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Free convection in a triangular cavity filled with a porous medium saturated by a nanofluid: Buongiorno's mathematical model

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Free convection in a triangular cavity filled with a porous medium saturated by a nanofluid

Buongiorno's mathematical model

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Abstract

Purpose – Steady-state free convection heat transfer in a right-angle triangular porous enclosure filled by a nanofluid using the mathematical nanofluid model proposed by Buongiorno has been numerically analyzed. The paper aims to discuss this issue.
Design/methodology/approach – The nanofluid model takes into account the Brownian diffusion and thermophoresis effects. The governing equations formulated in terms of the vorticity-stream function variables were solved by finite difference method.
Findings – It has been found that the average Nusselt number is an increasing function of the Rayleigh and Lewis numbers and a decreasing function of Brownian motion, buoyancy-ratio and thermophoresis parameters. At the same time the average Sherwood number is an increasing function of the Rayleigh and Lewis numbers, Brownian motion and thermophoresis parameters and a decreasing function of buoyancy-ratio parameter.
Originality/value – The present results are new and original for the heat transfer and fluid flow in a right-angle triangular porous enclosure filled by a nanofluid using the mathematical nanofluid model proposed by Buongiorno. The results would benefit scientists and engineers to become familiar with the flow behaviour of such nanofluids, and the way to predict the properties of this flow for possibility of using nanofluids in advanced nuclear systems, in industrial sectors including transportation, power generation, chemical sectors, ventilation, air-conditioning, etc.

Keywords Nanofluid, Free convection, Buongiorno's model, Numerical study, Triangular porous cavity
Paper type Research paper

Nomenclature

Roman letters

C	nanoparticle volume fraction	C_h	nanoparticle volume fraction of the hot wall
C_c	nanoparticle volume fraction of the cooled wall	D_B	Brownian diffusion coefficient ($m^2\ s^{-1}$)

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D_T thermophoretic diffusion coefficient (m^2 s^{-1})
g gravitational acceleration (m s^{-2})
k thermal conductivity (W m^{-1} K^{-1})
K permeability of the porous medium (m^2)
L length of the cavity (m)
Le Lewis number
$m_{w,0}$ mass transfer from hot wall (m s^{-1})
Nb Brownian motion parameter
Nr buoyancy-ratio parameter
Nt thermophoresis parameter
Nu local Nusselt number
$\overline{Nu}$ mean Nusselt number
$q_{w,0}$ heat flux from the hot wall (W m^{-2})
Ra Rayleigh number for the porous medium
Sh Sherwood number
$\overline{Sh}$ mean Sherwood number
T temperature of the fluid (K)
T_c temperature of the cooled wall (K)
T_h temperature of the hot wall (K)
$\bar{u}$, $\bar{v}$ dimensional velocity components along the axes $\bar{x}$, $\bar{y}$ (m s^{-1})
$\bar{x}$ dimensional coordinate measured along the bottom wall of the cavity (m)
$\bar{y}$ dimensional coordinate measured along the vertical wall (m)
x, y dimensionless Cartesian coordinates

Greek symbols

α_m thermal diffusivity of the porous medium (m^2 s^{-1})
β volumetric expansion coefficient of the fluid (K^{-1})
ε porosity of the porous medium
θ dimensionless temperature
μ dynamic viscosity (kg m^{-1} s^{-1})
ρ_f fluid density (kg m^{-3})
ρ_p nanoparticle mass density (kg m^{-3})
$(\rho C_p)_f$ heat capacity of the fluid (J K^{-1} m^{-3})
$(\rho C_p)_p$ effective heat capacity of the nanoparticle material (J K^{-1} m^{-3})
τ parameter defined by $\tau = (\rho C_p)_f/(\rho C_p)_p$
ϕ rescaled nanoparticle volume fraction
$\bar{\psi}$ dimensional stream function (m^2 s^{-1})
ψ dimensionless stream function

1. Introduction

Natural convective heat transfer in fluid-saturated porous media has occupied the centre stage in many fundamental heat transfer analyses and has received considerable attention over the last several decades. This interest is due to its wide range of applications in, for example, packed spherical beds, high performance insulation for buildings, chemical catalytic reactors, grain storage and such geophysical problems as frost heave. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors and to geothermal energy systems. Porous materials, such as sand and crushed rock, underground saturated with water, which, under the influence of local pressure gradients, migrates and transports energy through the material. Literature concerning convective flow in porous media is abundant. Representative studies in this are may be found in the recent books by Nield and Bejan (2013), Pop and Ingham (2001), Ingham and Pop (2005), Vafai (2005, 2010), Vadasz (2008) and de Lemos (2012).

Analysis of natural convection heat transfer and fluid flow in enclosures filled with viscous fluids or porous media has been extensively made using numerical techniques and experiments because of its wide applications and interest in engineering such as nuclear energy, double pane windows, heating and cooling of buildings, solar collectors, electronic cooling, micro-electromechanical systems, etc. (Bejan, 2013; Baytas and Pop, 1999; Kuznetsov and Sheremet, 2008).

An innovative technique to enhance heat transfer is by using nano-scale particles in the base fluid. Nanotechnology has been widely used in industry since materials with sizes of nanometers possess unique physical and chemical properties (Oztop and Abu-Nada, 2008). Nano-scale particles when added to fluids are called nanofluids, and this was first introduced by Choi (1995). Some numerical and experimental studies on nanofluids in cavities filled with water-based nanofluids have been first performed by many researchers. Khanafer *et al.* (2003) have analyzed the heat transfer performance of nanofluids inside an enclosure taking into account the solid particle dispersion. The transport equations are solved numerically using the finite-volume approach along with the alternating direct implicit procedure. The effect of suspended ultrafine metallic nanoparticles on the fluid flow and heat transfer processes within the enclosure is analyzed and effective thermal conductivity enhancement maps were developed for various controlling parameters. In addition, a heat transfer correlation of the average Nusselt number for various Grashof numbers and volume fractions has been presented. Further Tiwari and Das (2007) have numerically analyzed the behaviour of nanofluids inside a two-sided lid-driven differentially heated square cavity to gain insight into convective recirculation and flow processes induced by a nanofluid. A model is developed to analyze the behaviour of nanofluids taking into account the solid volume fraction χ. The left and the right moving walls are maintained at different constant temperatures, while the upper and the bottom walls are thermally insulated. Three cases were considered depending on the direction of the moving walls. The transport equations were solved numerically with finite volume approach using SIMPLE algorithm. It was found that both the Richardson number and the direction of the moving walls affect the fluid flow and heat transfer in the cavity. The mathematical nanofluid models proposed by Tiwari and Das (2007) has been used also by Oztop and Abu-Nada (2008), who have examined the natural convection heat transfer in a partially heated rectangular enclosure filled with nanofluids. Three different nanofluids as Cu (copper), Al_2O_3 (alumina) and TiO_2 (titania) were tested to investigate the effect of nanoparticles on natural convection flow and temperature fields. In a very interesting paper, Popa *et al.* (2010) presented a theoretical model based on the integral formalism approach for both laminar and turbulent external natural convection boundary layer flow along a vertical wall subjected to a uniform heat flux condition for water-Al_2O_3 and water-Cu nanofluids and particle volume fractions up to 10 percent. The approach is based on the assumption of single-phase homogeneous fluid model and the use of experimental data for the dynamical viscosity and the thermal conductivity nanofluids. It was shown that heat transfer strongly depends on the flow regime and on particle volume fraction. A clear degradation of heat transfer is observed using nanofluids while compared to that of the base-fluid. Moreover, the fact of increasing the particle volume fraction tends to delay the occurrence of the flow transition to turbulence. Sun and Pop (2014) considered the problem of steady free convection heat transfer and fluid flow in a tilted right-angle triangular enclosure filled with a porous medium and saturated by Cu-water nanofluid. The major purpose has been motivated by the need to determine the effects of pertinent parameters on the detailed flow and temperature characteristics as well as the local and average Nusselt numbers, such as the tilted angle of the cavity, Rayleigh number for a porous number and the solid volume fraction parameter of Cu-water nanofluid. To this end, we mention the papers by Roşca *et al.* (2014), Trambitas *et al.* (2014) and Patrulescu *et al.* (2014), where the steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled by a nanofluid (Roşca *et al.*, 2014), the steady mixed convection boundary layer flow of water nanofluids past a vertical needle to evaluate the influence of control parameters on the heat transfer characteristics of nanofluid (Trambitas *et al.*, 2014) and the development of the steady mixed convection boundary layer flow on a vertical impermeable frustum of a cone in a nanofluid. The problem has been formulated so that three different types of nanoparticles, namely, Cu, Al_2O_3, TiO_2 and water as a base fluid (Patrulescu *et al.*, 2014). In all these three papers the nanofluid mathematical model proposed by Tiwari and Das (2007) has been used. Detailed review studies on nanofluids are published in the book by Das *et al.* (2007) and the review papers by Buongiorno (2006), Kakaç and Pramuanjaroenkij (2009), Lee *et al.* (2010), Eagen *et al.* (2010), Wong and Leon (2010), Fan and Wang (2011), Mahian *et al.* (2013), etc. It is clear from the foregoing review that most of the studies have been performed considering water-based nanofluids in cavities using the mathematical nanofluid models proposed by Khanafer *et al.* (2003) and Tiwari and Das (2007).

