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When electric charge becomes also magnetic

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In nonlinear electrodynamics, QED included, we find a static solution to the field equations with an electric charge as its source, which is comprised of homogeneous parallel magnetic and electric fields, and a radial spherically nonsymmetric long-range magnetic field, whose magnetic charge is proportional to the electric charge and also depends on the homogeneous component of the solution.

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I. INTRODUCTION

Magnetoelectric effect in materials is well known theoretically and experimentally [1]. It has a variety of manifestations, where different schemes of interaction within the material lead either to linear or nonlinear dependence of magnetization of an applied electric field, or reciprocally, dependence of electrization on an applied magnetic field (see reviews in [2]). Calculation of the corresponding coefficients from the first principles is available [3] in the linear case.

In the vacuum the (nonlinear) magnetoelectric effect was reported within quantum electrodynamics (QED) in [4,5], and within noncommutative classical electrodynamics in [6] (There is an earlier indication of the magnetoelectric effect in noncommutative electrodynamics in Ref. [7]. The magnetic solution proposed there is, however, inadmissibly singular, see [6] for comments.) In these references the nonuniform magnetic field, associated with a static electric charge is that of a magnetic dipole, whose magnetic moment is quadratic in the charge. Here we find a magnetic field with its source being an equivalent magnetic charge linearly related to an applied static electric charge.

To be more precise, in a parity-conserving nonlinear electrodynamics of the vacuum, especially in QED, we demonstrate the existence of a static field configuration that possesses a magnetic charge. The magnetic field carrying the magnetic charge is long-range in the sense that it decreases with the distance $r = |\mathbf{x}|$ from the electric charge, which produces it, as r^{-2}, in contrast to the magnetic dipole field decreasing as r^{-3} from its center. The magnetic lines of force are directed along the radius-vector $\mathbf{x}$. The magnetic nonuniform part of the solution is necessarily accompanied by uniform constant electric and magnetic fields $\bar{\mathbf{E}}$ and $\bar{\mathbf{B}}$ (taken as parallel to each other in our consideration), being axial-symmetric (i.e., azimuth-depending) relative to the axis specified by the common direction of the uniform part of the solution. The magnetic charge defined as the surface integral of the magnetic flux around the electric charge depends on these constant field components, which are arbitrary, and it is proportional to the electric charge and to the pseudoscalar $\bar{\mathbf{E}} \cdot \bar{\mathbf{B}}$. When the electric charge is pointlike, the magnetic charge carried by it is pointlike, too. In other words, an electric monopole is also a magnetic monopole. The axial-symmetric solution depends also on a choice of the angular boundary. Its vector-potential is found to be singular, depending on a gauge, on either of the two half-axes (Dirac string) drawn through the charge parallel or antiparallel to the accompanying constant fields. For a very special choice of the boundary, generally depending on the accompanying fields, the Dirac string disappears leaving the axial-symmetric radial solution magnetically neutral.

In this paper we are paving the most straightforward way to a magnetically-charged solution of nonlinear Maxwell equations by omitting all the terms that may be under certain circumstances considered as inessential for the existence of the solution proposed. We take the nonlinearity into account in its simplest manifestation by keeping only the lowest nontrivial, third power of the fields in these equations, while the corresponding nonlinear self-coupling constant is considered to be small. When referring to a nonlinear electrodynamics we mostly keep in mind the nonlinearity of the Maxwell equations of QED stemming from the quantum phenomenon of self-interaction between electromagnetic fields. The approach, however, remains the same for any nonlinear electrodynamics with a local Lagrangian and also can be readily extended to include a parity-nonconserving contribution that may originate from weak interactions.

