\alpha}} k_\mu C_{1/2}\left(k^2/M^2\right) \tag{2.8}$$

$$\langle J(k) J(-k)\rangle = C_0\left(k^2/M^2\right) \tag{2.9}$$

$$\langle j_\alpha(k) j_\beta(-k)\rangle = M B_{1/2}(k^2/M^2), \tag{2.10}$$

where C_1, $C_{1/2}$ and C_0 are three real form factors, while $B_{1/2}$ is complex, and they are all dimensionless functions of the Lorentz-invariant combination k^2/M^2,

[3] When supersymmetry is broken spontaneously the supercharge is not well-defined, but the (anti-)commutators with local operators are defined and operators are still organized in multiplets.

M indicating a typical scale. These functions parametrize our ignorance of the details of the hidden sector, and their definition is sensible regardless of whether it is weakly or strongly coupled. The parametrization is the most general one compatible with Lorentz and current conservation. Notice that in presence of a conserved R-symmetry $B_{1/2} = 0$.

One may also want to consider the following additional one and two-point functions, which are not forbidden by Lorentz-invariance

$$\langle J \rangle = \zeta \tag{2.11}$$
$$\langle j_\mu(k) J(-k) \rangle = k_\mu \, H_0(k^2/M^2). \tag{2.12}$$

If we do not allow the global symmetry to be broken spontaneously, ζ can only be non-zero if the symmetry is abelian (i.e. for $U(1)_Y$). This term, like a Fayet-Iliopoulos term, can lead to tachyonic contributions to the scalar soft masses. Therefore, it is usually assumed to vanish, and in many models this condition can be enforced by the existence of a *messenger parity* discrete symmetry acting on the current superfield as $\mathcal{J} \to -\mathcal{J}$. As for the H_0 form factor, contracting the defining equation with k^μ and using the Ward identity for the conserved current, one finds

$$k^2 H_0(k^2/M^2) = \langle \delta J \rangle = 0, \tag{2.13}$$

where in the last equality we use that the symmetry is unbroken (and δJ is also vanishing as an operator, in the abelian case). Therefore $H_0 = 0$ in general.

If supersymmetry is unbroken, the following relations hold

$$\langle \{ \bar{Q}_{\dot\alpha}, (j_\alpha(k) J(-k)) \} \rangle = 0 \tag{2.14}$$
$$\langle \{ \bar{Q}_{\dot\alpha}, (j_\alpha(k) j_\mu(-k)) \} \rangle = 0 \tag{2.15}$$
$$\langle \{ Q_\alpha, (j_\beta(k) J(-k)) \} \rangle = 0 \tag{2.16}$$

which imply the following conditions on the form factors

$$C_0 = C_{1/2} = C_1 \,, \quad B_{1/2} = 0. \tag{2.17}$$

The gauging can be described by coupling the linear superfield to the vector superfield $\mathcal{V}$ containing the SM gauge bosons and its superpartners. We consider the case of an abelian symmetry for simplicity, the generalization being straightforward. At the linearized level the interaction is

$$2 \int d^4\theta \, g \, \mathcal{J} \, \mathcal{V} = g(DJ - \lambda \, j - \bar\lambda \, \bar{j} - A^\mu \, j_\mu). \tag{2.18}$$

Integrating out the hidden sector, the interactions with the vector multiplet can

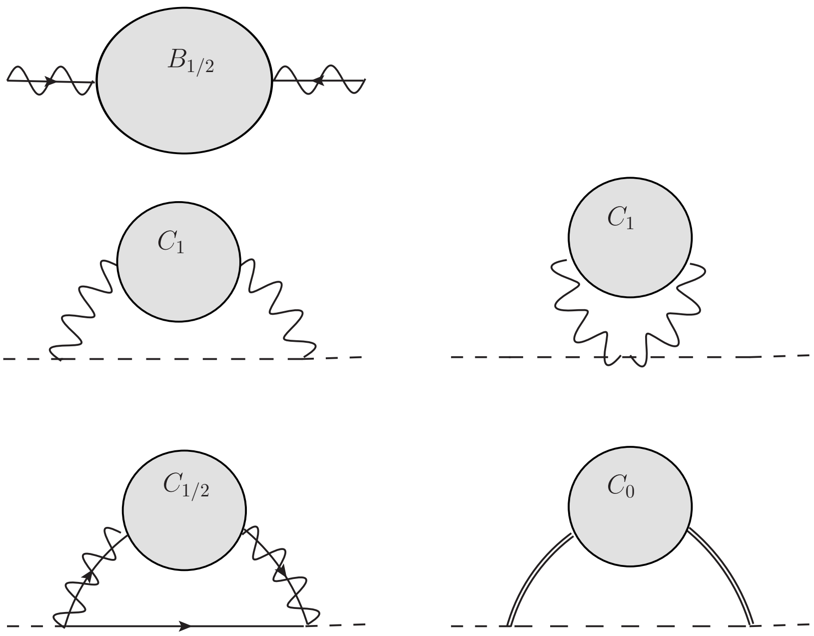

Figure 2.2: Diagrams generating the soft masses via insertion of the two-point functions of the hidden sector operators. The solid double line represents the auxiliary field D in the vector multiplet.

be encoded in an effective action, whose form at the quadratic level is completely determined in terms of the previous form factors in the following way

$$\frac{1}{g^2}\Big[-\frac{1}{4}\Big(1+g^2C_1(k^2/M^2)\Big)F^{\mu\nu}(k)F_{\mu\nu}(-k)+\Big(1+g^2C_{1/2}(k^2/M^2)\Big)\lambda(k)\sigma^\mu k_\mu\bar{\lambda}(-k)$$
$$+\Big(1+g^2C_0(k^2/M^2)\Big)D(k)D(-k)+\frac{1}{2}\Big(g^2MB_{1/2}(k^2/M^2)\lambda(k)\lambda(-k)+c.c.\Big)\Big]. \tag{2.19}$$

In the absence of charged massless particles in the hidden sector, all the form factors have smooth limits for $k^2 \to 0$, and we will assume this is the case for the rest of this section. We can use (2.19) to obtain the modified propagator of the gaugino

$$\langle\lambda_\alpha(k)\bar{\lambda}_{\dot{\alpha}}(-k)\rangle = -\frac{\big(1+g^2C_{1/2}(k^2/M^2)\big)\,\sigma^\mu_{\alpha\dot{\alpha}}k_\mu}{\big(1+g^2C_{1/2}(k^2/M^2)\big)^2k^2+g^4M^2|B_{1/2}(k^2/M^2)|^2} \tag{2.20}$$

$$\langle\lambda_\alpha(k)\lambda_\beta(-k)\rangle = -\frac{g^2MB_{1/2}(k^2/M^2)\,\epsilon_{\alpha\beta}}{\big(1+g^2C_{1/2}(k^2/M^2)\big)^2k^2+g^4M^2|B_{1/2}(k^2/M^2)|^2}. \tag{2.21}$$

From this formulas we see that the gaugino has a Majorana mass $M_{\tilde{g}}$ defined implicitily by the solution to the equation

$$\left[\left(1+g^2C_{1/2}(k^2/M^2)\right)^2k^2+g^4M^2|B_{1/2}(k^2/M^2)|^2\right]_{k^2=|M_{\tilde{g}}|^2}=0. \tag{2.22}$$

To leading order in the coupling g we get

$$M_{\tilde{g},r}=g_r^2M\,B_{1/2}^{(r)}(0), \tag{2.23}$$

where we inserted back the r-index running over $SU(3)$, $SU(2)$ and $U(1)_Y$. Consistently, this formula gives a vanishing result whenever supersymmetry, or any R-symmetry, is unbroken.

As for the squark/slepton masses, we can again use (2.19) to compute the loop correction to the two-point functions of the scalars (see Fig 2.2) and we get

$$m_{\tilde{f}}^2=g^2C_f\int\frac{d^4\,k}{(2\pi)^4}\frac{1}{k^2}\left(\frac{1}{1+g^2C_0(k^2/M^2)}-\frac{4}{1+g^2C_{1/2}(k^2/M^2)}+\frac{3}{1+g^2C_1(k^2/M^2)}\right), \tag{2.24}$$

which to leading order in g, and keeping track of the various gauge group factors, reduces to

$$\begin{aligned} m_{\tilde{f}}^2 &= \sum_r g_r^4C_f^rA_r \\ A_r &= \int\frac{d^4\,k}{(2\pi)^4}\frac{1}{k^2}\left(C_0^{(r)}(k^2/M^2)-4C_{1/2}^{(r)}(k^2/M^2)+3C_1^{(r)}(k^2/M^2)\right). \end{aligned} \tag{2.25}$$

Once again, as expected, we find a vanishing result if the supersymmetric relations between the form factors are satisfied.

As an example of the power of this general formulation, let us show that in all possible Gauge Mediation models a sum rule between the soft terms holds. This is easy to realize based on a counting of parameters: the soft masses in the MSSM Lagrangian are

$$M_{\tilde{g},1},\,M_{\tilde{g},2},\,M_{\tilde{g},3},\,m_{\tilde{Q}}^2,\,m_{\tilde{u}}^2,\,m_{\tilde{d}}^2,\,m_{\tilde{L}}^2,\,m_{\tilde{e}}^2, \tag{2.26}$$

which sum up to a total of 3 complex and 5 real parameters (assuming flavor universality), while the parameters in the hidden sector in terms of which the soft masses are expressed are

$$B_{1/2}^{(1)}(0),\,B_{1/2}^{(2)}(0),\,B_{1/2}^{(3)}(0),\,A_1,\,A_2,\,A_3, \tag{2.27}$$

that is 3 complex and 3 real parameters. Since there is a clear one-to-one correspondence between the complex parameters, this means that two sum-rules hold between the scalar soft masses. To derive it explicitly, recall that for each sparticle $\tilde{f}$ we have the corresponding SM particle f that is a chiral fermion. Therefore, if we consider a $U(1)$ symmetry which is flavor-blind (i.e. it acts in the same way on each of the three generations), the condition for it to be free of $U(1) - G_{SM}^2$ anomalies is

$$\sum_f Q_f C_f^r = 0, \tag{2.28}$$

where Q_f is the fermion charge under the $U(1)$. From formula (2.25), any such non-anomalous $U(1)$ gives rise to a sum-rule

$$\sum_f Q_f m_{\tilde{f}}^2 = 0. \tag{2.29}$$

There are indeed two non-anomalous $U(1)$ symmetries of the MSSM, $U(1)_Y$ and $U(1)_{B-L}$, and they give the two expected sum rules. Explicitly they read

$$0 = m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2 \tag{2.30}$$

$$0 = 2m_{\tilde{Q}}^2 - m_{\tilde{u}}^2 - m_{\tilde{d}}^2 - 2m_{\tilde{L}}^2 + m_{\tilde{e}}^2\,. \tag{2.31}$$

Notice that, strictly speaking, these sum rules hold only at the mediation scale, and then one needs an RG analysis to see to what extent they are corrected at energies relevant for experiments.

2.3.1 Massless particles in the Hidden Sector

In this section we consider a generalization of the previous setting to hidden sectors containing massless particles, along the lines of [27, 28]. This case has applications to model building, as we will discuss, and it is also useful for future reference.

In particular we want to study the case in which the presence of such particles is reflected in the appearance of $1/k^2$ poles in the form factors. This implies there are IR singularities in the effective action (2.19), which arise because the effective theory is ill-defined when we integrate over massless states. Still, one can fruitfully use equations (2.20-2.21-2.24) to calculate the contributions to the soft masses.

Before showing how the calculation goes, let us clarify what is the physics of these massless states. In the limit when the gauging is turned off, the hidden sector is a theory which dynamically generates the scale of supersymmetry breaking, and tipically this happens via the strongly coupled dynamics of a gauge theory which also has a mass gap. If massless particles are present, there is a symmetry protecting their masses. For massless scalars, the zero mass is protected if they are *Goldstone bosons* of a spontaneously broken global symmetry. As for massless fermions, their presence can be enforced by the matching of UV 't Hooft anomalies for an unbroken global symmetry, and in this case we will call these particles *'t Hooft fermions*. Let us analyze the two possibilities in turn, concentrating on the case of a $U(1)$ symmetry for definiteness.

The pole associated to a Goldstone boson arise in the two-point function of the conserved current

$$\langle j_\mu(k) j_\nu(-k)\rangle = -(\eta_{\mu\nu}k^2 - k_\mu k_\nu)\frac{v^2}{k^2} + \text{regular}, \tag{2.32}$$

so that we have

$$C_1 = \frac{v^2}{k^2} + \text{regular}. \tag{2.33}$$

Upon weakly gauging, the Goldstone boson gets eaten by the vector to form a massive vector, and the corresponding gauge symmetry is higgsed. Therefore this case applies to models with an extended gauge group at the mediation scale (possibly in a grand unification scenario), and some of the gauge symmetries get broken by the hidden sector (settings with some some higgsed extra $U(1)'$ are extensively studied in the literature, see for instance [29] and references therein). The massive vector still communicates supersymmetry breaking to the charged visible fields, and the resulting soft mass is understood as an additional contribution due to the higgsed part of the gauge group[4]. From equation (2.24), and concentrating on the singular part of C_1, we get

$$\delta m^2_{\tilde f} = g^2 C^f \int \frac{d^4k}{(2\pi)^4}\frac{3}{k^2 + g^2v^2}, \tag{2.34}$$

which has indeed the form of a loop with a massive gauge boson. Retaining only the pole is a valid approximation for C_1 only at low momenta (up to energies of the order of the vector mass), and therefore we can extract a sensible answer by putting a cutoff to the integral at some energy $\Lambda \gg gv$. Performing the integration we have

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + g^2v^2} = \frac{1}{16\pi^2} g^2 v^2 \log\left(\frac{g^2v^2}{\Lambda^2}\right). \tag{2.35}$$

The dependence on Λ will then disappear by taking into account the complete form of C_1 at large momenta. However, what is significant of the previous formula is the logarithmic dependence on the gauge coupling, which will not get corrected by the contributions of the regular part of C_1, because they are always analytic in g. Therefore, we can isolate the part due to the massive vector in the form

$$\delta m^2_{\tilde f} = \frac{3C^f v^2}{16\pi^2} g^4 \log(g^2). \tag{2.36}$$

Notice that due to the smallness of the gauge coupling, this contribution is negative and it is enhanced with respect to a usual GGM contribution both by the logarithm and by the presence of a 1-loop factor $1/16\pi^2$ as opposed to the common two-loops one $(1/16\pi^2)^2$ (reflecting the fact that indeed we have a single loop of a massive

[4] The case outlined here has to be contrasted with other similar scenarios of mediation through massive gauge bosons in which a visible field is responsible for the higgsing, because here the symmetry breaking is inherent to the hidden sector.

vector). Therefore this scenario is highly constrained.

The other possibility is a massless 't Hooft fermion, whose pole will arise in the two-point function of the fermionic operator

$$\langle j_\alpha(k)\bar{j}_{\dot\alpha}(-k)\rangle = -\sigma^\mu_{\alpha\dot\alpha}k_\mu\frac{v^2}{k^2} + \text{regular}, \tag{2.37}$$

implying the appearance of poles in the corresponding form factor

$$C_{1/2} = \frac{v^2}{k^2} + \text{regular}. \tag{2.38}$$

Notice that, for a $U(1)$-current multiplet, the operator j_α is charged only under R-symmetries. Consequently, the symmetry with non-trivial 't Hooft anomaly can only be an unbroken $U(1)_R$, so that the 't Hooft fermion carries the same quantum numbers as the current operator. What we said above on the contribution to the scalar soft masses applies also in this case, with the crucial difference that now it has a positive sign. In addition to that, now we also have a different result for the gaugino mass, as can be seen from the corrected propagator in equation (2.20). Recall that the unbroken R-symmetry implies $B_{1/2} = 0$. Then the contribution of the pole to (2.20) takes the form

$$\langle\lambda_\alpha(k)\bar\lambda_{\dot\alpha}(-k)\rangle = -\frac{\sigma^\mu_{\alpha\dot\alpha}k_\mu}{k^2+g^2v^2}. \tag{2.39}$$

Together with $\langle\lambda_\alpha(k)\lambda_\beta(-k)\rangle = 0$, this equation is telling that the gaugino has gained a Dirac mass gv. Since the gaugino on its own has not enough degrees of freedom to form a Dirac fermion, the interpretation of this result is that the massless 't Hooft fermion of the hidden sector mixes with the gaugino as a consequence of the gauging, so that they can form a Dirac fermion. It is easy to get convinced that it is indeed the case because equation (2.37) implies that the fermionic operator acts as an interpolating operator for the massless particle (call ψ_α the 't Hooft fermion field)

$$j_\alpha = v\,\psi_\alpha + \text{regular}, \tag{2.40}$$

so that the coupling due to gauging takes exactly the form of a Dirac mass

$$gv(\psi\lambda + c.c.). \tag{2.41}$$

The possibility of having a Dirac mass for gauginos is a widely studied subject in the literature (see [30] and references therein), also because it alleviates the difficulties with Majorana masses related to R-symmetries. Here we have seen a way to accommodate this scenario in the general formulation, the only input being equation (2.37).

As a little deviation from this scenario, one can consider adding a small source of R-symmetry breaking in the hidden sector, parametrized by $|B_{1/2}| \ll 1$. The previously massless 't Hooft fermion is expected to get a comparably small mass, and $C_{1/2}$ will be regular but with a large peak for $k = 0$, still giving the dominant (positive) contribution to the scalar masses. In this case the gaugino and the would-be t' Hooft fermion will be coupled in the mass matrix, which will be roughly of the form

$$\frac{1}{2} M B_{1/2}\, \psi\psi + gv\lambda\psi + \frac{1}{2} g^2 B_{1/2} M\, \lambda\lambda + c.c.\,, \tag{2.42}$$

so that the mass eigenstates are a mixing between the two fermions. The mass eigenvalues are given by the complete propagator as in equation (2.22).

As a conclusive remark, let us address the possibility that the massless Goldstino generates a pole in the two-point function of j_α. This would imply that j_α is a valid interpolating operator for the Goldstino field, so that at low energies

$$j_\alpha = v\chi_\alpha + \text{regular}, \tag{2.43}$$

where χ is the Goldstino field. Recalling that for the supercurrent operator $S_{\mu\alpha}$ one has

$$S_{\mu\alpha} = f(\sigma_\mu \bar{\chi})_\alpha + \text{regular}, \tag{2.44}$$

where f is the scale of supersymmetry breaking, one would obtain a non-transverse part in the correlator $\langle S_{\mu\alpha}(k) j_\beta(-k)\rangle$ mediated by the exchange of one Goldstino. On the other hand, by the Ward identities of supersymmetry

$$k^\mu \langle S_{\mu\alpha}(k) j_\beta(-k)\rangle = \langle \{Q_\alpha, j_\beta\}\rangle = 0, \tag{2.45}$$

where in the last equality we assumed that the one-point function of J is vanishing. Therefore we find an inconsistency, and we conclude that the Goldstino does not mix with the operator j_α.

2.3.2 The Higgs sector in Gauge Mediation

Our presentation until now has focused on the soft supersymmetry breaking masses for gauginos and sfermions, because their generation, as we have seen, is a direct consequence of the Gauge Mediation mechanism. However, in a supersymmetric extension of the SM, additional soft terms which involve the Higgs sector must be present. In this section, for completeness, we briefly describe how they can be generated in the context of Gauge Mediation, and we discuss the associated model-building issues that are raised.

Denoting by H_u and H_d the chiral superfields of the two Higgs doublets, and also their lowest scalar components, the soft terms take the form[5]

$$V_{soft}^{higgs} = m_u^2\,|H_u|^2 + m_d^2\,|H_d|^2 + (B_\mu\, H_u\, H_d + c.c.) + (A_u\, H_u\, F_u^\dagger + A_d\, H_d\, F_d^\dagger + c.c.). \tag{2.46}$$

Besides these supersymmetry breaking terms, a supersymmetric mass term can be added to the superpotential

$$\mathcal{W}_\mu = \int d^2\theta\, \mu\, H_u\, H_d \tag{2.47}$$

which both gives a (Dirac) mass to the higgsinos and contributes to the scalar masses.

A fundamental constraint on these masses comes from imposing that the scalar higgses are responsible for electro-weak symmetry breaking, which requires that μ^2, B_μ and $m_{u,d}^2$ are roughly of the same order, and close to the electro-weak scale. This requirement raises two problems, one which is general to supersymmetric extensions of the SM, and another one which is particularly severe in the context of Gauge Mediation.

The first one is the so-called μ-problem [31], and it is basically the question of why the μ parameter is small with respect to the UV cutoff. Indeed, since the μ coupling is supersymmetric, it is in principle unrelated to the dynamics responsible for breaking supersymmetry, and its natural value is very large compared to the electro-weak scale. This problem can be solved by invoking some symmetry of the visible sector that is broken by μ, so that its natural value is actually 0, and it can only be generated by the interaction with the hidden sector, just like the soft terms.

The second problem, which is specific to models of gauge-mediated supersymmetry breaking, is the question of why B_μ is of the same order of μ^2, and it is therefore called μ/B_μ problem [3]. Indeed, in a simple spurion model like the one we treated in section 2.1, assuming that both μ and B_μ are generated radiatively by the interaction of the higgs with the messengers, the typical values will be

$$\mu \sim \frac{\lambda_u\,\lambda_d}{16\pi^2}\frac{F}{M}, \quad B_\mu \sim \frac{\lambda_u\,\lambda_d}{16\pi^2}\frac{F^2}{M^2} \tag{2.48}$$

where $\lambda_{u,d}$ are the couplings of $H_{u,d}$ to the messenger fields, and F and M are respectively the scale of supersymmetry breaking and the scale of the messenger masses. Since $\lambda_{u,d}$ are small couplings, we see that we get the unacceptable relation $B_\mu \gg \mu^2$. Even if we displayed the phenomenon in a simple toy-model, it has a much more general validity, being ultimately related to the fact that μ and B_μ are both generated perturbatively and at the same order.

[5]Notice that we are not taking the most general form for the A-terms, which would involve all possible gauge-invariant trilinear couplings with sfermions. In this more general case, the A coefficients would be matrices in flavor space, thus introducing a flavor problem. In our case, instead, integrating out the F terms, one sees that the flavor mixing is proportional to the Yukawa couplings, and dangerous additional sources of flavor breaking are avoided.

To make this statement precise, one can formulate also the mediation of the soft terms in the higgs sector in a more general fashion [32], similar in spirit to the GGM formalism we reviewed. This amounts to enlarging the definition of GGM, to allow for direct couplings of the higgs to the hidden sector, for instance in the form

$$\int d^2\theta\,(\lambda_u\,H_u\,\mathcal{O}_u + \lambda_d\,H_d\mathcal{O}_d) + c.c.\,. \tag{2.49}$$

Just like in GGM the correlators of the current superfield dictate the form of the soft masses for gauginos and sfermions, here one can parametrize the correlators of the operators $\mathcal{O}_{u,d}$ in terms of form factors, and derive the soft terms of the higgs sector as functions of these form factors. The upshot of the analysis in [32] is that generically a too large B_μ is generated, but the problem can be solved if the μ term receives an additional contribution which does not vanish in the supersymmetric limit.

To conclude, let us mention that usually, in Gauge Mediation models, the A-terms are strictly 0 at the messenger scale, and are only generated radiatively. In the enlarged scenario with direct couplings between the Higgs and the hidden sector those terms can instead be non-vanishing. In this situation, one faces a $A_{u,d}/m^2_{u,d}$ problem, very similar in nature to the μ/B_μ problem we have just described. Recently there has been a lot of interest in models which can generate such terms, because supersymmetry generically predicts a too light higgs boson, and having sizable A-terms can help in raising the mass to the observed value [33–36].

Chapter 3

Weakly Coupled Hidden Sectors

In this chapter we are going to consider a particular class of hidden sectors, namely supersymmetry breaking theories with a bunch of chiral superfields and superpotential interactions, the prototypical example being the O'Raifeartaigh (O'R) model. Their simplicity allows to determine, via perturbation theory, the location of the supersymmetry breaking vacuum and the spectrum of fluctuations. We are not necessarily giving up with the requirement of dynamical breaking, because many examples of DSB are captured at low energies precisely by effective theories of this form [37–40]. Being tractable, O'R-like models are useful to explore to what extent one can cover the parameter space General Gauge Mediation in concrete models.

In particular, we will address the problem of breaking the R-symmetry and the related issue of the suppression of gaugino mass with respect to scalar soft masses. We will obtain conditions on O'R-like models so to avoid this suppression, paving the way for the next chapter, in which we will construct example of DSB theories reducing to these models at low energies.

3.1 O'Raifeartaigh model and its generalizations

The simplest model of F-term supersymmetry breaking is the *Polonyi model* [41], which is defined in term of a single chiral superfield with superpotential

$$\mathcal{W} = fX. \tag{3.1}$$

The auxiliary component of X gets a VEV and the vacuum energy density receives a positive contribution $|f|^2$. If the Kähler potential is canonical, this theory is free and the spectrum is composed by a massless Goldstino and two massless real scalars which we can take to be the modulus $|X|$ and the phase α of X (we use the same letter for the chiral superfield and its lowest component). There is a flat direction of degenerate supersymmetry breaking vacua parametrized by $|X|$. The superpotential has an R-symmetry with $R(X) = 2$, which is broken spontaneously in any vacuum with $|X| \neq 0$, α being the associated Goldstone boson, usually called R-axion. While α is massless for a symmetry reason, $|X|$ is not protected by any symmetry. Therefore, the vacuum degeneracy, contrarily to what happens when supersymmetry is exactly realized, is not expected to survive quantum corrections. As a consequences, X is referred to as a *pseudomodulus*, to contrast it with the case of a supersymmetric moduli space. One can add self-interactions of X by modifying the Kähler potential to the non-canonical form

$$K = |X|^2 \left(1 - \frac{|X|^2}{4M^2}\right), \tag{3.2}$$

so that the pseudomodulus is stabilized at the origin with mass

$$m_X^2 = \frac{|f|^2}{M^2}, \tag{3.3}$$

and the R-symmetry is unbroken.

This particular form of the non-canonical Kähler potential can arise as a loop correction, if one embeds the Polonyi model in an interacting theory with additional fields. This is realized in the simplest way in the *O'Raifeartaigh model* (O'R) [42], which includes two additional chiral superfields ϕ_1 and ϕ_2 with canonical Kähler potential and superpotential

$$\mathcal{W} = fX + m\phi_1\phi_2 + \frac{\lambda}{2}X\phi_1^2. \tag{3.4}$$

The previous R-symmetry is still present if one assigns $R(\phi_1) = 0$, $R(\phi_2) = 2$. As long as $m^2 < \lambda f$ (all parameters can be taken real and positive by appropriately shifting the fase of the fields) the classical potential has degenerate minima in

$$\phi_1 = \phi_2 = 0,\ X \text{ arbitrary}. \tag{3.5}$$

We see that even in this interacting model one has a pseudomodulus at the classical level, and only the vacuum with $X = 0$ preserves the R-symmetry. Quantum corrections lift the classical degeneracy: integrating out the massive fluctuations of $\phi_{1,2}$ around a vacuum with a certain VEV X, one can calculate the effective potential at 1-loop via the formula

$$V_{eff}^{(1-loop)} = \frac{1}{64\pi^2}\text{STr}\left[\mathcal{M}^4 \log\left(\frac{\mathcal{M}^2}{M_{UV}^2}\right)\right], \tag{3.6}$$

where $\mathcal{M}$ is the X-dependent mass matrix of the fluctuations, M_{UV} is the UV cutoff (whose dependence can be reabsorbed in the renormalization of the couplings), and STr indicates the weighted sum over bosons and fermions. For small values of X the result is

$$V_{eff}^{(1-loop)}(X) = V_0 + m_X^2|X|^2 + \mathcal{O}(|X|^4) \tag{3.7}$$

$$V_0 = f^2\left[1 + \frac{\lambda^2}{32\pi^2}\left(\log\frac{m^2}{M_{UV}^2} + \frac{3}{2} + \mathcal{O}\left(\lambda^2 f^2/m^4\right)\right) + \mathcal{O}(\lambda^4)\right] \tag{3.8}$$

$$m_X^2 = \frac{1}{32\pi^2}\frac{\lambda^4 f^2}{m^2}\left(\frac{2}{3} + \mathcal{O}(\lambda^2 f^2/m^4)\right) + \mathcal{O}(\lambda^4). \tag{3.9}$$

Since $m_X^2 > 0$, X is stabilized at the origin at 1-loop, and the R-symmetry is unbroken. This result is analogue to the one we obtained in the Polonyi model with non-trivial Kähler potential. In fact, in the limit of small supersymmetry breaking (as in the $\lambda f/m^2$ expansion), the 1-loop effective action is encoded in the supersymmetric correction to the Kähler potential due to the massive fields $\phi_{1,2}$, which gives the same result as equation (3.2) (see the discussion in the appendix of [43]). The effective potential for large X grows like a (positive) logarithm, reflecting the running of the vacuum energy with the scale of the masses. Since it can be shown to be monotonic, there are no additional local minima of the potential outside the origin.

Some of the features of the O'R model are generic consequences of F-term breaking, while some others are not. For instance, the existence of a pseudomodulus (at least one) which is a flat direction of the potential at the classical level, is completely general and holds in every model of chiral superfields with supersymmetry breaking [44]. This can be seen by considering the masses of the bosonic fields at tree level

$$\begin{pmatrix}\phi^\dagger & \phi^T\end{pmatrix}\mathcal{M}_B^2\begin{pmatrix}\phi \\ \phi^*\end{pmatrix} = \begin{pmatrix}\phi^\dagger & \phi^T\end{pmatrix}\begin{pmatrix}\mathcal{M}_F^2 & \mathcal{F}^\dagger \\ \mathcal{F} & \mathcal{M}_F^2\end{pmatrix}\begin{pmatrix}\phi \\ \phi^*\end{pmatrix}, \tag{3.10}$$

where $\mathcal{M}_F$ is the fermion mass matrix and the complex matrix $\mathcal{F}$ is generated by supersymmetry breaking. Whenever the fermionic mass matrix has some massless eigenvector v, the formula just displayed gives

$$\begin{pmatrix}v^\dagger & v^T\end{pmatrix}\mathcal{M}_{bos}^2\begin{pmatrix}v \\ v^*\end{pmatrix} = v^T\mathcal{F}v + c.c.\,. \tag{3.11}$$

The left-hand side of this equation is a non-negatively defined function of v, while the right-hand side is not, unless it is zero. For consistency, we conclude that $v^T\mathcal{F}v = 0$. Therefore, at tree level, the existence of a massless fermion implies the existence of a complex massless boson. Since there is always a massless Goldstino in a supersymmetry breaking theory, there must also always be a classically massless scalar, namely the pseudomodulus.