On the other hand, it seems that Nield and Kuznetsov (2009, 2011) are the first who have studied the Cheng and Minkowycz's (1977) problem for natural convective boundary layer flow over a vertical flat plate embedded in a porous medium filled with a nanofluid using the mathematical nanofluid model proposed by Buongiorno (2006). In another two papers, Kuznetsov and Nield (2010, 2011) have provided a numerical solution to the problem of natural convective heat transfer in the boundary layer flow of a nanofluid past a vertical flat plate embedded in a viscous (Newtonian) fluid using the same Buongiorno's (2006) model. Buongiorno (2006) noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity (that he calls the slip velocity). He considered in turn seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity settling. It is worth pointing out here that very recently, Kuznetsov and Nield (2013) and Nield and Kuznetsov (2014) have revisited their previous model and extended it to the case when the nanofluid particle fraction on the boundary is passively rather than actively controlled. This makes the model physically more realistic than the previous model considered in the papers by Nield and Kuznetsov (2009, 2011) as well as the models employed by other authors simulating nanofluid flow in porous media. These authors have assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix. On the other hand, it is very important to explain how nanofluid flow is possible in a porous medium. Thus, it should be pointed out that without special precautions, nanoparticles will be simply absorbed by the porous matrix. Basically, the porous matrix will work as a filter for nanoparticles. This situation has been described, explained and modelled in the recent papers by Wu *et al.* (2010, 2011). The physical situation described in these papers show that the work on porous media filled by nanofluids are not just a mathematical exercise, but are based on deep physical understanding of nanofluid flows. This demonstrates that we are simulating here a real physics problem of free convection flow and heat transfer in a porous cavity filled by a nanofluid. However, relative little research has been performed on convective flow in porous cavities filled with nanofluids using the mathematical model proposed by Buongiorno (2006). In this respect, we mention the very recently published papers by Sheremet *et al.* (2014) and Sheremet and Pop (2014).

In the present study, we investigate numerically the problem of steady free convection heat transfer in a triangular cavity filled with a porous medium saturated with water-based nanofluid using Bongiorno's (2006) model in combination with

Darcy's law for the flow in the porous medium and the Boussinesq approximation for the convective forces. We notice to the end the published papers by Varol *et al.* (2006), Oztop *et al.* (2009) and Yesiloz and Aydin (2013) on convective flow in right-angled triangular cavities.

2. Basic equations

Consider the free convection in a two-dimensional porous triangular cavity filled with a nanofluid based on water and nanoparticles. A schematic geometry of the problem under investigation is shown in Figure 1, where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates and L is the bottom wall length and H is the height of the vertical wall. It is assumed that the vertical wall is maintain at temperature T_h and the constant nanoparticle volume fraction C_h, while the inclined wall is kept at a temperature T_c and nanoparticle volume fraction C_c, where we assume that $T_h > T_c$ and $C_h > C_c$, respectively. It is also assumed that the bottom wall of the cavity is adiabatic. Using the Darcy-Boussinesq approximation, and following the nanofluid model proposed by Buongiorno (2006), the basic equations are given by (see Nield and Bejan, 2013):

$$\frac{\partial^2\bar{\psi}}{\partial\bar{x}^2}+\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}=-\frac{(1-C_c)\rho_f gK\beta}{\mu}\frac{\partial T}{\partial\bar{x}}+\frac{\rho_p-\rho_f}{\mu}gK\frac{\partial C}{\partial\bar{x}} \tag{1}$$

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial T}{\partial\bar{x}}-\frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial T}{\partial\bar{y}}=\alpha_m\left(\frac{\partial^2 T}{\partial\bar{x}^2}+\frac{\partial^2 T}{\partial\bar{y}^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial\bar{x}}\frac{\partial T}{\partial\bar{x}}+\frac{\partial C}{\partial\bar{y}}\frac{\partial T}{\partial\bar{y}}\right)+\frac{D_T}{T_c}\left[\left(\frac{\partial T}{\partial\bar{x}}\right)^2+\left(\frac{\partial T}{\partial\bar{y}}\right)^2\right]\right\} \tag{2}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial C}{\partial\bar{x}}-\frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial C}{\partial\bar{y}}\right)=D_B\left(\frac{\partial^2 C}{\partial\bar{x}^2}+\frac{\partial^2 C}{\partial\bar{y}^2}\right)+\frac{D_T}{T_c}\left[\frac{\partial^2 T}{\partial\bar{x}^2}+\frac{\partial^2 T}{\partial\bar{y}^2}\right] \tag{3}$$

here T is the nanofluid temperature, C is the nanoparticle volume fraction, K is the permeability of the porous medium, ε is the porosity, $\bar{\psi}$ is the stream function which is defined as $\bar{u}=\partial\bar{\psi}/\partial\bar{y}$ and $\bar{v}=-\partial\bar{\psi}/\partial\bar{x}$, and the physical meaning of the other quantities is given in Nomenclature.

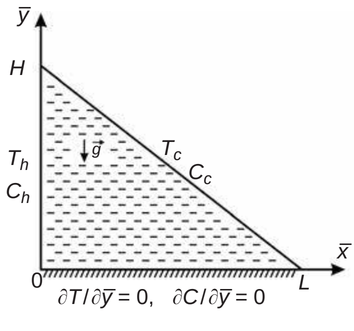


Figure 1.
Physical model and coordinate system

Introducing the following dimensionless variables:

$$x = \overline{x}/L, \quad y = \overline{y}/L, \quad \psi = \overline{\psi}/\alpha_m, \quad \theta = (T - T_c)/\Delta T, \quad \phi = (C - C_c)/\Delta C \tag{4}$$

where $\Delta T = T_h - T_c$, $\Delta C = C_h - C_c$, and substituting (4) into Equations (1)-(3), we obtain:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -Ra\frac{\partial \theta}{\partial x} + Ra \cdot Nr\frac{\partial \phi}{\partial x} \tag{5}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Nb\left(\frac{\partial \phi}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial \phi}{\partial y}\frac{\partial \theta}{\partial y}\right) + Nt\left[\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right] \tag{6}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = \frac{1}{Le}\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right] + \frac{1}{Le}\frac{Nt}{Nb}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] \tag{7}$$

where $Ra = (1 - C_c)gK\rho_f\beta\Delta TL/(\alpha_m\mu)$ is the Rayleigh number. The corresponding boundary conditions of these equations are given by:

$$\begin{aligned} &\psi = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{on} \quad x = 0 \\ &\psi = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{on} \quad x + y = 1 \\ &\psi = 0, \quad \tfrac{\partial \theta}{\partial y} = 0, \quad \tfrac{\partial \phi}{\partial y} = 0 \quad \text{on} \quad y = 0 \end{aligned} \tag{8}$$

here the four parameters Nr, Nb, Nt and Le denote a buoyancy-ratio parameter, a Brownian motion parameter, a thermophoresis parameter and a Lewis number, respectively, which are defined as:

$$Nr = \frac{(\rho_p - \rho_f)\Delta C}{\rho_f \beta \Delta T(1 - C_c)}, \quad Nb = \frac{\tau D_B \Delta C}{\alpha_m}, \quad Nt = \frac{\tau D_T \Delta T}{\alpha_m T_c}, \quad Le = \frac{\alpha_m}{\varepsilon D_B} \tag{9}$$

It should be noticed that for $Nr = Nb = Nt = 0$ (regular fluid), Equations (5) and (6) reduce to those of Walker and Homsy (1978), Bejan (1979), Beckermann *et al.* (1986), Gross *et al.* (1986), Moya *et al.* (1987) and Manole and Lage (1992).

The physical quantities of interest are the local Nusselt number Nu, the local Sherwood number Sh and the mean Nusselt $\overline{Nu}$ and Sherwood $\overline{Sh}$ numbers. The local Nusselt and Sherwood numbers are defined as:

$$Nu = \frac{Lq_{w,0}}{k(T_h - T_c)}, \quad Sh = \frac{Lm_{w,0}}{D_B(C_h - C_c)} \tag{10}$$

where the heat and mass transfer from the vertical wall $q_{w,0}$ and $m_{w,0}$ are given by:

$$q_{w,0} = -k\left(\frac{\partial T}{\partial \overline{x}}\right)_{\overline{x}=0}, \quad m_{w,0} = -D_B\left(\frac{\partial C}{\partial \overline{x}}\right)_{\overline{x}=0} \tag{11}$$

Using (4), (10) and (11), we get:

$$Nu = -\left(\frac{\partial \theta}{\partial x}\right)_{x=0}, \quad Sh = -\left(\frac{\partial \phi}{\partial x}\right)_{x=0}, \quad \overline{Nu} = \int_0^1 Nu\, dx, \quad \overline{Sh} = \int_0^1 Sh\, dx \tag{12}$$

3. Numerical method

Finite difference method is used to solve governing Equations (5)-(7) subject to the boundary conditions (8). Central difference method is applied for discretization of equations. The solution of linear algebraic equations was performed using successive under relaxation method. As convergence criteria, 10^{-10} is chosen for all dependent variables and value of 0.1 is taken for under-relaxation parameter (Varol *et al.*, 2006). Regular grid distribution is used in this study. The inclined wall was approximated with staircase-like zigzag lines. The uppermost grid-point on each vertical grid line coincided with top wall of the triangular enclosure as indicated by Oztop *et al.* (2009).