II. NONLINEAR MAXWELL EQUATIONS

Let the Lagrangian of a nonlinear theory be a function of the field invariants $\mathfrak{F} = (B^2 - E^2)/2$, $\mathfrak{G} = -\mathbf{E} \cdot \mathbf{B}$, and let it depend on the space-time coordinate x^μ only through the fields and not contain their space-time derivatives

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$$L = -\mathfrak{F}(x) + \mathcal{L}(\mathfrak{F}(x), \mathfrak{G}(x)),$$

which implies that the action $S = \int L(x')d^4x'$ is a local functional. The first term here is the Lagrangian of the standard linear classical electrodynamics. The principle of correspondence with the classical theory requires that the nonlinear addition $\mathcal{L}$ should not contain any correction to it in the weak-field limit $B \to 0$, $E \to 0$. Therefore it will be accepted that the first derivative of $\mathcal{L}$ disappears when taken at zero values of the field invariants: $\mathcal{L}_{\mathfrak{F}} = \frac{\partial \mathfrak{L}}{\partial \mathfrak{F}}|_{\mathfrak{F}=\mathfrak{G}=0} = 0$. In a theory with parity conservation, such as QED, the Lagrangian may depend only on even powers of the pseudoscalar $\mathfrak{G}$. Hence, we shall accept that $\mathcal{L}_{\mathfrak{G}} = \frac{\partial \mathfrak{L}}{\partial \mathfrak{G}}|_{\mathfrak{F}=\mathfrak{G}=0} = 0$, $\mathcal{L}_{\mathfrak{F}\mathfrak{G}} = \frac{\partial^2 \mathfrak{L}}{\partial \mathfrak{F}\partial \mathfrak{G}}|_{\mathfrak{F}=\mathfrak{G}=0} = 0$. These quantities may be kept when needed. Under $\mathcal{L}$ we shall mostly mean the effective Lagrangian of QED in the local (infrared) limit, given in the one-loop approximation by the so-called Euler-Heisenberg Lagrangian [8]. Our approach covers, however, other nonlinear local Lagrangians irrespective of their origin. The nonlinear static Maxwell equations generated via the minimum action principle $\frac{\delta S}{\delta A_\mu(x)} = j^\mu(x)$, where $A_\mu(x)$ is the vector-potential, with the Lagrangian $\mathcal{L}$ truncated at the second power of its Taylor expansion in the field invariants—after the time-dependence has been dropped from their form derived in [9]—read

$$\nabla \cdot \mathbf{E}(\mathbf{x}) = j_0 + j_0^{\rm nl}, \qquad [\nabla \times \mathbf{B}(\mathbf{x})] = \mathbf{j} + \mathbf{j}^{\rm nl}. \tag{1}$$

Here j_μ are external current components, while the nonlinear current $j_\mu^{\rm nl}$, cubic in the fields, is the one induced by the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields:

$$j_0^{\rm nl} = \nabla \cdot (\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\mathfrak{F}\mathbf{E} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}}\mathfrak{G}\mathbf{B}), \tag{2}$$

$$\mathbf{j}^{\rm nl} = [\nabla \times (\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\mathfrak{F}\mathbf{B} - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}\mathfrak{G}\mathbf{E})]. \tag{3}$$

(It is understood that E, B, $\mathfrak{F}$, and $\mathfrak{G}$ depend on $\mathbf{x}$. The ∇'s act on everything to the right of them). Here $\mathcal{L}_{\mathfrak{F}\mathfrak{F}}$ and $\mathcal{L}_{\mathfrak{G}\mathfrak{G}}$ are time- and space-independent:

$$\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \frac{\partial^2 \mathfrak{L}}{\partial \mathfrak{F}^2}\bigg|_{\mathfrak{F}=\mathfrak{G}=0}, \qquad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \frac{\partial^2 \mathfrak{L}}{\partial \mathfrak{G}^2}\bigg|_{\mathfrak{F}=\mathfrak{G}=0}, \tag{4}$$

taken at zero values of the fields, in other words the background, against which the expansion of the Lagrangian has been developed, is empty. Then, as far as QED is concerned, $\mathcal{L}_{\mathfrak{F}\mathfrak{F}}$ and $\mathcal{L}_{\mathfrak{G}\mathfrak{G}}$ are quadratic with respect to the fine-structure constant [8] $\alpha = e^2/4\pi \simeq (137)^{-1}$,

$$\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \frac{4\alpha}{45\pi}\left(\frac{e}{m^2}\right)^2, \qquad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \frac{7\alpha}{45\pi}\left(\frac{e}{m^2}\right)^2, \tag{5}$$

where m and e are electron mass and charge, respectively.

