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The choice of the optimal approximation in the kinetic description of the vacuum creation of electron-positron plasma in strong laser fields

Konstantin Y. Kravtsov[a], Vadim V. Dmitriev[a], Sergey A. Levenets[a], Anatoly D. Panfyorov[a], Stanislav A. Smolyansky[a,b], Lukash Yukhnovsky[c], and David B. Blaschke[c,d,e]

[a]Saratov State University,Saratov, Russia
[b]Tomsk State University,Tomsk, Russia
[c]University of Wroclaw, Wroclaw, Poland
[d]Bogoliubov Laboratory for Theoretical Physics,JINR,Dubna, Russia
[e]National Research Nuclear University (MEPhI), Moscow, Russia

ABSTRACT

The paper justifies E^2-similarity of kinetic equation solutions to describe vacuum emergence of electron-positron plasma under the effect of strong laser fields, where $E(t)$ is the intensity of the strong time-dependant laser field. The boundaries of existence of this similarity were studied.

Keywords: Vacuum creation,kinetic equation, approximation choice, strong laser fields, electronpositron plasma, E^2-similarity, Schwinger effect.

1.Introduction

In connection with the planned experiments on detection of Sauter-Schwinger dynamic effect in the crossed fields of superpower lasers[1–4] the interest to theoretic studies of this effect based on nonperturbative approach in QED of strong fields[5,6] is growing. The corresponding kinetic equations (KE) are fairy complex for analysis, thus the efforts on the search of approximate methods of their solution (e.g.,[7,8]) are justified. This paper concerns this problem. It mainly describes the so-called E^2-similarity of KE solutions found as a result of numerical calculations in,[9]], but not yet theoretically justified. It is shown below that the result of[9] follows the assumption on a rather slow change of the external field compared to rapid vacuum oscillations. In particular, this means that E^2-similarity must be true both in the region of rather weak fields $E \ll E_c = m^2/e$, and for supercritical fields $E \gtrsim E_c$, , which may be relevant in the theory of vacuum emergence of quark-gluon plasm (e.g.,[10]). However, the decrease of the effective field frequency causes non-Markovian processes violating E^2-similarity, which leads to a significant narrowing of the region where this phenomenon manifests itself.

2.Kinetic equation

When describing vacuum emergence of electron-positron plasma (EPP) in the strong electromagnetic fields, KE,[5,6,9] is often used today. This KE is the exact result of QED in the model of linearly polarized spatially uniform time-dependant electric field. This KE in integro-differential form is as follows

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\lambda(\mathbf{p},t)\int^{t} dt^{'}\lambda(\mathbf{p},t^{'})\left[1-2f(\mathbf{p},t^{'})\right]\cos\Theta(t,t^{'}), \quad (1)$$

Where the amplitude of vacuum transitions $\lambda(\mathbf{p},t)$and the phase of high-frequency vacuum oscillations $\Theta(t,t^{'})$ are as follows

$$\lambda(\mathbf{p},t) = \frac{eE(t)\varepsilon_{\perp}(\mathbf{p}_{\perp})}{\varepsilon^{2}(\mathbf{p},t)}, \quad (2)$$

Further author information: (Send correspondence to K.Y.K.)
K.Y.K.: E-mail: kosta6808@mail.ru

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$$\Theta(t,t') = 2\int_{t'}^{t} dt'' \varepsilon(t''), \tag{3}$$

Where quasi-energy $\varepsilon(\mathbf{p},t)$, transversal energy $\varepsilon_\perp(\mathbf{p}_\perp)$ and quasi-momentum $P(\mathbf{p}_{\shortparallel},t)$ are, respectively,

$$\varepsilon(\mathbf{p},t) = \sqrt{\varepsilon_\perp^2(\mathbf{p}_\perp) + P^2(\mathbf{p}_{\shortparallel},t)}, \tag{4}$$

$$\varepsilon_\perp(\mathbf{p}_\perp) = \sqrt{m^2 + \mathbf{p}_\perp^2}, \tag{5}$$

$$P(\mathbf{p}_{\shortparallel},t) = \mathbf{p}_{\shortparallel} - eA(t). \tag{6}$$