For the purpose of obtaining grid independent solution, a grid sensitivity analysis is performed. Six cases of a uniform grid are tested: 100×100, 200×200, 300×300, 400×400, 500×500 and 600×600 points. The grid independent solution was performed by preparing the solution for natural convection in a triangular enclosure filled by a classical (Darcian) porous medium when $Nr=Nb=Nt=0$ and $Ra=10^3$. Table I shows an effect of the mesh on the average Nusselt number of the hot wall.

On the basis of the conducted verifications the uniform grid of 500×500 points has been selected for the following analysis.

Several tests were made to compare the results obtained by the present code with some of earlier studies. The first test was the classical natural convection heat transfer problem in a differently heated triangular clear enclosure filled with the pure fluid. The obtained results for the streamlines and isotherms were compared with experimental and numerical data of Yesiloz and Aydin (2013) as shown in Figures 2 and 3. It can be seen that the results obtained here are in good agreement with the results that have been obtained by Yesiloz and Aydin (2013).

The second test was performed to make a comparison with data of Oztop *et al.* (2009) and Basak *et al.* (2008) (see Figure 4). It was found that the streamlines and isotherms contours are almost the same as those given by Oztop *et al.* (2009) and Basak *et al.* (2008). Therefore, these results provide confidence to the accuracy of the present numerical method.

Table II compares the accuracy of the average Nusselt number in case of a square porous cavity for different values of the Rayleigh number at $Nr=Nb=Nt=0$ (regular fluid) when the mass transfer is absent with some numerical solutions reported by different authors (Baytas and Pop, 1999; Walker and Homsy, 1978; Bejan, 1979; Beckermann *et al.*, 1986; Gross *et al.*, 1986; Manole and Lage, 1992; Moya *et al.*, 1987).

4. Results and discussion

Numerical investigation of the boundary value problem (5)-(8) has been carried out at the following values of dimensionless complexes: $Ra=100$-500; $Le=1$-10; $Nb=0.1$-0.4; $Nt=0.1$-0.4; $Nr=0.1$-0.4; $A=H/L=1.0$. Particular efforts have been focussed on the

Table I. Variations of the average Nusselt number of the hot wall with the uniform grid

Uniform grids	Nu	$\Delta=\frac{\lvert Nu_{i\times j}-Nu_{500\times500}\rvert}{Nu_{i\times j}}\times100\%$
100×100	17.5945	12.34
200×200	16.2794	5.26
300×300	15.7874	2.30
400×400	15.5532	0.83
500×500	15.4239	–
600×600	15.3451	0.51

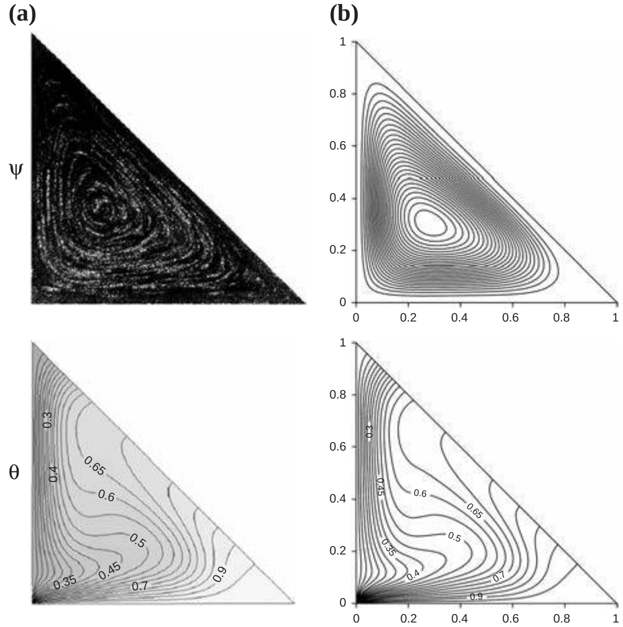


Figure 2.
Experimental streamlines ψ and numerical isotherms θ by Yesiloz and Aydin (2013) (a) and obtained results (b) at $Ra = 10^5$

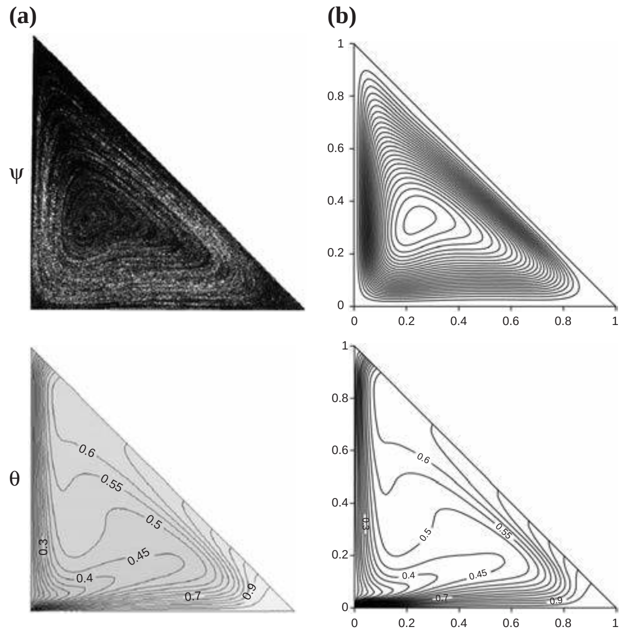


Figure 3.
Experimental streamlines ψ and numerical isotherms θ by Yesiloz and Aydin (2013) (a) and obtained results (b) at $Ra = 10^6$

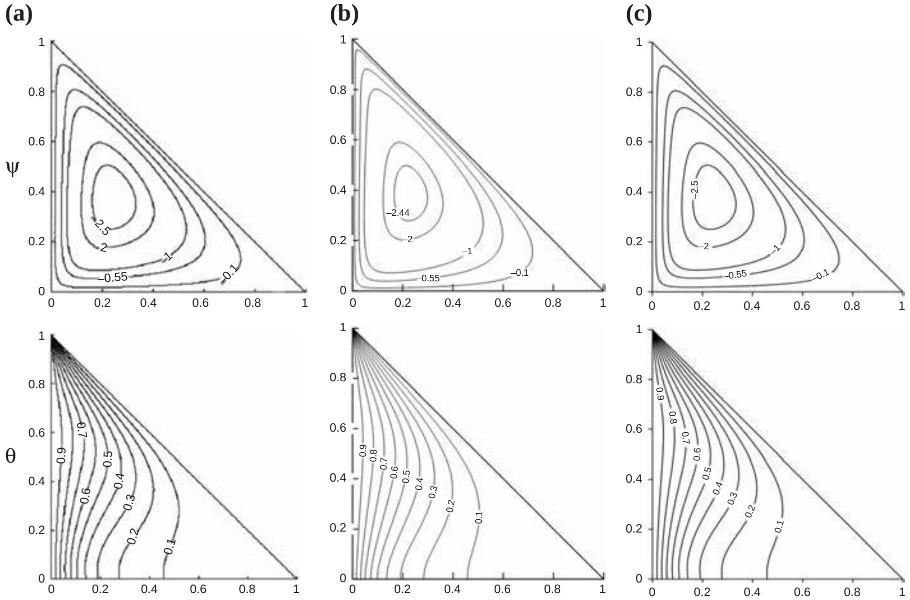


Figure 4. Streamlines ψ and isotherms θ for $Da = 10^{-3}$, $Ra = 8\times10^{4}$, $Pr = 0.7$

Notes: (a) Numerical data of Basak *et al*. (2008); (b) numerical data of Oztop *et al*. (2009); (c) present results

Table II. Comparison of the average Nusselt number of the hot wall

Authors	*Ra* 10	100	1,000	10,000
Baytas and Pop (1999)	1.079	3.16	14.06	48.33
Walker and Homsy (1978)	–	3.097	12.96	51.0
Bejan (1979)	–	4.2	15.8	50.8
Beckermann *et al.* (1986)	–	3.113	–	48.9
Gross *et al.* (1986)	–	3.141	13.448	42.583
Moya *et al.* (1987)	1.065	2.801	–	–
Manole and Lage (1992)	–	3.118	13.637	48.117
Present results	1.071	3.104	13.839	49.253

effects of five types of influential factors such as the Rayleigh and Lewis numbers, the buoyancy-ratio parameter, the Brownian motion parameter and the thermophoresis parameter on the fluid flow and heat transfer.