Equations (1) should be completed with the other pair of static Maxwell equations

$$[\nabla \times \mathbf{E}(\mathbf{x})] = 0, \qquad \nabla \cdot \mathbf{B}(\mathbf{x}) = 0, \tag{6}$$

which are intact to the nonlinearity as long as the fields are given by a 4-vector-potential.

III. MAGNETIC SOLUTION FOR THE FIELD OF ELECTRIC CHARGE

In the present approach the electric field will be that produced by a spherically-symmetric external charge $j_0 \neq 0$. The Maxwell equations (1) will be treated perturbatively with respect to the small self-coupling contained in the coefficients $\mathcal{L}_{\mathfrak{F}\mathfrak{F}}$ and $\mathcal{L}_{\mathfrak{G}\mathfrak{G}}$. For this reason, the nonlinearly induced charge density $j_0^{\rm nl}$ (2) is neglected, $j_0^{\rm nl} = 0$, as giving rise only to higher-order contribution. The magnetic field will not be supported by any external source $\mathbf{j}$: the latter will be kept equal to zero throughout, $\mathbf{j} = 0$. The only source of the magnetic field will be the current $\mathbf{j}^{\rm nl}$, Eq. (3), formed by the electric and magnetic fields themselves.

An additional approximation adopted in solving the nonlinear Maxwell equations (1) is stemming from the fact that constant fields $\mathbf{E} = \bar{\mathbf{E}} = \text{const}$, $\mathbf{B} = \bar{\mathbf{B}} = \text{const}$ identically satisfy the Maxwell equations (1) with no external currents, $j_0 = \mathbf{j} = 0$, needed to support them, as it is immediately seen from Eqs. (1), (2), (3), (6). This is an approximation-independent manifestation of the gauge invariance, since the effective Lagrangian depends only on the field strength, what makes any constant field a solution. We shall be solving the second equation in (1) together with the second equation in (6) for the magnetic deviation $\delta\mathbf{B}(\mathbf{x}) = \bar{\mathbf{B}} - \mathbf{B}(\mathbf{x})$ considered to be small as compared to its constant part $\delta B \ll \bar{B}$. As for the electric field, its deviation from the constant field $\delta\mathbf{E}(\mathbf{x}) = \bar{\mathbf{E}} - \mathbf{E}(\mathbf{x})$ will be taken, in correspondence with the above-said, as the field produced by a charge via the standard linear equations $\nabla \cdot \mathbf{E}(\mathbf{x}) = j_0$, $[\nabla \times \mathbf{E}(\mathbf{x})] = 0$ without nonlinearity. We also assume that $\delta E \ll \bar{E}$. We configure our solution with the fields $\bar{\mathbf{B}}$ and $\bar{\mathbf{E}}$ parallel or antiparallel to each other (in the Lorentz frame, where the electric charge is at rest). Their arbitrary common direction is presented by a unit (pseudo)vector $\boldsymbol{\mu} = \frac{\bar{\mathbf{B}}}{\bar{B}}$, $|\boldsymbol{\mu}| = 1$.