An alternative form of KE (1) is the system of ordinary differential equations

$$\begin{aligned} \dot{f}(\mathbf{p},t) &= \frac{1}{2}\lambda(\mathbf{p},t)u(\mathbf{p},t), \\ \dot{u}(\mathbf{p},t) &= \lambda(\mathbf{p},t)\left[1 - 2f(\mathbf{p},t)\right] - 2\varepsilon(\mathbf{p},t)v(\mathbf{p},t), \\ \dot{v}(\mathbf{p},t) &= 2\varepsilon(\mathbf{p},t)u(\mathbf{p},t). \end{aligned} \tag{7}$$

In (1) and (7) $f(\mathbf{p},t)$ is the distribution function of quasi-particle excitations of EPP, and functions $u(\mathbf{p},t)$ and $v(\mathbf{p},t)$ describe the evolution of vacuum polarization.

Further, the fields with Gaussian distribution are used in the paper for numerical simulation.

$$E(t) = E_0 \cos(\omega t) e^{-t^2/2\tau^2}, \tag{8}$$

where E_0 is the maximum field amplitude, τ is the characteristic scale of the field change (field pulse duration), ω is the field frequency.

3.E^2-similarity

In[9] as a result of numerical solution of KE as the system of ordinary differential equations (7) in the region of weak fields $E \ll E_c$ E^2-similarity of EPP distribution function was found

$$f(\mathbf{p},t) = E^2(t)\Phi(\mathbf{p},t), \tag{9}$$

Where function $\Phi(\mathbf{p},t)$ weakly depends on time in the quasiparticle region of EPP evolution (Fig. 1). The weak field causes low level of EPP excitation, which corresponds to the low density approximation $f(\mathbf{p},t) \ll 1$. Write

Figure 1. The graph of the distribution function $f(\mathbf{p}=0,t)$, combined with the graph of the square of the external field $E^2(t)$ (left),and the graph of the similarity function $\Phi(\mathbf{p},t)$(on the right) for the case of a field with a maximum amplitude $E_0 = 0.05E_c$, the pulse duration $\tau = 25\tau_C$ and the field frequency $\omega = 1/5\tau_C$. Narrow outbursts of the similarity function $\Phi(\mathbf{p},t)$ in the right figure are associated with a decrease in the field strength to zero, while the value of the distribution function $f(\mathbf{p},t)$ remains finite

KE (1) using this approximation

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\lambda(\mathbf{p},t)\int^{t} dt' \lambda(\mathbf{p},t') \cos\Theta(t,t') \tag{10}$$

and the system of KEs (7)

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\lambda(\mathbf{p},t)u(\mathbf{p},t),$$
$$\dot{u}(\mathbf{p},t) = \lambda(\mathbf{p},t) - 2\varepsilon(\mathbf{p},t)v(\mathbf{p},t), \tag{11}$$
$$\dot{v}(\mathbf{p},t) = 2\varepsilon(\mathbf{p},t)u(\mathbf{p},t).$$

Consider E^2-similarity using this approximation.

3.1. Case of $E \ll E_c$

Note that according to (2) $\lambda \sim E$, i. e

$$\lambda(\mathbf{p},t) = E(t)\Lambda(\mathbf{p},t), \quad \Lambda(\mathbf{p},t) = \frac{e\varepsilon_\perp(\mathbf{p}_\perp)}{\varepsilon^2(\mathbf{p},t)}. \tag{12}$$

Further, it is substantial that two time scales are present in KEs (1) and (7): characteristic scale τ of the change of external field and compton time $\tau_C = 2\pi/m$.

Consider the case of slow fields $\tau \gg \tau_C$ (Fig. 1). In this case, field $E(t)$) under the sign of integration in KE