Calling X the pseudomodulus, the superpotential of the theory can always be put in the form

$$\mathcal{W} = fX + \frac{1}{2}\left(m_{ij} + X\lambda_{ij}\right)\phi_i\phi_j + \frac{1}{6}h_{ijk}\phi_i\phi_j\phi_k, \tag{3.12}$$

where all the other fields have been collected in ϕ_i. We will refer to these theories as *generalized O'Raifeartaigh models* [15]. The superpotential may admit an R-symmetry depending on the precise form of the couplings[1]. As in the O'R model, any R-symmetry satisfies $R(X) = 2$, so that it is spontaneously broken in a generic point of the pseudomoduli space, and a question arises about the fate of the R-symmetry after loop corrections are considered. The answer found in the O'R model is not the most generic one, as we will explain in the next section: there exist generalized O'R models in which the radiative spontaneous R-breaking does occur.

3.2 R-symmetry and its breaking

Let us review the special role played by the R-symmetry in models of supersymmetry breaking. It is a consequence of the tension between the following two basic facts

- *Nelson-Seiberg criterion* [45]: if a theory with a generic superpotential (i.e. containing all terms compatible with global symmetries) breaks supersymmetry spontaneously in a stable vacuum, then the theroy must posses an R-symmetry. The argument for this criterion is a simple counting of the number of equations for the supersymmetric vacua compared to the number of variables. Since the F-term equations take the form $\partial_i\mathcal{W} = 0$, there are as many equations as variables, and generically a solution always exists. Nevertheless, in presence of an R-symmetry, there is the additional constraint $2\mathcal{W} = \sum_i \phi_i\,\partial_i\mathcal{W}\,R(\phi_i)$ which implies $\mathcal{W} = 0$. Therefore, in this case (and only in this case) the equations are one more than the number of variables, and there is room for supersymmetry breaking.

- *Majorana Gaugino masses*: any unbroken R-symmetry forbids a Majorana mass for the gauginos.

In order to avoid the existence of an R-symmetry one has to allow the existence of supersymmetric vacua, so that the supersymmetry breaking happens in a metastable but sufficiently long-lived vacuum [43]. The prototypical example of a dynamical meta-stable supersymmetry breaking is given by the ISS model [37]. Still, in concrete models one typically finds that even in this case the theory enjoys an approximate R-symmetry [37–39]. This can be heuristically understood as follows: from the point of view of the effective theory in the supersymmetry breaking vacuum, the absence

[1] According to the Nelson-Seiberg criterion it always admits at least one if it is generic (i.e. all couplings which are allowed by simmetries are present), see below.

of the R-symmetry can be parametrized by adding a deformation term which gives an explicit breaking. By the Nelson-Seiberg criterion, this deformation will re-introduce supersymmetric vacuum solutions, whose distance in field space will be parametrically large when the deformation is small. As a consequence, the amount of explicit breaking of the R-symmetry is proportional to the small parameter that controls the lifetime of the vacuum. The Majorana mass of the gaugino in that vacuum will be suppressed by the same small parameter, unless additional sources of breaking are present.

Therefore, a natural question to ask is whether it is possible to break spontaneously R-symmetry, and how generic such breaking can be.[2] A suitable arena to address this question are the weakly coupled theories of chiral superfields discussed above, both for their simplicity and because they may capture the IR of a dynamical theory. In this class of models the question can be specified as follows: can the pseudomodulus direction, which is flat at the classical level, be stabilized at the loop level in $|X| \neq 0$? The analysis of [47] revealed that in order to achieve spontaneous breaking, it is necessary to have some of the fields with R-charge different from 0 or 2. We will refer to this result as *Shih's theorem.*

Let us briefly review how the argument of [47] goes. Concentrating on the flat direction parametrized by X with all other fields vanishing $\phi_i = 0$, one can integrate out the massive fluctuations contained in ϕ_i around one of the classical vacua with generic value for X, thus obtaining the Coleman-Weinberg potential as a function of X. This potential can be recast in the form of a matrix integral

$$V_{eff}^{(1-loop)} = -\frac{1}{32\pi^2}\,\mathrm{Tr}\int_0^{\Lambda} dv\, v^5 \left(\frac{1}{v^2+\mathcal{M}_B^2} - \frac{1}{v^2+\mathcal{M}_F^2}\right), \tag{3.13}$$

where the mass matrices of fermionic and bosonic fluctuations depend on the matrices m_{ij} and λ_{ij} entering the superpotential as follows

$$\mathcal{M}_F = \begin{pmatrix} 0 & m^\dagger + \lambda^\dagger X \\ m + \lambda X & 0 \end{pmatrix}, \tag{3.14}$$

$$\mathcal{M}_B^2 = \mathcal{M}_F^2 + \begin{pmatrix} 0 & \lambda^\dagger f \\ \lambda f & 0 \end{pmatrix}. \tag{3.15}$$

[2]Notice that if the breaking is spontaneous there is an R-axion in the spectrum, which would only get a tiny mass when the theory is embedded in supergravity. Such light scalars are severely constrained phenomenologically [46], and this is an additional motivation to introduce a source of explicit R-symmetry breaking in the theory.

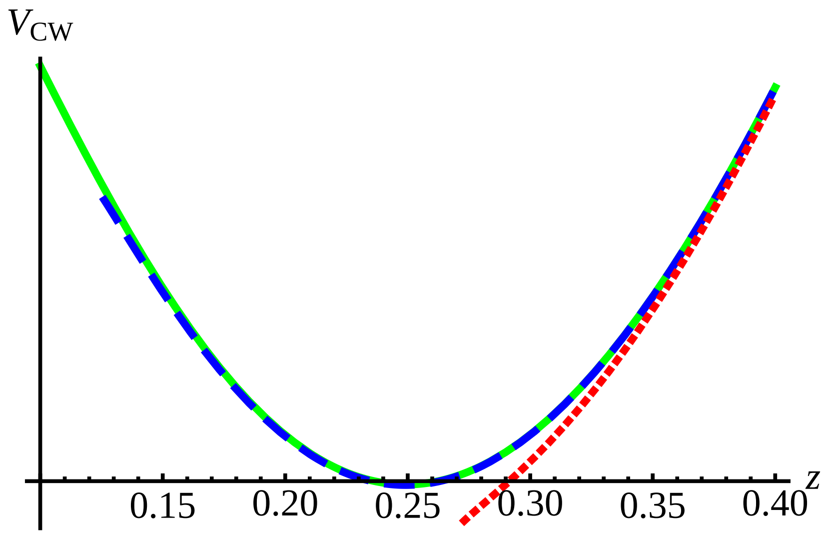

Figure 3.1: The effective potential normalized in units of $m^4 y^2$, and $y = 10^{-2}$ (dotted line), 10^{-3} (dashed line) and 10^{-4} (solid line). Each plot ends on the left at the corresponding $z_{min} = z_{min}(y)$. For $y \gtrsim 10^{-3}$ the would-be minimum would falls into the unstable region of the classical pseudomoduli space and the theory does not have a metastable vacuum. For smaller values of y the minumum exists, and a potential barrier develops against decay toward the supersymmetric runaway vacua at $S_0 \to 0$.

Expanding the integral around $X = 0$, one finds a closed formula for the squared-mass of the pseudomodulus

$$m_X^2 = M_1^2 - M_2^2\,, \tag{3.16}$$

$$M_1^2 = \frac{1}{16\pi^2 f^2}\int_0^\Lambda dv\, v^5\, \mathrm{Tr}\left[\frac{\mathcal{F}^4}{1-\mathcal{F}^2}\right]\,, \tag{3.17}$$

$$M_2^2 = \frac{1}{8\pi^2 f^2}\int_0^\Lambda dv\, v^3\, \mathrm{Tr}\left[\left(\frac{\mathcal{F}^2}{1-\mathcal{F}^2}\mathcal{M}_F|_{X=0}\right)^2\right]\,, \tag{3.18}$$

$$\mathcal{F} = (v^2+\mathcal{M}_F^2)^{-1/2}(\mathcal{M}_B^2-\mathcal{M}_F^2)(v^2+\mathcal{M}_F^2)^{-1/2}|_{X=0}. \tag{3.19}$$

The sign of the mass is what determines whether the R-preserving vacuum is destabilized, and consequently the R-symmetry is spontaneously broken. The argument goes on by showing that in a theory with R-charges only 0 or 2, the particular form of the superpotential couplings m and λ is such that $M_2^2 = 0$ and M_1^2 is positively defined. Therefore in any such theory the R-symmetry is unbroken. Curiously, the typical assignement of charge arising from dynamical model is exactly such that all fields have charge 0 or 2, and this is why the possibility of breaking spontaneously R-symmetry in generalized O'R models is a relatively recent discovery.

We will now give an example [48,49] that indeed realizes the spontaneous breaking. Possibly the simplest generalized O'R with charges other than 0 or 2 is defined by the superpotential

$$\mathcal{W} = fX + \frac{m}{2}\phi_1^2 + \lambda X \phi_1 \phi_2, \tag{3.20}$$

whose unique R-symmetry has $R(X) = 2$, $R(\phi_1) = -R(\phi_2) = 1$. Notice that having fields with charges other than 0 or 2 that enter the superpotential interactions, one is also forced to assign negative R-charges to some of the fields. Requiring the potential to be stationary one finds a pseudomoduli space of supersymmetry breaking vacua parametrized by

$$\phi_1 = \phi_2 = 0,\ X\,\text{arbitrary}, \tag{3.21}$$

whose vacuum-energy density is $|f|^2$. However these are not absolute minima, because there is a runaway direction

$$\phi_1 = \pm\frac{\sqrt{fm}}{\lambda}X^{-\frac{1}{2}}\ ,\ \phi_2 = \mp\sqrt{\frac{f}{m}}X^{\frac{1}{2}}\ ,\ |X| \to 0, \tag{3.22}$$

signaling the existence of supersymmetric vacua at infinity in field space. One can easily compute the masses of the fluctuations $\delta\phi_1$ and $\delta\phi_2$ around the pseudomodulus direction in terms of the convenient parameters $y = \frac{\lambda f}{m^2}$ and $z = \frac{\lambda X}{m}$. The result is

$$\mathcal{M}_B^2 = m^2 \begin{pmatrix} z^2 & z & 0 & y \\ z & 1+z^2 & y & 0 \\ 0 & y & z^2 & z \\ y & 0 & z & 1+z^2 \end{pmatrix},\ \mathcal{M}_F^2 = m^2 \begin{pmatrix} z^2 & z & 0 & 0 \\ z & 1+z^2 & 0 & 0 \\ 0 & 0 & z^2 & z \\ 0 & 0 & z & 1+z^2 \end{pmatrix}. \tag{3.23}$$

Two of the eigenvalues of the bosonic mass matrix are always positive, while the other two get negative when z becomes smaller than a critical value $z_*^{\pm}$, which is a function of y. In a small y expansion one has

$$z_*^+ \simeq \frac{y}{2}(1 + \mathcal{O}(y^2))\ , \quad z_*^- \simeq (2y)^{\frac{1}{3}}(1 + \mathcal{O}(y^{\frac{2}{3}})). \tag{3.24}$$

As a consequence, the pseudomoduli space is a local minimum of the potential only away from the origin, precisely for $z > z_{min} = \text{maz}\{z_*^+,\, z_*^-\}$. If a local minimum exists at all at 1- loop, R-symmetry is spontaneously broken there. The Coleman-Weinberg potential in a small y expansion is given by

$$V_{eff}^{(1-loop)} = \frac{m^4 y^2}{32\pi^2}\left[2\text{log}\left(\frac{\Lambda^2}{m^2}\right) + g(z)\right] + \mathcal{O}(y^4) \tag{3.25}$$

$$g(z) = \frac{1+12z^2}{1+4z^2} + 4\log z + \frac{1+2z^2}{(1+4z^2)^{\frac{3}{2}}}\log\frac{1+2z^2+\sqrt{1+4z^2}}{1+2z^2-\sqrt{1+4z^2}} \tag{3.26}$$

and has a local minimum, which can be found numerically, at

$$\bar{z} \simeq 0.249 + \mathcal{O}(y^2). \tag{3.27}$$

In order for the vacuum to exist, the parameter y has to be small enough to ensure $\bar{z} > z_{min}$.

A rough estimate of the parametric dependence of the lifetime can be given by noticing that, keeping fixed the vacuum energy density f^2, the barrier width scales like

$$\bar{z} - z_f \sim \frac{\sqrt{\lambda f}}{m}(0.249 - y^{\frac{1}{3}})y^{-\frac{1}{2}} \sim y^{-\frac{1}{2}}, \tag{3.28}$$

where z_f is the point where the potential along the runaway direction becomes equal to f^2, the energy density of the metastable vacuum. This indicates that the lifetime is parametrically long in the limit of small y.

We have shown this example as an "existence proof", and for future reference. In what follows (next section and next chapter) we will address the natural questions that follow: what is the result if we use one of this simple models as hidden sectors in a gauge mediation model? Is there a dynamical theory which reduces at low energy to an O'R model which breaks R-symmetry?

Before going on, some remarks are in in order on possible loopholes to Shih's theorem. First, in an O'R model there could be more than one supersymmetry breaking flat direction at tree level, each of them parametrized by some pseudomolus Y_a, $a = 1\ldots, n$ with $R(Y_a) = 2$. Therefore, as noticed in [50,51], even if all R-charges are 0 or 2 and $\langle X \rangle = 0$ at 1 loop, one could envision the possibility that some Y_a is instead radiatively stabilized in an R-breaking vacuum. This loophole was closed in [52], where the theorem was extended to an arbitrary number n of pseudomoduli. In this work, it also shown that some such pseudomoduli can remain massless at the 1-loop level, leaving open the possibility for R-breaking triggered by higher order perturbative corrections (see also [53]). Secondly, even if the origin is stable at loop level, one could ask whether additional metastable vacua exist away from the origin. Since the Coleman-Weinberg potential grows logarithmically for large VEVs, this vacuum could only exist in a narrow region between the origin and the scale of the largest mass parameter, and it is arguably difficult to get a sufficiently long lifetime. Still, this possibility remains open.

3.3 Direct Mediation and Suppression of Gaugino Mass

In this section we consider the gross features of a gauge mediation model in which the hidden sectors is a generalized O'R model. For definiteness we take the fields in a vector-like representation $(\phi^i, \tilde{\phi}^i) \in \rho \oplus \bar{\rho}$ of $SU(5)$, with $i = 1, \ldots, N_m$, the pseudomolus being a singlet, with a superpotential of the form

$$\mathcal{W} = FX + (m_{ij} + \lambda_{ij}X)\phi^i\tilde{\phi}^j. \tag{3.29}$$

Models of this type where studied systematically in [48], and dubbed *Extraordinary Gauge Mediation* (EOGM). Since $(\phi^i,\, \tilde{\phi}^i)$ play the role of the messenger fields, and they also take part in the breaking of supersymmetry, this is a model of direct gauge mediation. The leading perturbative contributions to the soft masses of sfermions and gauginos can be calculated in a fashion similar to the Minimal model discussed in the previous chapter, and the result is almost identical in form

$$M_{\tilde{g},r} = \frac{g_r^2}{16\pi^2}\Lambda_G\,, \quad m_{\tilde{f}}^2 = 2\sum_r C_{\tilde{f}}^r \left(\frac{g_r^2}{16\pi^2}\right)^2 \Lambda_S^2\,, \tag{3.30}$$

$$\Lambda_G = F\frac{\partial}{\partial X}\log\det(m+\lambda X)\,, \quad \Lambda_S^2 = \frac{1}{2}|F|^2\frac{\partial^2}{\partial X \partial X^*}\mathrm{tr}\left[(\log|m+\lambda X|^2)^2\right]\,, \tag{3.31}$$

$$N_{eff}(\lambda,\, m,\, X) = \frac{\Lambda_G^2}{\Lambda_S^2} \tag{3.32}$$

the difference being that the dimensionless parameter $N_{eff}(\lambda,\, m,\, X)$ in this case need not to coincide with the number $N = n_\rho N_m$, where n_ρ is twice the Dynkin index of the representation ρ. Instead, it is a function of the couplings λ, m and of the VEV X, bounded by N from above.

Much more can be said on the scale Λ_G if one assumes that the superpotential enjoys an R-symmetry, because this requirement constrains the matrices λ and m and a more explicit expression for the determinant can be found. One can then classify the model according to the behavior of the matrices, and the following results hold

- **type I**: taking $\det m \neq 0$, then

$$\det(m+\lambda X) = \det m. \tag{3.33}$$

  Therefore,

$$\Lambda_G = 0, \tag{3.34}$$

  and the leading contribution to gaugino masses vanishes,

- **type II**: in this case $\det m = 0$ and $\det\lambda \neq 0$, which implies

$$\det(m+\lambda X) = X^N \det\lambda. \tag{3.35}$$

  Therefore, in this case

$$\Lambda_G = N\frac{F}{M}, \tag{3.36}$$

  in analogy with the minimal model of the previous chapter,

- **type III**: the remaining case is $\det\lambda = 0 = \det m$, giving

$$\det(m+\lambda X) = X^n\, G(\lambda, m)\,, \quad 0 < n < N. \tag{3.37}$$

The striking consequence is that even if the R-symmetry is spontaneously broken, the Majorana mass of the gauginos can turn out to be suppressed with respect to the scalar soft masses. This is the case in all models of type I, where the Majorana mass is not generated at the leading order in the small-F expansion. This phenomenon is a generic feature of direct mediation models, and was first noticed in [7] where it was dubbed *gaugino screening*. Examples of models in which this phenomenon occurrs are [14, 16, 54–57]. In the context of semi-direct mediation, in [16, 17] the suppression was shown to persist beyond the small F approximation, but only at leading order in the hidden-sector gauge coupling. The problem is circumvented by O'R hidden sectors of type II or III. However, the typical O'R which arises at low energies in a dynamical theory is of type I, besides having usually only R-charges 0 or 2.

In building models of Direct Mediation based on such dynamical theories one therefore faces two difficulties in order to get a sizable Majorana mass for the gaugino: the first is introducing some interacting field with R-charge other than 0 or 2, to make R-symmetry breaking possible; the second is to modify the interactions in order to get type II or III. The next chapter will be devoted to show a method to achieve this goal, which was developed in [49].

As a conclusive observation, let us notice that the problem of gaugino screening in O'R models can also be related to the classical stability of the pseudomodulus. Indeed, in [15] it was shown that whenever the mass-squared matrix of fluctuations is positive definite for any value of the VEV X, then

$$\frac{\partial}{\partial X}\det(m + \lambda X) = 0. \tag{3.38}$$

This observation fits with the classification of O'R models described above, because the models with a pseudomodulus which is everywhere stable can only be of type I. Models of type II always develop an instability near to the origin of the pseudomoduli space, typically driving the scalars to a runaway direction, so that the only sensible vacua are for $X > X_{min}$. Notice that the example we gave in the previous section falls in this category and has in fact this kind of instability. Also models of type I can develop an instability, but only for large X, so that in general one has to restrict to $X < X_{max}$. The situation in type III is intermediate between the previous two, and an instability typically arises both for large and small values of the pseudomodulus, so that the interesting vacua are limited in a region $X_{min} < X < X_{max}$. The location of the runaway in the different cases can be heuristically understood by considering the F-term equation for ϕ

$$(m + \lambda X)\phi = 0, \tag{3.39}$$

which, regarded as a linear equation for the vector ϕ, must have some non-zero solution in order for supersymmetry to be unbroken. At finite value of the fields this is impossible because det $(m+\lambda X) \neq 0$, but approaching $X \to 0$ or $X \to \infty$ one can use that m or λ are not invertible to find a solution. The behavior of the ϕ fields along the runaway direction is dictated by the R-charge assignment: for instance, when $X \to 0$, there will be some ϕ field of negative R-charge which approaches infinity as the appropriate negative power of X.

The classification of generalized O'R models is summarized in figure 3.2.

3.4 Covering GGM parameters space at Weak Coupling

In the previous chapter, we reviewed how to give a model-independent definition of Gauge Mediation, which led us to the elegant formulation of GGM. In this framework, we saw that the soft masses are expressed in terms of few parameters, three real numbers A_r and and three complex numbers $B^{(r)}_{1/2}(0)$, which are univocally determined by the hidden sector.

While the nice feature of this point of view is exactly that it does not need a specification of the theory we use as hidden sector, it leaves open an interesting, constructive question, namely whether all possible values of A_r and $B^{(r)}_{1/2}(0)$ are indeed produced by some concrete theory. This is a relevant problem to study, because if the parameters are found to obey relations which restrict the allowed space, then such relations would constitute additional predictions of Gauge Mediation models.

The first thing to do to try and answer this question is to consider a class of simple and flexible examples, and see if they cover the whole parameter space. The weakly coupled model of chiral superfields that we studied in this chapter are promising candidate in this respect. Indeed, once extended to allow for non-zero D-terms, they were used to show that the whole parameter space of GGM can be covered in [25], completing a study initiated in [22]. While in [22] it is shown that all the different parameters are indeed generated by just considering F-term supersymmetry breaking, resulting in a splitting in the messenger mass as in equation 3.15, the analysis of [25] revealed that an additional splitting generated by some non-zero D-term is necessary in order to cover the whole parameter space, resulting in a messenger mass of the form

$$\mathcal{M}_B^2 = \mathcal{M}_F^2 + \begin{pmatrix} \xi & F \\ F^\dagger & \tilde{\xi} \end{pmatrix} \tag{3.40}$$

where ξ and $\tilde{\xi}$ are hermitian mass splittings originated by some D-term. Their presence is necessary because the F splitting alone gives a positive definite contribution to the sfermion soft masses, which cannot be rendered arbitrarily small with respect to gaugino soft masses, thus precluding a region of parameter space.

In these papers, it is also argued that some restriction on the parameters is neces-

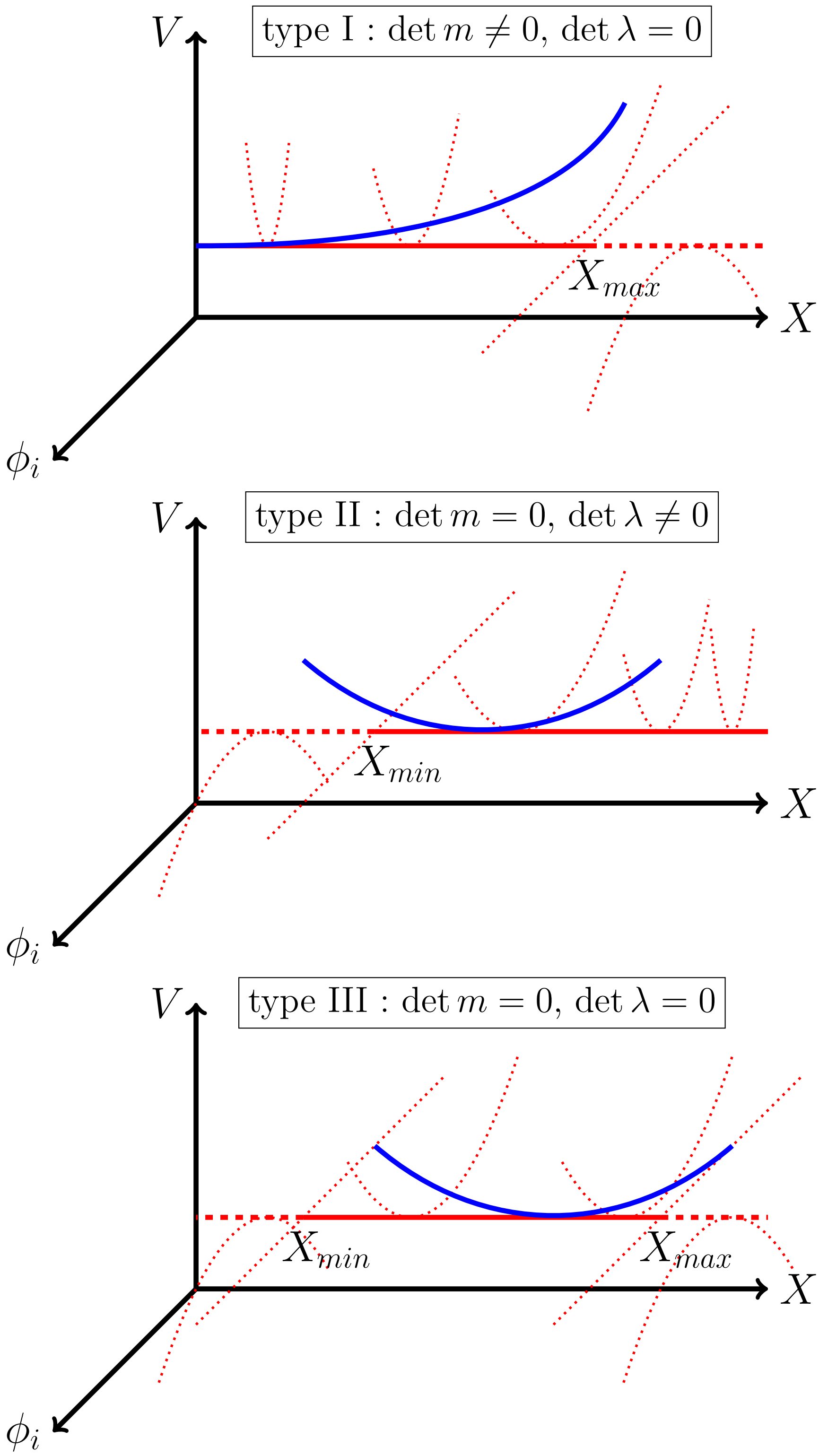

Figure 3.2: The classification of generalized O'R with an R-symmetry. In red, the shape of the tree-level potential is shown. There are regions in which the pseudo-moduli space is stable (solid red line), and others in which the ϕ fluctuations become tachyonic (dashed red line). In blue the possible form of the Coleman-Weinberg potential along the X direction is shown. While in type I it can be stabilized at the origin (thus preserving the R-symmetry), in type II or III the stable vacuum (if any) breaks R-symmetry spontaneously.

sary in order to satisfy phenomenological constraints. For instance, the parameters $B^{(r)}_{1/2}(0)$ should be taken real to avoid too large CP violation effects. Moreover, as we already mentioned, a discrete symmetry acting on the messenger fields must be imposed in order not to generate a large D-term for $U(1)_Y$.[3] These requirements amount to some constaints on the form of the messenger mass matrix, namely one has to take F, ξ and $\tilde{\xi}$ to be real, and also $F^T = F$, $\xi = \tilde{\xi}$.

Keeping this important result in mind, we will come back to the problem of covering the GGM parameter space in chapter 7, where we will show an additional example of a flexible and calculable class of hidden sectors that can be used to reach the same conclusion. Differently to the case we have just treated, they will be strongly coupled theories, and calculability will be provided by holography.

[3] Alternatively, in [58], viable models with an enlarged parameter space are obtained by invoking some mechanism which avoids the generation of the D-term at leading order.

Chapter 4

Dynamical Completions of Generalized O'Raifeartaigh Models

As discussed in the previous chapter, O'R-like models of type II or III are the most interesting ones from a phenomenological view point, since, when used in the context of Direct Mediation, they avoid the suppression of the gaugino mass. It would be desiderable to find realizations for such theories as effective DSB models (as well as a guiding principle towards their construction). This is the topic of the present chapter, which closely follows the work [49].

The models proposed in [49] are simple deformations of DSB theories with quantum deformed moduli space, the prototype example being the ITIY model [38, 39], a supersymmetric $SU(2)$ gauge theory with 2 flavors plus singlets. The strategy is similar to the one pursued in [59] (see also [60, 61]), where a completion of a type I model is proposed. As we will see, suitable deformations of ITIY models with gauge group $USp(2N)$ can provide completions of a large class of models of type I, II or III. Hence, once embedded into direct gauge mediation, these models can give a soft spectrum with either suppressed or unsuppressed gaugino mass.

In fact, the problem of getting R-symmetry breaking vacua has been addressed by many authors in the context of ISS-like models (see e.g. [53, 62–72]). One basic difference with the models we are presenting that we would like to emphasize, is that in those constructions the R-symmetry of the UV theory is explicitly broken by mass terms, and an approximate R-symmetry emerges in the low energy effective theory. The latter is then spontaneously or explicitly broken thanks to suitable modifications of the original ISS Lagrangian. Instead, in [49] the starting point is a gauge theory admitting a non anomalous R-symmetry, and this is hence the same R-symmetry enjoyed in the IR, which then happens to be spontaneously broken in the full theory.

The remainder of this chapter is organized as follows. In section 4.1 we briefly review ITIY models with symplectic gauge groups and the mechanism by which supersymmetry gets dynamically broken. At low energy, these models reduce to O'R models with all fields having R-charges either 0 or 2, and hence unbroken R-symmetry. In section 4.2 we outline the general strategy one should follow in order to get at low energy O'R models with negative R-charges and type II or III. Basically, this amounts to add tree level superpotential deformations which partially break the global symmetry of the original ITIY model (keeping, still, the UV theory generic and renormalizable). To make our discussion concrete, in section 4.3 we focus on a specific class of deformations and analyze the corresponding theory in full detail, showing explicitly how our strategy works. As DSB models, our models are uncalculable, in the sense that there does not exist a region of the parameter space where Kähler corrections can be computed exactly, as much as the original ITIY model. However, following [73], we will see that there exists a region of the parameter space where uncalculable Kähler corrections are suppressed with respect to those coming from the one loop effective potential. Remarkably, this region coincides with that for which the lifetime of the supersymmetry breaking vacua is parametrically large, hence making the full construction self-consistent. Section 4.4 contains a summary and discusses possible future directions.