Further on, we would like to express the physical reasons for changing of each key parameter. So, an increase in the Rayleigh number is caused with an increase in the buoyancy force effect owing to temperature differences. A variation of the Lewis number is caused with a variation of physical parameters of the nanofluid. Parameter *Nr* characterizes an effect of the buoyancy force due to concentration difference in comparison with the buoyancy force due to temperature difference. Therefore the product $Ra{\cdot}Nr$ characterizes an effect of the buoyancy force owing to concentration differences. Brownian motion parameter *Nb* defines the random motion of nanoparticles within the base fluid (Brownian motion) and characterizes an effect of the nanofluid temperature and the nanoparticles diameter. At the same time the thermophoresis parameter Nt characterizes an effect of a temperature gradient on the particles diffusion and depends on the thermal conductivity of the fluid and particle materials. It is worth noting here that when turbulent effects are not important (like in the present analysis) Brownian diffusion and thermophoresis become important as slip mechanisms (Buongiorno, 2006; Haddad *et al.*, 2012).

Figures 5-8 illustrate streamlines, isotherms and isoconcentrations at different values of key parameters.

An increase in the Rayleigh number leads to changes in all characteristics (streamlines, isotherms and isoconcentrations). One can see from these figures a decrease in the thermal and concentration boundary layers thickness, and an intensification of the convective flow in the cavity judging by the maximum absolute value of the stream function, with Ra. At the same time an increase in Ra leads to changes in the convective cell core, namely, at small Rayleigh number like $Ra = 100$ (Figures 5 and 6) the core was expanded along the vertical axis but at high Ra like $Ra = 500$ (Figures 7 and 8) the core has the form of the isosceles spherical triangle. The main reason for such modification is a stratification of the flow, temperature and concentration.

It is interesting to note a nonmonotonic effect of the Rayleigh number on the local Nusselt and Sherwood numbers in Figure 9. An increment in Ra leads to a growth of Nu at $0 \leqslant Y \leqslant 0.7$ but for $0.7 < Y < 1$, one can find a decrease in Nu with Ra that can be explained by a proximity of the cold wall. A decrease in Nu with $Y \leqslant 0.7$ is due to an increase in the thermal boundary layer thickness and an increase in Nu with $0.7 < Y$ is due to an interaction of the thermal boundary layers close to the vertical and inclined walls. A similar behavior to the presented above one can find with the local Sherwood number. The main differences are both a threshold value of $Y = 0.5$ after that Sh increases with Y and a presence of the local maximum value of Sh (Figure 9(b)) close to the bottom wall owing to a decrease in the nanoparticle volume fraction in the zone $0 < Y < 0.1$. The latter can be explained by an effect of Brownian motion and thermophoresis.

The average Nusselt and Sherwood numbers increase with Ra with the exception of $Nr = 0.4$ and $100 < Ra < 200$ for the average Sherwood number.

An increment in the Lewis number leads to both an intensification of the convective flow judging by the maximum absolute value of the stream function and essential changes in the isoconcentrations. The latter is related to a decrease in the concentration boundary layer thickness. It is well known that an increase in the Lewis number leads to a decrease in the thickness of concentration boundary layers at vertical walls. It physically means that flow with large Lewis number prevent spreading the nanoparticle in the nanofluid. The isotherms change insignificantly that has been also reflected on the distributions of the local Nusselt number (Figure 10(a)). While the local Sherwood number increases with Le along the vertical wall with the exception of the top part of this wall. At the same time the average Nusselt number increases with Le at high Rayleigh numbers and the average Sherwood number significantly increases with Le regardless of the Rayleigh number value.

One can find from Figures 5-8 that an increase in the buoyancy-ratio parameter Nr leads to an attenuation of the convective flow taking into account the maximum absolute value of the stream function. Such behavior leads to the reduction of the heat and mass transfer rate. It is worth pointing out a decrease in the average Nusselt and Sherwood numbers with Nr (Figure 11) While Nu and Sh are nonmonotonical functions

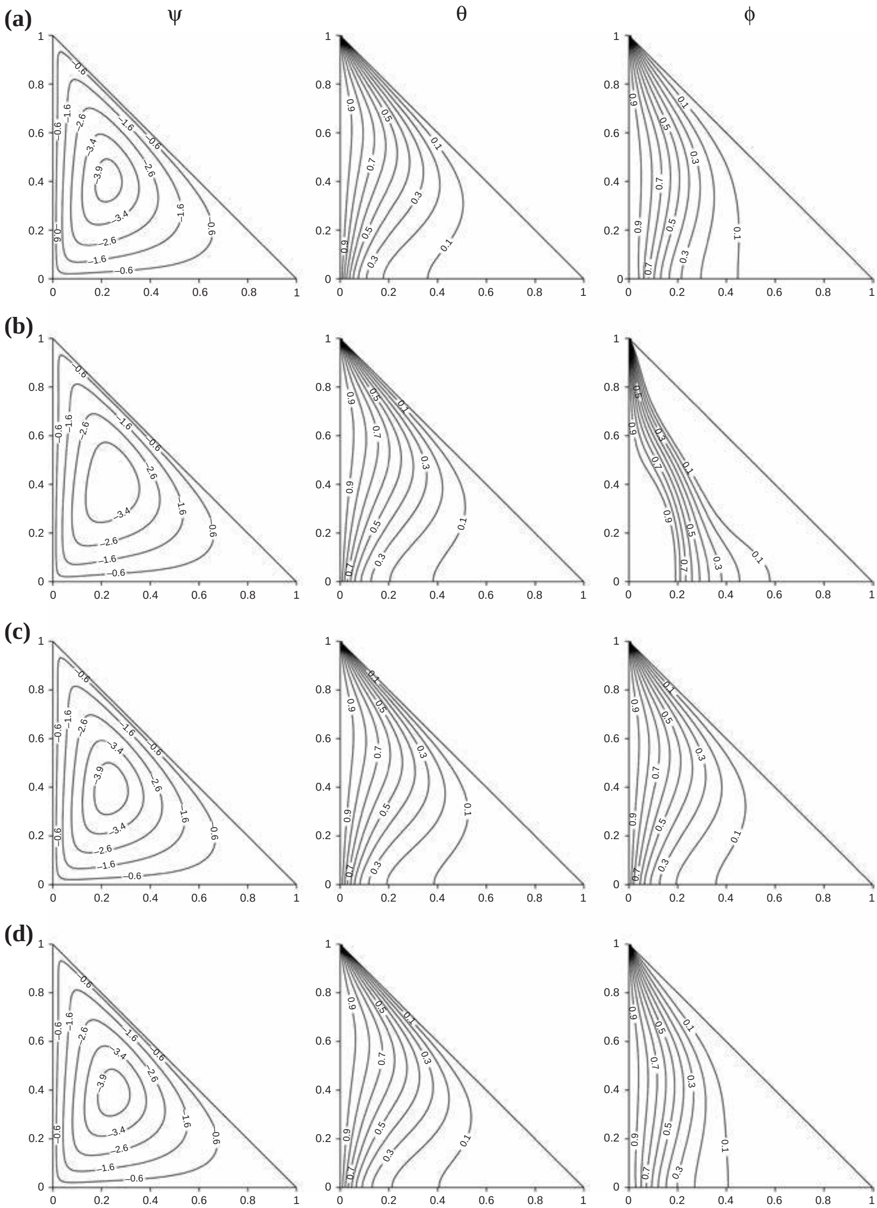
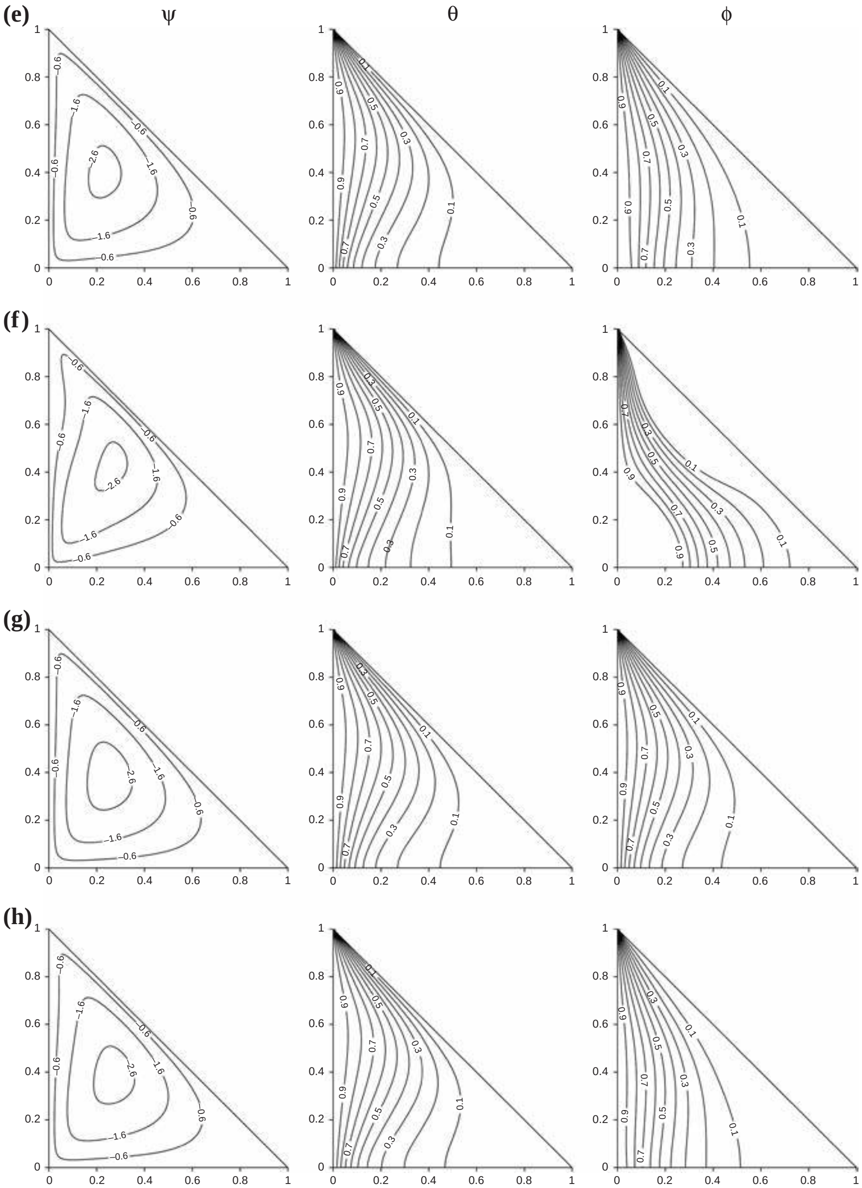