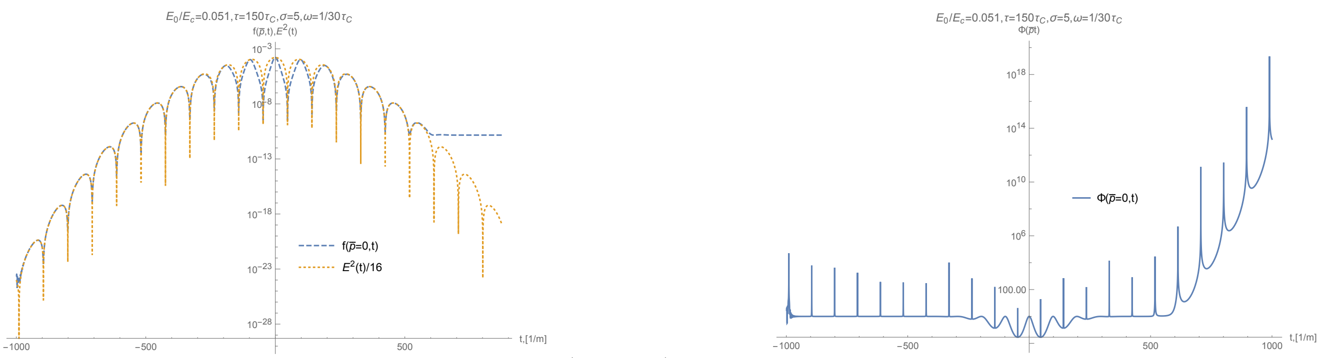

Figure 2. The graph of the distribution function $f(\mathbf{p}=0,t)$, combined with the graph of the square of the external field$E^2(t)$ (left), and the graph of the similarity function $\Phi(\mathbf{p},t)$(on the right) for the case of a field with a maximum amplitude $E_0 = 0.051E_c$, the pulse duration $\tau = 150\tau_C$ and the field frequency $\omega = 1/30\tau_C$

(1) remains almost constant for the period of the change in high-frequency phase (3) and thus may be taken out of the sign of integration. Integrate KE (10) by time and take field $E(t)$ out of the sign of integration. This will result in equation (9) with the similarity function

$$\Phi(\mathbf{p},t) = \frac{1}{2}\int^t dt^{'}\Lambda(\mathbf{p},t^{'})\int^{t^{'}} dt^{''}\Lambda(\mathbf{p},t^{''})\cos\Theta(t^{'},t^{''}). \tag{13}$$

E^2-similarity can also be found by analyzing the system of KEs (7) using low density approximation $f(\mathbf{p},t) \ll 1$. Represent polarization function $u(\mathbf{p},t)$ as

$$u(\mathbf{p},t) = E(t)U(\mathbf{p},t). \tag{14}$$

This immediately allows deriving (9). Now to assure validity of (14), exclude polarization function $v(\mathbf{p},t)$ in the last two equations of the system (7) . We derive

$$\dot{u}(\mathbf{p},t) = E(t)\Lambda(\mathbf{p},t) - 4\varepsilon(\mathbf{p},t)\int^t dt^{'}\varepsilon(\mathbf{p},t^{'})u(\mathbf{p},t^{'}),$$

wherefrom, in case of slowly changing field $E(t)$, in the form (14) follows

$$\dot{U}(\mathbf{p},t) = \Lambda(\mathbf{p},t) - 4\varepsilon(\mathbf{p},t)\int^t dt^{'}\varepsilon(\mathbf{p},t^{'})U(\mathbf{p},t^{'}), \tag{15}$$

which proves quasilinear representation of (14).

It results in impressive efficiency of E^2-similarity (the left side of fig. 1). In this case, the similarity function is almost constant (right part of Fig. 1).). Howerer, the decrease in the external field frequency (Fig. 2) violates E^2similarity both in the central region (near $t = 0$) of quasistatic evolution, and especially when generating the residual EPP.

3.2. Strong slow fields

It follows from the above arguments that in the justification of the E^2 -similarity, the assumption is that the field is sufficiently slow to vary. In other words, the E^2 -similarity similarity should also be valid in the case of strong slow fields, when the low-density approximation is inapplicable. Indeed, in the case of a strong slow field