4.1 Review of generalized ITIY models

In this section we briefly review the structure of ITIY models with symplectic gauge group, following [73]. Let us consider a supersymmetric gauge theory with gauge group $USp(2N)$, $F = N + 1$ fundamental flavors Q_i, $i = 1, \ldots, 2F$, and an antisymmetric singlet S^{ij} with tree level superpotential

$$W_{tree} = hS^{ij}Q_iQ_j \ , \tag{4.1}$$

which respects the $SU(2F)$ flavor symmetry. When $h = 0$ the classical moduli space is parametrized by gauge invariant operators $V_{ij} = Q_iQ_j$ subject to the constraint $\mathrm{Pf}V = 0$. For $h \neq 0$ the mesonic flat directions are lifted, and one is left with a moduli space spanned by S^{ij} with $V = 0$. The modified constraint due to non perturbative gauge dynamics,

$$\mathrm{Pf}V = \Lambda^{2F}, \tag{4.2}$$

is therefore incompatible with $h \neq 0$ and supersymmetry is broken.

If $h\langle S\rangle \ll \Lambda$, quarks are light at tree level and the low energy theory is rewritten in terms of the meson matrix V. The independent degrees of freedom are determined by solving the quantum constraint (4.2), and they can be identified with the fluctuations around a point of the quantum moduli space. At a generic point, up to gauge transformations and global $SU(2F)$ symmetry

$$V = \begin{pmatrix} v_1\,\epsilon & & \\ & \ddots & \\ & & v_F\,\epsilon \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{4.3}$$

so that $SU(2F)$ is broken to $SU(2)^F$, but there are submanifolds of enhanced symmetry when some of the v_i's coincide. Since the superpotential deformation does not break the global symmetry, the point (if any) where the theory is (meta)stabilized, once the susy breaking mechanism is taken into account, must be a special one in this moduli space. Therefore, we consider a point belonging to the compact submanifold of maximal symmetry $USp(2F)$, which corresponds to taking $v_1 = \cdots = v_F$, and solve the constraint in an expansion around it

$$V = \Lambda(V_0 J + V')\,, \quad S = \frac{1}{\sqrt{2F}} S_0 J + S'\ , \tag{4.4}$$

where $J = \mathbf{1}_{F\times F} \otimes \epsilon$ is the $USp(2F)$ invariant tensor and V', S' satisfy $\mathrm{tr}\,[JV'] = 0 = \mathrm{tr}\,[JS']$. The factor of Λ in the definition above is to make the dimension of V_0 and V' fields equal to one. The solution of the quantum constraint for V_0 in a small V' expansion is

$$V_0 = \Lambda\left(1 - \frac{1}{4F\Lambda^2}\mathrm{tr}\,\left[JV'JV'\right] + \mathcal{O}\left(\frac{V'^3}{\Lambda^3}\right)\right). \tag{4.5}$$

giving us the following low energy superpotential to quadratic order in V'

$$W_{eff} = fS_0 + \tilde{h}\, S_0 \mathrm{Tr}\left[JV'JV'\right] + M\,\mathrm{Tr}\left[S'V'\right]\ , \tag{4.6}$$

where

$$f = \sqrt{2F} h\Lambda^2\ , \quad \tilde{h} = -\frac{1}{2\sqrt{2F}} h \ \text{ and } \ M = -h\Lambda. \tag{4.7}$$

Upon the identification $S_0 \equiv X, (V',\, S') \equiv \phi_i$ we see that we get an O'R model of the form (3.12), with $\det m \neq 0$, $USp(2F) \times U(1)_R$ global symmetry and all R-charges equal 0 or 2. The pseudomoduli space is hence stabilized at the origin of field space by the one loop potential. As discussed in [73], these perturbative corrections are dominant with respect to uncalculable Kähler contributions at least near the origin of the pseudomoduli space, making the existence of the supersymmetry breaking minimum (which is a global one, in this case) reliable.

4.2 Modified ITIY models

In what follows we want to discuss which modifications one could make on the model described above to obtain more general O'R models at low energy. From the arguments discussed in the previous chapter, we expect the supersymmetry breaking minimum to be at most metastable, as lower energy vacua are expected to emerge in more general models. We will find indeed runaway vacua in the effective theories, which, as we will see, may or may not be real runaways in the full theory.

In this section we outline the general strategy one should follow in order to accomplish this task. In section 4.3 we will put the general recipe at work, focusing on some (classes of) models and discussing in an explicit example the full dynamics in detail.

As discussed in [59], a simple way to obtain fields with R-charge different from 0 and 2 is to break the global symmetry and mix the original R-symmetry with some broken $U(1)$ generator of the flavor symmetry group. To this end, one can add explicit symmetry breaking terms in the superpotential and/or reduce the field content of the theory. Suppose that the global symmetry is broken according to the pattern

$$SU(2F) \times U(1)_R \to G \times U(1)_{R'} \ , \tag{4.8}$$

where G is a subgroup of the residual $USp(2F)$ around the enhanced symmetry point of the moduli space. Recall that V' is in an irreducible representation of $USp(2F)$ which we denote by $\boldsymbol{r}$ and S' is in the conjugate representation $\bar{\boldsymbol{r}} = (\boldsymbol{r}^T)^{-1}$. Such representations split in G representations as

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{r_1} \oplus \cdots \oplus \boldsymbol{r_k} \ , \qquad V' = (V_1, \ldots, V_k) \\ \bar{\boldsymbol{r}} &= \bar{\boldsymbol{r}}_{\boldsymbol{1}} \oplus \cdots \oplus \bar{\boldsymbol{r}}_{\boldsymbol{k}} \ , \qquad S' = (S_1, \ldots, S_k) \ . \end{aligned} \tag{4.9}$$

The two representations are also equivalent, hence each block in $\boldsymbol{r}$ decomposition is equivalent to a certain one in $\bar{\boldsymbol{r}}$ decomposition. Since $JV'J$ is in the same $\bar{\boldsymbol{r}}$ representation of S', the upshot is that the $USp(2F)$ invariant quadratic terms in the V' and S' fields are rewritten as

$$\begin{aligned} \mathrm{tr}\ \left[S'V'\right] &= \sum_{I=1}^{k} S_I V_I \ , \\ \tilde{h}\,\mathrm{tr}\ \left[JV'JV'\right] &= \sum_{I,J=1}^{k} C^{IJ} V_I V_J \ , \end{aligned} \tag{4.10}$$

where contractions of the representations are understood, and the matrix C acts by swapping some couples of indices, i.e. it takes the form

$$C = \left(\begin{array}{c|c} C_1 & 0 \\ \hline 0 & C_2 \end{array} \right) , \tag{4.11}$$

with

$$C_1 = \mathrm{diag}(c_1^{(1)}, \ldots, c_1^{(p)}) \ , \ C_2 = \mathrm{diag}(c_2^{(1)}, \ldots, c_2^{(q)}) \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{4.12}$$

By genericity, also the coupling h must be split into $k+1$ different couplings $h_0, \ldots, h_k$. This introduces a first (trivial) source of explicit breaking and, naively, one could think this is enough to our purposes. So, as a first step, let us suppose that this is the only explicit breaking source. If we collect the fields S' and V' in a vector

$$\phi^T \equiv (S_1, \ldots, S_k, V_1, \ldots, V_k) \ , \tag{4.13}$$

and repeat the same analysis of the previous section, we end up at low energy with the following O'R model

$$W_{eff} = fS_0 + S_0 \sum_{I,J=1}^{k} C^{IJ} V_I V_J - \sum_{I=1}^{k} h_I \Lambda S_I V_I \tag{4.14}$$

whose mass and Yukawa matrices are

$$m = -\Lambda \, \mathrm{diag}(h_1, \ldots, h_k) \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \lambda = C \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} . \tag{4.15}$$

This corresponds, again, to a theory with $\det m \neq 0, \det \lambda = 0$ *and* R-charges equal to 0 or 2 only. Hence, less trivial deformations are needed, to reach our goal.

Let us first notice that since $R'(S_0) = R(S_0) = 2$, whenever a field V_I enters the C_1 block, hence appearing quadratically in the Yukawa coupling, then $R'(V_I) = R(V_I) = 0$ and $R'(S_I) = R(S_I) = 2$. For such fields, once G and hence C are fixed, there is no possible definition of $U(1)_{R'}$ allowing R' charges other than 0 or 2, independently of possible deformations of the superpotential. As a consequence our deformations will focus on the C_2 block, only.

Let us suppose there exists an $\boldsymbol{r_I}$ in the C_2 block with $\mathbf{1} \subset \boldsymbol{r_I} \otimes \boldsymbol{r_I}$ and let $\boldsymbol{r_J}$ be the representation coupled to $\boldsymbol{r_I}$ by C_2. We consider two possible modifications of the ITIY theory:

(a) Give a (large) mass to the singlet S_I by adding a superpotential term

$$\Delta W_{tree} = \frac{m_I}{2} {S_I}^2 \ , \ m_I \gtrsim \Lambda \ . \tag{4.16}$$

The effect of this term is to make $R'(S_I) = R'(V_I) = 1$, $R'(S_J) = 3$ and $R'(V_J) = -1$. At low energy S_I can be integrated out and a quadratic term for V_I is generated

$$\Delta W_{eff} = -\frac{h_I^2 \Lambda^2}{2m_I} V_I^2 \tag{4.17}$$

(notice that the mass of V_I is smaller than Λ). This way, we get at low energy a O'R model having some field with R-charge different from 0 and 2. On the other hand, since the invertibility of the m matrix is not affected by the above deformation, we still have $\det m \neq 0$ (and $\det \lambda = 0$). Hence, deformations of this type give models of type I.

(b) Perform deformation (4.16) for the index I *and* eliminate altogether the singlet S_J from the field theory content. One can easily see that this modification gives a different theory with $\det m = 0$, hence models of type II or III.

Notice that the same effect can obtained by setting $h_J = 0$. This way, the field S_J would remain as a free field, and hence it would not enter the dynamics. However, the theory would loose genericity, since there would be no symmetry reasons for the coupling $S_J V_J$ to be absent. Dropping the field from the theory, instead, while giving rise to the same low energy efferctive dynamics, keeps the UV theory generic.

The correspondence between the above deformations of the UV theory and the type of resulting generalized O'R models one gets at low energy, can be further clarified if one considers a configuration of parameters such that all couples (S_K, V_K) which do not undergo any deformation are integrated out. This can be achieved taking the corresponding h_K couplings sufficiently large. The resulting theory in terms of the light fields is

- type I, if we perform only modifications (a),
- type II (i.e. $\det \lambda \neq 0$), if we perform only modifications (b),
- type III (i.e. $\det \lambda = 0$), if we perform both (on different, independent indices).

The first question one should worry about is whether these deformations preserve the supersymmetry breaking mechanism and/or they give rise to supersymmetric vacua. Both deformations may allow, in principle, for a non zero VEV for the corresponding mesonic fields, and supersymmetric vacua are restored whenever these VEVs can be arranged to solve the quantum constraint (4.2). Indeed, an analysis of the full set of the F-term equations reveals that

- Any (a)-modification introduces a runaway at $S_0 \to \infty$,
- any (b)-modification introduces a runaway at $S_0 \to 0$.

This result matches the type of the resulting O'R. In fact, the runaway directions are exactly those expected in an O'R of type I (for (a) modifications), type II (for (b) modifications) or type III (for both), which where discussed in the previous chapter. Notice that the runaways are found using the superpotential obtained in the small V' approximation. Hence, one has to check in the full theory, by solving the D-term equations along the putative runaway mesonic directions, whether these are real runaways or they lie at finite distance in field space.

In concrete examples, one has first to determine which region of the pseudomoduli space is classically stable. Then, one should see where (and if) supersymmetry breaking vacua are stabilized by quantum corrections, and finally check whether their lifetime is sufficiently long, as well as to what extent Kähler corrections coming from gauge dynamics may influence the whole analysis.

4.3 Breaking the flavor symmetry

In this section we would like to put the outlined strategy at work, and consider some concrete examples in detail. We will consider a specific global symmetry breaking pattern (a group G), implement an (a) or (b)-modification, and look at the low energy effective theory, once the confining gauge dynamics has taken place. We start by analyzing the case where the surviving global symmetry group G is the $SO(F)$ subgroup of $USp(2F)$ specified by the embedding

$$SO(F) \ni O \to \left(\begin{array}{c|c} O & 0 \\ \hline 0 & O \end{array}\right) \in USp(2F) \ . \tag{4.18}$$

This is possibly the simplest non-trivial choice one can make, but it is rich enough to let us address many of the issues outlined in the previous section. Moreover, it is a convenient first step for possible phenomenological applications, since one could easily embed a GUT group into $SO(F)$. In the second part of this section we will discuss other possibilities for G.

4.3.1 SO(F) flavor symmetry

Under the $SO(F)$ defined by the embedding (4.18) the S' and V' fields defined in eq.s (4.9) decompose according to

$$S' = \left(\begin{array}{c|c} S_1 & S_3 \\ \hline -{S_3}^T & S_2 \end{array}\right) , \ V' = \left(\begin{array}{c|c} V_1 & V_3 \\ \hline -V_3^T & V_2 \end{array}\right) \ , \tag{4.19}$$

where S_1, S_2, V_1 and V_2 are antisymmetric tensors of $SO(F)$, S_3, V_3 are traceless tensors, and we have chosen a basis in which

$$J = \left(\begin{array}{c|c} 0 & -\mathbf{1}_F \\ \hline \mathbf{1}_F & 0 \end{array} \right) . \tag{4.20}$$

The $USp(2F)$ quadratic invariant is rewritten as

$$\mathrm{tr}\ \left[JV'JV'\right] = 2\left(V_3^2 - V_1V_2\right) \tag{4.21}$$

where traces on $SO(F)$ indices are understood, so that in the basis (V_1, V_2, V_3) we have

$$C = \tilde{h} \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} . \tag{4.22}$$

There are now two paths we can follow. This form of the C matrix allows one to consider either modification (a) or (b) on the index 1 (equivalently the index 2; being $C^{33} \neq 0$ instead, no deformations can be introduced for the index 3). In the former case we obtain the tree level superpotential

$$W_{tree} = h_0\Lambda S_0V_0 + h_1\Lambda S_1V_1 + h_2\Lambda S_2V_2 + h_3\Lambda S_3V_3 + \frac{m_1}{2}{S_1}^2 , \tag{4.23}$$

which is generic under the $SO(F) \times U(1)_{R'}$ global symmetry, with R' charge assignment

	S_0	S_1	S_2	S_3	V_0	V_1	V_2	V_3
R'	2	1	3	2	0	1	-1	0

As specified in the previous section, we consider $m_1 \gtrsim \Lambda$. Moreover, since V_3 is forced to have 0 R-charge and cannot undergo any deformation, for simplicity we will take $h_3 \gg h_{1,2}$. This way, we can integrate out S_1, S_3, V_3.

Solving the quantum constraint, at energies below the scale Λ one gets the effective superpotential

$$W_{eff} = fS_0 - 2\tilde{h}\, S_0V_1V_2 + h_2\Lambda S_2V_2 - \frac{h_1^2\Lambda^2}{2m_1}V_1^2 . \tag{4.24}$$

This is an O'R-like superpotential of the general form (3.12), with S_0 playing the role of the pseudomodulus. Collecting the other low energy fields in the vector $\phi^T \equiv (S_2, V_1, V_2)$ we get

$$m = \begin{pmatrix} 0 & 0 & \frac{1}{2}h_2\Lambda \\ 0 & -\frac{h_1^2\Lambda^2}{2m_1} & 0 \\ \frac{1}{2}h_2\Lambda & 0 & 0 \end{pmatrix} , \ \lambda = -\tilde{h} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{4.25}$$

Hence, we end up with $\det m \neq 0, \det \lambda = 0$ and R-charges other than 0 or 2, that is a type I model. At the classical level the pseudomodulus is locally stable in a finite region around the origin and there is a runaway for $S_0 \to \infty$. A simple version of this superpotential with no flavor symmetry was studied in [47].

It is perhaps more interesting to choose the other option. If we perform a type (b) deformation on the index 1, we obtain the following superpotential

$$W_{tree} = h_0 \Lambda S_0 V_0 + h_1 \Lambda S_1 V_1 + \lambda_3 \Lambda S_3 V_3 + \frac{m_1}{2} {S_1}^2. \tag{4.26}$$

Under the same assumptions as before, we are now led to the effective superpotential

$$W_{eff} = f S_0 - 2\tilde{h}\, S_0 V_1 V_2 - \frac{h_1^2 \Lambda^2}{2m_1} V_1^2, \tag{4.27}$$

which is again of the form (3.12), but now we are left with one field less and the matrices m and λ take the form

$$m = \begin{pmatrix} -\frac{h_1^2\Lambda^2}{2m_1} & 0 \\ 0 & 0 \end{pmatrix}, \; \lambda = -\tilde{h} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4.28}$$

This is a model with $\det m = 0$ and $\det \lambda \neq 0$, hence a type II model, with S_0 the pseudomodulus. This modified superpotential leads to runaway supersymmetric vacua at $V_2 \to \infty$, $S_0 \to 0$ and the pseudomoduli space is classically stable everywhere but in a neighborhood of the origin. An analysis of D-terms equations in terms of the original electric variables reveals that, along the D-flat direction V_2, an ADS superpotential is generated by the dynamics of the unbroken gauge group. Therefore, in this case the approximation of small V' gives a result which is reliable even in the complete theory.

In summary, choosing $G = SO(F)$, we see one can construct models of both type I and II (the symmetry breaking pattern is too simple to allow for independent deformations of type (a) and (b) so, in order to get type III models one should look for less simple global symmetry breaking patterns). The question of the actual existence of the local supersymmetry breaking vacua and their lifetime can be addressed with a calculation of the Coleman-Weinberg potential, and by evaluating, possibly, the magnitude of uncalculable Kähler corrections around such minima. In the following we address these two issues in turn.

Loop corrections and the metastable vacuum

In order to show that the model (4.27) developes a parametrically long lived, R-symmetry breaking metastable vacuum at one loop, we can simply refer to the example of an R-breaking O'R model that we discussed in section 3.2. Indeed the superpotential (4.27) is analogous to the superpotential (3.20) encountered in that case, the only difference being the promotion of ϕ_1 and ϕ_2 to multiplets of the flavor symmetry. Up to an overall normalization and to the obvious mapping of parameters, the Coleman-Weinberg potential for the pseudomodulus is the same. Therefore, the discussion about the existence of metastable vacua and their lifetime can be directly read from there, and we repeat here the conclusions: in the limit of small $y = \frac{\tilde{h}f}{\tilde{m}^2} \lesssim 10^{-3}$, where $\tilde{m} = \frac{h_1^2\Lambda^2}{m_1}$ is the mass of the V_1 field, there exists a parametrically long-lived metastable and R-symmetry breaking vacuum at $S_0 \simeq \frac{\tilde{m}}{2\tilde{h}}(0.249 + \mathcal{O}(y^2))$.

Kähler corrections and calculability

The ITIY model, as well as any of the deformations we presented in section 4.2, is an instance of uncalculable DSB model. Therefore, Kähler potential corrections coming from gauge theory dynamics at scale $\gtrsim \Lambda$ could in principle affect the low energy effective theory, and spoil the quantum analysis performed above. Following the discussion of [73], we want to estimate such corrections and compare them to the one loop effective potential contributions. If the latter are dominant, at least in some region of parameter space, then the calculation performed in terms of the low energy degrees of freedom is reliable and the metastable vacuum survives the embedding in the UV theory.

For definiteness, we keep on focusing on the example (4.27), but most of the present considerations have wider applicability. Since we are interested in the quantum lifting of the tree-level pseudo-flat direction, we can restrict the Kähler potential to the pseudomoduli space, after the massive fields S_1, S_3 and V_3 have been integrated out. First, notice that the holomorphic decoupling of such fields is expected to produce non-canonicity of the effective Kähler potential. However, these corrections are largely suppressed in the hierarchical regime

$$h_3 \gg h_1 \ , \ m_1 \gg h_1\Lambda \ , \tag{4.29}$$

in which those fields can be integrated out. The Kähler potential for the remaining fields is constrained by the global symmetry to have the form (recall that we have chosen a point of maximal symmetry on the moduli space, which constrains the Kähler potential to be diagonal in the effective fields)

$$K = {S_0}^\dagger S_0 + V_1^\dagger V_1 + \ V_2^\dagger V_2 + \Lambda^2 \ \mathcal{G}(\tilde{h}S_0/\Lambda, \tilde{h}{S_0}^\dagger/\Lambda) \ , \tag{4.30}$$

where

- the real function $\mathcal{G}$ is parametrizing our ignorance of the gauge loop corrections, and depends only on S_0 since we are restricting to the pseudomoduli space;
- the prefactor Λ^2 gives vanishing corrections in the classical limit $\Lambda \to 0$;

- the combination $\tilde{h}S_0/\Lambda$ appears because the only way gauge interactions know of the singlet is through the tree level quark masses $\sim \tilde{h}\langle S_0\rangle$.

This shows that the first corrections are of the form

$$\mathcal{G} \sim \frac{h^4 (S_0 {S_0}^\dagger)^2}{\Lambda^4} + \mathcal{O}\left(\frac{\tilde{h}^6 (S_0 {S_0}^\dagger)^3}{\Lambda^6}\right) \ , \tag{4.31}$$

giving a term in the effective potential of order

$$\Delta V = -\Lambda^2 \left(\partial_{S_0}\partial_{{S_0}^\dagger}\mathcal{G}\right) |\partial_{S_0} W_{eff}|^2 \sim \Lambda^2 \tilde{h}^6 |S_0|^2 \ . \tag{4.32}$$

On the other hand, the CW contribution is of order $\sim m^4$ where m is the typical mass of the light IR degrees of freedom entering the loops. In the present case, these masses are given by $\tilde{m} = h_1^2\Lambda^2/m_1$ and $h\langle S_0\rangle$. Therefore, suppression of uncalculable corrections requires

$$\tilde{h}\Lambda \ll \langle S_0\rangle \ll \frac{\tilde{m}^2}{\tilde{h}^3\Lambda} = \frac{h_1^4\Lambda^3}{4\tilde{h}^3 m_1^2} \tag{4.33}$$

which, in terms of dimensionless parameters $y = \frac{\tilde{h}f}{\tilde{m}^2}$ and $z = \tilde{h}S_0/\tilde{m}$, recalling the definitions (4.7), reads

$$\tilde{h}\sqrt{y} \ll z \ll \frac{1}{\tilde{h}\sqrt{y}} \ . \tag{4.34}$$

Since the local minimum seats at $\bar{z} \simeq 0.249 + \mathcal{O}(y^2)$ these inequalities are trivially satisfied in the limit of small y and h. It is amusing to notice that the limit of small y provides both a long lifetime and small Kähler corrections.

Let us recap the discussion above and take a closer look to the hierarchies we need, in order to have a safe local minimum. First, when giving a mass to the singlet S_1, we have chosen $m_1 \gtrsim \Lambda$. This ensures that at low energy S_1 can be integrated out and the corresponding mesonic field V_1 gets a small mass $\tilde{m}_1 = h_1\Lambda^2/2m_1$ (we will always consider $h_I < 1$ so that all dynamically generated masses $h_I\Lambda$ are below the dynamical scale). Then, for simplicity, we have chosen the field pair which is not modified, (S_3, V_3), to be much heavier than the other pairs, and this can be accomplished by a larger value for the corresponding coupling, $h_3 \gg h_{1,2}$. As we have seen, the existence of a local minimum with a long lifetime and suppressed uncalculable corrections are both controlled by the smallness of one single parameter,

$$y = \frac{f\tilde{h}}{\tilde{m}^2} = 2\left(\frac{h_0}{h_1^2}\right)^2\left(\frac{m_1}{\Lambda}\right)^2 \ . \tag{4.35}$$

The requirement of small y forces a small value for $\sqrt{h_0}$: for instance, if $m_1/\Lambda \sim 10$ then $y \lesssim 10^{-3}$ implies $\sqrt{h_0} \lesssim 10^{-1.25}h_1$. Notice that as for any dynamical model, in this model both the supersymmetry breaking scale $f = \sqrt{2F}h_0\Lambda^2$ and the masses of the low energy O'R model $h_I\Lambda$ are related to one and the same dynamical scale Λ. Therefore, it is not surprising that a (modest) tuning between dimensionless parameters is necessary to obtain metastable supersymmetry breaking. The limit of small y can indeed be simply reinterpreted as the limit of small vacuum energy with respect to the scale set by the masses, and, from the expressions of f itself, it is clear that this requires $\sqrt{h_0}$ to be small compared to all other h's.

4.3.2 Other breaking patterns

While the class of models we discussed above is general enough to make the strategy manifest, it is clearly not the most general option one can think of. For instance, as we have already noticed, the possibility of making independent (a) and (b) deformations requires a more involved symmetry breaking pattern. Moreover, in view of phenomenological applications, having SU global symmetry groups, besides SO groups, might also be interesting. In what follows, we want to make a few comments on both these options. We will not discuss the vacuum structure in any detail, nor the dynamics around the supersymmetry breaking minima, but just limit ourselves to display the basic structure of the emerging low energy effective theories.

• $SO(n)\times SO(F-n)$

The simplest step we can take beyond the $SO(F)$ models we analyzed before, is to consider the group G to be $G=SO(n)\times SO(F-n)$, with $1<n<F$. Under such G, the $SO(F)$ components of V and S defined in eqs.(4.9) decompose as follows

$$V_I=\left(\begin{array}{c|c} V_I^{(n)} & W_I \\ \hline -W_I^T & V_I^{(F-n)} \end{array}\right)\ ,\quad S_I=\left(\begin{array}{c|c} S_I{}^{(n)} & T_I \\ \hline -T_I^T & S_I{}^{(F-n)} \end{array}\right)\ ,\quad I=1,\,2,\,3\ , \tag{4.36}$$

where W_I and T_I are $n\times(F-n)$ "bi-vectors" of the two SO factors. In the basis

$$(V_1^{(n)},\,V_2^{(n)},\,V_1^{(F-n)},\,V_2^{(F-n)},\,W_1,\,W_2), \tag{4.37}$$

the matrix C_2 takes the form

$$C_2=\tilde{h}\left(\begin{array}{cc|cc|cc} 0 & -1 & & & & \\ -1 & 0 & & & & \\ \hline & & 0 & -1 & & \\ & & -1 & 0 & & \\ \hline & & & & 0 & 1 \\ & & & & 1 & 0 \end{array}\right). \tag{4.38}$$

Clearly, there are now much more options for the modified theory: each off-diagonal two-by-two block can undergo deformations (a) or (b). Let us consider a possibility which was not available in the simpler case $G = SO(F)$, and make a deformation (a) for the first two blocks together with a deformation (b) for the last one. The starting point is therefore a generic tree level superpotential of the form

$$W_{tree} = h_0 \Lambda S_0 V_0 + \Lambda \sum_{I=1}^{3} \left[h_I^{(n)} V_I^{(n)} S_I{}^{(n)} + h_I^{(F-n)} V_I^{(F-n)} S_I{}^{(F-n)} \right]$$
$$+ h_1' \Lambda T_1 W_1 + h_3' \Lambda T_3 W_3 + \frac{m_1^{(n)}}{2} S_1{}^{(n)2} + \frac{m_1^{(F-n)}}{2} S_1{}^{(F-n)2} + \frac{m_1'}{2} T_1^2 \ . \qquad (4.39)$$

Just like in the simpler $SO(F)$ case, no deformations can be made for the $I = 3$ fields and, for simplicity, we choose the parameters so that these fields can be integrated out. Solving the quantum constraint and integrating out all other heavy fields, one finally gets the effective superpotential

$$W_{eff} = f S_0 + 2\tilde{h} S_0 \left[W_1 W_2 - V_1^{(n)} V_2^{(n)} - V_1^{(F-n)} V_2^{(F-n)} \right]$$
$$+ h_2^{(n)} \Lambda S_2{}^{(n)} V_2^{(n)} + h_2^{(F-n)} \Lambda S_2{}^{(F-n)} V_2^{(F-n)}$$
$$- \frac{h_1^{(n)2} \Lambda^2}{2 m_1^{(n)}} V_1^{(n)2} - \frac{h_1^{(F-n)2} \Lambda^2}{2 m_1^{(F-n)}} V_1^{(F-n)2} - \frac{h'^2 \Lambda^2}{2m'} W_1^2 \ , \qquad (4.40)$$

which describes a model with $\det m, \det \lambda = 0$, i.e. a type III model. Notice, in passing, that the number of light fields typically diminishes, the more deformations (a) and/or (b) one does. This might be a welcome feature for phenomenological applications.

• $SU(F-1)$

Here we consider a particular breaking pattern leading to theories with an $SU(F-1)$ flavor symmetry. First we introduce the embedding in $USp(2F)$

$$SU(F-1) \ni U \to \begin{pmatrix} \mathbf{1_2} & 0 & 0 \\ 0 & U & 0 \\ 0 & 0 & U^* \end{pmatrix} . \qquad (4.41)$$

The field content can be arranged in self-conjugate (reducible) representations in the following way

(4.42)

	$SU(F-1)$
V_1, S_1	$\bullet$
V_2, V_2	$\left(\mathbf{F-1} \oplus \overline{\mathbf{F-1}}\right) \otimes_{\mathbf{A}} \left(\mathbf{F-1} \oplus \overline{\mathbf{F-1}}\right)$
V_3, S_3	$\mathbf{F-1} \oplus \overline{\mathbf{F-1}}$
V_4, $S4$	$\mathbf{F-1} \oplus \overline{\mathbf{F-1}}$.

The quadratic invariant is

$$\mathrm{tr}_{2\mathrm{F}}\left[J_{2F}V'J_{2F}V'\right] = 2V_1^2 + V_2^2 + 2V_4V_3 - 2V_3V_4 = 2V_1^2 + V_2^2 + 4V_4V_3 \ , \qquad (4.43)$$

and from it one can see the form of the matrix C.

The only possible deformation involves the fields $3,4$ and it can be of type (a) or (b). This deformation breaks $USp(2F-2)\times SU(2)\to SU(F-1)$ and forces the R-charge of $V_{3(4)}$ to be $1(-1)$, whereas the other V fields remain uncharged.