We shall now handle the second equation in (1) with $\mathbf{j} = 0$, which may be equivalently written as

$$(1 - \mathfrak{F}\mathcal{L}_{\mathfrak{F}\mathfrak{F}})[\nabla \times \mathbf{B}] + \mathcal{L}_{\mathfrak{F}\mathfrak{F}}[\mathbf{B} \times \nabla]\mathfrak{F} - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}[\mathbf{E} \times \nabla]\mathfrak{G} = 0,$$

because $[\nabla \times \mathbf{E}] = 0$ due to (6). Omitting the deviations squared and neglecting $\bar{\mathfrak{F}}\mathcal{L}_{\mathfrak{F}\mathfrak{F}} \ll 1$ in the first term, this becomes the linearized equation for $\delta\mathbf{B}$

$$[\nabla\times\delta\mathbf{B}] = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}[\bar{\mathbf{B}}\times\nabla](\bar{\mathbf{E}}\cdot\delta\mathbf{E}-\bar{\mathbf{B}}\cdot\delta\mathbf{B}) - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}[\bar{\mathbf{E}}\times\nabla](\bar{\mathbf{B}}\cdot\delta\mathbf{E}+\bar{\mathbf{E}}\cdot\delta\mathbf{B}). \quad (7)$$

We shall seek for solutions to Eq. (7) in the class of axial-symmetric magnetic fields directed in the same way as the radius-vector, $\delta\mathbf{B} = \mathbf{x}b(r,z)$, where $r = |\mathbf{x}|$, and $z = (\boldsymbol{\mu}\cdot\mathbf{x}) = r\cos\theta$ is the coordinate component along the vector $\boldsymbol{\mu}$. Within this class, the second equation in (6) can be satisfied only if the representation

$$\delta\mathbf{B} = \mathbf{x}\frac{1}{r^3}f\left(\frac{z}{r}\right), \quad (8)$$

is true (everywhere except for the singularity in $x = 0$).

Out of all central-symmetric electric fields of the form $\delta\mathbf{E} = \mathbf{x}\mathcal{E}(r)$, Eq. (7) is only compatible with (8) provided that the field is Coulombic, $\delta\mathbf{E} = \frac{c\mathbf{x}}{r^3}$, with c being an arbitrary constant. But this is just the solution to the first equation (1) with j_0^{nl} set equal to zero, as argued above, and with c chosen as $c = \frac{q}{4\pi}$, where q is the charge, $q = \int j_0(\mathbf{x})d^3\mathbf{x}$. Here the integration runs over the volume occupied by the charge centered in the origin. (When the charge is distributed in a spherically symmetric way over a sphere $r = R$ or it is pointlike, $R = 0$, it should be understood that we are working outside the charge, $r > R$. If the charge is not spherically symmetric, our equations are valid far from the region occupied by it.) Then the ansatz (8) turns Eq. (7) to the first-order linear inhomogeneous differential equation for the function $f(\zeta)$, $\zeta = \frac{z}{r} = \cos\theta$ (after cancellation of the overall vector factor $[\boldsymbol{\mu}\times\mathbf{x}]/r^4$)

$$(1+b\zeta^2)f' + 3bf\zeta + g\zeta = 0, \quad (9)$$

$$g = \frac{3q}{4\pi}\bar{\mathfrak{G}}(\mathcal{L}_{\mathfrak{G}\mathfrak{G}} - \mathcal{L}_{\mathfrak{F}\mathfrak{F}}), \quad b = -\mathcal{L}_{\mathfrak{G}\mathfrak{G}}\bar{E}^2 - \mathcal{L}_{\mathfrak{F}\mathfrak{F}}\bar{B}^2, \quad (10)$$

where $\bar{\mathfrak{G}} = -\bar{\mathbf{B}}\cdot\bar{\mathbf{E}}$. The general solution of (9) is

$$f(\zeta) = \left(\frac{1+b\zeta_0^2}{1+b\zeta^2}\right)^{\frac{3}{2}}\left[f(\zeta_0) + \frac{g}{3b}\right] - \frac{g}{3b}. \quad (11)$$

Here ζ_0 is the asimuth point, where a boundary $f(\zeta_0)$ value for the solution is to be set. The singularity at $\zeta^2 = -1/b$ may lie within the physical interval $\zeta^2 \le 1$ only for sufficiently large fields $\bar{E}, \bar{B}$, which are beyond the scope of the present truncated approximation.