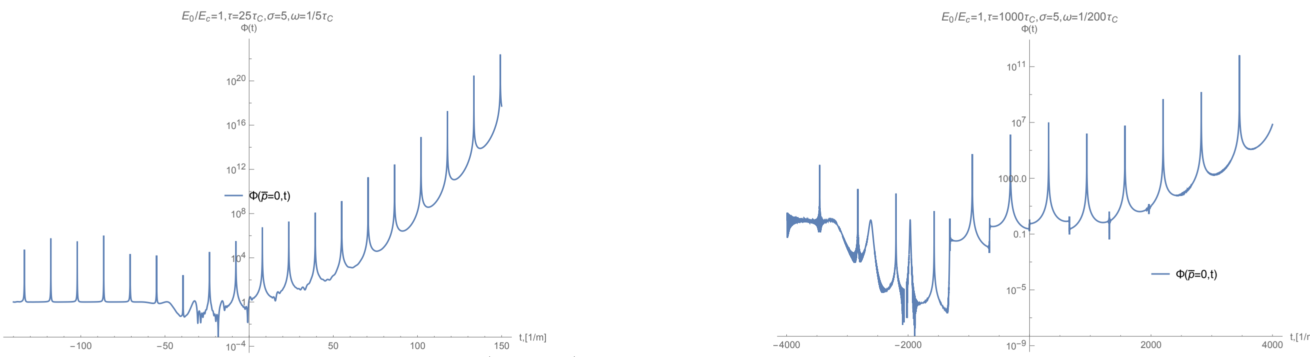

Figure 3. Graphs of the similarity functions $\Phi(\mathbf{p} = 0, t)$ for the case of Gaussian fields with a maximum amplitude $E_0 = E_c$, pulse duration $\tau = 25\tau_C$, the frequency $\omega = 1/5\tau_C$ (left) and the pulse duration $\tau = 1000\tau_C$, frequency $\omega = 1/200\tau_C$ (right)

one can use the representation (12) and take $E(t)$ as the integral sign in the KE (1), which leads to the integral equation

$$f(\mathbf{p}, t) = \frac{1}{2} E^2(t) \int^t dt^{'} \Lambda(\mathbf{p}, t^{'}) \int^{t^{'}} dt^{''} \Lambda(\mathbf{p}, t^{''}) \left[1 - 2f(\mathbf{p}, t^{''})\right] \cos\Theta(t^{'}, t^{''}). \tag{16}$$

This implies the formula (9) with the similarity function $\Phi(\mathbf{p}, t)$ functionally dependent on $f(\mathbf{p}, t)$. This introduces an additional veiling dependence on time, in contrast to the low-density approximation, where the similarity function is weakly time-dependent in the quasiparticle region of the evolution of the EPP.

A typical situation is illustrated in Fig. 3. Here it is seen that in the region of sufficiently high frequencies (the left-hand side of the figure) E^2 -similarity, it is satisfied only at the initial stage of evolution, and in the transition to slower fields (the right-hand side of the figure), the E^2 -similarity similarity is destroyed at all stages of evolution.

Thus, as can be seen from Fig. 1 - 3, the effect considered is manifested in the region of sufficiently high frequencies of the external field. When the frequency is lowered, deviations from the E^2 -similarity are observed, which increase with decreasing frequency and an increase in the field amplitude. The cause of such a violation are non-Markovian effects in the KE. The main role here is played by an increase in the vector of the potential $A(t)$ entering quasienergy (4), which leads to the decrease in the amplitude of the vacuum transitions (2) and the growth of the phase (3). Figures 4 and 5 illustrate this conclusion using the example of the univalent Eckart-Sauter field model $E(t) = E_0 \cosh^{-2}(t/\tau)$. Here the parameter $1/\tau$ plays the role of frequency.

4.Conclusions

In this paper, we substantiate the E^2 -similarity of the solutions of the simplest KE, which is intended to describe the vacuum production of EPP under the influence of strong laser fields.[9] The analysis was limited to a model of a linearly polarized "laser" field with a Gaussian envelope. This made it possible to study the region of manifestation of the detected effect, which turned out to be quite limited in frequency, in the amplitude of the external field, and also in the duration of the quasiparticle evolution period.