Figure 5.
Streamlines ψ, isotherms θ and isoconcentrations ϕ for $Ra = 100$, $Le = 1.0$

(continued)



Notes: (a) $Nr = 0.1$, $Nb = 0.1$, $Nt = 0.1$; (b) $Nr = 0.1$, $Nb = 0.1$, $Nt = 0.4$; (c) $Nr = 0.1$, $Nb = 0.4$, $Nt = 0.1$; (d) $Nr = 0.1$, $Nb = 0.4$, $Nt = 0.4$; (e) $Nr = 0.4$, $Nb = 0.1$, $Nt = 0.1$; (f) $Nr = 0.4$, $Nb = 0.1$, $Nt = 0.4$; (g) $Nr = 0.4$, $Nb = 0.4$, $Nt = 0.1$; (h) $Nr = 0.4$, $Nb = 0.4$, $Nt = 0.4$

Figure 5.

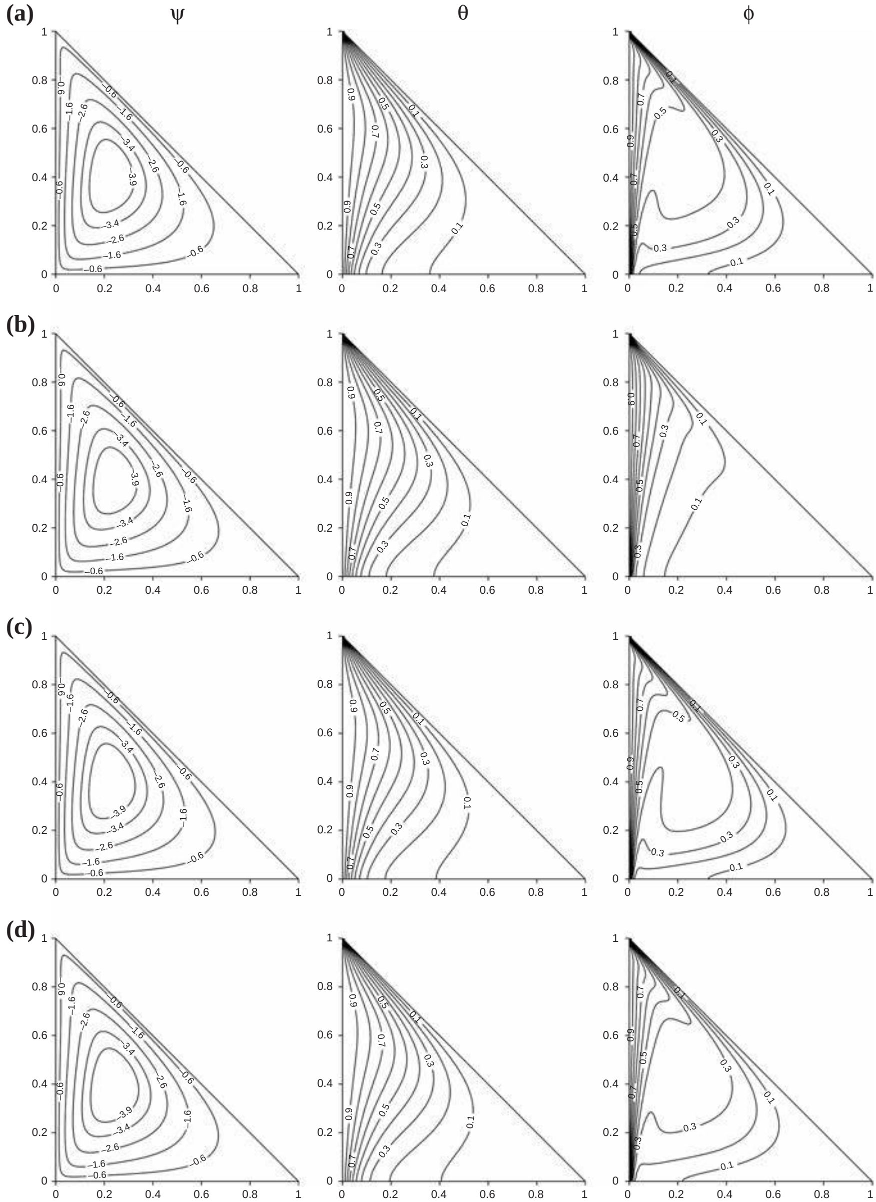
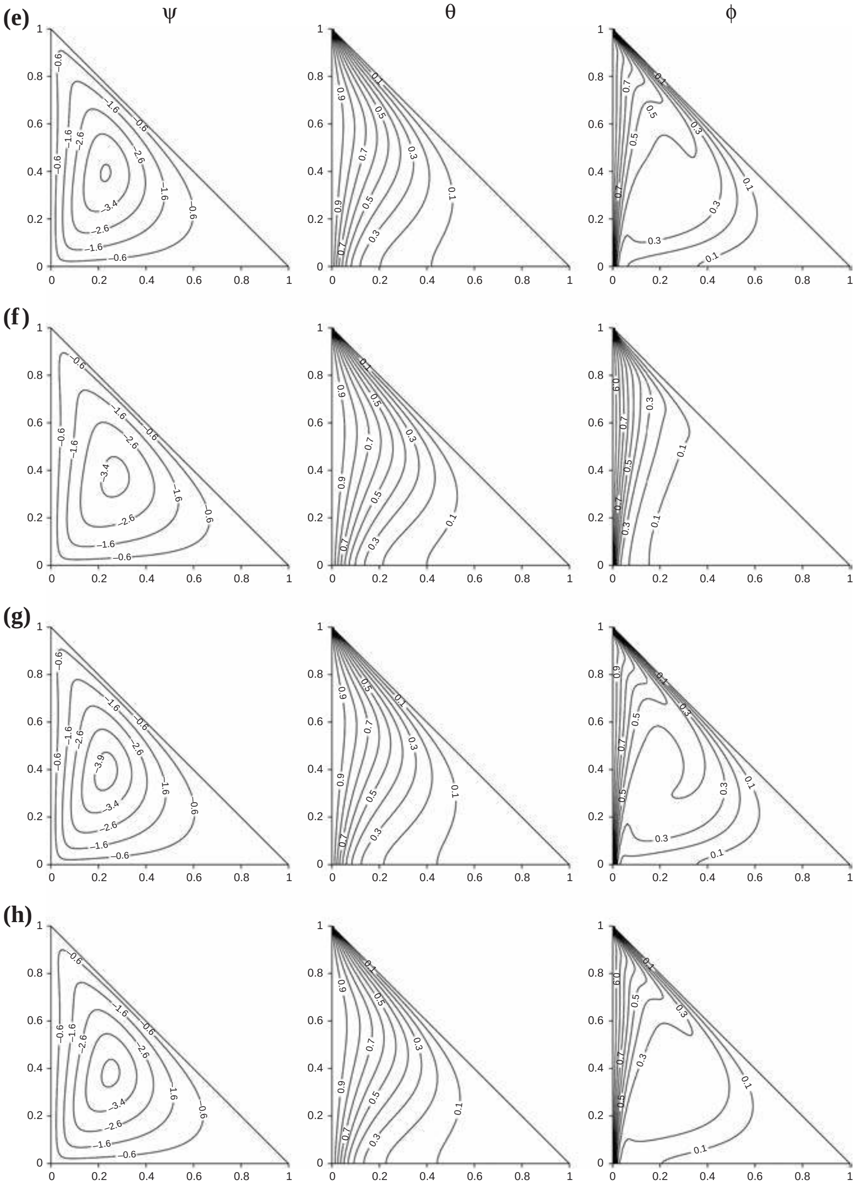


Figure 6.
Streamlines ψ, isotherms θ and isoconcentrations ϕ for $Ra = 100$, $Le = 10.0$

(continued)



Notes: (a) $Nr=0.1$, $Nb=0.1$, $Nt=0.1$; (b) $Nr=0.1$, $Nb=0.1$, $Nt=0.4$; (c) $Nr=0.1$, $Nb=0.4$, $Nt=0.1$; (d) $Nr=0.1$, $Nb=0.4$, $Nt=0.4$; (e) $Nr=0.4$, $Nb=0.1$, $Nt=0.1$; (f) $Nr=0.4$, $Nb=0.1$, $Nt=0.4$; (g) $Nr=0.4$, $Nb=0.4$, $Nt=0.1$; (h) $Nr=0.4$, $Nb=0.4$, $Nt=0.4$

Figure 6.