In order to implement the deformation, we write in this case a quadratic term for S_3 which breaks $USp(2F-2)$ while preserving a $SU(F-1)$. The choice is unique (up to a multiplicative factor) and reads

$$S_3^2 \equiv {S_3}^T \begin{pmatrix} 0 & \mathbf{1_{F-1}} \\ \mathbf{1_{F-1}} & 0 \end{pmatrix} S_3 \ . \qquad (4.44)$$

Adding a large mass term of this form to the superpotential we obtain

$$W_{tree} = h_0\, S_0V_0 + h_I\, S_IV_I + \frac{m}{2}S_3^2. \qquad (4.45)$$

After integrating out S_3, S_1, S_2, V_1, V_2 we end up with a type I O'R superpotential

$$W_{eff} = f\, S_0 + 4h\, S_0V_4V_3 + \Lambda h_4\, S_4V_4 + \frac{\Lambda^2 h_3^2}{m}V_3^2 \ , \qquad (4.46)$$

or a type II superpotential

$$W_{eff} = f\, S_0 + 4\tilde{h}\, S_0V_4V_3 + \frac{\Lambda^2 h_3^2}{m}V_3^2 \ , \qquad (4.47)$$

if we perform a (b) deformation, instead. In terms of fields transforming in irreducible representations, the last equation reads

$$W_{eff} = f\, S_0 + 4\tilde{h}\, S_0\left(\tilde{V}_4V_3 - \tilde{V}_3V_4\right) + 2\frac{\Lambda^2 h_3^2}{m}\tilde{V}_3V_3 \ , \qquad (4.48)$$

where tilded fields transform in the anti-fundamental and untilded in the fundamental of $SU(F-1)$.

4.4 Discussion

In this chapter we have constructed DSB models which at low energy reduce to O'R-like models admitting supersymmetry breaking vacua where also the R-symmetry is spontaneously broken. Starting from well known generalizations of the ITIY model, we have explained a precise pattern to modify the microscopic theory so to get at low energy models falling in all three classes of EOGM [48], which was proposed in [49]. In the second part of the chapter we focused on a concrete (class of) example(s) and discussed in some detail the low energy effective theory both at classical and quantum level. Interestingly, the same window which allows for a long-lived metastable vacuum, makes the perturbative analysis of the effective theory reliable against corrections coming from Λ-dependent Kähler potential contributions.

These models are interesting in that they are SQCD-like theories which realize dynamical supersymmetry breaking in a metastable vacuum, similarly to the ISS model but in a different phase. Namely, ISS vacuum is in the free-magnetic phase of the theory, while the vacua we have discussed arise in the case of quantum-modified moduli space (the analogue of $N_f = N_c$ for $SU(N_c)$ gauge group) and they are enforced by the presence of additional singlet fields in the theory. To our knowledge, this is the first example of DSB in a metastable vacuum occurring in this phase.

On a more phenomenological side, these models are promising as direct gauge mediation models. Of course, when it comes to construct fully fledged phenomenological models, one should take care of many issues. For instance, in models of direct mediation one thing to worry about are Landau poles. A possible direction could be to consider a breaking pattern with e.g. $G = SO(F-n) \times SO(n)$, weakly gauge the $SO(n)$ GUT group (take for definiteness $n = 10$) and implement enough deformations of type (a) and/or (b) so not to have too many messengers around. Seemingly, one could consider to break the original global symmetry group to several unitary groups, e.g. $G = SU(F-1-n) \times SU(n)$ (take now $n = 5$). Notice that since the UV theory is non-chiral, all messengers would come into real representations so one should not worry about $SU(5)$ gauge anomalies.

The construction described here seems flexible enough to let one cover a sizable region of the parameter space of supersymmetry breaking models admitting a weakly coupled low energy description of the type discussed in the previous chapter. The basic phenomenological potential outcome is to provide models of direct gauge mediation with spontaneous R-symmetry breaking and unsuppressed gaugino mass.

Chapter 5

Strongly Coupled Hidden Sectors via Holography

In the first part of this thesis we discussed the General formulation of Gauge Mediation models, and we explored their basic properties by means of calculable, weakly coupled hidden sectors, which can eventually be promoted to dynamical theories, along the lines that we described. In what follows, we will address the possibility that the hidden sector is an inherently strongly coupled field theory and assume that it can be described by a dual gravitational theory. Via holography, we will be able to explore the main features of the spectrum in this setup, and to answer important questions such as to what extent the parameter space of GGM is covered by the set of strongly coupled hidden sectors.

We will rely on the fact that, in GGM, all information needed about the hidden sector is encoded in two-point functions of the multiplet of a conserved current. Holography gives indeed a precise and practical prescription to compute correlators of gauge-invariant operators in a strongly coupled field theory, and can therefore be used to obtain the GGM current correlators even when the hidden sector is not treatable with standard field theory techniques. In this approach, which goes under the name of Holographic General Gauge Mediation (HGGM) [74], the details of the theory, and the mechanism which breaks supersymmetry, are rephrased in terms of the geometry in the gravity dual.

Constructing fully viable models of this sort, at the phenomenological level, is beyond the scope of the present treatment. The main obstacle, in that respect, is that theories that are described by a gravity dual must have some sort of large N limit, and this large number of degrees of freedom is problematic when the hidden sector is coupled to the visible one. On the other hand, since N enters the interesting correlators only as an overall normalization, one can still extract physically meaningful information by considering their ratios, and the qualitative features are expected to be trustable beyond the large N limit. Moreover, the analysis that we will perform has a validity and a theoretical interest that goes beyond the application to Gauge Mediation. In fact, it rests on the general question of studying the behavior of operators related by supersymmetry, and in particular how supersymmetry breaking enters their correlators. In this approach, one can see such correlators as simple and calculable probes of the supersymmetry breaking dynamics, whose nature is not always transparent in the geometry.

This chapter is organized as follows: we will start with a quick review of the idea of holography, and of the prescription to compute correlators of gauge invariant operators via the gravity dual; then, we will explain why the particular gravity theory of interest for our analysis is an $\mathcal{N} = 2$ gauged supergravity in $5D$, whose building blocks and gross features we will review; finally, we will deepen about the calculation of correlators, outline the procedure of holographic renormalization, and apply it to the case of interest, namely to a massless vector multiplet of the supergravity theory.

The application of this machinery to concrete supersymmetry breaking solutions, and the analysis of the outcome, is postponed to the next chapters.

5.1 The holographic correspondence

The statement of holography [75, 76] is that a certain quantum gravity theory in a $D + 1$ dimensional space-time with a boundary, is equivalent to a quantum theory *without* gravity living on the D dimensional boundary, and it originated from the investigation of a microscopic explanation to the area-law for the entropy of black holes. A precise formulation can be given if the gravity theory lives on a space-time that asymptotically has the geometry of AdS_{D+1}. In this case, the space-time has a time-like conformal boudary which is conformally equivalent to a Minkowski (flat) space-time. In order for the dynamical problem to be well-defined in such space, the fields in the gravity theory must be assigned a fixed value on the boundary, for all times. This may sound strange compared to more usual evolution problems in flat space, that require initial values to be specified on a fixed time space-like surface, and then determine the behavior at subsequent times. However, the dependence of the gravity theory on these boundary values is actually at the core of the correspondence.

Indeed, for the reason just explained, the observables in the quantum gravity theory, and in particular the partition function, will be functional of these fields defined on the boundary Minkowski space-time. The field content may vary depending on the theory, but one field in particular must be present, namely the $D+1$ dimensional metric giving the graviton, whose boundary value is a metric in D dimensions. On the other hand, a quantum field theory in a D dimensional space also naturally defines functionals of D-dimensional fields. For instance, the generator of correlation functions is a functional of the external sources of the operators. An operator which is universally defined in QFT is the energy-momentum tensor, hence the generator will always depend on its source, which is a metric in D dimensions. The holographic correspondence in this case can be formulated as an identification between the two functionals defined in the two quantum theories [77, 78][1]

$$Z_{grav}[g_{ij}, J_a] = \left\langle e^{-\int d^D x \sqrt{g}\left(g_{ij}T^{ij} + \sum_a J_a \mathcal{O}_a\right)} \right\rangle_{QFT}, \tag{5.1}$$

where $i, j = 1, \ldots, D$ and we have schematically indicated by $\mathcal{O}_a$ the sets of operators in the QFT and with J_a the corresponding source. Already from this general formula we can derive some properties of the way the correspondence works: QFT correlators reflect the response of the gravity partition function to a change of the boundary conditions. For any gauge-invariant local operator $\mathcal{O}_a$ in the quantum field theory, there is a corresponding field on the gravity side whose boundary value is the source J_a. For instance, a global symmetry of the quantum field theory entails a conserved-current operator, whose source is a gauge field, and there must exist a gauge boson in the $D + 1$ theory whose boundary value is the source of the current. Therefore, a global symmetry on the field theory side gets mapped to a gauge symmetry on the gravity side. In the same spirit, a space-time symmetry of the QFT corresponds to an isometry on the gravity side.

What explained so far is still rather abstract, both because neither of the sides of the correspondence has been specified, and also because neither of the two functionals is calculable without resorting to some approximation scheme (i.e. perturbation theory in some small parameter or semiclassical limit). The first and well-known example in which the correspondence is at work, is that between type IIB string theory on $AdS_5 \times S^5$, with N units of F_5 flux on S^5, and $\mathcal{N} = 4$, $SU(N)$ Super Yang-Mills (SYM) theory. In this case the correspondence can be motivated starting from type IIB string theory in flat $10D$ space-time, with a stack of N parallel $D3$ branes [79]. The low-energy theory living on the stack of branes is $\mathcal{N} = 4$, $U(N)$ SYM theory. The additional $U(1)$ in the gauge group is related to the overall position of the branes, and it decouples from the rest of the dynamics (moreover it can be disregarded in the large N limit that we are going to consider soon). On the other hand, one can see the stack of branes as a black-brane solution in type IIB supergravity. Hence, in the low energy limit, taking into account the redshift caused by the localized objects, one is just left with the string modes which live in the near-horizon geometry of the black-brane solution, this geometry being exactly $AdS_5 \times S^5$. In this specific example, both sides of the correspondence come with parameters which make the theory under control in some regime.

On the field theory side, we have the gauge coupling g_{YM}, associated to the

[1]Here, for simplicity, we consider just the formulation in Euclidean signature, so that the natural observables in the QFT side are correlation functions of operators, and there is no global issue in the choice of coordinates for AdS.

usual perturbative expansion,[2] and the number of colors N, associated to the large N expansion. All fields live in the adjoint representation of the gauge group $SU(N)$, i.e. they are $N \times N$ matrices, and every gauge-invariant operator built out of such fields will have the form of a trace of products of matrices, or of products of such traces

$$\mathrm{Tr}[\Phi_1 \dots \Phi_n]\,, \quad \mathrm{Tr}[\Phi_1 \dots \Phi_k]\,\mathrm{Tr}[\Phi_{k+1} \dots \Phi_m], \quad \dots \tag{5.2}$$

Therefore, gauge-invariant operators can be classified as single-trace, double-trace, and so on. In the large N limit with the 't Hooft coupling $\lambda = g_{YM}^2 N$ finite, correlators of single-trace operators factorize as products of one-point functions, so that the limit can be interpreted as a classical one (different from the usual, free-theory limit $g_{YM} \to 0$). Moreover, insertions of multiple-trace operators are suppressed in this limit. The diagrammatic expansion can be organized as a sum over surfaces of different topologies, weighted by a factor of N^{2-2g}, where g is the genus of the surface, so that the leading contribution comes from planar diagrams, and increasingly complex topologies give more and more negligible contribution. The surface is defined by the fact that the diagram can be drawn on it without self-intersections.

On the string theory side, the parameters are given by the string coupling constant g_s and by two dimensionful parameters, the string length l_s and the curvature radius of the background R. The coupling g_s controls the loop expansion, which closely resembles the one we have just described for the field theory diagrams in the large N limit: higher loops corrections in the string amplitude imply higher genus of the corresponding world-sheet, and each diagram comes with a factor $g_s^{-\chi}$. This fact suggests that a sensible correspondence between parameters should map the small g_s expansion on one side with the large N expansion on the other side. Since the Yang-Mills interactions on the world-volume of the $D3$ branes is due to the zero-modes of open strings ending on them, one has the identification

$$g_{YM}^2 = 4\pi g_s. \tag{5.3}$$

Recalling that in the large N limit one keeps the 't Hooft coupling fixed, we can write

$$\frac{\lambda}{4\pi}\,\frac{1}{N} = g_s, \tag{5.4}$$

so that large N corresponds to small g_s, and indeed the two expansions are mapped into each other.

In the black-brane solution, the curvature radius R (i.e. the common radius of the five-sphere and of AdS_5) is fixed in terms of the string length and of the

[2] Notice that the theory we are considering is exactly conformal even at the quantum level, so that the gauge coupling is really a parameter that we can choose, and is not traded with a scale at the quantum level, as would be the case for an asymptotically free theory.

Ramond-Ramond flux by the relation

$$R^4 = 4\pi g_s N l_s^4, \tag{5.5}$$

which implies

$$\lambda = N g_{YM}^2 = \left(\frac{R}{l_s}\right)^4 . \tag{5.6}$$

Here we see that when the gauge theory is in the perturbative regime, $\lambda \ll 1$, the geometry where strings propagate is highly curved, and it is not known how to calculate the complete spectrum of string excitations, much less how to quantize the theory. On the other hand, when the field theory is strongly coupled, $\lambda \gg 1$, the string length is negligible with respect to the typical scale of the geometry on which strings are propagating. In this regime, string theory should be captured by a field theory approximation, meaning that we can just keep the zero-modes and neglect higher excitations, whose mass-squared will be of order $l_s^{-2}(1 + \mathcal{O}(l_s^2/R^2))$. The resulting theory is type IIB supergravity on $AdS_5 \times S^5$. In this case, to leading order in g_s, the partition function on the gravity side can be evaluated by a saddle-point approximation, in terms of the on-shell action for the supergravity fields with the appropriate boundary conditions

$$Z_{grav}[g_{ij}, J_a] \approx e^{-S^{o.s.}_{sugra}|_{G_{\mu\nu}\to g_{ij}, \mathcal{J}_a \to J_a}} , \tag{5.7}$$

where $G_{\mu\nu}$ is the $D+1$-dimensional metric and $\mathcal{J}_a$ indicates the supergravity field corresponding to a certain operator $\mathcal{O}_a$. Notice that, in the gravity theory, the answer will depend on which solution of the equations of motion we choose. In the dual field theory this ambiguity reflects the choice of the vacuum in which correlators are calculated.

To summarize, we first take the limit $g_s \to 0$, $N \to \infty$ with λ fixed. This leaves us with a free theory of strings propagating on $AdS_5 \times S^5$ on the gravity side, and with a free theory (due to factorization) of matrices of infinite-size on the field theory side. Notice that the correspondence is telling us something very non-trivial at first glance, namely that the classical configuration which dominates the path integral of the field theory at large N is a theory of ten-dimensional strings. However, neither of the two theories, despite being free at leading order, is tractable for generic values of λ. In the field theory, we know how to characterize the operators, their anomalous dimensions and OPE coefficients only when λ is small. In the string theory, we know the spectrum of excitations and their interactions only when λ is large. Therefore, the correspondence takes the form of a weak/strong duality between the two theories. The direction of the correspondence which is of interest for our applications is to consider $\lambda \gg 1$, and use a supergravity action to calculate field-theory correlators.

Let us just mention that in the last decade a great advancement has been achieved in extending the test of the correspondence to finite values of λ, by using integrability techniques (see the review [80] and reference therein).

5.1.1 Generalizations

A natural question at this point is whether other examples of the holographic correspondence exist, and which of the features we described can have more general validity. The previous example was motivated by considering a stack of parallel $D3$ branes in flat $10D$ space-time: in this case, before considering backreaction, the 6 dimensions transverse to the world-volume of the branes are flat and homogeneous. It turns out that a first extension arises if one allows the existence of singularities at some point in the 6 transverse dimensions. If the branes are located at this special point, both the low-energy gauge theory living on their world-volume and their near horizon geometry get modified. Therefore, following the same logic we outlined in the previous section, one can derive a holographic correspondence between different pairs of theories [81–85]. For instance, if the geometry of the transverse dimensions is a Calabi-Yau cone over a compact $5D$ Sasaki-Einstein manifold X_5, the near horizon geometry of the branes located at the tip of the cone is $AdS_5 \times X_5$, and the number of conserved supercharges in both the dual theories is reduced in general from 32 to 8.

One can also consider a simplified version of the correspondence involving a $5D$ gravity theory on AdS_5. This can be justified starting from type IIB supergravity on $AdS_5 \times X_5$, by truncating in a consistent way the gravity theory so to keep only a finite number of Kaluza-Klein modes of the $10D$ fields. When the compact manifold is S^5, if one just keeps the lowest modes, the resulting theory is the maximally supersymmetric gravity theory on AdS_5, namely $\mathcal{N} = 8$ gauged supergravity [86,87]. This theory, in turn, can be further consistently truncated to less supersymmetric theories with reduced field content. In the dual field theory, a truncation corresponds to restricting to a certain subset of operators. Another way to get less supersymmetric theories in $5D$ is to start with a more general Sasaki-Einstein manifold X_5 replacing the five-sphere, giving rise to an $\mathcal{N} = 2$ gauged supergravity in $5D$.

What we have briefly described until now are examples motivated by brane dynamics in string theory. However, it is believed that a holographic correspondence exists in a broader class of theories. Indeed, nowadays it is often applied in this more general context, possibly in cases where only one of the two dual theories is known in detail. Therefore, let us mention, even more generally, what requirements are believed to be necessary in order for a field theory to admit a gravity dual [88], on the basis of the known examples. First, a large N limit is necessary in order to get a weakly-coupled gravitational theory, and suppress quantum effects. The possibility to distinguish single-particle and multi-particle states in the weakly-coupled gravity theory is reflected in the classification of operators as single-trace or multiple-trace.

Secondly, in order to be described in terms of a finite and possibly small number of fields with a local Lagrangian in the gravity dual, the field theory should have a large gap in the operator dimensions, with a finite set of operator with small dimension which dominate the dynamics. In the case we discussed, the parameter λ provides such gap, by giving large anomalous dimension $\sim \lambda^{1/4}$ to operators which are not protected by supersymmetry. Indeed, exactly the limit of large λ permits to neglect the tower of string excitations, keeping only the supergravity modes.

When the $5D$ background is AdS_5, whose isometry group $SO(4,2)$ is the conformal group in $4D$, the dual field theory enjoys conformal symmetry, the dilations being mapped to translations in the extra-dimension of the gravity theory. Since we want to describe theories which dynamically break supersymmetry, and in particular are not conformal, it will be necessary to relax the homogeneity in the extra-dimension by adding scalar profiles to the geometry [89, 90]. From the field theory point of view, this amounts to turning on relevant perturbations of the interacting fixed point, by adding operators to the Lagrangian which are dual to the given non-trivial scalars in the geometry. Alternatively, the translational symmetry in the bulk coordinate $z \in \mathbb{R}^+$ can be broken by truncating the geometry at some value z_*. The gravity fields must be assigned additional boundary conditions on the "wall" $z = z_*$. This class of models, going under the name of hard wall models, have the advantage of being easily calculable, but their interpretation in terms of the field theory is often less transparent.

For our scope, instead of deriving the correspondence in a systematic way by starting with a brane construction and reducing consistently the resulting gravity theory, it will suffice to follow a more effective approach, by focussing on symmetry requirements. The $4D$ field theory we would like to describe has $\mathcal{N} = 1$ supersymmetry (which is then spontaneously broken, but this does not affect the counting of supercharges) and therefore has 4 conserved supercharges, which are enhanced to 8 supercharges in the deep UV due to superconformal symmetry. Therefore, the $5D$ gravity theory must also have 8 supercharges, making it an $\mathcal{N} = 2$ supergravity theory. Only half of them will be preserved by the the breaking of the conformal symmetry. We are going to consider both options to break conformality, namely hard wall models and non-trivial backgrounds of some scalar fields. In addition, since we want to model a hidden sector in a Gauge Mediation scenario, the field theory should have a global symmetry, which is then gauged to couple the theory to the visible sector. Consequently, the gravity dual should have a corresponding gauge symmetry. The multiplet of the global current, whose correlators we are going to calculate, will be mapped to a vector multiplet of the supergravity theory (see the next section). Finally, in order to pick a supersymmetry breaking vacuum in the field theory, in equation (5.7) we have to evaluate the on-shell supergravity action on a supersymmetry breaking solution.

5.2 $\mathcal{N} = 2$, $5D$ gauged supergravity

Since the gravity theory we will use to construct holographic hidden sectors is an $\mathcal{N} = 2$ gauged supergravity theory in $5D$, we will now recall the basic structure of this theory, referring to the literature for a more detailed account [91, 92]. The superalgebra in $5D$ with eight supercharges has an $SU(2)_R$ automorphism related to the symplectic-Majorana nature of irreducible spinors in $5D$, which acts as an R-symmetry on the theory and helps organizing the representations, similarly to what happens in the perhaps more familiar case of $\mathcal{N} = 2$ theory in $4D$. The representations of interest for constructing a gauged supergravity theory coupled to matter will be obviously that containing the metric, the one containing a vector boson, and the one with only scalars and their fermionic partners. Therefore, the supersymmetric multiplets we will use to build the theory consist of[3]

- one *graviton multiplet*
$$\{e_\mu^a,\, \psi_\mu^i,\, A_\mu\} \tag{5.8}$$
containing the graviton (5 real d.o.f.), one $SU(2)_R$ symplectic Majorana gravitino (8 real d.o.f.) and one vector (called the *graviphoton*, 3 real d.o.f.), a is a flat spacetime index, and $i = 1, 2$ is an $SU(2)_R$ index.

- n_V vector multiplets
$$\{A_\mu,\, \lambda^i,\, \phi\} \tag{5.9}$$
each containing a vector (3 real d.o.f.), an $SU(2)_R$ symplectic Majorana fermion (4 real d.o.f.) and one real scalar field.

- n_H hypermultiplets
$$\{\zeta^A,\, q^X\} \tag{5.10}$$
containing an $SU(2)$ symplectic Majorana fermion (4 real d.o.f.) and four real scalars, $A = 1, 2$ being an $SU(2)$ index and $X = 1, \ldots, 4$. Notice that in this case the $SU(2)$ used to implement the symplectic-Majorana condition is not the $SU(2)_R$ but a different one, inherent to the hypermultiplet representation. When there are n_H such multiplets together, this $SU(2)^{n_H}$ can get enhanced up to a $USp(2n_H)$.

Taking into account all the multiplets together and organizing the fields according to the spin, the theory we are considering contains, besides the graviton and the gravitinos

- $n_V + 1$ vector fields collectively denoted A_μ^I, $I = 0, \ldots, n_v$,

- n_V real scalars ϕ^x, $x = 1, \ldots, n_V$,

[3] We consider here only the on-shell degrees of freedom.

- $4n_H$ real scalars q^X, $X=1,\dots,4n_H$,
- n_V doublets of fermions λ^{ix}, $i=1,2$,
- and $2n_H$ fermions ζ^A, $A=1,\dots,2n_H$.

The interactions are governed by a σ-model with target scalar manifold

$$\mathcal{M}=\mathcal{S}(n_V)\otimes\mathcal{Q}(n_H), \tag{5.11}$$

with $dim_{\mathbb{R}}\mathcal{S}=n_V$, and it is parametrized by the coordinate ϕ^x, and $dim_{\mathbb{R}}\mathcal{Q}=4n_H$, parametrized by the coordinate q^X.

The scalar target manifold of vector multiplets $\mathcal{S}$ is a *very special manifold*, it is usually described in term of a cubic hypersurface

$$C_{IJK}h^Ih^Jh^K=1 \tag{5.12}$$

of an ambient space parametrized by coordinates $h^I=h^I(\phi)$, where C is a completely symmetric constant tensor. The metric on the ambient space is given in terms of the constant symmetric tensor as

$$a_{IJ}\equiv-2C_{IJK}h^K+3h_Ih_J,\quad h_I\equiv C_{IJK}h^Jh^K. \tag{5.13}$$

The tensor a_{IJ} and its inverse a^{IJ} are used to lower and raise indices. The induced metric on $\mathcal{S}$ is then given by

$$g_{xy}\equiv h_x^Ih_y^Ja_{IJ},\quad h_x^I\equiv-\sqrt{\frac{3}{2}}\partial_xh^I. \tag{5.14}$$

The hyperscalars span a quaternionic manifold $\mathcal{Q}$, the holonomy group of such manifold is a direct product of $SU(2)_R$ and some subgroup of $USp(2n_H)$. Thus one can introduce the vielbeins f_X^{iA} to pass to flat indices transforming under $SU(2)_R\times USp(2n_H)$. The metric on this manifold is then given by

$$g_{XY}\equiv f_X^{iA}f_Y^{jB}\epsilon_{ij}C_{AB}, \tag{5.15}$$

where ϵ and C are the symplectic metrics of $SU(2)_R$ and $USp(2n_H)$, respectively, and are also used to raise and lower indices. One can introduce an $SU(2)_R$ connection ${\omega_{Xi}}^j$ whose curvature is given by

$$\mathcal{R}^r=\mathrm{d}\omega^r-\epsilon^{rst}\omega^s\omega^t\,,\quad {\omega_i}^j\equiv\mathrm{i}\omega^r{(\sigma_r)_i}^j. \tag{5.16}$$

The introduction of gauge interactions is achieved by identifying the gauge group with a subgroup of the isometries of the manifold $\mathcal{M}$. Let us consider, for simplicity, the case of an abelian symmetry. Since the fields in the vector multiplets cannot be charged under an abelian symmetry, we will discuss the gauging of an $U(1)^{n_V+1}$ acting on the hyperscalars manifold only. Of course, for this to be done, the $\mathcal{Q}$ manifold should have at least a $U(1)^{n_V+1}$ isometry group.

The gauging now proceeds by introducing n_V+1 Killing vectors acting on $\mathcal{Q}$ as

$$q^X \to q^X + \epsilon^I K_I^X(q) \tag{5.17}$$

for infinitesimal parameters ϵ^I. Due to the quaternionic structure, these Killing vectors can be obtained from an $SU(2)$ triplet of real prepotentials $P_I^r(q)$ defined by the relation

$$\mathcal{R}^r_{XY} K_I^Y = D_X P_I^r, \quad D_X P_I^r \equiv \partial_X P_I^r + 2\epsilon^{rst}\omega_X^s P_I^t. \tag{5.18}$$

These prepotentials also satisfy the constraint

$$\frac{1}{2}\mathcal{R}^r_{XY} K_I^X K_J^Y = \epsilon^{rst} P_I^s P_J^t. \tag{5.19}$$

Note that equation (5.19) is valid just for abelian isometries. In the generic case it would pick additional contributions from the structure constants.

The theory is now gauged by promoting the derivatives of the underlying σ-model to gauge-covariant ones. For the scalars and the gravitino then we have

$$\mathcal{D}_\mu q^X = D_\mu q^x + g A_\mu^I K_I^X, \tag{5.20}$$

$$\mathcal{D}_\mu \psi_\nu^i = D_\mu \psi_\nu^i + \partial_\mu q^X \omega_{Xj}{}^i\, \psi_\nu^j + g A_\mu^I P_{Ij}{}^i\, \psi_\nu^j, \tag{5.21}$$

where D_μ is the covariant derivative with respect to local Lorentz transformations. The gauging also introduces a potential for the scalars which can be written as

$$\mathcal{V} = -4P^r P^r + 2P_x^r P_y^r g^{xy} + 2\mathcal{N}_{iA}\mathcal{N}^{iA}, \tag{5.22}$$

where

$$P^r \equiv h^I P_I^r, \quad P_x^r \equiv h_x^I P_I^r, \tag{5.23}$$

$$\mathcal{N}_i^A \equiv \frac{\sqrt{6}}{4} f_{iX}^A h^I K_I^X. \tag{5.24}$$

Finally let us introduce the *superpotential* function

$$W \equiv \sqrt{\frac{2}{3}P^r P^r}, \tag{5.25}$$

such that, if the costraint $\partial_x(\frac{Pr}{W}) = 0$ is satisfied, the potential can be expressed as

$$\mathcal{V} = -6W^2 + \frac{9}{2} g^{\Lambda\Sigma} \partial_\Lambda W \partial_\Sigma W, \tag{5.26}$$

where $g_{\Lambda\Sigma}$ is the metric on $\mathcal{M}$ and we denoted with Λ, Σ indices spanning the whole scalar manifold.

The detailed form of the Lagrangian, containing also four-fermions interactions and extended to include the possibility of tensor multiplets, can be found in [91].

5.3 Two-point functions and Holographic Renormalization

In this section we will see more explicitly how the holographic prescription can be used to actually calculate two-point functions. The infinities that we will encounter in the calculation, will lead us to the topic of Holographic Renormalization. Instead of doing a general treatment, we will consider an example [77, 93–96] which should make the procedure clear.

Let us consider a scalar operator $\mathcal{O}$ of dimension Δ in the field theory, and call ϕ the corresponding scalar field in the gravity theory. We will assume that the geometry is asymptotically AdS_{D+1}, meaning that it coincides with AdS_{D+1} near the boundary, but can differ from it in the bulk, due to scalar profiles or to a hard wall, as already explained in section 5.1.1.

We pick coordinates $x^\mu = (z, x^i)$ such that the asymptotic AdS_{D+1} metric is (we are taking the AdS radius to be 1 for simplicity)

$$ds^2 = G_{\mu\nu}\, dx^\mu\, dx^\nu \underset{z\to 0}{\simeq} \frac{1}{z^2} \left((dx^i)^2 + dz^2 \right) \tag{5.27}$$

where $i = 1, \ldots, D$ and $z \to 0$ at the conformal boundary, and x^i can be thought of as coordinates of the space where the dual field theory lives. The isometry of the asymptotic metric which is related to scale transformation in field theory is

$$z \to \lambda z,\; x^i \to \lambda x^i,\; \lambda \in \mathbb{R}^+. \tag{5.28}$$

The action for the scalar coupled to the curved background is

$$S = \frac{1}{2} \int d^D x\, dz\, \sqrt{G} \left(G^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi + m^2 \phi^2 \right) \tag{5.29}$$

where the mass m, as we will see, depends on the dimension of the dual operator.