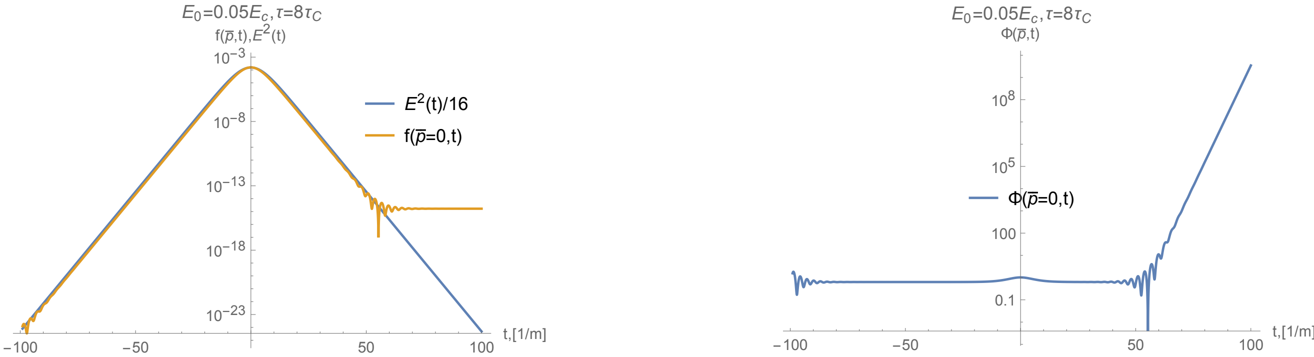

Figure 4. The graph of the distribution function $f(\mathbf{p}=0,t)$, combined with the graph of the square of the external field $E^2(t)$ (left), and the graph of the similarity function $\Phi(\mathbf{p}\ ,t)$ (on the right) for the case of univalent the Eckart-Sauter field model with the amplitude $E_0 = 0.05E_c$ and the pulse duration $\tau = 8\tau_C$

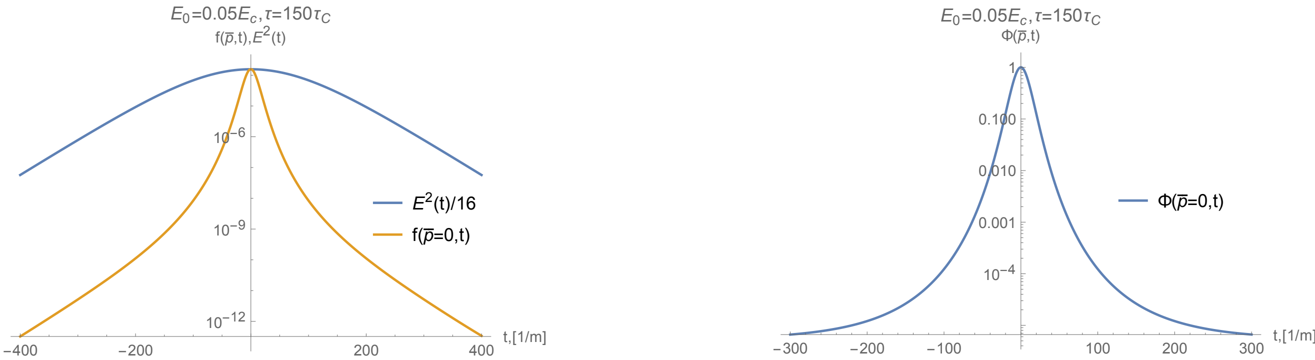

Figure 5. The graph of the distribution function $f(\mathbf{p}=0,t)$, combined with the graph of the square of the external field $E^2(t)$ (left), and the graph of the similarity function $\Phi(\mathbf{p}\ ,t)$ (on the right) for the case of univalent the Eckart-Sauter field model with the amplitude $E_0 = 0.05E_c$ and the pulse duration $\tau = 150\tau_C$

REFERENCES

[1] http://www.clf.stfc.ac.uk/CLF

[2] http://www.eli-beams.eu

[3] http://www.hiper-laser.org

[4] http://www.ipfran.ru/english/science/las.phis.html

[5] I. Bialynicky-Birula, R. Gornicki, J. Rafelski. *Phys. Rev.*, **D44**, 1825 (1991).

[6] S.M. Schmidt, D.B. Blaschke, G. Röpke, S.A. Smolyansky, A.V. Prozorkevich and V.D. Toneev. *Int. J. Mod. Phys.*, **E7**, 709 (1998).

[7] V.V. Dmitriev, S.A. Smolyansky, R.M. Yakibaev. *News of high schools "PND"*, **23**, 27 (2015).

[8] S.A. Smolyansky, A.V. Prozorkevich ,V.V. Dmitriev, A.V. Tarakanov. *Int. J. Mod. Phys.*, **E23**, 1450068 (2014).

[9] L. Juchnowski, D. Blaschke, S.A. Smolyansky, A.D. Panferov, A. Otto, B. Kaempfer. *Vacuum pair creation in strong fields as a field-induced phase transition*, The XXXIX Max Born Symposium: 5th International Symposium on Optics, Wroclaw (2017).

[10] S.A. Smolyansky. *Project-HMEC*, 2305207v0, p.85 (2017).