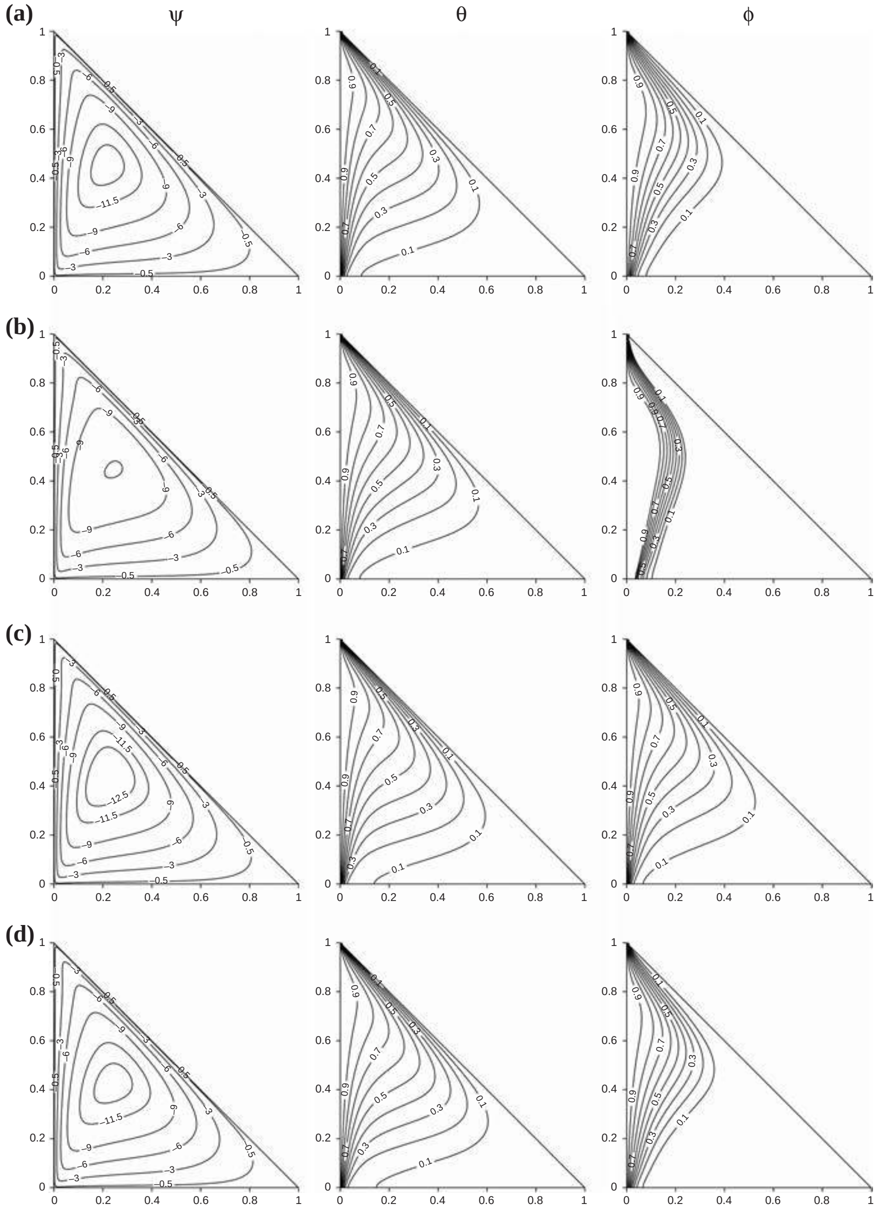
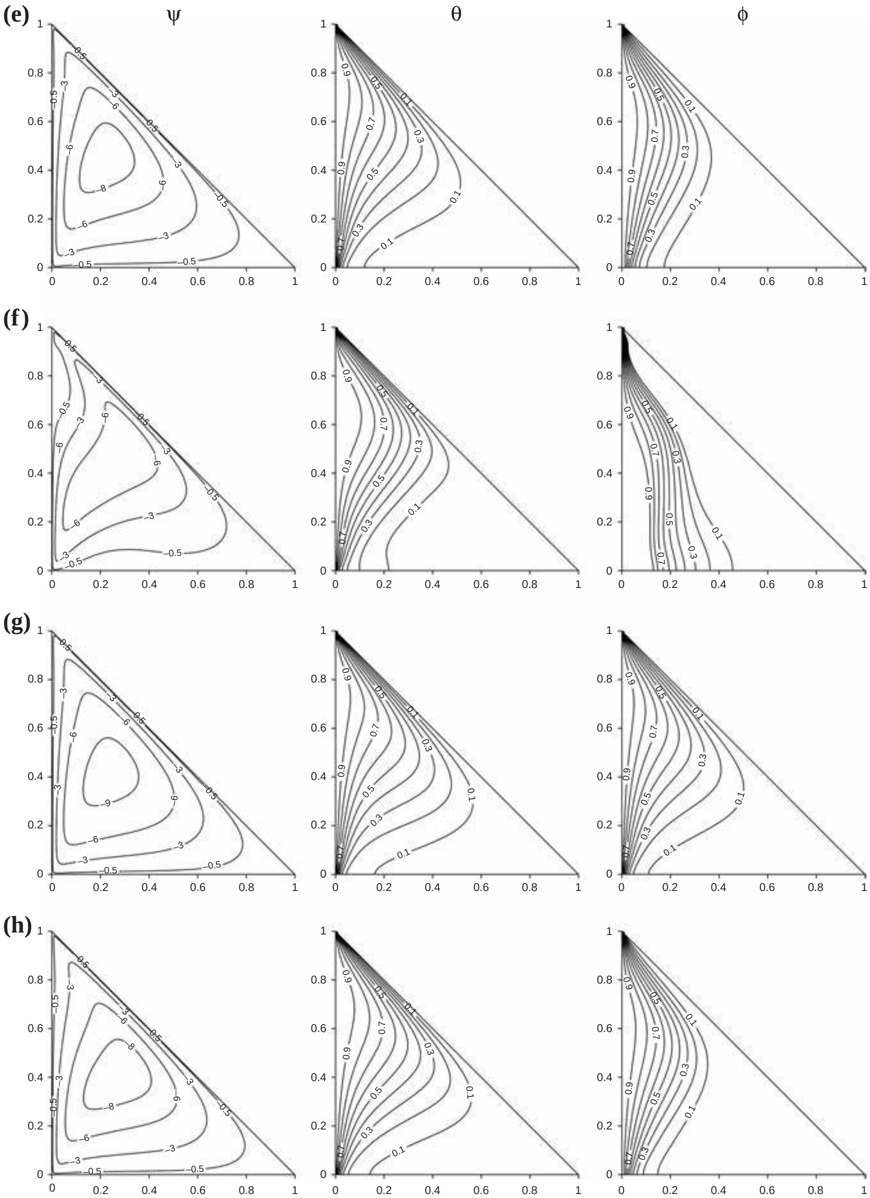


Figure 7.
Streamlines ψ, isotherms θ and isoconcentrations ϕ for $Ra = 500$, $Le = 1.0$

(continued)



Notes: (a) $Nr=0.1$, $Nb=0.1$, $Nt=0.1$; (b) $Nr=0.1$, $Nb=0.1$, $Nt=0.4$; (c) $Nr=0.1$, $Nb=0.4$, $Nt=0.1$; (d) $Nr=0.1$, $Nb=0.4$, $Nt=0.4$; (e) $Nr=0.4$, $Nb=0.1$, $Nt=0.1$; (f) $Nr=0.4$, $Nb=0.1$, $Nt=0.4$; (g) $Nr=0.4$, $Nb=0.4$, $Nt=0.1$; (h) $Nr=0.4$, $Nb=0.4$, $Nt=0.4$

Figure 7.