The equation of motion resulting from the action takes the form

$$0 = \left(\Box_G - m^2\right)\phi \underset{z\to 0}{\simeq} \left(z^2\,\partial_z^2 - (D-1)\,z\,\partial_z + z^2\,(\partial_i)^2 - m^2\right)\phi. \tag{5.30}$$

Plugging in the ansatz

$$\phi \underset{z\to 0}{\simeq} z^\delta \tag{5.31}$$

we get a quadratic equation for the coefficient δ

$$\delta(\delta - D) = m^2 \Rightarrow \delta_\pm = \frac{D}{2} \pm \sqrt{\frac{D^2}{4} + m^2} \tag{5.32}$$

meaning that the scalar has two possible behaviors approaching the boundary, which we can parametrize with two coefficients

$$\phi \underset{z\to 0}{\simeq} \phi_+ z^{\delta_+} + \phi_- z^{\delta_-}. \tag{5.33}$$

Notice that $\delta_+ > \delta_-$, so that z^{δ_-} is the leading mode at the boundary, and we are assuming $m^2 < 0$, so that both modes are finite, which, as we will soon see, corresponds to consider a relevant deformation in field theory. Having negative mass-squared is compatible with stability in AdS_{D+1} backgrounds, as long as it is above the Breitenlohner-Freedman (BF) bound, $m^2 \geq -\frac{D^2}{4}$ [97, 98]. The arbitrariness in the coefficients is fixed by imposing

a) a boundary condition at $z = 0$ on the leading mode ϕ_-,

b) a regularity condition in the bulk, which singles out the full solution of the equation and, in particular, fixes ϕ_+.[4]

The latter depends on the specific form of the background in the interior, so that it cannot be captured by the near-boundary analysis. The former is exactly what is needed to make contact with the dual field theory. Recalling the prescription (5.1-5.7), ϕ_- plays the role of the source for the boundary operator $\mathcal{O}$,[5] namely the field theory action is perturbed by adding

$$\int d^D x\, \phi_-\, \mathcal{O}. \tag{5.34}$$

Notice that the coefficients $\phi_\pm$ pick a factor under the dilation isometry (5.28)

$$\phi_\pm \to \lambda^{\delta_\pm} \phi_\pm. \tag{5.35}$$

[4]Since the equation is homogeneous, if we rescale ϕ_- by a coefficient, ϕ_+ gets rescaled by the same coefficient. Therefore, we can think of the regularity condition as fixing the ratio ϕ_+/ϕ_-.

[5]In (5.1-5.7), for the sake of simplicity, we were a bit imprecise by saying that the source is given by the limit of the field on the boundary, because only massless fields can have constant limits for $z \to 0$. Here we see the more precise statement: the source is the coefficient of the leading mode.

Since the dimension of $\mathcal{O}$ is Δ, we see that in order for ϕ_- to behave correctly we must impose

$$\Delta = D - \delta_- = \delta_+ . \tag{5.36}$$

This gives the well-known relation $m^2 = \Delta(\Delta - D)$ which shows that, as anticipated, a negative mass-squared corresponds to a relevant deformation. Moreover, once the correct scaling for ϕ_- is fixed, ϕ_+ has automatically the correct scaling to be interpreted as the VEV of the operator $\langle\mathcal{O}\rangle$.

One can go beyond the leading modes and further expand the solution near the boundary[6]

$$\phi \underset{z\to 0}{\simeq} z^{\delta_-}\Big(\phi_-^{(0)} + \sum_{n\geq 1}\phi_-^{(2n)} z^{2n}\Big) + z^{\delta_+}\Big(\phi_+^{(0)} + \psi_+^{(0)}\log(z\Lambda) + \sum_{n\geq 1}\Big(\phi_+^{(2n)} + \psi_+^{(2n)}\log(z\Lambda)\Big) z^{2n}\Big). \tag{5.37}$$

The coefficients of this expansion can be then related recursively to the leading modes $\phi_+^{(0)}$ and $\phi_-^{(0)}$ by using the equation of motion

$$\left[(\delta_\pm + 2n)(\delta_\pm + 2n - D) - m^2\right]\phi_\pm^{(2n)} = k^2 \phi_\pm^{(2n-2)} \tag{5.38}$$

$$(2\delta_+ - D)\psi_+^{(0)} = k^2 \phi_-^{(\delta_+ - \delta_- - 2)}, \tag{5.39}$$

$$\left[(\delta_+ + 2n)(\delta_+ + 2n - D) - m^2\right]\psi_+^{(2n)} = k^2 \psi_+^{(2n-2)} \tag{5.40}$$

where we have Fourier transformed the coefficient to D-dimensional momentum space, the momentum being denoted by k.

Notice that logarithmic modes can be present in the expansion of the subleading solution. The coefficient $\psi_+^{(0)}$ of the leading logarithmic mode, and consequently all the $\psi_+^{(2n)}$s, can be non-zero only when $\delta_+ - \delta_-$ is an even integer, which implies

$$m^2 = -\frac{D^2}{4} + n^2 \iff \Delta = \frac{D}{2} + n\,, \quad n = 0, 1, \ldots \tag{5.41}$$

Precisely in this situation, the recursion (5.38) for the modes $\phi_-^{(2n)}$ of the leading solution becomes singular, so that all the modes with $2n \geq \delta_+ - \delta_-$ remain undetermined and should not be included in the expansion. Therefore, one can think of the logarithmic modes as replacing those $\phi_-^{(2n)}$ terms, in this particular case in which the series expansions of the two solutions are merged in a unique series. Since the logarithm shifts under the dilation isometry, the presence of these coefficients gives an additional, unexpected contribution to the transformation of the $\phi_+^{(2n)}$s under dilations. Indeed, $\psi_+^{(0)}$ can be shown to be the holographic counterpart of the contribution to the Weyl anomaly generated by the non-trivial source ϕ_- for the composite operator $\mathcal{O}$ [99].

This procedure of finding a solution in form of a series expansion around a sin-

[6] In order to define the logarithmic mode in the bulk we have to introduce a scale Λ.

gular point of a second order differential equation goes under the name of *Frobenius method* in mathematics. The interesting point in (5.38-5.39), from a physical point of view, is that most of the solution can be reconstructed from the near boundary analysis, with all the coefficients given in terms of the leading mode $\phi_-^{(0)}$, up to a relation between $\phi_+^{(0)}$ and $\phi_-^{(0)}$ which is left undetermined. This is a welcome feature, because relations (5.38-5.40) are polynomial in momenta (i.e. local), and the non-polynomial structure that one expects in the field theory correlators can only come from the relation between $\phi_+^{(0)}$ and $\phi_-^{(0)}$, which will be dictated by some regularity condition in the interior of the geometry: we need to probe the whole geometry to get the correlators. Moreover, notice that the coefficients of the anomalous logarithmic terms are also determined as a local function of the source.

A special treatment is needed for the logarithmic mode in the particular case $\delta_+ = \delta_- = \frac{D}{2}$ in which $\psi_+^{(0)}$ remains undetermined by (5.39). In this case the equation for the exponent δ has a double zero, and one needs to add a logarithmic mode from the start, which turns out to be the leading one (i.e. the source term). In this case the BF bound is saturated. Curiously, having $\Delta = \frac{D}{2}$, this is the relevant case for the scalar operator in the multiplet of a conserved current in $D = 4$, so we will encounter precisely this situation in the following.

We are now ready to plug the solution back in the supergravity action and calculate the two-point function. Integrating by parts we have

$$S = \frac{1}{2}\int d^{D+1}x\sqrt{G}\left(\phi(-\Box_G + m^2)\phi\right) + \frac{1}{2}\lim_{\epsilon\to 0}\int_{z=\epsilon} d^Dx\sqrt{G}G^{zz}\,\phi\,\partial_z\phi. \qquad (5.42)$$

The first term vanishes on-shell, and we are left with the boundary term

$$S^{o.s.} = \frac{1}{2}\lim_{\epsilon\to 0}\int_{z=\epsilon} d^Dx\,\epsilon^{1-D}\,\phi\,\partial_z\phi. \qquad (5.43)$$

If we naively plug the expansion (5.37) in the previous formula, we get a divergent result. This divergence reflects the UV divergences that one usually encounters in field theory. They are mapped into IR singularities in supergravity, which arise due to the infinite distance of the boundary which sets the integration limit. Inspired by the procedure to remove divergences in field theory, by shifting the coupling of the regulated Lagrangian, here we will take an analogous route. First, we keep ϵ fixed, playing the role of an IR regulator in the integral, and we write down all the finite and divergent terms in the regulated action

$$S^{reg} = S^{fin} + S^{div}\,, \qquad (5.44)$$

where

$$S^{fin} = \frac{1}{2}\int_{z=\epsilon} d^Dx\left[D\,\phi_+^{(0)}\,\phi_-^{(0)} + \psi_+^{(0)}\phi_-^{(0)}\right]\,, \qquad (5.45)$$

and

$$S^{div} = \frac{1}{2}\int_{z=\epsilon} d^D x \left[\delta_- \, \phi_-^{(0)} \, \phi_-^{(0)} \, \epsilon^{\delta_- - \delta_+} + \sum_{1 \leq n < \delta_+ - \delta_-/2} (\delta_- + 2n) \, \phi_-^{(2n)} \phi_-^{(0)} \, \epsilon^{\delta_- - \delta_+ + 2n} \right.$$
$$\left. + D \, \psi_+^{(0)} \, \phi_-^{(0)} \log(\Lambda\epsilon) \right] . \qquad (5.46)$$

In accordance with the locality of divergences, the coefficients of the divergent terms are all given by local functions of the source, as derived in equation (5.38-5.39). Therefore, they can be subtracted by modifying the regulated action by the addition of local counterterms. Since the counterterms will be given by a Lagrangian density at $z = \epsilon$, the renormalization procedure is clearly non-covariant with respect to the (approximated) dilation isometry of the (asymptotic) background. On the other hand, we will always require the counterterms to be covariant with respect to D-dimensional isometries. This is the holographic counterpart of the fact that the renormalization procedure requires the introduction of a scale in field theory, so that the classical scale invariance is violated if we insist in preserving Lorentz invariance [100].

Taking into account the relations (5.38), it is straightforward to write a D-dimensionally covariant action at $z = \epsilon$ which exactly riproduces the structure of divergences in S^{div}

$$S^{c.t.} = \frac{1}{2}\int_{z=\epsilon} d^D x \, \epsilon^{-2\delta_-} \left[\delta_- \, \phi(x,\, \epsilon) \, \phi(x,\, \epsilon) \, \epsilon^{\delta_- - \delta_+} \right.$$
$$\left. + \sum_{1 \leq n < \delta_+ - \delta_-/2} C_{2n} \, \phi(x,\, \epsilon) \partial_i^{2n} \phi(x,\, \epsilon) \, \epsilon^{\delta_- - \delta_+ + 2n} + C_{\delta_+ - \delta_-} \, \phi(x,\, \epsilon) \partial_i^{\delta_+ - \delta_-} \phi(x,\, \epsilon) \log(\Lambda\epsilon) \right] . \qquad (5.47)$$

To put this action in an explicitly covariant form, we can reabsorb all the factors of ϵ in the D-dimensional metric at $z = \epsilon$, that is $\gamma_{ij} = \epsilon^{-2}\eta_{ij}$, also used to contract the derivatives, and the result is

$$S^{c.t.} = \frac{1}{2}\int_{z=\epsilon} d^D x \, \sqrt{\gamma} \left[\delta_- \, \phi(x,\, \epsilon) \, \phi(x,\, \epsilon) \right.$$
$$\left. + \sum_{1 \leq n < \delta_+ - \delta_-/2} C_{2n} \, \phi(x,\, \epsilon) (\partial_i^{\gamma})^{2n} \phi(x,\, \epsilon) + C_{\delta_+ - \delta_-} \, \phi(x,\, \epsilon) (\partial_i^{\gamma})^{\delta_+ - \delta_-} \phi(x,\, \epsilon) \log(\Lambda\epsilon) \right] . \qquad (5.48)$$

The factors C_{2n} and $C_{\delta_+-\delta_-}$ are easily calculated but quite cumbersome to write down in general, so we will leave them implicit. Recall that the logarithmic term is present only when $\delta_+ - \delta_-$ is an even integer, and as a result this action is local. Subtracting this terms from the regulated action we can finally take the limit $\epsilon \to 0$ and obtain a finite renormalized answer

$$S^{ren} = \lim_{\epsilon\to 0}(S^{reg} - S^{c.t.}) = \frac{1}{2}\int_{z=\epsilon} d^D x \left[(D - 2\,\delta_-)\,\phi_+^{(0)}\,\phi_-^{(0)} + \psi_+^{(0)}\phi_-^{(0)} \right]. \tag{5.49}$$

Notice that the answer differs from the finite term S^{fin}, because $S^{c.t.}$, besides subtracting the divergences, also contributes to the finite part (namely, the $\delta_-\,\phi(x,\,\epsilon)^2$ counterterm shifts the coefficient of $\phi_+^{(0)}\phi_-^{(0)}$ by $2\delta_-$). Let us stress that here the scheme, and therefore the finite terms in the renormalized action, is fixed by requiring that the counterterms are written as a covariant $4D$ action.

Now that we have properly cured the divergences, we can take the result for the on-shell supergravity action and derive it with respect to the source $\phi_-^{(0)}$ to obtain the correlators of the operator. First we see that, as anticipated by the scaling argument, $\phi_+^{(0)}$ gives the VEV of the operator (with background sources turned on)

$$\langle \mathcal{O} \rangle = \frac{\delta S^{ren}}{\delta \phi_-^{(0)}} = (D - 2\delta_-)\,\phi_+^{(0)} = (2\Delta - D)\,\phi_+^{(0)}. \tag{5.50}$$

The holographic renormalization procedure was essential to get the correct coefficient in the last equation. Here we have neglected $\psi_+^{(0)}$ because, as already emphasized, this term is local in the source, and therefore it gives a scheme-dependent contribution. Then, we can go on to finally get the two-point function

$$\langle \mathcal{O}(k)\mathcal{O}(-k) \rangle = -\frac{\delta^2 S^{ren}}{\delta\phi_-^{(0)}(k)\delta\phi_-^{(0)}(-k)} = -(2\Delta - D)\,\frac{\delta\phi_+^{(0)}(-k)}{\delta\phi_-^{(0)}(k)}. \tag{5.51}$$

We see that the two-point function is encoded in the dependence of the subleading mode from the leading one. If the equation is homogenous, as in our simple example, than the functional derivative can be replaced by a ratio.

Let us use this example to outline the procedure which is needed in general to compute D-dimensional two-point functions from a (super)gravity theory in $D + 1$ dimensions. Given a certain operator $\mathcal{O}$ (not necessarily a scalar one) whose dual field is some field ϕ, one has to

1) Choose a solution of the (super)gravity theory for the metric and for all the additional scalars fields which are possibly turned on in the background; this corresponds to choosing the vacuum of the field theory where we want to compute the two-point function.

2) Consider the action for the field ϕ coupled to the selected background. Since

$4D$ op.	Δ	$5D$ field	Leading Boundary Mode	AdS mass
$J(x)$	$\Delta_0 = 2$	$D(z,x)$	$D(z,x) \simeq z^2 \log z \, d_0(x)$	$m_D^2 = \Delta_0(\Delta_0 - 4) = -4$
$j_\alpha(x)$	$\Delta_{1/2} = 5/2$	$\lambda(z,x)$	$\lambda(z,x) \simeq z^{3/2}\lambda_0(x)$	$\lvert m_\lambda \rvert = \Delta_{1/2} - 2 = 1/2$
$j_i(x)$	$\Delta_1 = 3$	$A_\mu(z,x)$	$A_\mu(z,x) \simeq a_{0\mu}(x)$	$m_A = (\Delta_1 - 2)^2 - 1 = 0$

Table 5.1: $4D$ $\mathcal{N} = 1$ current multiplet and dual supergravity fields

we want to obtain a two-point function, we only need this action at quadratic order in ϕ, which means that we linearize its equations of motion.

3) Find a complete solution of the linearized equation for the field ϕ, by imposing a boundary condition at $z = 0$ which fixes the source, and by requiring a regularity condition in the bulk.

4) Go through the near-boundary analysis and the holographic renormalization procedure to determine which mode enters in the expression of the two-point function, so to also fix the coefficient.

5) Use the exact solution to extract the relevant mode, and take the appropriate ratio to the source term to get the two-point function.

In the next section, following [74], we will describe holographic renormalization in the case which is of direct interest for the application we have in mind, namely that of a vector multiplet in $\mathcal{N} = 2$ supergravity in $D + 1 = 5$.

5.4 The case of a Vector Multiplet

As discussed in the review of $\mathcal{N} = 2$ $5D$ supergravity, a (massless) vector multiplet contains, besides the gauge boson A_μ, a symplectic Majorana fermion λ and a real scalar D (we change notation for the scalar, so that it coincides with the notation for the sources of the $4D$ theory). According to the holographic field/operator correspondence, this is the multiplet dual to a current supermultiplet $\mathcal{J}$, as detailed in Table 5.1 (the scaling at the boundary is directly related to the mass of supergravity fields).

The leading boundary mode for the scalar, d_0, can be directly identified with the source for the operator J. Similarly for the vector, with the gauge choice $A_z = 0$, the leading boundary mode a_{0i} is the source for the conserved vector current of the boundary theory. As for the spinor, it can always be written in terms of $4D$ Weyl spinors of opposite chirality [101]

$$\lambda = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix} \ . \tag{5.52}$$

Choosing the sign of the mass $|m_\lambda| = 1/2$ we are fixing the chirality of the leading mode in the near boundary expansion. As we will review, choosing $m_\lambda = -1/2$ the leading mode at the boundary has negative chirality $\lambda \simeq z^{3/2}\bar{\xi}_0$.

The GGM functions, as defined in equation (2.7-2.10), can then be determined from the renormalized boundary action as

$$C_0(k^2) = -\frac{\delta^2 S_{\rm ren}}{\delta d_0 \delta d_0} \ , \quad C_{1/2}(k^2) = -\frac{\bar{\sigma}_i^{\dot{\alpha}\alpha} k^i}{2k^2} \frac{\delta^2 S_{\rm ren}}{\delta \xi_0^\alpha \delta \bar{\xi}_0^{\dot{\alpha}}} \ , \quad C_1(k^2) = \frac{\eta_{ij}}{3k^2} \frac{\delta^2 S_{\rm ren}}{\delta a_{0i} \delta a_{j0}} \ , \tag{5.53}$$

$$B_{1/2}(k^2) = -\frac{\epsilon^{\alpha\beta}}{2} \frac{\delta^2 S_{\rm ren}}{\delta \xi_0^\alpha \delta \xi_0^\beta} \ . \tag{5.54}$$

In order to derive more explicit formulas in terms of the boundary modes, the starting point is given by the interactions of the vector multiplet with the background. We will assume that the profiles of possible background scalars vanish sufficiently fast, so that the only interaction with the background comes from the minimal coupling to the metric, given by the following (Euclidean) quadratic Lagrangian

$$L_{\rm quad} \underset{z\to 0}{\simeq} L_{\rm min} = \frac{1}{2}(G^{\mu\nu}\partial_\mu D \partial_\nu D - 4D^2) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\bar{\lambda}\not{D}\lambda + c.c.) - \frac{1}{2}\bar{\lambda}\lambda \ . \tag{5.55}$$

As in the previous example, we take the metric to be asymptotically AdS_5, hence the equations approach the AdS form near the boundary

$$(\Box_{AdS} + 4)D \underset{z\to 0}{\simeq} 0 \ , \tag{5.56}$$

$$({\rm Max})_{AdS} A_i \underset{z\to 0}{\simeq} 0 \ , \tag{5.57}$$

$$(\not{D}_{AdS} - \frac{1}{2})\lambda \underset{z\to 0}{\simeq} 0 \ . \tag{5.58}$$

where we fixed the $5D$ Coulomb gauge in which $A_z = 0$ and the Lorentz gauge $\partial^i A_i = 0$[7], and the differential operators above take the form

$$\Box_{AdS} = z^2\partial_z^2 - 3z\partial_z + z^2(\partial_i)^2 \ , \tag{5.59}$$

$$({\rm Max})_{AdS} = z^2\partial_z^2 - z\partial_z + z^2(\partial_i)^2 \ , \tag{5.60}$$

$$\not{D}_{AdS} = z\gamma^z\partial_z - 2\gamma_z + z\gamma^i\partial_i \ . \tag{5.61}$$

The spinor equation can be rewritten in terms of Weyl components as

$$\begin{cases} z\partial_z\chi + iz\sigma^i\partial_i\bar{\xi} - \frac{5}{2}\chi \underset{z\to 0}{\simeq} 0 \\ -z\partial_z\bar{\xi} + iz\bar{\sigma}^i\partial_i\chi + \frac{3}{2}\bar{\xi} \underset{z\to 0}{\simeq} 0 \ . \end{cases} \tag{5.62}$$

[7]Notice that with this gauge choice we can compute only the coefficient of η_{ij} in the vector current correlator; the additional term can obviously be inferred by current conservation.

Correspondingly, the asymptotic behavior of the supergravity fields takes the following form

$$D(z,k) \underset{z\to 0}{\simeq} z^2 \left(d_0(k)\log(z\Lambda) + \tilde{d}_0 + O(z^2)\right) \ , \tag{5.63}$$

$$A_i(z,k) \underset{z\to 0}{\simeq} a_{i0}(k) + z^2(\tilde{a}_{i2}(k) + a_{i2}(k)\log(z\Lambda)) + O(z^4) \ , \tag{5.64}$$

$$\begin{cases} \bar{\xi}(z,k) \underset{z\to 0}{\simeq} z^{3/2}\left(\bar{\xi}_0(k) + z^2(\bar{\tilde{\xi}}_2(k) + \bar{\xi}_2(k)\log(z\Lambda)) + O(z^4)\right) \ , \\ \chi(z,k) \underset{z\to 0}{\simeq} z^{5/2}\left(\tilde{\chi}_1(k) + \chi_1(k)\log(z\Lambda) + O(z^2)\right) \ . \end{cases} \tag{5.65}$$

Note that, as anticipated, in the scalar case the leading term at the boundary has a logarithmic scaling. This is a peculiar feature related to the fact that $m_D^2 = -4$ saturates the BF bound in AdS_5 [97,98]. In the fermionic case the choice $m_\lambda = -1/2$ implies that the leading mode at the boundary has negative chirality, as already observed.

Every coefficient in the expansion is a function of the momentum k, and the variational principle of the supergravity theory is defined by fixing the leading modes at the boundary (d_0, $\bar{\xi}_0$, a_{i0}) and letting all other coefficients free to vary independently. Substituting the ansatz in the asymptotic equations of motion (5.59)-(5.61) one can determine the on-shell values of all coefficients but tilded ones as local (i.e. polynomial in k) functions of the leading modes (d_0, $\bar{\xi}_0$, a_{i0}), as we have showed in the previous section in equations (5.38-5.39). In this case we find

$$a_{i2} = \frac{k^2}{2}a_{i0}(k) \ , \qquad \bar{\xi}_2 = \frac{k^2}{2}\bar{\xi}_0(k) \ , \qquad \chi_1 = -\sigma^i k_i \bar{\xi}_0(k) \ . \tag{5.66}$$

Conversely, the subleading modes in the near boundary expansion are not determined by the near boundary analysis and in general will be non-local functions of the external momenta which can be derived from the exact solutions of the equations of motion.

When evaluated on-shell, the supergravity action (5.55) reduces to the boundary terms, which are in general divergent in the limit $z \to 0$, and have to be regularized. As explained in the previous section, this can be done by considering a cutoff surface $z = \epsilon$, so that, after Fourier transformation on the $4D$ coordinates, the boundary terms become

$$S_{\text{reg}} = -\int_{z=\epsilon} \frac{d^4k}{(2\pi)^4}\frac{1}{2}\left[\epsilon^{-4}(Dz\partial_z D)_{z=\epsilon} + \epsilon^{-2}(A_i z\partial_z A^i)_{z=\epsilon} - \epsilon^{-4}(\xi\chi + \bar{\chi}\bar{\xi})_{z=\epsilon}\right] \ . \tag{5.67}$$

Note that the fermionic boundary term [102, 103] is reminiscent of a Dirac mass term.

Plugging the near boundary expansion and taking into account the on-shell relations between the various coefficients, we can collect both a divergent and a finite contribution in the regularized action (5.67)

$$S_{\rm reg}|_{\rm div} = -\int\limits_{z=\epsilon} \frac{d^4k}{(2\pi)^4}\frac{1}{2}\log(\epsilon\Lambda)[2\log(\epsilon\Lambda)d_0^2 + 4D_0\tilde{d}_0 + d_0^2 + a_0^i k^2 a_{0i} + 2\xi_0\sigma^i k_i\bar{\xi}_0] \ ,$$

$$S_{\rm reg}|_{\rm finite} = -\int \frac{d^4k}{(2\pi)^4}\frac{1}{2}[2\tilde{d}_0^2 + d_0\tilde{d}_0 + 2a_0^i\tilde{a}_{i2} + a_0^i\frac{k^2}{2}a_{i0} - (\xi_0\tilde{\chi}_1 + \bar{\tilde{\chi}}_1\bar{\xi}_0)] \ .$$

The divergent terms can be subtracted by means of the following covariant counterterms

$$S_{\rm ct} = -\frac{1}{2}\int\limits_{z=\epsilon} \frac{d^4k}{(2\pi)^4}\sqrt{\gamma}[2D^2 + \frac{D^2}{\log(\epsilon\Lambda)} - \frac{1}{2}\log(\epsilon\Lambda)F_{ij}F^{ij} + 2\log(\epsilon\Lambda)\bar{\lambda}\gamma^i k_i\lambda] \ . \quad (5.68)$$

The counterterms for the scalar components contribute also to the finite part of the boundary action so we finally get the result

$$S_{\rm ren} = \frac{N^2}{8\pi^2}\int \frac{d^4k}{(2\pi)^4}[d_0\tilde{d}_0 - 2a_0^i\tilde{a}_{i2} - \frac{1}{2}a_0^i k^2 a_{i0} + \xi_0\tilde{\chi}_1 + \bar{\tilde{\chi}}_1\bar{\xi}_0] \ , \quad (5.69)$$

where we restored the normalization of the action which was neglected so far. The coefficient is due to the identification $\frac{1}{8\pi G_5} = \frac{N^2}{4\pi^2}$, on which we will return in the next chapter when treating the pure AdS example.

Twice differentiating with respect to the sources we finally get for the two-point functions

$$\langle J(k)J(-k)\rangle = \frac{N^2}{8\pi^2}\left(-2\frac{\delta\tilde{d}_0}{\delta d_0}\right) \ , \quad (5.70)$$

$$\langle j_i(k)j_j(-k)\rangle = \frac{N^2}{8\pi^2}\left(2\frac{\delta\tilde{a}_{i2}}{\delta a_0^j} + 2\frac{\delta\tilde{a}_{j2}}{\delta a_0^i} + k^2\eta_{ij}\right) \ , \quad (5.71)$$

$$\langle j_\alpha(k)\bar{j}_{\dot{\alpha}}(-k)\rangle = \frac{N^2}{8\pi^2}\left(\frac{\delta\tilde{\chi}_{1\alpha}}{\delta\bar{\xi}_0^{\dot{\alpha}}} + \frac{\delta\bar{\tilde{\chi}}_{1\dot{\alpha}}}{\delta\xi_0^\alpha}\right) \ , \quad (5.72)$$

$$\langle j_\alpha(k)j_\beta(-k)\rangle = \frac{N^2}{8\pi^2}\left(\frac{\delta\tilde{\chi}_{1\alpha}}{\delta\xi_0^\beta} - \frac{\delta\tilde{\chi}_{1\beta}}{\delta\xi_0^\alpha}\right) \ . \quad (5.73)$$

An important comment about the fermionic correlators is in order. From the structure of the spinor equation of motion one can notice that the subleading mode $\tilde{\chi}_1$, which is determined by the full bulk equation, will always have a non-trivial dependence on the leading mode of opposite chirality $\bar{\xi}_0$, ensuring a non-zero value for the function $C_{1/2}$. On the other hand, already at this very general stage, we see that the only way to obtain a non-zero $B_{1/2}$ is to have the mode $\tilde{\chi}_1$ to depend also on the source ξ_0. In the next chapter, when discussing the couplings of the vector to the background, as dictated by the supergravity action, we will show that this is closely related to the presence of Majorana-like couplings in the $5D$ action.

These couplings, in turn, can arise only in the presence of a non-trivial profile for scalars charged under the R-symmetry, that is mapped holographically to the $U(1)$ symmetry gauged by the graviphoton. This result should be expected, since a non-zero $B_{1/2}$ requires R-symmetry to be broken. We will see under which conditions non-trivial Majorana-like couplings of the bulk fermions can be produced.

Chapter 6

Models in $5D$ supergravity

We are now ready to apply the techniques described in the previous chapter to a concrete model of HGGM. Following [74], we will fix a concrete supergravity theory, with the minimal field content which can allow for supersymmetry breaking solutions. Once a supersymmetry breaking background is found, we will fluctuate the vector multiplet on it, and apply the (properly renormalized) holographic prescription to compute the GGM form factors (other works using a similar philosophy are [104, 105]). A theme which we already encountered in the weakly coupled models of chapters 3-4 will be relevant also in this case, namely the special role played by the R-symmetry. This is ultimately related to the fact that a Majorana mass for the gaugino can only be generated when R-symmetry is broken. In the holographic setup, the R-symmetry is mapped to the gauge symmetry associated to the graviphoton (i.e. the vector belonging to the graviton multiplet). As a result, in order to get $B_{\frac{1}{2}} \neq 0$, we will need this gauge symmetry to be higgsed by a non-trivial profile of some charged scalar. A surprising result we will find is that, even when the R-symmetry remains unbroken, a mass for the gaugino, of Dirac type, is generated. As we will see, this is due to the presence of massless fermionic degrees of freedom in the hidden sector [27, 28], a mechanism we described in section 2.3.1. Let us now outline in more detail the various steps we will follow in this chapter.

We will start by addressing the problem of choosing the field content and solving for the background. An interesting class of backgrounds can be obtained by considering a gauged supergravity Lagrangian describing the interaction of the $\mathcal{N} = 2$ gravity multiplet with one hypermultiplet. The latter, known as the universal hypermultiplet, contains a neutral complex scalar with $m^2 = 0$, and another one which has $m^2 = -3$ (in AdS units) and is charged under the R-symmetry. Using a terminology inspired by reduction from $10D$, we will refer to the first scalar as the *axio-dilaton*, and to the second one as *squashing mode*. This model is the simplest possible model containing all necessary ingredients to let us treat several qualitatively different examples, all arising as consistent solutions of the same 5D equations of motion.