One can see that $f(\zeta) = -\frac{g}{3b}$ is the ζ-independent solution in immediate agreement with (9) with $f' = 0$. When substituted to (8), it would give rise to a magnetic monopole field in its standard center-symmetric form. This solution, however, should rather be disregarded, since it falls beyond the accuracy of the adopted approximation, as not being small in proportion to the smallness of nonlinearity. It disappears by going away to infinity in the limit $b \to 0$. We shall concentrate on angle-dependent solutions, free of this disadvantage. We are interested only in such solutions of the inhomogeneous equation (7), which are due to the electric charge q and vanish when $g = q = 0$. To separate them we impose the zero boundary condition $f(\zeta_0) = 0$ (that also excludes the above angle-independent solution). With this condition satisfied, the solution still depends on the point ζ_0, where it is imposed, i.e., on the couple of the asimuth directions $\theta_0 = \arccos\zeta_0$ and $\pi - \arccos\zeta_0$, along which the magnetic lines of force are not emitted from the charge. To fix the boundary, note that ζ_0 is a constant pseudoscalar, and there may be no other choice for it except zero, since there is no field-independent, intrinsic pseudoscalar at our disposal, and we should not introduce it once we are interested in the magnetic field produced only by the electric charge and by no other magnetically charged sources. The point $\zeta_0 = 0$, $\theta_0 = \frac{\pi}{2}$ is the only point invariant under the space reflection, because it is mapped to itself, since $\theta_0 = \pi - \theta_0$, when $\theta_0 = \frac{\pi}{2}$. With the choice $\zeta_0 = 0$ advocated above the magnetic field (8) is

$$\delta\mathbf{B} = \mathbf{x}\frac{1}{r^3}\frac{g}{3b}[(1+b\cos^2\theta)^{-\frac{3}{2}} - 1]. \quad (12)$$

Note that this expression, as well as all other expressions in the rest of the paper, survives the interesting limit $b = 0$, which originates from dropping the homogeneous terms $\mathcal{L}_{\mathfrak{F}\mathfrak{F}}[\bar{\mathbf{B}}\times\nabla](\bar{\mathbf{B}}\cdot\delta\mathbf{B})$ and $-\mathcal{L}_{\mathfrak{G}\mathfrak{G}}[\bar{\mathbf{E}}\times\nabla](\bar{\mathbf{E}}\cdot\delta\mathbf{B})$ from Eq. (7) and corresponds to the neglect of linear magnetization.

The magnetic charge q_M is determined by integrating $\delta\mathbf{B}$ over the surface of a sphere with its radius large enough to embrace the whole charge and not to violate the conditions $\delta B \ll \bar{B}$ and $\delta E \ll \bar{E}$. The result is

$$q_M = \frac{4\pi g}{3b}[(1+b)^{-\frac{1}{2}} - 1]. \quad (13)$$

Hence, for the pointlike electric charge, its magnetic charge density is $\nabla\cdot\delta\mathbf{B} = q_M\delta^3(\mathbf{x})$. The magnetic charge (13) is proportional to the electric charge q via Eq. (10).

IV. DIRAC STRING

The vector potential generating the field (12) via the relation $\delta\mathbf{B} = [\nabla\times\mathbf{A}]$ can be taken in the form

$$\mathbf{A} = \frac{[\boldsymbol{\mu}\times\mathbf{x}]}{r^2}\omega(\zeta),$$

$$\omega(\zeta) = \frac{g}{3b}\frac{1}{\zeta^2-1}\left[\frac{\zeta}{\sqrt{1+b\zeta^2}} - \frac{\tilde{\zeta}}{\sqrt{1+b\tilde{\zeta}^2}} + \tilde{\zeta} - \zeta\right]. \quad (14)$$