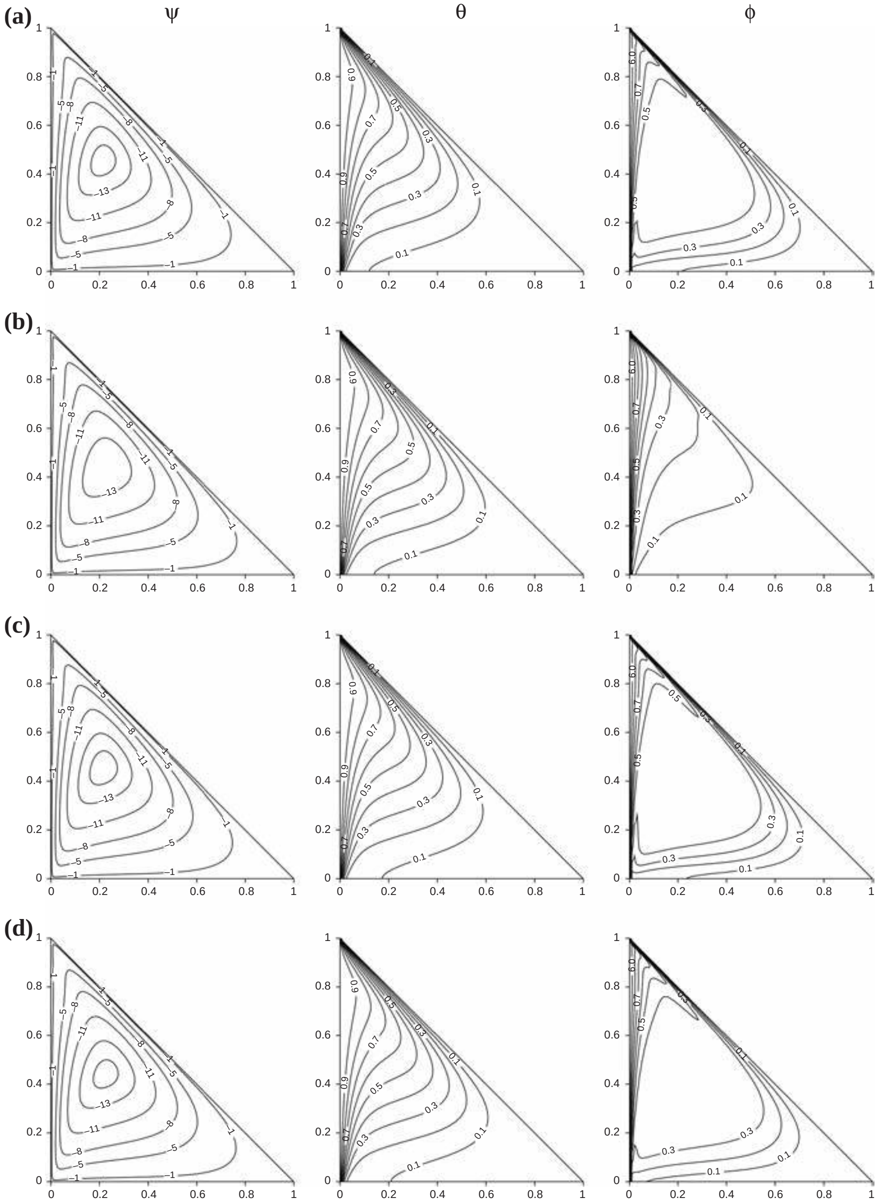
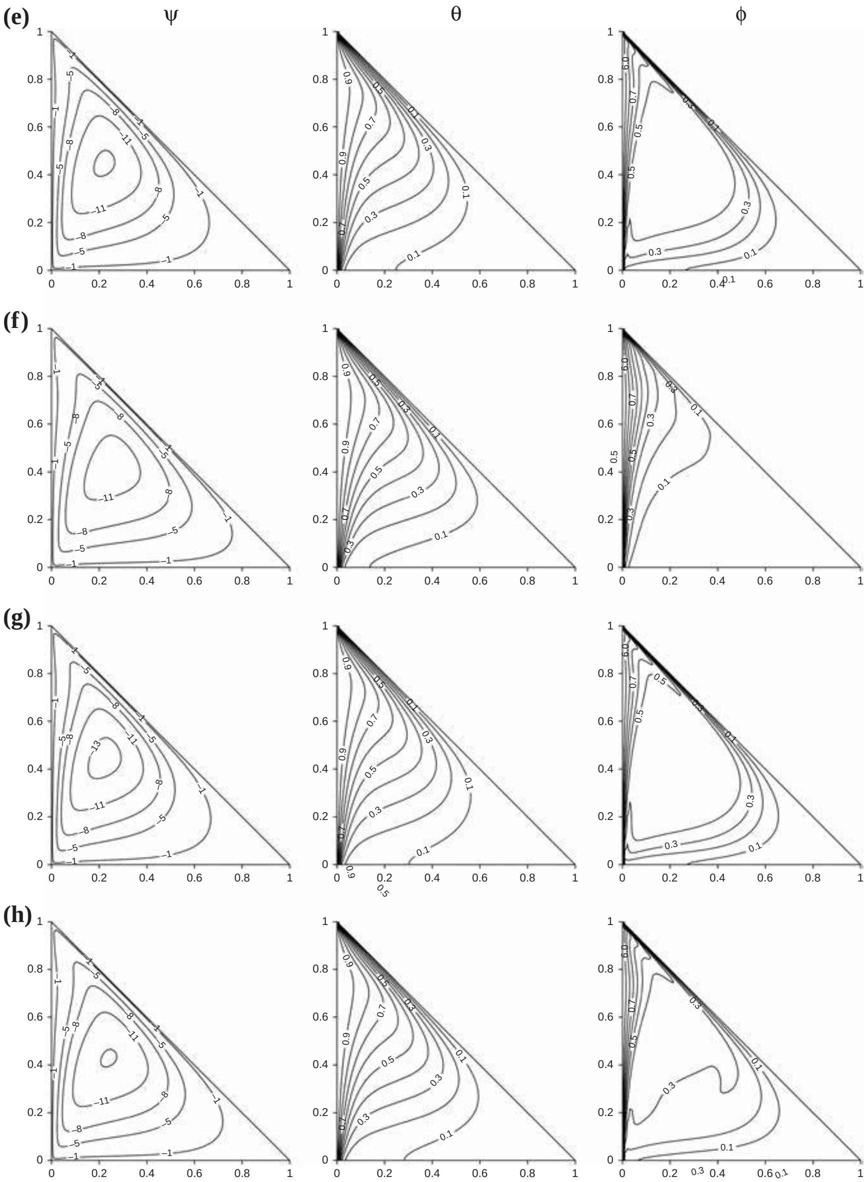


Figure 8.
Streamlines ψ, isotherms θ and isoconcentrations φ for $Ra = 500$, $Le = 10.0$

(continued)



Notes: (a) $Nr=0.1$, $Nb=0.1$, $Nt=0.1$; (b) $Nr=0.1$, $Nb=0.1$, $Nt=0.4$; (c) $Nr=0.1$, $Nb=0.4$, $Nt=0.1$; (d) $Nr=0.1$, $Nb=0.4$, $Nt=0.4$; (e) $Nr=0.4$, $Nb=0.1$, $Nt=0.1$; (f) $Nr=0.4$, $Nb=0.1$, $Nt=0.4$; (g) $Nr=0.4$, $Nb=0.4$, $Nt=0.1$; (h) $Nr=0.4$, $Nb=0.4$, $Nt=0.4$

Figure 8.

of Nr with a threshold value of Y. The threshold value of Y is similar to ones in case of the Rayleigh number effect.