As a warm-up, we will consider a pure (and hence supersymmetric) AdS background. The holographic GGM functions are those of a supersymmetric (and conformal) field theory. We use the holographic computation to check the validity of our approach, and as useful reference for the more interesting examples we consider afterwards.

We will then turn to a dilaton-domain wall background, originally found in [106, 107] (see also [108]) in the context of type IIB supergravity, used at that time as a candidate gravitational dual for confining theories. The background breaks all supersymmetry but preserves the R-symmetry. We find non-vanishing values for the sfermion masses while, consistently, SSM gauginos do not acquire a Majorana mass term. As anticipated, we will find a pole at zero momentum for one fermionic correlator, which signals the presence of a Dirac-like (hence R- symmetry preserving) mass for the gauginos. Such Dirac mass contribution arises as a consequence of strong dynamics in the hidden sector.

Finally, we will turn-on a small R-symmetry breaking scalar profile, that we treat linearly and without backreaction on the dilaton-domain wall background. The linear approximation is sufficient to show how things change significantly. Indeed, we show that in this case a Majorana mass for the gauginos is generated. Moreover, the pole at zero momentum of the fermionic correlator, responsible for a Dirac- like contribution to gaugino masses in the pure dilatonic background, automatically disappears, in remarkable agreement with field theory expectations (see section 2.3.1).

6.1 The gauged supergravity theory

The $5D$ gravity theory, besides the graviton multiplet, must contain at least one $\mathcal{N} = 2$ vector multiplet, which is dual to the multiplet of the conserved current of the boundary theory. Here we are taking the simplifying assumption that the global symmetry of the hidden sector used to mediate supersymmetry breaking is just a $U(1)$, clearly in more realistic models one should enlarge it to the full Standard Model gauge group (we postpone to the last section a discussion of how this goal can be achieved). As a necessary condition for the theory to be a consistent truncation of $10D$ type IIB supergravity, the matter content must include also one hypermultiplet which is the $\mathcal{N} = 2$ multiplet of the $10D$ dilaton, usually called universal hypermultiplet. In fact, enlarging the matter content to include an hypermultiplet is also necessary to the aim of finding interesting backgrounds, as we will see. Therefore, the minimal $5D$ content one should consider consists of $\mathcal{N} = 2$ supergravity coupled to a vector multiplet and a hypermultiplet.

We consider a class of gauged supergravity theories studied in [92] which actually contains precisely the minimal field content described above.[1] Recalling the basic ingredients of the theory that we described in section 5.2, in this case we have that the scalar manifold is

$$\mathcal{M} = O(1,1) \times \frac{SU(2,1)}{U(2)}. \tag{6.1}$$

$\mathcal{S}$ is the very special manifold parametrized by the scalar D in the vector multiplet, with metric

$$ds_1^2 = dD^2, \tag{6.2}$$

$\mathcal{Q}$ is the quaternionic manifold parametrized by the four real scalars in the universal hypermultiplet $q^X = (\phi,\, C_0,\, \eta,\, \alpha)$, with metric $ds_2^2 = g_{XY} dq^X dq^Y$ given by

$$\frac{1}{2}\cosh^2(\eta) d\phi^2 + \frac{1}{2}(2\sinh^2(\eta) d\alpha + \mathrm{e}^{\phi}\cosh^2(\eta) dC_0)^2 + 2\sinh^2(\eta) d\alpha^2 + 2d\eta^2 \tag{6.3}$$

where $\eta \geq 0$ and $\alpha \in [0, 2\pi]$. The scalar $\eta e^{i\alpha}$ is the squashing mode, related to a squashing parameter of the internal compactification manifold in the context of reduction from $10D$. The scalar $\phi + iC_0$ is the so-called axio-dilaton, its real part ϕ being the dilaton.

The isometries of the scalar manifold have a $U(2)$ maximal compact subgroup acting on $\mathcal{Q}$. Since the theory contains two vectors, one in the gravity multiplet and the other one in the vector multiplet, the maximal subgroup we can gauge is a $U(1) \times U(1)$. As a minimal set up we choose to gauge just the $U(1)$ corresponding to the shift symmetry

$$\alpha \to \alpha + c \tag{6.4}$$

of the above metric, which is a compact isometry because the scalar α is a phase.

The vector field of this $U(1)$ which acts non trivially on the scalar manifold is the graviphoton in the gravity multiplet, so that this gauge symmetry is dual to the R-symmetry of the boundary theory. On the other hand, in our simplified setting the $U(1)$ gauged by the vector belonging to the vector multiplet acts trivially on all supergravity fields (this does not mean that the vector multiplet is free, because of the unavoidable interaction with gravity). Notice that the axio-dilaton is neutral under both $U(1)$'s while the squashing mode is charged under the symmetry gauged in the bulk by the graviphoton. Therefore, a background with a non-trivial profile for the dilaton preserves the R-symmetry, while a non-trivial profile for η breaks it. For later reference let us notice that while the axio- dilaton is massless, and holographycally dual to the hidden sector $\mathrm{Tr}F_{ij}^2$ operator, the squashing mode has $m^2 = -3$ and it is dual to the hidden sector gaugino bilinear. Hence, the leading mode for this field at the boundary would provide an explicit mass to the

[1] This class of theories has the virtue that, for some choices of the gauging, the resulting theory is believed to be a consistent truncation of the maximally gauged $\mathcal{N} = 8$ supergravity in $5D$ (and therefore of $10D$ type IIB compactified on a sphere).

hidden gauginos (hence an explicit R-symmetry breaking term), while a subleading term would correspond to a VEV for the gaugino bilinear (hence a spontaneous R-symmetry breaking term).

Starting from the $5D$ Lagrangian of this theory, along the lines described in section 5.3, there are basically two steps one should perform:

- First, we should find a non-supersymmetric background configuration with just the metric and some of the hyperscalars turned on. In order to do this we will truncate the Lagrangian to the relevant field content (provided this is consistent with the full set of equations) and extract the equations of motion which the background must satisfy.

- Second, we need to extract the linearized differential equations for the vector multiplet fluctuating on the background that we will find. To this aim, we will perform a different truncation of the Lagrangian setting all fields but the vector multiplet to their background values, and retain only the couplings which are no more than quadratic in the vector multiplet fields.

We will now present the explicit form of these truncated Lagrangians.

6.1.1 Lagrangian for the background

Let us start by setting to zero the whole vector multiplet, as well as the gravitino, the graviphoton and the fermions of the hypermultiplet. The phase α can be gauge-fixed to zero. The resulting truncated (Euclidean) action reads

$$S_{\mathrm{b.g.}} = \int d^5x\sqrt{G}\left[-\frac{1}{2}R + L_{\mathrm{kin}} + \mathcal{V}\right] \tag{6.5}$$

where the kinetic term is given in term of the metric (6.3) by $L_{\mathrm{kin}} = \frac{1}{2}g_{XY}\partial_\mu q^X\partial^\mu q^Y$, that is

$$L_{\mathrm{kin}} = \frac{1}{4}\left[4\partial_\mu\eta\partial^\mu\eta + \cosh^2(\eta)\partial_\mu\phi\partial^\mu\phi + \mathrm{e}^{2\phi}\cosh^4(\eta)\,\partial_\mu C_0\partial^\mu C_0\right] . \tag{6.6}$$

As a consequence of the gauging we have a non-trivial potential given by

$$\mathcal{V} = \frac{3}{4}\left(\cosh^2(2\eta) - 4\cosh(2\eta) - 5\right) . \tag{6.7}$$

We end up with the following system of differential equations

$$R_{\mu\nu} = \frac{2}{3}\mathcal{V}G_{\mu\nu} + 2\left(\partial_\mu\eta\partial_\nu\eta + \frac{1}{4}\cosh^2(\eta)\,\partial_\mu\phi\partial_\nu\phi\right) , \tag{6.8}$$

$$\Box_G\,\eta = \frac{1}{2}\frac{\partial\mathcal{V}}{\partial\eta} + \frac{1}{8}\sinh(2\eta)\,\partial_\mu\phi\partial^\mu\phi, \tag{6.9}$$

$$\Box_G\,\phi = -2\tanh(\eta)\,\partial_\mu\eta\partial^\mu\phi, \tag{6.10}$$

where $\Box_G$ is the usual Klein-Gordon operator on a curved space

$$\Box_G = \frac{1}{\sqrt{G}}\partial_\mu(\sqrt{G}G^{\mu\nu}\partial_\nu) \ . \tag{6.11}$$

The condition of asymptotically AdS-ness can be phrased by taking a metric of the form

$$ds_5^2 = \frac{1}{z^2}\left(dz^2 + F(z)(dx^i)^2\right) \tag{6.12}$$

with $F(z)$ approaching 1 at the boundary $z \to 0$. Therefore the solution to the above equations determine the three unknown functions ϕ, η and F of the radial coordinate z.

In the case of unbroken R-symmetry, $\eta = 0$, the above system of equations reduces exactly to the one considered in [106], and admits both a supersymmetric AdS solution with constant dilaton, as well as a singular dilaton domain-wall solution [106, 107]. The latter breaks both conformal invariance and (all) supersymmetry. Another interesting background is one where also the charged scalar η has a non-trivial profile. We will consider all these examples in turn.

6.1.2 Quadratic Lagrangian for the vector multiplet

We now turn to the action describing the coupling of vector multiplet fluctuations to the background. To this end we fix F, ϕ and η to their (z-dependent) background value into the full Lagrangian, and retain only those terms involving the vector multiplet up to second order. The resulting (Euclidean) action can be divided in two pieces

$$S_{\text{quad}} = \int d^5x\sqrt{G}\,[L_{\text{min}} + L_{\text{int}}] \, . \tag{6.13}$$

The first one contains kinetic terms and mass terms for the fluctuations, and it is uniquely fixed by the dimensions of the dual operators and their minimal coupling to the metric to be L_{min}, eq. (5.55). The second one contains interactions with the scalars ϕ and η and takes the form

$$\begin{aligned} L_{\text{int}} &= \frac{1}{2}\delta M^2 D^2 - \delta m_D \bar{\lambda}\lambda \\ &- \frac{1}{2}\left(m_M \bar{\lambda}\lambda^c + v_M \bar{\lambda}(\partial\!\!\!/\eta)\lambda^c + \tilde{v}_M \bar{\lambda}(\partial\!\!\!/\phi)\lambda^c + c.c.\right) . \end{aligned} \tag{6.14}$$

where

$$\delta M^2 = 2(\cosh^2(2\eta) - \cosh(2\eta)) \ , \ \delta m_D = -\frac{1}{2}\sinh^2(\eta) \tag{6.15}$$

$$m_M = i\sinh(\eta) \ , \ v_M = -\frac{i}{\cosh(\eta)} \ , \ \tilde{v}_M = \frac{i}{2}\sinh(\eta). \tag{6.16}$$

In the first line there are (z-dependent) shifts for scalar mass squared and Dirac fermion mass, whereas in the second line there are a Majorana mass term and additional Majorana-like couplings. We wrote the couplings in a $5D$ covariant manner, but one should bear in mind that η and ϕ are background values which actually can depend only on the radial coordinate, so that the additional terms are equivalent to $4D$ covariant terms constructed with a γ_5 matrix. Notice that all couplings (6.15-6.16) vanish if η is identically zero in the background.

From the action (6.13) we get the equations of motions

$$(\Box_G + 4 - \delta M^2)D = 0 \ , \tag{6.17}$$

$$\frac{1}{\sqrt{G}}\partial_\mu(\sqrt{G}G^{\mu\rho}G^{\nu\sigma}F_{\rho\sigma}) = 0 \ , \tag{6.18}$$

$$(\not{D} - \frac{1}{2} - \delta m_D)\lambda - (m_M + v_M \not{\partial}\eta + \tilde{v}_M \not{\partial}\phi)\lambda^c = 0 \ , \tag{6.19}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \tag{6.20}$$

$$\not{D} = e^\mu_a \gamma^a \left(\partial_\mu + \frac{1}{8}\omega^{cb}_\mu[\gamma_b, \gamma_c]\right) . \tag{6.21}$$

As already noticed, the $5D$ spinor has the same form as a $4D$ Dirac spinor and it is often useful to rewrite its equation of motions in terms of chirality eigenstates, that is

$$\lambda = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix} , \qquad \bar{\lambda} = -\begin{pmatrix} \xi & \bar{\chi} \end{pmatrix} , \qquad \lambda^c = -\begin{pmatrix} \xi \\ \bar{\chi} \end{pmatrix} . \tag{6.22}$$

In terms of Weyl components χ and ξ, eq. (6.19) becomes

$$(z\partial_z - \frac{5}{2} + z\frac{F'}{F} - \delta m_D)\chi + i\frac{z}{\sqrt{F}}\sigma^i\partial_i\bar{\xi} - (m_M + v_M z\eta' + \tilde{v}_M z\phi')\xi = 0 \ , \tag{6.23}$$

$$(z\partial_z - \frac{3}{2} + z\frac{F'}{F} + \delta m_D)\bar{\xi} - i\frac{z}{\sqrt{F}}\bar{\sigma}^i\partial_i\chi - (m_M - v_M z\eta' - \tilde{v}_M z\phi')\bar{\chi} = 0 \ . \tag{6.24}$$

As can be seen from above equations, when Majorana-like couplings are turned on, not only $\bar{\xi}$ but also ξ appears in the equation for χ, and vice-versa. As we concluded in section 5.4, we thus see that it is necessary to turn on a background for the scalar η in order to have correlators with a non-zero $B_{1/2}$.

6.1.3 Renormalized action with a non-trivial η

When the scalar η has a non-trivial profile and has non-vanishing leading boundary behavior, the renormalized action for the vector multiplet should be slightly modified with respect to what discussed in 5.4.

The scalar η has $m^2 = -3$, and therefore its leading and subleading boundary behavior is

$$\eta \underset{z\to 0}{\simeq} \eta_0 z + \tilde{\eta}_2 z^3 + \dots \tag{6.25}$$

As the numerical analysis in the following sections will show, whenever the leading mode η_0 is present (a source term for the corresponding $\Delta = 3$ boundary operator, the hidden gaugino bilinear), the renormalized boundary action (5.69) should be modified by the following term

$$S^{\eta}_{\text{ren}} = \frac{N^2}{8\pi^2} \int \frac{d^4k}{(2\pi)^4} [i\eta_0(\xi_0\xi_0 - \bar{\xi}_0\bar{\xi}_0)] \ . \tag{6.26}$$

Accordingly, the expression for the correlator (5.73) is modified to

$$\langle j_\alpha(k) j_\beta(-k)\rangle_\eta = \frac{N^2}{8\pi^2}\left(\frac{\delta\tilde{\chi}_{1\alpha}}{\delta\xi_{0\beta}} - \frac{\delta\tilde{\chi}_{1\beta}}{\delta\xi_{0\alpha}} + 2i\epsilon_{\alpha\beta}\eta_0\right) . \tag{6.27}$$

The corrected expression (6.27) is necessary to ensure that the fermionic correlator properly goes to zero at large momenta, as dictated by supersymmetry restoration at high energy. The ultra-local term (6.26) can be seen as a counterterm which we add to the boundary action in order to reabsorb an unwanted contact term in the correlator. This countertem only depends on quantities that are held fixed in the variational principle.

Notice that if the η profile has a leading boundary behavior proportional to $\tilde{\eta}_2$, which is holographically dual to a purely dynamical generation of an R-symmetry breaking VEV, no modification in the renormalized boundary action occurs. Still, having η a non trivial profile, $\tilde{\chi}_1$ would depend on ξ_0, and hence the correlator (5.73) would be in general different from zero.

The origin of the additional term (6.26) can alternatively be motivated as follows. The interaction Lagrangian (6.14) at linear order in η reads

$$L^{\text{lin}}_{\text{int}} = \frac{1}{2}\left(-i\eta\bar{\lambda}\lambda^c + i\bar{\lambda}(\partial\!\!\!/\eta)\lambda^c - \frac{i}{2}\eta\bar{\lambda}(\partial\!\!\!/\phi)\lambda^c + c.c.\right) \ , \tag{6.28}$$

where we can actually neglect the third term, since in a background with a non-trivial dilaton profile, which necessarily behaves as $z\partial_z\phi = \mathcal{O}(z^4)$, this cannot contribute to the boundary action.

The key observation is that the following boundary term

$$S^{\eta}_{\text{reg}} = \int_{z=\epsilon} \frac{d^4k}{(2\pi)^4}\frac{i}{2}\epsilon^{-4}\left[\eta(\xi\xi - \bar{\xi}\bar{\xi} - \chi\chi + \bar{\chi}\bar{\chi})\right]_{z=\epsilon} \tag{6.29}$$

is obtained if one integrates by parts the second term in (6.28). We note that this boundary term is now Majorana-like, in contrast with the usual one, eq. (5.67), which is Dirac-like. The term bilinear in χ is always vanishing at the boundary, but we notice that when $\eta \sim \epsilon$ the term bilinear in ξ is actually finite, and is exactly the term (6.26) after we restore the proper normalization.

6.2 Holographic correlators in AdS

As a warm-up exercise we want to compute the GGM two-point functions for a pure AdS background, which is a solution of eqs.(6.8-6.10) with $\phi = \eta = 0$. This exercise has several motivations. First of all it will enable us to verify that the machinery we have described correctly reproduces what we expect for a conformal and supersymmetric case. Second, the values for the correlators that we find in AdS will be the reference to confront with, when considering other backgrounds. In particular, each correlator will have to asymptote to those of the pure AdS case, at large momenta. Finally, the computations we perform in this section can be of interest in a different context, that is when conformality and supersymmetry breaking are implemented by a hard wall in AdS [109], which we will discuss in detail in the next chapter.

The pure AdS solution is a trivial solution of our $5D$ effective model. However, in order to fix the overall normalization of correlators, it is useful to uplift it to the $AdS_5 \times S^5$ solution of $10D$ type IIB supergravity, which reads (see e.g. [106])

$$ds^2_{10} = \frac{L^2}{z^2}(dz^2 + (dx^i)^2) + L^2 d\Omega_5^2 \ , \tag{6.30}$$

$$F_5 = \frac{N\sqrt{\pi}}{2\pi^3}(\mathrm{vol}(S_5) + \frac{1}{z^5} d^4x \wedge dz) \ , \tag{6.31}$$

where the radius of AdS_5 is fix to be $L^4 = \frac{k_{10}N}{2\pi^{5/2}}$ by $10D$ Einstein equations. The overall constant in front of the $10D$ action is $1/2k_{10}^2$ so that, substituting the value of the $10D$ Newton constant in terms of the string theory parameters $k_{10} = \sqrt{8\pi G_{10}} = 8\pi^{7/2} g_s \alpha'^2$, we get $L^4 = 4\pi g_s N \alpha'^2$. Taking $L = \alpha' = 1$ we find $G_5 = \frac{\pi}{2N^2}$ and the overall constant in front of the $5D$ effective action is $1/8\pi G_5 = \frac{N^2}{4\pi^2}$.

In pure AdS the equations of motion (5.56), (5.57) and (5.62) are related to standard Bessel equations, and therefore it is possible to get analytic solutions for the fields. In fact, taking

$$D = z^2 d \ , \ A_i = z\alpha_i \ , \ \bar{\xi} = z^{5/2}\Xi \ , \ \chi = z^{5/2} X \ , \tag{6.32}$$

we can substitute and get the equations for the bosonic fluctuations in the form

$$(z^2\partial_z^2 + z\partial_z - z^2k^2)d = 0 \ , \tag{6.33}$$

$$(z^2\partial_z^2 + z\partial_z - (z^2k^2 + 1))\alpha_i = 0 \ , \tag{6.34}$$

while the system of equations for the gaugino is

$$\begin{cases} (z^2\partial_z^2 + z\partial_z - (z^2k^2+1))\bar{\Xi} = 0 \ , \\ z\bar{\sigma}^i k_i X = (z\partial_z + 1)\bar{\Xi} \, . \end{cases} \tag{6.35}$$

The general solutions to these equations for $k^2 \geq 0$ are

$$d(z,k) = c_1(k)I_0(kz) + c_2(k)K_0(kz) \ , \tag{6.36}$$

$$\alpha_i(z,k) = \alpha_{i1}(k)I_1(kz) + \alpha_{i2}(k)K_1(kz) \ , \tag{6.37}$$

$$\bar{\Xi}(z,k) = \bar{\theta}_1(k)I_1(kz) + \bar{\theta}_2(k)K_1(kz) \ . \tag{6.38}$$

The general solutions of second order differential equations depend on two arbitrary constants, that in our case can be arbitrary functions of the $4D$ momentum k.

In order to impose Dirichlet boundary conditions at $z \to 0$, which identify the sources for the boundary operators, we expand the solutions (6.36-6.38) using the limiting behavior of the I Bessel functions

$$\begin{aligned} I_0(x) &\underset{x\to 0}{\simeq} 1 + \frac{1}{4}x^2 + \ldots, \\ I_1(x) &\underset{x\to 0}{\simeq} \frac{1}{2}x + \frac{1}{16}x^3 + \ldots, \end{aligned} \tag{6.39}$$

and similarly for K

$$\begin{aligned} K_0(x) &\underset{x\to 0}{\simeq} -\log x + \log 2 - \gamma + \ldots, \\ K_1(x) &\underset{x\to 0}{\simeq} \frac{1}{x} + \frac{1}{2}x\left(\log x - \log 2 - \frac{1}{2} + \gamma\right) + \ldots . \end{aligned} \tag{6.40}$$

Comparing with (5.63-5.65) we get

$$c_2(k) = -d_0(k) \ , \ \alpha_{i2} = ka_{i0}(k) \ , \ \bar{\theta}_2 = k\bar{\xi}_0(k) \ . \tag{6.41}$$

This leaves us with three arbitrary functions $c_1(k), a_{i1}(k), \theta_1(k)$. In order to have a regular solution in the full domain we have to impose the following conditions in the deep interior

$$\lim_{z\to\infty} D(z,k) = 0 \ , \ \lim_{z\to\infty} A_i(z,k) = 0 \ , \ \lim_{z\to\infty} \xi(z,k) = 0 \ . \tag{6.42}$$

Using the expansions

$$I_{0,1}(x) \underset{x\to\infty}{\simeq} \frac{e^x}{(2\pi x)^{1/2}} \, , \quad K_{0,1}(x) \underset{x\to\infty}{\simeq} \left(\frac{\pi}{2x}\right)^{1/2} e^{-x} \ , \tag{6.43}$$

we see that (6.42) implies $c_1(k) = a_{i1}(k) = \theta_1(k) = 0$. We note that the regularity conditions in the bulk are crucial to single out the solution to the fluctuation equations. Therefore, the AdS_5 solutions are

$$D^{AdS}(z,k) = -z^2 K_0(kz) d_0(k) \ , \tag{6.44}$$

$$A_i^{AdS}(z,k) = zkK_1(kz) a_{i0}(k) \ , \tag{6.45}$$

$$\bar{\xi}^{AdS}(z,k) = z^{5/2} k K_1(kz) \bar{\xi}_0(k) \ , \tag{6.46}$$

$$\bar{\chi}^{AdS}(z,k) = z^{5/2} K_0(kz) \sigma^i k_i \bar{\xi}_0(k) \ , \tag{6.47}$$

where, in order to derive the last equation, we used the relation

$$x\partial_x K_1(x) + K_1(x) = -xK_0(x). \tag{6.48}$$

We can expand the solutions near the boundary, using again (6.40), and get the following results for the subleading modes

$$\tilde{d}_0(k) = \left[-\frac{1}{2}\log\left(\frac{\Lambda^2}{k^2}\right) - \log 2 + \gamma\right] d_0(k) \ , \tag{6.49}$$

$$\tilde{a}_{i2}(k) = \frac{k^2}{2}\left[-\frac{1}{2}\log\left(\frac{\Lambda^2}{k^2}\right) - \log 2 + \gamma - \frac{1}{2}\right] a_{i0}(k) \ , \tag{6.50}$$

$$\tilde{\bar{\xi}}_2(k) = \frac{k^2}{2}\left[-\frac{1}{2}\log\left(\frac{\Lambda^2}{k^2}\right) - \log 2 + \gamma - \frac{1}{2}\right] \bar{\xi}_0(k) \ , \tag{6.51}$$

$$\tilde{\chi}_1(k) = \left[-\frac{1}{2}\log\left(\frac{\Lambda^2}{k^2}\right) - \log 2 + \gamma\right] \sigma^i k_i \bar{\xi}_0(k) \ . \tag{6.52}$$

Substituting these expressions into eqs.(5.70)-(5.73), we finally get the two-point functions

$$\langle J(k)J(-k)\rangle = \frac{N^2}{8\pi^2} C_0^{AdS}(k^2) = \frac{N^2}{8\pi^2}\left[\log\left(\frac{\Lambda^2}{k^2}\right) + 2\log 2 - 2\gamma\right] \ , \tag{6.53}$$

$$\langle j_i(k)j_j(-k)\rangle = -\frac{N^2}{8\pi^2}\delta_{ij}k^2 C_1^{AdS}(k^2) = -\frac{N^2}{8\pi^2}\delta_{ij}k^2\left[\log\left(\frac{\Lambda^2}{k^2}\right) + 2\log 2 - 2\gamma\right] \ , \tag{6.54}$$

$$\langle j_\alpha(k)\bar{j}_{\dot{\alpha}}(-k)\rangle = -\frac{N^2}{8\pi^2}\sigma^i k_i C_{1/2}^{AdS}(k^2) = -\frac{N^2}{8\pi^2}\sigma^i k_i\left[\log\left(\frac{\Lambda^2}{k^2}\right) + 2\log 2 - 2\gamma\right] \ , \tag{6.55}$$

$$\langle j_\alpha(k)j_\beta(-k)\rangle = 0 \ , \tag{6.56}$$

where we can always take in (6.54) $\delta_{ij} \to \delta_{ij} - \frac{k_i k_j}{k^2}$ because of current conservation.[2]

These results are in agreement with CFT computations [110,111]. Note that we can always subtract the constant contribution $\log 2 - \gamma$ to the two-point functions

[2] As was already noticed the holographic prescription performed with the transverse gauge fixing allows us to compute only the part of the vector current two-point function which is proportional to δ_{ij}.

by means of finite counterterms which preserve the $\mathcal{N} = 2$ supersymmetry of the bulk action, so these terms are inessential and will be ignored in what follows.

As expected for a supersymmetric background we find that the relations (2.17) are satisfied, and thus that both gaugino (2.23) and sfermion masses (2.25) are identically zero. In a superconformal theory the OPE of the conserved current satisfies some general constraints which were studied in general in [112] and applied to the GGM formalism in [113]. In particular, if the hidden sector is exactly superconformal, as it is the case for $\mathcal{N} = 4$ SYM, in the OPE of $J(x)J(0)$ only the unit operator can have an expectation value, leading to

$$C_0(x) = C_{1/2}(x) = C_1(x) \sim \frac{1}{x^4} \to C_0(k^2) = C_{1/2}(k^2) = C_1(k^2) \sim \log\left(\frac{\Lambda^2}{k^2}\right). \tag{6.57}$$

The precise coefficient appearing in the two-point functions, which is usually called τ, has been exactly determined from 't Hooft anomaly in [114]. It gives the contribution of the CFT matter to the beta function associated to the gauge coupling constant of the $U(1)$ subgroup of $SO(6)$ that we are gauging. For the AdS_5 case, our result gives $\tau = 2N^2$. We note here that such a large number would be in contrast with keeping the SSM gauge couplings perturbative before unification. We have alredy mentioned this problem in the introduction and will not comment on this further, besides saying that we are really trying to extract from this holographic approach qualitative features of correlators in strongly coupled hidden sectors, that we assume are a good approximation even outside the large N limit.

6.3 Holographic correlators in a dilaton-domain wall

In this section we will consider supersymmetry breaking solution for the background, and indeed we will obtain non-trivial results for the soft masses. First we will keep a trivial profile for the squashing mode, $\eta = 0$, but allow for a non-trivial dilaton profile. We will see how the IR behavior of the correlators will change drastically with respect to their conformal expressions found in the previous section.

The dilaton-domain wall is, in fact, a solution of the full $10D$ type IIB supergravity found in [106,107]. This is a singular solution with a non-trivial background for the dilaton ϕ which preserves the full $SO(6)$ R-symmetry. Upon dimensional reduction on S^5 we get the following $5D$ background

$$ds_5^2 = \frac{1}{z^2}(dz^2 + \sqrt{1 - z^8}\,(dx^i)^2) \,, \tag{6.58}$$

$$\phi(z) = \phi_\infty + \sqrt{6}\,\mathrm{arctanh}(z^4) \,. \tag{6.59}$$

The metric goes to AdS_5 at the boundary $z \to 0$ and presents a naked singularity in the deep interior of the bulk, which we have set to $z = 1$ by adjusting one of the constants of integration. At the singularity the dilaton diverges

$$\lim_{z\to 1}\phi(z)=\infty\ . \tag{6.60}$$

The presence of the naked singularity signals a breakdown of the supergravity approximation and therefore the holographic interpretation of this background as a well-defined field theory could be problematic. It appears that this particular singularity is physically acceptable according to the two criteria of [115] and [116]. Respectively, its scalar potential is bounded from above (it is exactly zero), and g_{tt} (of the $10D$ solution) is monotonously decreasing towards the singularity. A possible physical interpretation of this background was discussed in [106, 108]. Suffices here to say that it describes a vacuum of a theory which in the UV coincides with $\mathcal{N}=4$ SYM, where however a non-trivial VEV for $\mathrm{tr}\,F_{ij}^2$ is turned on triggering confinement and SUSY breaking. In the following we will probe some of its features by the explicit computation of the GGM correlators. This background is interesting for our program because it breaks, besides conformality, all the supersymmetries (as one can see from the supersymmetry transformation of the dilatino) and it preserves the $SO(6)$ symmetry, so that we can consider an $\mathcal{N}=2$ vector multiplet gauging a $U(1)\subset SO(6)$.

The effective action at the linearized level for the $\mathcal{N}=2$ vector multiplet in the dilaton-domain wall is of the form (5.55), and the resulting equations of motion will take the schematic form

$$(\Box_{DW}-4)D\equiv\left(z^2\partial_z^2-\left(\frac{3+5z^8}{1-z^8}\right)z\partial_z+\frac{z^2(\partial_i)^2}{\sqrt{1-z^8}}-4\right)D=0\ , \tag{6.61}$$

$$(\mathrm{Max})_{DW}A_i\equiv\left(z^2\partial_z^2-\left(\frac{1+3z^8}{1-z^8}\right)z\partial_z+\frac{z^2(\partial_i)^2}{\sqrt{1-z^8}}\right)A_i=0\ , \tag{6.62}$$

$$(\not{D}_{DW}-\frac{1}{2})\lambda\equiv\left(z\gamma_z\partial_z-2\frac{1+z^8}{1-z^8}\gamma_z+\frac{z}{(1-z^8)^{1/4}}\gamma^i\partial_i-\frac{1}{2}\right)\lambda=0\ . \tag{6.63}$$

We note that the AdS equations are modified by terms of $O(z^8)$ in a near boundary expansion.

The second order equations for the fluctuations of the supergravity fields can be solved once two boundary conditions are specified.[3] One boundary condition will always determine the leading term at the boundary, fixing the overall normalization of the solution, the second condition should be a regularity condition in the bulk, as we explained in section 5.3. In this case, since the geometry terminates at a finite value of z, the regularity condition in the interior amounts to fixing the behavior near the singular point $z=1$.

[3]For the sake of the argument that follows, we can convert the two first order equations for the spinors χ and $\bar{\xi}$ into a single second order equation for $\bar{\xi}$.

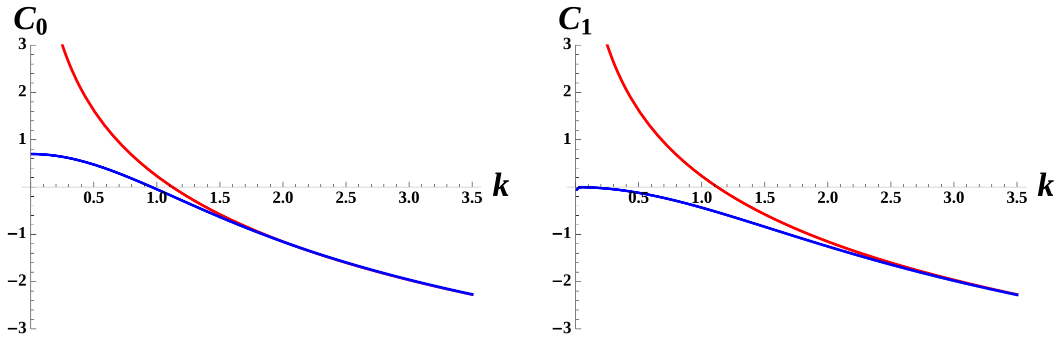

Figure 6.1: C_0 and C_1 functions: in red the AdS logarithm, in blue the dilaton domain wall result.

Expanding eqs. (6.61-6.63) to the leading order in $1 - z \equiv y \to 0$ we get

$$(y^2\partial_y^2 + y\partial_y)D = 0 \ , \tag{6.64}$$

$$(y^2\partial_y^2 + \frac{1}{2}y\partial_y)A_i = 0 \ , \tag{6.65}$$

$$(y^2\partial_y^2 + \frac{5}{4}y\partial_y - \frac{1}{8})\bar{\xi} = 0 \ . \tag{6.66}$$

whose solutions are given in terms of two undetermined coefficients α and β as

$$D \underset{y\to 0}{\simeq} \alpha_0 \log y + \beta_0 \ , \tag{6.67}$$

$$A_i \underset{y\to 0}{\simeq} \alpha_{i1} + \beta_{i1} y^{1/2} \ , \tag{6.68}$$

$$\bar{\xi} \underset{y\to 0}{\simeq} \alpha_{1/2} y^{-1/2} + \beta_{1/2} y^{1/4} \ . \tag{6.69}$$

The differential equations are well posed if we require, for all of the three fields, that a linear combination of α and β vanishes.[4] A condition giving a unequivocal choice for all of the three fields is requiring that both the field and its derivative are finite at the singularity. This condition can be satisfied for all of the three fields and their first derivatives, except for the first derivative of the fermion, which will diverge in any case. We thus select the choice of parameters $\alpha_0 = \beta_1 = \alpha_{1/2} = 0$.[5]

Once we specify the boundary conditions, a solution to eqs. (6.61-6.63) can be found numerically for any value of the parameter k corresponding to the $4D$ momentum. By using the holographic formulas (5.70 - 5.72) we can then plot the C_s functions.

We show the plots in figures 6.1, and 6.2. In each graph we plot both the result for the supersymmetric AdS case, as well that for the dilaton domain wall solution.

[4]For instance $D = 0$ or $\partial\bar{\xi} =$ const. at the singularity are not suitable boundary conditions because they would kill both the coefficients.

[5]More general choices of the boundary conditions are in principle allowed (in bottom-up approaches for instance), and would give rise to different physics. We will see more on this perspective in the next chapter when discussing hard wall models.

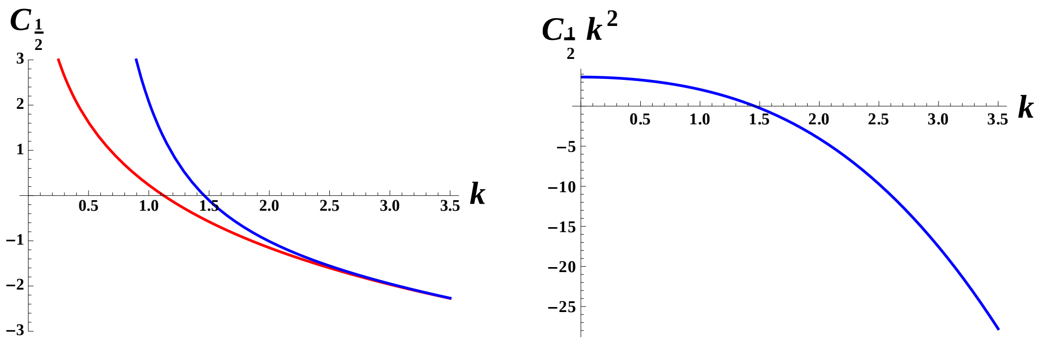

Figure 6.2: $C_{1/2}$ function: the plot on the left shows in red the AdS logarithm, in blue the dilaton domain wall result; the plot on the right shows the $1/k^2$ pole for low momenta

Notice that, as expected, the AdS tails for the three graphs correctly coincide and tend to their supersymmetric value.

One of the interesting results of the plots is the k^{-2} IR behavior of the fermionic correlator $C_{1/2}$. In figure 6.2 we plot $k^2 C_{1/2}$, which clearly shows this correlator has a $1/k^2$ pole at zero momentum. This kind of behavior is related to the existence of massless excitations carrying the same quantum numbers of the corresponding current. For the fermionic current j_α, this signals the existence of massless fermions, tipically 't Hooft fermions, that compensate the global anomaly of the unbroken $U(1)_R$-symmetry [27]. Note, in passing, that imposing the “wrong” boundary condition for the vector field fluctuations, namely $\alpha_1 = 0$, we would have gotten a k^{-2} pole also for C_1. For the vector current this would be a massless Goldstone boson. This would imply the existence of Goldstone bosons associated to some broken global symmetry, which cannot be the case here since the original $10D$ background preserves the full $SO(6)$ (and hence our $U(1) \times U(1)_R$) symmetry.

While we cannot prove that there are indeed R-charged 't Hooft fermions in our strongly coupled theory, and just observe that the holographic analysis suggests them to be there, it is useful to refer to the full $10D$ background to get some more confidence about our result. From the $10D$ perspective there is a whole $SO(6)$ symmetry which the background preserves. Hence, at every scale there must exist massless fermions in the spectrum so to match the UV global anomaly. The UV fixed point is $\mathcal{N} = 4$ SYM, which has indeed a non-zero global anomaly for the $SO(6)$ current. At this point one may think that the $U(1)$ global symmetry of the hidden sector which we are eventually weakly gauging and identifying with the (simplified) SM gauge group is anomalous. This has not to be the case because having a non-zero $SO(6)^3$ anomaly still allows to consider a non-anomalous $U(1)$ subgroup inside $SO(6)$. On the contrary, our result suggests that (part of) the $SU(4)$ anomaly is transmitted to the $U(1)_R$ current. Let us emphasize that any other anomalous global symmetry would not provide a pole to the fermionic correlator $C_{1/2}$, which is neutral under any global symmetry but the R-symmetry. Hence, field theory expectations would suggest that when the R-symmetry is broken, R-charged 't Hooft fermions would not exist, and the pole in the fermionic correlator should vanish. We will come back to this point in the next section.

The Majorana gaugino mass, determined by $B_{1/2}$ through (2.23), vanishes because of unbroken R-symmetry. However, the pole in $C_{1/2}$ provides for a Dirac mass for the SSM gaugino (see the discussion in section 2.3.1). This is very similar to any other model of R-symmetric Dirac gaugino masses, except that the massless fermion in the adjoint that must couple bilinearly with the gaugino is here a composite fermion generated at strong coupling. The soft spectrum, in this situation, is very much reminiscent of that of gaugino mediation models. See [28, 30] for a discussion of Dirac gaugino masses in General Gauge Mediation.

Let us finally notice how different are the C_s in the dilaton domain wall background with respect to the ones in AdS, at large momentum. Numerically we find that

$$C_0 - 4C_{1/2} + 3C_1 \sim O(k^{-8}), \quad k \to \infty \ . \tag{6.70}$$

This is due to the fact that the correction of the domain wall metric with respect to the AdS one near the boundary is of $O(z^8)$. Note that since the dilaton does not enter the equations for the vector multiplet fluctuation, its $O(z^4)$ behaviour near the boundary does not influence the C_s. Another nice feature of the asymptotic behaviour (6.70) is that it makes the integral (2.25) nicely convergent in the UV.

6.4 Holographic correlators in a dilaton/η-domain wall

Let us discuss another example, and look for a solution of eqs.(6.8-6.10) with a non-trivial profile for both the dilaton *and* the squashing mode. The latter breaks the R-symmetry so one should expect a very different behavior for the correlators.

In fact, in what follows we will only turn on a perturbative profile for the R-symmetry breaking scalar η, that is we consider only the linearized equation for η on the dilaton domain-wall background, and neglect the backreaction of such a profile on the dilaton and the metric. As we are going to show, this will still be enough to provide a drastic change in the holographic correlators (nicely matching, again, field theory expectations).

The linearized equation for η is most conveniently written and solved using the following parametrization of the asymptotic AdS metric (with boundary at $r \to \infty$)

$$ds^2 = \left(dr^2 + e^{2r}(dx^i)^2\right) \tag{6.71}$$

and reads

$$\eta''(r) + 4\coth(4r)\eta'(r) + 3\eta(r) - \frac{3}{2(\sinh(4r))^2}\eta(r) = 0. \tag{6.72}$$

The solution depends on two integration constants A and B and is given by

$$\eta(r) = (e^{8r}-1)^{\frac{1}{4}\sqrt{\frac{3}{2}}}\left[A\ {}_2F_1\left(\frac{2+\sqrt{6}}{8},\frac{4+\sqrt{6}}{8},\frac{3}{4},e^{8r}\right)\right.$$
$$\left. +B\ {}_2F_1\left(\frac{4+\sqrt{6}}{8},\frac{6+\sqrt{6}}{8},\frac{5}{4},e^{8r}\right)\right] , \qquad (6.73)$$

where ${}_2F_1$ is the hypergeometric function.

Changing variables to the usual $z = e^{-r}$ radial coordinate, one can verify that indeed this solution has the expected behavior (6.25) near the boundary, with η_0 and $\tilde{\eta}_2$ expressed as linear combinations of A and B. On the other hand, studying the equation near the singularity $y = 1 - z \to 0$ one finds the following behavior

$$\eta \underset{y\to 0}{\simeq} \alpha y^{\frac{1}{4}\sqrt{\frac{3}{2}}} + \beta y^{-\frac{1}{4}\sqrt{\frac{3}{2}}} , \qquad (6.74)$$

with α and β which are in turn linear combinations of A and B. If one imposes the boundary condition at the singularity so to meet the criterion on the boundedness of the potential [115], that is $\beta = 0$, one finds a relation between A and B which imposes both η_0 and $\tilde{\eta}_2$ to be turned on at the boundary (indicating that R-symmetry is broken explicitly in the hidden sector). This implies that in doing the holographic renormalization procedure one should bear in mind the discussion in section 6.1.3 and augment the boundary action by the term (6.26).

Plugging our results in the formulas for the holographic correlators (5.53) and (5.54), it is easy to see that C_0 and C_1 are unaffected. On the other hand, both fermionic correlators are modified. As shown in figure 6.3 the correlator $B_{1/2}$ has now a non-trivial dependence on the momenta. Consistently with expectations, it reaches a finite value at zero momentum (hence providing non-vanishing Majorana mass to SSM gauginos), and falls off to zero at $k \to \infty$. On the other hand, the pole at $k^2 = 0$ in $C_{1/2}$ has now disappeared (see figure 6.3). This is consistent with field theory intuition: in fact this is precisely the holographic realization of the case with 't Hooft fermions and an R-symmetry breaking perturbation, that we discussed in section 2.3.1.

6.5 Possible generalizations

In this chapter we have been working in the context of $5D$ consistent truncations of type IIB string theory and focused our attention on supersymmetry breaking asymptotically AdS backgrounds. The same techniques can be applied, in principle, to several other models and can also find applications in different contexts. The approach of using a bottom-up set up, in particular hard wall models, will be discussed

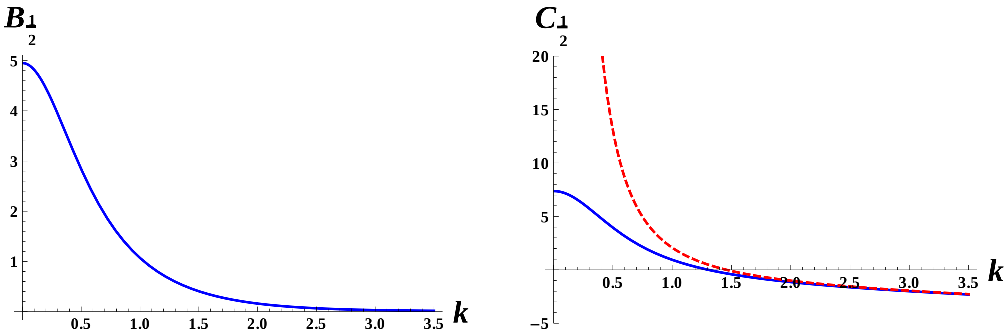

Figure 6.3: $B_{1/2}$ and $C_{1/2}$ functions in presence of a non-trivial profile for η: the plot on the left shows that a non-zero Majorana mass for the gaugino is generated in this case; the red dashed line, in the plot on the right, shows the result for $\eta = 0$.

in the next chapter. Here we would like to mention possible directions to generalize the construction while keeping a top-down approach.

A natural next step is to try and generalize the present holographic approach to backgrounds which are not asymptotically AdS. Indeed, a superconformal theory cannot break supersymmetry spontaneously (because of the tracelessness of the stress-energy tensor). Thus, the cases considered here and in [109] are nice toy-models but cannot be the full story for a genuine SUSY breaking hidden sector. It would be interesting, in this respect, to extend our analysis to cascading backgrounds, as those considered in [117,118]. Alternatively, one would need at least two distinct scalar profiles to be turned-on in the background, that are independently responsible for the explicit breaking of conformality and the spontaneous breaking of supersymmetry.

Eventually, the addition of probe or backreacting D7 branes to represent the SSM gauge groups will also be a necessary ingredient, especially if one wants to make contact with the original set up of holographic gauge mediation [117, 118]. Indeed, in $5D$ backgrounds descending from string theory without explicit D-brane sources, the maximal global symmetry one can have is $SO(6) \simeq SU(4)$, i.e. the rank is not big enough to match that of the SSM gauge group. Adding D7 branes is thus necessary in this top-down approach, even though it makes the mediation of SUSY breaking less direct and neat.

Chapter 7

Hard-Wall Models

In the concrete models of HGGM we discussed in the previous chapter, the constraints of supergravity dictated the precise form of the interactions, and there was no free parameter left to play with. While this can be a welcome feature from the point of view of the predictivity of the model, it would be interesting to have more flexible, bottom-up examples of HGGM, in the same spirit of the toy-models for the hidden sector which, in a purely field-theoretical context, we treated in chapter 3. Given such class of examples, one could hope to address interesting questions such as: how large a portion of the GGM parameter space can holographic models of gauge mediation cover? Are there any restrictions and/or preferred patterns? These questions are difficult to answer within fully-fledged top-down models, given also the poor number of concrete and sufficiently explicit string theory supersymmetry breaking solutions available in the literature.

In this chapter we will present the results in this direction obtained in [109]. We will consider supersymmetry breaking models which do not have necessarily a completion in string theory, but on the other hand allow for more flexibility and analytical power, enabling to try and answer the above questions. The simple backgrounds we will focus on are so-called hard wall (HW) models [119–121], which consists of a pure AdS background with a sharp IR cut-off in the bulk. In this setup the correlation functions can be established analytically; moreover, at the price of loosing any clear string embedding, there is more freedom in specifying the details of the background (for instance, in contrast to what we discussed in section 6.4, in this case one can allow a profile for η providing a VEV for the hidden gauginos bilinear and not a source term).

The features of this setup are defined by the boundary conditions on the fluctuating fields at the IR cut-off. A scan of all possible boundary conditions, which can be parameterized in terms of a number of (*a priori* free) parameters, gives the following results:

- Generically, that is if one does not tune any of the parameters, the resulting soft spectrum is that of gauge messengers mediation scenario. Gauginos acquire Dirac masses, while sfermions are loop suppressed, hence the soft spectrum turns out to be similar to that of gaugino mediation models. These features arise due to the presence of massless (composite) bosonic and fermionic modes in the hidden sector.

There are two possible ways out of this scenario, as we will explain in detail:

- Tuning some of the parameters one can eliminate hidden sector massless modes: in this case one can scan any value of sfermion and gaugino masses, and obtain a spectrum which can range from gaugino mediation to minimal gauge mediation or scenarios with suppressed gaugino masses, hence covering all of GGM parameter space.

- A more constrained but attractive possibility, relies on adding dynamical degrees of freedom in the bulk. Within this approach, the hidden sector massless fermionic modes disappear automatically, and standard Majorana masses for gauginos are generated, without any necessary tuning of bulk field boundary conditions.

It is worth to point out that this construction is reminiscent of GGM realizations in warped geometries, considered in [122–124] (a realization similar in spirit but with a flat extra dimension was presented in [125]). The essential difference between [122–124] and the present approach can be expressed, in the terminology of [126, 127], by saying that the latter uses a holographic picture, while the former rely on a Kaluza-Klein picture, or 5D picture, in which the visible fields actually live in a universe with a compact and warped extra-dimension, namely a slice of AdS. While the physics of the two models is essentially the same, the advantage of the holographic picture is that one can concentrate on the hidden sector, disregarding the details of the mediation mechanism, so to directly obtain closed formulas for the GGM form factors, which make more transparent the range of possibilities that can be covered by changing the IR boundary conditions.

The chapter is organized as follows. We will start by introducing the hard wall model, and by showing that one can impose a variety of different boundary conditions on the bulk fields at the IR wall. We will then discuss the phenomenological consequences of this freedom, and see when and how the general results anticipated above can actually be achieved. A somewhat unpleasant feature of this way of proceeding is that, in order to cover large portions of GGM parameter space, and in fact allow for non-vanishing Majorana gaugino masses to begin with, *ad hoc* boundary conditions should be imposed. Therefore, we will then consider examples in which, by turning on a background profile for a scalar whose dual operator is charged under the R-symmetry, gaugino masses can in fact be generated in a more dynamical way, following a similar strategy as that of the previous chapter. We will conclude by showing the derivation of some analytic results about the behavior of GGM correlators at low momenta which we will be useful along the way.

7.1 Description of the setup

Let us now consider the model that will be our primary interest in this chapter, the hard wall model. This is just AdS_5 in which the geometry ends abruptly in the interior by putting a sharp IR cut-off at $z = 1/\mu$. This model was originally studied as a toy model of a confining gauge theory because it provides an holographic dual for theories with a gapped and discrete spectrum [119–121]. When discussing correlators in the hard-wall model, it is useful to keep in mind the AdS case of section 6.2.

Indeed, in the case of an hard wall background the general solution of the equations of motion for the fluctuations in the vector multiplet is exactly the same as for pure AdS, eqs. (6.36-6.38), and depends on six integration constants (two for each field). Also the UV boundary conditions (6.41) remain the same, and can be simply understood as fixing the source of the boundary operator, leaving only three constants undetermined. However, we are now solving the differential equations in the open set $(0, 1/\mu)$, and the regularity conditions are replaced by some IR boundary conditions at $z = 1/\mu$. These conditions can be solved for the three remaining constants, in order to fix the functional dependence of the subleading modes on the leading ones.

The generic form of the solutions is

$$D(z,k) = D^{AdS}(z,k) + c_1(k) z^2 I_0(kz) \ , \tag{7.1}$$

$$A_i(z,k) = A_i^{AdS}(z,k) + \alpha_{1i}(k) z I_1(kz) \ , \tag{7.2}$$

$$\bar{\xi}(z,k) = \bar{\xi}^{AdS}(z,k) + \bar{\theta}_1(k) z^{5/2} I_1(kz) \ , \tag{7.3}$$

where c_1, α_{i1} and $\bar{\theta}_1$ are integration constants determined by the IR boundary conditions. Expanding these expressions near the UV boundary, one can easily find that the correlators are modified with respect to the AdS case in the following way

$$C_0(k^2) = C^{AdS}(k^2) - 2\frac{\delta c_1}{\delta d_0} \ , \tag{7.4}$$

$$C_1(k^2) = C^{AdS}(k^2) - \frac{1}{2k}\delta^i_j \frac{\delta \alpha_{1i}}{\delta a_{0j}} \ , \tag{7.5}$$

$$C_{1/2}(k^2) = C^{AdS}(k^2) - \left(\frac{1}{2k}\delta^{\dot{\alpha}}_{\dot{\beta}} \frac{\delta \bar{\theta}_1^{\dot{\beta}}}{\delta \bar{\xi}_0^{\dot{\alpha}}} + c.c. \right) \ , \tag{7.6}$$

$$B_{1/2}(k^2) = -\frac{\sigma^i_{\alpha\dot{\alpha}} k_i}{k} \frac{\delta \bar{\theta}_1^{\dot{\alpha}}}{\delta \xi_{0\alpha}}. \tag{7.7}$$

7.1.1 Homogeneous IR boundary conditions

We start by taking general homogeneous boundary conditions at the IR cut-off

$$(D(z,k) + \rho_0 z\partial_z D(z,k))|_{z=1/\mu} = 0 \ , \tag{7.8}$$
$$(A_i(z,k) + \rho_1 z\partial_z A_i(z,k))|_{z=1/\mu} = 0 \ , \tag{7.9}$$
$$(\bar{\xi}(z,k) + \rho_{1/2} z\partial_z \bar{\xi}(z,k))|_{z=1/\mu} = 0 \ , \tag{7.10}$$

which depend on three independent coefficients ρ_s. As we will see, in order to cover all of GGM parameter space it will be necessary to turn on also inhomogeneous terms in the above equations, something we will do next.

As it befits coefficients computed with homogeneous boundary conditions, the coefficients c_1, α_{1i} and $\bar{\theta}_1$ in (7.1-7.3) are all proportional to the source terms. The resulting GGM functions are

$$C_0^{(h)}(k^2) = C^{AdS}(k^2) + 2\frac{-(1+2\rho_0)K_0(\frac{k}{\mu}) + \rho_0\frac{k}{\mu}K_1(\frac{k}{\mu})}{(1+2\rho_0)I_0(\frac{k}{\mu}) + \rho_0\frac{k}{\mu}I_1(\frac{k}{\mu})} \ , \tag{7.11}$$
$$C_1^{(h)}(k^2) = C^{AdS}(k^2) + 2\frac{K_1(\frac{k}{\mu}) - \rho_1\frac{k}{\mu}K_0(\frac{k}{\mu})}{I_1(\frac{k}{\mu}) + \rho_1\frac{k}{\mu}I_0(\frac{k}{\mu})} \ , \tag{7.12}$$
$$C_{1/2}^{(h)}(k^2) = C^{AdS}(k^2) + 2\frac{(1+\frac{3}{2}\rho_{1/2})K_1(\frac{k}{\mu}) - \rho_{1/2}\frac{k}{\mu}K_0(\frac{k}{\mu})}{(1+\frac{3}{2}\rho_{1/2})I_1(\frac{k}{\mu}) + \rho_{1/2}\frac{k}{\mu}I_0(\frac{k}{\mu})} \ , \tag{7.13}$$
$$B_{1/2}^{(h)}(k^2) = 0 \ . \tag{7.14}$$

The analysis of the boundary condition-dependent soft spectrum emerging from the correlators (7.11-7.13) is postponed to section 7.2. For future reference we would instead like to comment here on the behavior in the IR and UV. Making use of the asymptotic expansion for $x \ll 1$ for the Bessel functions (6.39-6.40) we find the correlators at low momentum to behave as

$$C_0^{(h)}(k^2) \underset{k\to 0}{\simeq} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \frac{2\rho_0}{1+2\rho_0} \ , \tag{7.15}$$
$$C_1^{(h)}(k^2) \underset{k\to 0}{\simeq} \frac{4}{1+2\rho_1}\frac{\mu^2}{k^2} + \log\left(\frac{\Lambda^2}{\mu^2}\right) - \frac{3+8\rho_1}{2(1+2\rho_1)^2} \ , \tag{7.16}$$
$$C_{1/2}^{(h)}(k^2) \underset{k\to 0}{\simeq} 4\frac{2+3\rho_{1/2}}{2+7\rho_{1/2}}\frac{\mu^2}{k^2} + \log\left(\frac{\Lambda^2}{\mu^2}\right) - \frac{(2+3\rho_{1/2})(6+25\rho_{1/2})}{2(2+7\rho_{1/2})^2} \ . \tag{7.17}$$

As for the UV limit, given the large x behavior of Bessel functions (6.43), we can see that all the $C_s^{(h)}$ functions approach the supersymmetric AdS value with exponential rate at large momentum

$$C_0^{(h)}(k^2) \sim C_{1/2}^{(h)}(k^2) \sim C_1^{(h)}(k^2) \underset{k\to\infty}{\simeq} C^{AdS}(k^2) - 2\pi e^{-\sqrt{\frac{k^2}{\mu^2}}} \ . \tag{7.18}$$

From the field theory point of view, the exponential suppression in the UV suggests that the breaking of supersymmetry in a hidden sector described by a hard wall is not induced by any operator, which generically would appear in the OPE of the GGM correlation functions with a scaling behavior in k^2 fixed by its dimension.

Two additional remarks are in order at this point. The first is that one can of course compute the above functions also using the numerical approach pursued in the previous chapter, finding perfect agreement with the analytic computation above. The second comment is that the above functions can be continued to negative values of k^2. It is easy to convince oneself that they will then display an infinite sequence of poles on the negative k^2 axis, corresponding to the glueball towers for each spin sector. They return the same values that can be obtained through the more traditional holographic approach of computing glueball masses, i.e. finding normalizable fluctuations for each field.

7.1.2 Inhomogeneous IR boundary conditions

Let us now consider the possibility of having inhomogeneous boundary conditions in the IR. We thus take general boundary conditions at the IR cut-off depending on three more arbitrary terms, namely

$$(D(z,k) + \rho_0 z\partial_z D(z,k))|_{z=1/\mu} = \Sigma_0(k) \ , \tag{7.19}$$

$$(A_i(z,k) + \rho_1 z\partial_z A_i(z,k))|_{z=1/\mu} = \Sigma_{i1}(k) \ , \tag{7.20}$$

$$(\bar{\xi}(z,k) + \rho_{1/2} z\partial_z \bar{\xi}(z,k))|_{z=1/\mu} = \bar{\Sigma}_{1/2}(k) \ . \tag{7.21}$$

The coefficients c_1, α_{i1} and $\bar{\theta}_1$ in eqs. (7.1-7.3) will pick up an additional contribution, linear in the Σ_s. Since these coefficients enter the GGM functions only through the first derivative with respect to the source, the inhomogeneous terms can contribute only if we allow them to be dependent on the source, with the result that the condition at $z = 1/\mu$ involves both IR and UV data of the function. In particular, from eq. (7.7), a dependence of $\bar{\Sigma}_{1/2}(k)$ on the source ξ_0 can give a non-vanishing $B_{1/2}$, as opposed to the case of homogenous boundary conditions (7.14). Therefore, such a dependence implies that the boundary condition (7.21) explicitly breaks the R-symmetry.

Since, in any case, only the first derivative enters eqs.(7.4-7.7), it is enough to let the Σ_s depend linearly on the sources $d_0(k), a_{i0}(k)$ and $\xi_0(k)$. Taking into account

Lorentz invariance, a reasonable choice is

$$\Sigma_0(k^2) = -\frac{1}{\mu^2}E_0 d_0(k) \ , \tag{7.22}$$

$$\Sigma_{i1}(k^2) = -E_1 a_{i0}(k) \ , \tag{7.23}$$

$$\bar{\Sigma}^{\dot{\alpha}}_{1/2}(k^2) = -\frac{1}{\mu^{3/2}}E_{1/2}\bar{\xi}^{\dot{\alpha}}_0(k) - H_{1/2}\frac{1}{\mu^{7/2}}\bar{\sigma}_i^{\dot{\alpha}\alpha}k^i\xi_{\alpha 0}(k) \ , \tag{7.24}$$

where the E's and H are coefficients which do not depend on the momentum. Hence we are left with 4 new parameters due to the inhomogeneous boundary conditions.