An increase in the Brownian motion parameter Nb leads to both an intensification of the convective flow and mass transfer while the temperature field changes insignificantly. It should be noted that the above mentioned changes are more pronounced for $Le = 10$ and $Nr = 0.4$. The local Nusselt number is a weakly decreasing function of Nb along the vertical wall while Sh is a nonmonotonic function of Nb, namely, an increasing function at $Y < 0.78$ and a decreasing function at $Y > 0.78$ (Figure 12). It is worthpointing out that an increase in Nb leads to a disappearance of a zone close to the bottom wall where one can find a local maximum of Sh like it was at $Nb = 0.1$. On the basis of the mentioned features the average Nusselt number is a decreasing function and the average Sherwood number is an increasing function of Nb. On the basis of an analysis of Figures 5-8 one can notice a contrary effect of the thermophoresis parameter Nt in comparison with an influence of Nb. An increase in

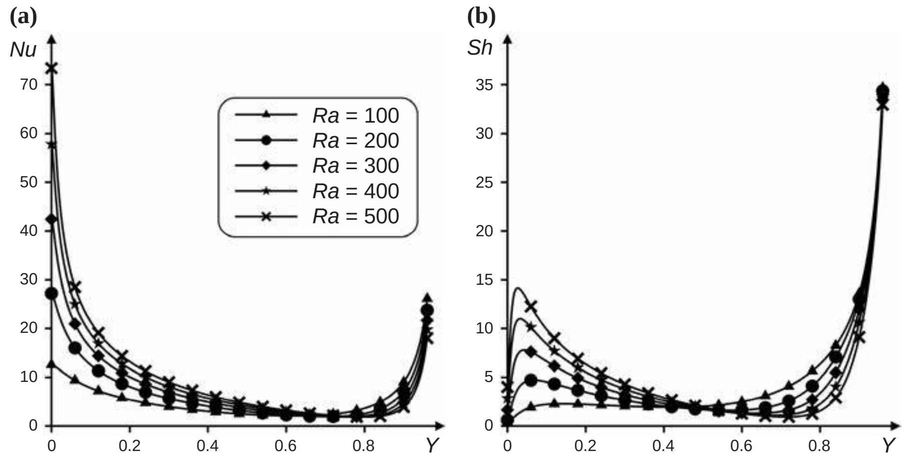


Figure 9.
Variation of local Nusselt number (a) and local Sherwood number (b) at vertical wall for $Le = 1.0$, $Nr = Nb = Nt = 0.1$ and different values of Rayleigh number

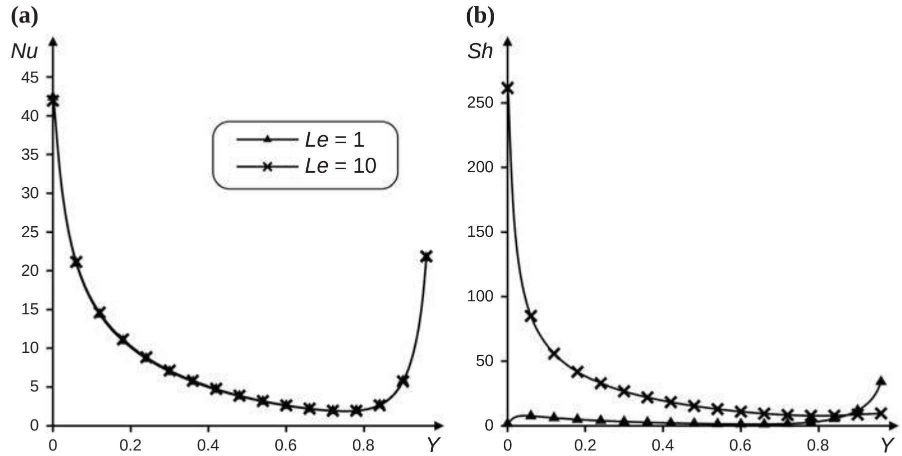


Figure 10.
Variation of local Nusselt number (a) and local Sherwood number (b) at vertical wall for $Ra = 300$, $Nr = Nb = Nt = 0.1$ and different values of Lewis number

Nt leads to an attenuation of the convective flow taking into account the maximum absolute value of the stream function. At the same time the mass transfer rate increases while the temperature field changes insignificantly. Behavior of the local Nusselt and Sherwood numbers vs *Nt* is similar to an effect of the Brownian motion parameter. Variations of the average numbers presented in Figure 13 are similar to variations of the local numbers.

5. Conclusions

Numerical analysis of the steady free convection flow and heat transfer in a triangular porous cavity filled by a nanofluid using the nanofluid model proposed by Buongiorno has been carried out. Distributions of streamlines, isotherms and isoconcentrations at a wide range of key parameters such as $Ra = 100\text{-}500$, $Le = 1\text{-}10$, $Nb = 0.1\text{-}0.4$, $Nt = 0.1\text{-}0.4$, $Nr = 0.1\text{-}0.4$, $A = H/L = 1.0$ have been obtained. It has been found that the average Nusselt

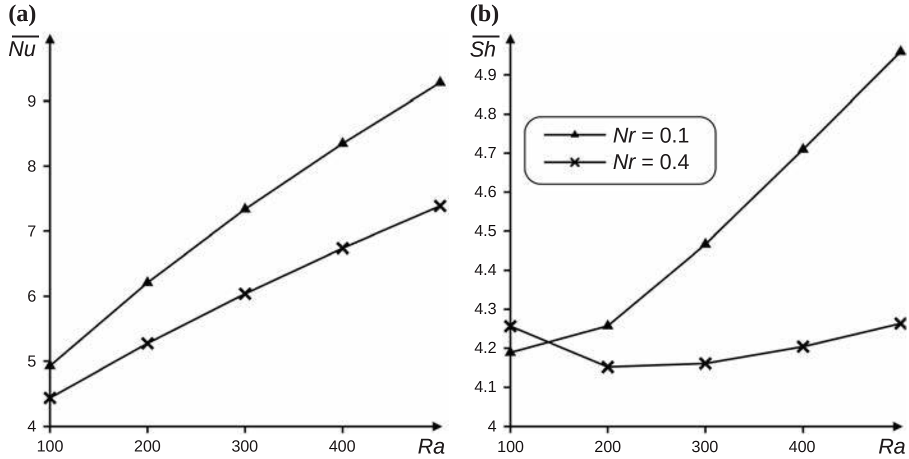


Figure 11. Variation of mean Nusselt number (a) and mean Sherwood number (b) at vertical wall with different Rayleigh numbers for $Le = 1.0$, $Nb = Nt = 0.1$ and different values of Nr

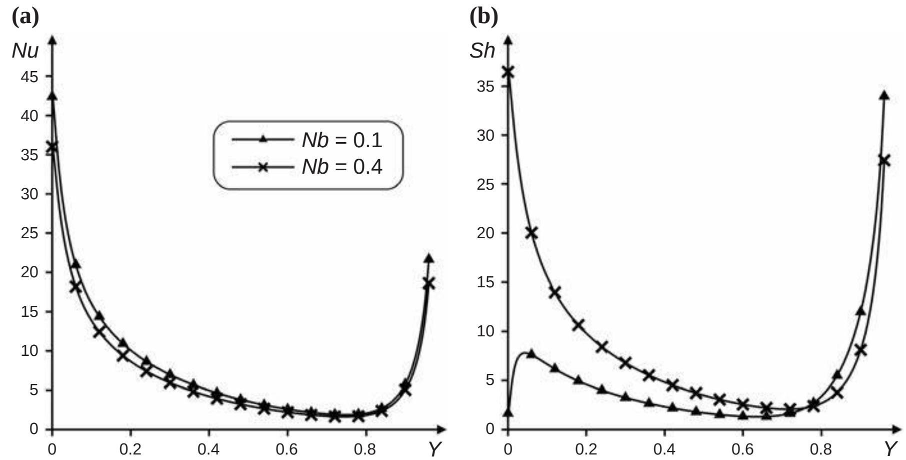


Figure 12. Variation of local Nusselt number (a) and local Sherwood number (b) at vertical wall for $Ra = 300$, $Le = 1.0$, $Nr = Nt = 0.1$ and different values of Nb

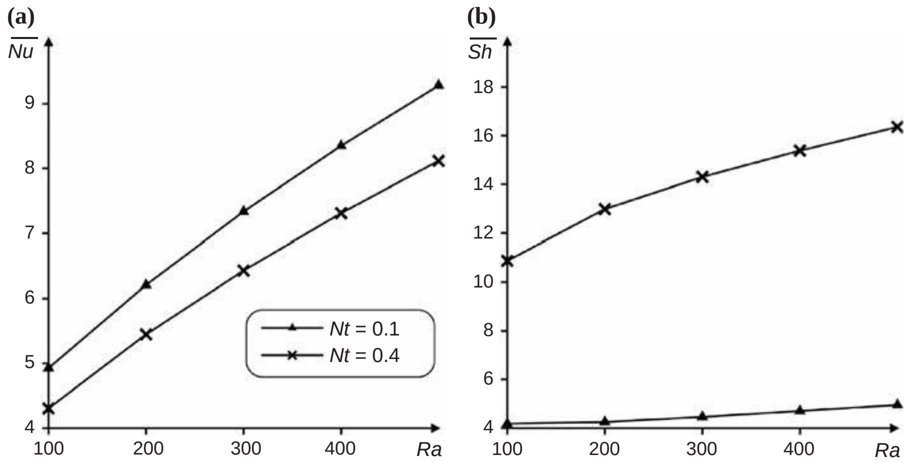


Figure 13. Variation of mean Nusselt number (a) and mean Sherwood number (b) at vertical wall with different Rayleigh numbers for $Le = 1.0$, $Nr = Nb = 0.1$ and different values of Nt

number is an increasing function of *Ra*, *Le*, and a decreasing function of *Nr*, *Nb*, *Nt*. At the same time the average Sherwood number is an increasing function of *Ra*, *Le*, *Nb*, *Nt* and a decreasing function of *Nr*. It has been shown that the local Nusselt and Sherwood numbers along the vertical wall decrease with $Y < Y^*$ due to an increase in the boundary layer thickness and increase with $Y > Y^*$ due to an interaction of the boundary layers close to the vertical and inclined walls.

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