The GGM C functions in this case take the form

$$C_0^{(nh)}(k^2) = C^{AdS}(k^2) + 2\frac{-(1+2\rho_0)K_0(\frac{k}{\mu}) + \rho_0\frac{k}{\mu}K_1(\frac{k}{\mu}) + E_0}{(1+2\rho_0)I_0(\frac{k}{\mu}) + \rho_0\frac{k}{\mu}I_1(\frac{k}{\mu})} \ , \tag{7.25}$$

$$C_1^{(nh)}(k^2) = C^{AdS}(k^2) + 2\frac{K_1(\frac{k}{\mu}) - \rho_1\frac{k}{\mu}K_0(\frac{k}{\mu}) + \frac{\mu}{k}E_1}{I_1(\frac{k}{\mu}) + \rho_1\frac{k}{\mu}I_0(\frac{k}{\mu})} \ , \tag{7.26}$$

$$C_{1/2}^{(nh)}(k^2) = C^{AdS}(k^2) + 2\frac{(1+\frac{3}{2}\rho_{1/2})K_1(\frac{k}{\mu}) - \rho_{1/2}\frac{k}{\mu}K_0(\frac{k}{\mu}) + \frac{\mu}{k}E_{1/2}}{(1+\frac{3}{2}\rho_{1/2})I_1(\frac{k}{\mu}) + \rho_{1/2}\frac{k}{\mu}I_0(\frac{k}{\mu})} \ , \tag{7.27}$$

and we also get a non-zero result for the B function, that is

$$B_{1/2}^{(nh)}(k^2) = 2\frac{\frac{k}{\mu}H_{1/2}}{(1+\frac{3}{2}\rho_{1/2})I_1(\frac{k}{\mu}) + \rho_{1/2}\frac{k}{\mu}I_0(\frac{k}{\mu})} \ . \tag{7.28}$$

Indeed, the boundary condition (7.24) explicitly breaks the R-symmetry when $H_{1/2} \neq 0$. The result with homogeneous boundary condition is simply recovered by setting the E's and H to zero.

The inhomogeneous terms contribute to the IR behavior as follows

$$C_0^{(nh)}(k^2) - C_0^{(h)}(k^2) \underset{k\to 0}{\simeq} \frac{2}{1+2\rho_0}E_0 \ , \tag{7.29}$$

$$C_1^{(nh)}(k^2) - C_1^{(h)}(k^2) \underset{k\to 0}{\simeq} \frac{4}{1+2\rho_1}\frac{\mu^2}{k^2}E_1 \ , \tag{7.30}$$

$$C_{1/2}^{(nh)}(k^2) - C_{1/2}^{(h)}(k^2) \underset{k\to 0}{\simeq} \frac{8}{2+7\rho_{1/2}}\frac{\mu^2}{k^2}E_{1/2} \ , \tag{7.31}$$

$$B_{1/2}^{(nh)}(k^2) \underset{k\to 0}{\simeq} \frac{8}{2+7\rho_{1/2}}H_{1/2} \ . \tag{7.32}$$

As for the UV asymptotic, the large x behavior of the Bessel functions (6.43) tells us that the exponential approach to the supersymmetric limit remains valid in this case, also for $B_{1/2}(k^2)$ that asymptotes to 0. So we see that, consistently, the inhomogeneous boundary conditions do not modify the UV behavior.

7.2 Analysis of the soft spectrum

We now discuss the physical interpretation, in terms of soft supersymmetry breaking masses, of the C_s and B functions we found in the previous section.

Let us start with a very basic requirement: since the correlators happen to have non-trivial denominators which depend on the momentum, we should exclude the possibility that tachyonic poles are developed. The denominators are linear combinations of two Bessel functions evaluated at $x = k/\mu$, and studying their monotonicity properties and their limits for $x \to 0$ and $x \to \infty$ one can easily see that the poles are excluded if and only if the coefficients of the linear combination have the same sign. This condition results in the following inequalities

$$\{\rho_0 \leq -\frac{1}{2}\} \cup \{\rho_0 \geq 0\} \ , \{\rho_1 \geq 0\} \ , \{\rho_{1/2} \leq -\frac{2}{3}\} \cup \{\rho_{1/2} \geq 0\} \ . \tag{7.33}$$

The IR behavior of the C_s functions, in particular the expressions given in eqs. (7.15-7.17), shows that the theory described holographically by the hard wall has a threshold μ for the production of two particle states and possibly a certain number of massless poles which depends on the choice of the boundary conditions. Below we analyze the cases of homogeneous and inhomogeneous boundary conditions in turn.

7.2.1 Homogeneous boundary conditions

For generic choices of ρ_s parameters, we see from eqs. (7.15-7.17) that C_1 and $C_{1/2}$ have poles at $k^2 = 0$ while C_0 has not. The interpretation of such poles is that they arise from the exchange of a massless state with the same quantum numbers of the corresponding operator, as we discussed in section 2.3.1.

In that section we have already described the consequence on the soft spectrum of poles in the correlators C_1 and/or $C_{1/2}$, which was studied in [27, 28], and can be summarized as follows: the gauge boson gets a mass due to the would-be Goldstone boson in C_1 , the gaugino acquires a Dirac mass by mixing with the would-be massless fermion in $C_{1/2}$ (recall that a Majorana mass is forbidden by the unbroken R-symmetry), and the integral giving the sfermion masses is dominated by the contribution of the poles. Comparing with the usual result in General Gauge Mediation without IR singularities, the sfermion soft mass is enhanced by a logarithm of the gauge coupling. Notice that the pole in C_1 ($C_{1/2}$) contributes with a negative (positive) sign, so that generically one can get a tachyonic contribution to the sfermion mass-squared. In formulae

$$m_{\tilde{g}} = gM_{1/2} \ , \tag{7.34}$$

$$m^2_{\tilde{f}} \simeq \frac{g^4}{(4\pi)^2}\left(\log\frac{1}{g^2}\right)(4M^2_{1/2} - 3M^2_1) \ , \tag{7.35}$$

where g is the gauge coupling, $m_{\tilde{g}}$ is the Dirac mass of the gaugino, $m_{\tilde{f}}$ is the sfermion mass, and M_s^2 is the residue of the massless pole $C_s \simeq M_s^2/k^2$. From eqs. (7.16-7.17) we see that in the present case[1]

$$M_1^2 = 4\mu^2 \frac{1}{1+2\rho_1} \ , \ M_{1/2}^2 = 4\mu^2 \frac{1+\frac{3}{2}\rho_{1/2}}{1+\frac{7}{2}\rho_{1/2}}. \tag{7.36}$$

Notice that in the tachyon-free range (7.33) the two residues are always positive. If we further impose the contribution to the sfermion mass-squared (7.35) to be positive, we get the additional inequality

$$\rho_1 \geq -\frac{1}{8} \, \frac{1-\frac{9}{2}\rho_{1/2}}{1+\frac{3}{2}\rho_{1/2}}. \tag{7.37}$$

We see from eqs. (7.34-7.35) that in this scenario the sfermions are somewhat lighter that the gaugino. This is typical of Dirac gaugino scenarios [30], though in our model the Dirac partner of the gaugino is a strongly coupled composite fermion.

Tuning the ρ_s parameters

We now briefly mention different possibilities to evade the generic scenario presented above, which can be realized by choosing specific values for the ρ_s parameters.

1. As a first possibility, consider the case in which $M_1^2 = M_{1/2}^2$, that is

$$\rho_1 = \frac{\rho_{1/2}}{1+\frac{3}{2}\rho_{1/2}} \ , \tag{7.38}$$

   while ρ_0 is kept generic. We are still in a scenario in which the global symmetry is spontaneously broken in the hidden sector, and the soft spectrum is described by the same formulae as before (notice however that the contribution to the sfermion mass-squared is positive, now). Nevertheless, in this case we can argue a different interpretation of the physics in the hidden sector, the reason stemming from a somehow surprising fact: the condition (7.38) that makes the two residues coincide, actually renders the whole C_1 and $C_{1/2}$ functions (7.12) and (7.13) equal for all values of k^2. As a consequence, one is led to interpret the massless fermion as the partner of the Goldstone boson associated to the broken global symmetry, rather than a 't Hooft fermion. Since C_0 differs from $C_1 = C_{1/2}$ for generic ρ_0, supersymmetry is still broken in the hidden sector, but mildly enough so not to lift the fermionic partner of the Goldstone boson.

[1]Here and in the following we tacitly assume that the prefactor $N^2/8\pi^2$ can be set to unity. For this value the overall normalization of the physical correlation functions (2.7-2.10) coincides with the one that we have used throughout (5.70-5.73).

2. As a subcase of 1, consider in addition to tune the ρ_0 parameter to $\rho_0 = -1/2$. In this case the low momentum expansion (7.15) is not valid, and by repeating the analysis one finds that also C_0 develops a $1/k^2$ pole, with residue $M_0^2 = 4\mu^2$. As explained in [27, 28], a pole in C_0 is unphysical, unless the hidden sector breaks the global symmetry in a supersymmetric manner, so that $C_0 = C_{1/2} = C_1$ and a massless Goldstone mode is present in all three functions[2]. Indeed, if we require $M_0^2 = M_{1/2}^2 = M_1^2$, that is $\rho_1 = \rho_{1/2} = 0$, we find from eqs. (7.11-7.13) that this condition is sufficient to ensure $C_0 = C_{1/2} = C_1$ for all values of k^2, supporting the interpretation of a supersymmetric global symmetry breaking in the hidden sector.

3. Finally, ρ_1 and $\rho_{1/2}$ can also be (independently) tuned in such a way to eliminate the massless pole in C_1 and $C_{1/2}$ respectively, the specific values being $\rho_1 = \infty$ [3] and $\rho_{1/2} = -2/3$. If only one of the two parameters is tuned, the soft masses and the interpretation of the physics in the hidden sector remains the same as in the previous section, with the only difference that M_1^2 or $M_{1/2}^2$ are tuned to 0. It is therefore more interesting to consider the possibility that both parameters are tuned: in this case none of the C_s has an IR singularity and we are in a situation similar to ordinary GGM, as far as sfermion masses are concerned (the gaugino remains massless because the hidden sector does not break the R-symmetry). Since at large k all the C_s approach their supersymmetric value exponentially, the weighted sum $-(C_0 - 4C_{1/2} + 3C_{1/2})$ goes to zero at the same rate, so that we can determine the sign of the sfermion mass-squared by studying its IR limit. From eqs. (7.15-7.17) we see that the leading term, with the present values of ρ_1 and $\rho_{1/2}$, is given by

$$-(C_0 - 4C_{1/2} + 3C_1) \underset{k\to 0}{\simeq} -\frac{2\rho_0}{1+2\rho_0} \ . \tag{7.39}$$

In the tachyon-free range (7.33) this expression is negative. Therefore, in this tuned scenario we find vanishing gaugino mass and tachyonic sfermion mass. We will see later that both this unwanted features can be overcome: one way, which is somehow more *ad-hoc*, consists in enlarging the parameter space by considering inhomogeneous boundary conditions; the other, which is more dynamical, consists in turning on a R-breaking scalar on top of the hard wall background. Most of what follows will therefore consist in improvements of this setting with tuned ρ_1 and $\rho_{1/2}$.

[2] In the simple example of a $U(1)$ broken by the VEV of a charged chiral superfield the pole in C_0 is related to the modulus of the complex scalar.

[3] A global parametrization which avoids infinities could be conveniently given in terms of angles α_s, the change of variable being $\rho_s = \mathrm{tg}(\alpha_s)$.

7.2.2 Inhomogeneous boundary conditions

Let us proceed by considering the functions (7.25-7.27), which we obtained adding source-dependent inhomogeneous terms in the boundary conditions. Besides the ρ_s, we have now four additional real parameters to play with, namely the dimensionless E_s and the R-breaking parameter $H_{1/2}$, which has dimension of a mass.

For generic values of the parameters the situation is analogous to the one with homogeneous boundary conditions, so that the E_s parameters appear to be somehow redundant: C_1 and $C_{1/2}$ have a massless pole, while C_0 has not. The major difference with respect to the previously considered case is that now $H_{1/2}$ gives a non-zero Majorana mass to the gaugino,

$$m_{\tilde{g}} = \frac{8}{2 + 7\rho_{1/2}} H_{1/2} \ . \tag{7.40}$$

Since now R-symmetry is broken, the pole in $C_{1/2}$ cannot be interpreted as due to a 't Hooft fermion, and it seems unphysical. In order to get more interesting and reasonable results, eliminating the poles at $k^2 = 0$ in $C_1^{(nh)}$ and $C_{1/2}^{(nh)}$, we can take $E_1 = -1$ and $E_{1/2} = -(1 + \frac{3}{2}\rho_{1/2})$, see eqs. (7.30) and (7.31). As opposed to eq. (7.39), the IR limit of the weighted sum $-(C_0 - 4C_{1/2} + 3C_{1/2})$ depends now on four parameters, the ρ_s and E_0, so that one can easily obtain a positive mass-squared for the sfermions. For definiteness and for an easier comparison with eq. (7.39), consider taking $\rho_1 = \infty$ and $\rho_{1/2} = -2/3$, so that

$$-(C_0 - 4C_{1/2} + 3C_1) \underset{k \to 0}{\simeq} -\frac{2\rho_0 + 2E_0}{1 + 2\rho_0} \ , \tag{7.41}$$

which can be positive if $E_0 < -\rho_0$ (assuming a positive ρ_0). The sfermion masses can then be even bigger than the Majorana gaugino mass if $H_{1/2}$ is somewhat smaller than $\sqrt{|E_0|}\mu$.

The punchline of the above analysis is that tuning appropriately the boundary conditions, one can realize holographically any scenario between pure gaugino mediation [128–135] to minimal gauge mediation [3–5] as well as scenarios with suppressed gaugino masses [7, 14, 16, 54–57] which would fit into a split supersymmetry scenario [18, 19]. Hence, hard wall models can actually cover all of GGM parameter space. In fact, it is not entirely satisfactory that a necessary ingredient for all this amounts to introduce two parameters, $H_{1/2}$ and E_0, which are directly proportional to gaugino and sfermions masses, respectively. This is reminiscent of minimal benchmark points. It would thus be desirable to try and obtain both Majorana gaugino masses and positive squared sfermions masses by enriching the dynamics in the bulk instead of introducing inhomogeneous terms in the IR boundary conditions. In the next section we will describe how this goal can indeed be achieved, by turning on a linear profile for an R-charged scalar, similarly to the appraoach we had in the previous chapter.

Let us finally mention that, as noticed in [136], a positive value for $C_1 - C_0$ is a desirable feature, in that it helps raising the mass of the Higgs in gauge mediation scenarios. In present models, this is achieved by the same conditions which make the right hand side of (7.41) positive.

7.3 Hard wall with R-symmetry breaking mode

In this section we would like to construct a simple scenario in which the R-symmetry is broken (and gaugino masses generated) dynamically. We will follow the same logic of chapter 6, where it was observed that considering only the minimal action (5.55) for the vector supermultiplet it is impossible to break the R-symmetry by bulk dynamics, and get non-zero gaugino Majorana masses. As in the top-down model considered there, we will see that the dynamical breaking of R-symmetry implies automatically the absence of massless modes in $C_{1/2}$. Notice that this physical consistency condition had instead to be imposed by hand, in the previous section.

We introduce a new dynamical scalar field η in the bulk, with $m^2 = -3$, and treat it as a linear fluctuation around the hard wall metric.

The action for η at the linearized level is completely determined by its mass, while the precise values of its couplings with the vector multiplet can be guessed by analogy with the $N = 2$ supergravity embedding considered in chapter 6, based on the general results of [91, 92]

$$S_{\text{kin}} = \frac{N^2}{4\pi^2} \int d^5x \sqrt{G} (G^{\mu\nu} \partial_\mu \eta \partial_\nu \eta - 3\eta^2) \ , \tag{7.42}$$

$$S_{\text{int}} = \frac{N^2}{4\pi^2} \int d^5x \frac{\sqrt{G}}{2} \left[(\eta + z\partial_z \eta)(\chi\chi + \bar{\chi}\bar{\chi}) + (\eta - z\partial_z \eta)(\xi\xi + \bar{\xi}\bar{\xi}) \right] \ . \tag{7.43}$$

One might think that, in view of the possibility of constructing more general bottom-up models, it might be interesting to see what happens if we take arbitrary coefficients in the interactions term. On the other hand, asking for a gravity dual of a supersymmetric field theory (which then breaks supersymmetry spontaneously or by a soft deformation) puts severe constraints on the possible interactions. In fact, precisely the constraints dictated by supergravity. One can check that choices other than the interactions above do not give the right supersymmetric result in the deep UV.

We demand the R-symmetry breaking mode $\eta(z, x)$ to have a non-trivial profile in the vacuum which is independent on the boundary space-time directions in order to preserve Poincaré invariance of the dual field theory. The most general solution

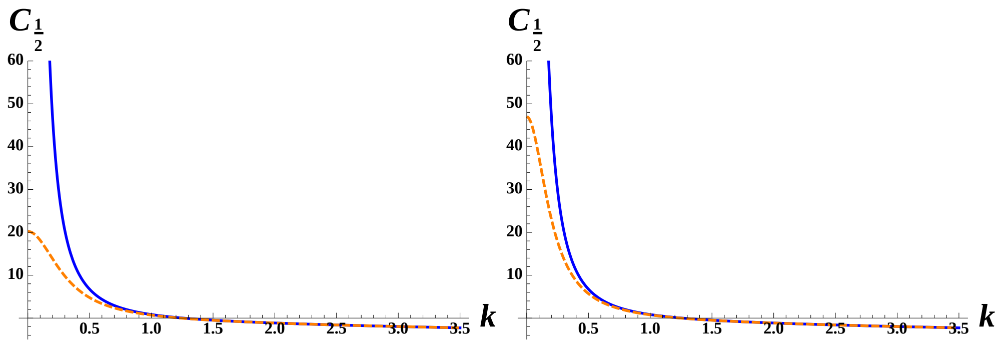

Figure 7.1: The solid blue line is for $\eta = 0$, the dashed one on the left figure is for $\eta = 0.1\ z^3$ and on the right figure for $\eta = 0.1\ z$. In these plots $\rho_{1/2} = 3$ and $\mu = 1$. Notice that turning on η the massless pole disappears.

to the resulting equations of motion for η without k dependence is

$$\eta(z) = z\eta_0 + z^3\tilde{\eta}_2 \ , \tag{7.44}$$

where η_0 and $\tilde{\eta}_2$ are two arbitrary constants. These two constants can be fixed imposing, as usual, boundary conditions at $z = 0$ and at the IR cut-off $z = 1/\mu$. This strictly amounts to considering them as free parameters, which we will do in the following.

The equations of motion for λ are modified by the presence of the extra contribution (7.43) and become

$$\begin{cases} (z\partial_z - \frac{5}{2})\chi + z\sigma^i k_i \bar{\xi} + (\eta - z\partial_z\eta)\xi = 0 \ , \\ (-z\partial_z + \frac{3}{2})\bar{\xi} + z\bar{\sigma}^i k_i \chi + (\eta + z\partial_z\eta)\bar{\chi} = 0 \ . \end{cases} \tag{7.45}$$

Comparing with eqs. (5.62), one can easily conclude that in the present case the leading boundary behavior of the fermionic field is not modified with respect to the minimal case (5.65). On the other hand, as discussed in section 6.1.3, whenever $\eta_0 \neq 0$, we have to modify the definition of the fermionic correlator defining $B_{1/2}$ according to

$$\langle j_\alpha(k) j_\beta(-k)\rangle = \frac{\delta\tilde{\chi}_{1\alpha}}{\delta\xi_0^\beta} - \frac{\delta\tilde{\chi}_{1\beta}}{\delta\xi_0^\alpha} + 2\eta_0\epsilon_{\alpha\beta} \ , \tag{7.46}$$

while the expression for the non-chiral fermionic correlator (5.72) remains unchanged.

We now need to solve eqs. (7.45) by imposing (homogeneous) boundary conditions in the IR (that for generic $\rho_{1/2}$ would give a massless pole when $\eta = 0$). Unfortunately, this cannot be done analytically, and we have to resort to numerics. Figures 7.1 and 7.2 contain our results.

It is remarkable to see that when the R-symmetry is broken by a scalar profile, the pole in $C_{1/2}$ disappears automatically. We note that the sfermion mass-squared

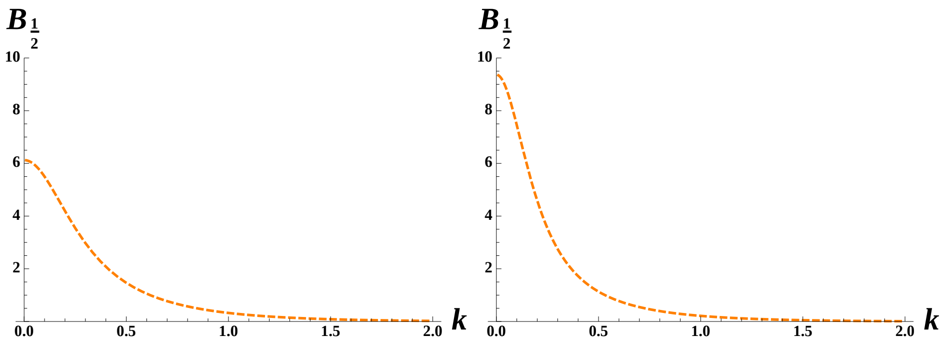

Figure 7.2: On the left $\eta = 0.1\ z^3$, on the right $\eta = 0.1\ z$. In these plots $\rho_{1/2} = 3$ and $\mu = 1$. When $\eta \neq 0$ a non-zero $B_{1/2}$ is generated.

is driven positive by the fact that $C_{1/2}$ is still quite large near $k = 0$, at least as far as η is a perturbation. If we stick to this model without playing with inhomogeneous boundary conditions in the IR, it can be seen that we are able to explore a smaller region of parameter space. (Possibly, a larger portion of parameter space can be reached by playing with $\rho_{1/2}$.)

While the above analysis is done numerically, it would be nicer to have some analytical control on (at least) the low momenta behavior of the correlators, to see, for instance, how the pole in $C_{1/2}$ disappears when the R-charged scalar is turned on. This analysis turns out to be possible if we also take the parameters η_0 and $\tilde{\eta}_2$ parametrically small, and we obtain

$$C_{1/2} \simeq \frac{1 + \frac{3}{2}\rho_{1/2}}{1 + \frac{7}{2}\rho_{1/2}} \frac{4\mu^2}{k^2 + 4M^2_{\eta_0,\tilde{\eta}_2}} \ , \tag{7.47}$$

$$B_{1/2} \simeq \frac{1 + \frac{3}{2}\rho_{1/2}}{1 + \frac{7}{2}\rho_{1/2}} \frac{8\mu^2 M_{\eta_0,\tilde{\eta}_2}}{k^2 + 4M^2_{\eta_0,\tilde{\eta}_2}} \ , \tag{7.48}$$

where

$$M_{\eta_0,\tilde{\eta}_2} = \eta_0 + \frac{1 + \frac{11}{2}\rho_{1/2}}{1 + \frac{7}{2}\rho_{1/2}} \frac{\tilde{\eta}_2}{\mu^2} \ . \tag{7.49}$$

Notice that these formulae agree with the numerical plots in figures 7.1 and 7.2. The details of this computation are postponed to the next section.

7.4 The IR limit of correlation functions

In the previous section we have shown that the presence of a non-trivial profile for an R-charged scalar field, η, while providing a non-vanishing value for the R-breaking fermionic correlator $B_{1/2}$, consistently removes the pole from the non-chiral fermionic correlator $C_{1/2}$. The analysis was done by numerical methods. Before concluding, in this section we take a brief detour to discuss the techniques that allow to study the IR behavior of holographic correlators analytically.

We are interested in analyzing the behavior of the correlation functions for small k. In the example of the model we treated in this chapter, the relevant quantity is $k/\mu \ll 1$, where $z = 1/\mu$ is the position of the IR wall, so that the limit can also be seen as moving the wall closer to the boundary. This suggests that if we just need to evaluate the behavior of the C_s functions at low momenta, i.e. (7.15-7.17), we can impose the IR boundary conditions directly on the near-boundary expansion of the solutions, keeping only terms up to a mode high enough to match the order in k^2 at which we need the C_s. Indeed, in previous sections we have seen that this limit is very easy to obtain when one has exact solutions, since it involves expanding Bessel functions near the origin, i.e. keeping only the near-boundary expansion.

Let us illustrate this procedure with C_0 with homogenous IR boundary conditions. We just need to substitute the expansion (5.63) in the boundary conditions (7.8). We get

$$\frac{1}{\mu^2}(d_0 \log(\Lambda/\mu) + \tilde{d}_0) + \rho_0 \frac{1}{\mu^2}(2d_0 \log(\Lambda/\mu) + d_0 + 2\tilde{d}_0) = 0 \ , \tag{7.50}$$

that is

$$\tilde{d}_0 = -d_0 \left(\log(\Lambda/\mu) + \frac{\rho_0}{1 + 2\rho_0} \right) \ . \tag{7.51}$$

Applying

$$C_0 = -2 \frac{\delta \tilde{d}_0}{\delta d_0}, \tag{7.52}$$

we obtain (7.15) right away and effortlessly. In order to reproduce eqs. (7.16) and (7.17), the only added difficulty is that we have to go one order higher in the expansion, if interested in both the $1/k^2$ pole and the finite term.

Notice that this procedure works because the equations of motion themselves are not modified with respect to the AdS ones. If we had $\mathcal{O}(\mu)$ corrections to the metric (as in the example used in chapter 6), it would be impossible to take $1/\mu$ small without introducing large corrections to the background metric and thus to the equations for the fluctuations.

The case of an AdS hard wall with a scalar profile turned is a particular case. In order to prove that the pole in $C_{1/2}$ disappears when $\eta = \eta_0 z + \tilde{\eta}_2 z^3$ is turned on, we should take the limit $k \to 0$ in such a way to keep terms of the form $(k^2 + \eta_0^2)^{-1}$ or $(k^2 + \mu^{-4}\tilde{\eta}_2^2)^{-1}$. Therefore, the correct scaling is

$$\eta_0/\mu \sim \tilde{\eta}_2/\mu^3 \sim k/\mu = \epsilon \to 0 \ , \tag{7.53}$$

and we should focus on the order ϵ^{-2} in the small ϵ expansion of $C_{1/2}$. Keeping η small we also ensure that we can still use the AdS near boundary expansion for the fluctuations. The same kind of expansion can be done for the $B_{1/2}$ correlator, with the difference that it starts from the ϵ^{-1} order. In both cases, the leading terms in the ϵ expansion receive a non-trivial contribution both from $\eta_0 \neq 0$ and from $\tilde{\eta}_2 \neq 0$ and they are determined by keeping the near-boundary expansion

$$\xi(z,x) = z^{3/2}\left[\xi_0 + \sum_{n=1}^{\infty}(\tilde{\xi}_{2n} + \xi_{2n}\log(z\Lambda))z^{2n}\right] \tag{7.54}$$

up to $n = 1$ and $n = 2$, respectively. The results for the order ϵ^{-2} of $C_{1/2}$ and the order ϵ^{-1} in $B_{1/2}$ are reported in the previous section. If one wants to go to the next order in ϵ, which is order ϵ^0 for $C_{1/2}$ and order ϵ for $B_{1/2}$, one should keep terms up to $n = 3$ in the near-boundary expansion. Let us stress that this ϵ expansion is different from a simple expansion for small momenta. For instance, the finite $k = 0$ term will receive contribution from arbitrary high orders in ϵ, which in turn would require to keep arbitrary high terms in the near boundary expansion. Nevertheless, as long as η_0 and $\tilde{\eta}_2$ are kept small, the approximations (7.47-7.48) give a reliable information about the finite value at $k = 0$, as can be checked with the numerical results plotted in figures 7.1 and 7.2.

Chapter 8

Conclusions and outlook

In this thesis we treated various aspects of Gauge Mediation model building, with a particular focus on holographic realizations. Let us pinpoint the main results we have obtained:

- We showed how to construct theories of dynamical supersymmetry breaking which reduce at low energy to any class of generalized O'Raifeartaigh models. In particular, our constructions provide a dynamical completions of the sub-set of weakly coupled theories which are more promising as hidden sectors, in that they spontaneously break the R-symmetry and generate a sizable Majorana mass for gauginos. Moreover, our theories show that ISS-like metastable vacua can exist also outside the free-magnetic phase.

- We have constructed top-down models of holographic hidden sectors, working at the level of an $\mathcal{N} = 2$ $5D$ supergravity theory which, at least for some choice of parameters, can be derived from $10D$ string theory. The main phenomenological features of these models were studied, and it was shown that they can realize both scenarios with Dirac and Majorana gauginos.

- We considered bottom-up holographic hidden sectors on a hard wall background. We showed that within this class of models, by varying the boundary conditions, one can cover the whole parameter space of GGM, and discussed how different scenarios can be more or less generic.

We will conclude by describing some possible future perspectives of this line of research.

One possible direction is to consider more sophisticated top-down models, with the aim of relaxing some of the simplifying assumptions we had to make, and also to have a broader range of examples that may allow to draw more general conclusions on the predictions of these holographic models. As we have explained in section 6.5, particularly interesting extensions could consist in adding $D7$ branes to the background, or considering cascading backgrounds instead of the more manageable asymptotically-AdS ones.

Another direction would be to try and extend the holographic hidden sectors beyond the strict definition of Gauge Mediation models, so to allow also for direct couplings of the hidden sector with the Higgs sector of the SM, making contact to what we explained in section 2.3.2. If the Higgs couples linearly to a certain composite operator, correlators of the latter will determine the form of the soft terms. In holographic hidden sectors, the operator is mapped to a gravity multiplet with the same quantum numbers of the Higgs, and the usual holographic prescription can be used to extract the relevant form factors, in complete analogy to what we have done for ordinary GGM. This construction may in principle be realized in a top-down setup, though it seems more suited for a bottom-up analysis, aimed to scan the parameter space.

Finally, an additional perspective consists in considering the correlators as a probe of the gravity solution. In the whole set of two-point functions of operators sitting in a supersymmetric multiplet, the difference between form factors which are related by supersymmetry carries non-trivial information about the mechanism of supersymmetry breaking. This suggests to extend the present treatment to other multiplets of operators. A particularly interesting possibility, because of its universality and of its direct connection with the dynamics of supersymmetry (and conformal symmetry) breaking, would be to consider the supersymmetric multiplet of the energy-momentum tensor.

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