The latter function is subject to the condition $\omega(\tilde{\zeta}) = 0$. Arbitrariness in choosing the constant $\tilde{\zeta}$ is the gauge arbitrariness. The potential (14) is singular along the axis $\zeta^2 - 1 = 0$. Two special gauges, $\tilde{\zeta} = \pm 1$, however, exist, with which the axial singularity is restricted only to the positive half-axis (the lower sign) or to the negative half-axis (the upper sign). With these two choices $\omega(\zeta)$ becomes

$$\omega^{\pm}(\zeta) = \frac{g}{3b}\frac{1}{\zeta^2 - 1}\left[\frac{\zeta}{\sqrt{1+b\zeta^2}} \mp \frac{1}{\sqrt{1+b}} - (\zeta \mp 1)\right].$$

It is seen by expanding the first term in the brackets in powers of ζ^2 near the point $\zeta^2 = 1$ that $(\zeta \mp 1)$ cancels the same factor in the denominator if the latter is represented as $\zeta^2 - 1 = (\zeta \mp 1)(\zeta \pm 1)$. The resulting singularity $(\zeta \pm 1)^{-1}$ stretches along the axis passing through the charge either parallel or antiparallel to the common direction of the constant part of the solution, the fields $\bar{\mathbf{B}}$ and $\bar{\mathbf{E}}$. When the modulus of the vector-potential is calculated following (14), the factor $|\boldsymbol{\mu} \times \mathbf{x}| = \sqrt{1-\zeta^2}$ appears, but we are still left with the singularity $\pm\sqrt{\frac{1\mp\zeta}{1\pm\zeta}}$ on either of the two half-axes. This is the Dirac string [10], whose direction depends on a gauge, but cannot be eliminated by any choice of it.

It is worth noting that a special choice of the boundary point $\zeta_0^2 = \frac{\sqrt[3]{1+b}-1}{b}$ exists that eliminates the Dirac string and simultaneously nullifies the magnetic charge. This value depends on the fields $\bar{B}$ and $\bar{E}$, but in the limit $b \to 0$ it is just $\zeta_0 = \frac{\pm 1}{\sqrt{3}}$. One may think that introducing the boundary ζ_0 other than zero leads to an additional magnetic charge that neutralizes the magnetic charge of the electric charge and results in the radial, but spherically nonsymmetric magnetic field with the incoming and outgoing magnetic fluxes compensating each other.

V. CONCLUDING REMARKS

With the QED values (5) and the choice of the boundary $\theta_0 = \frac{\pi}{2}$, and also neglecting b as compared to unity the magnetic charge is

$$q_M = q\frac{\alpha}{30\pi}\frac{\bar{E}}{E_0}\frac{\bar{B}}{B_0},$$

where E_0 and B_0 are the characteristic QED values $B_0 = \frac{m^2}{e} = 4.4 \times 10^{13}$ G, $E_0 = \frac{m^2}{e} = 1.3 \times 10^{16}$ V/cm (in CGSE units). Therefore, not too close to the electric charge, all conditions assumed in the course of derivation of the present result, including the requirement that $\bar{\mathfrak{F}}$ and $\bar{\mathfrak{G}}$ be smaller than $(m^2/e)^2$, needed to justify the truncation of the effective action, are met. (The latter restriction seems to be only technical and may be overcome by expanding the action against the constant field background to be kept in (4). In that instance also the field-depending derivative $\mathcal{L}_{\mathfrak{F}\mathfrak{G}}$ will contribute). So, for the astrophysical-scale values $\bar{B} \sim \bar{E} \lesssim \frac{m^2}{e}$ the magnetic charge makes up the 8×10^{-5}th part of its electric charge value.

It is important that the coefficient between the electric and magnetic charges is a pseudoscalar, $\bar{\mathfrak{G}}$, whose presence in the solution is necessary for the magnetoelectric effect described. Any possible field configurations carrying magnetoelectric effects different from ours must contain this unique pseudoscalar, too.

Note that in the limit $\bar{\mathbf{E}} = 0$, solution (12) disappears not to turn into the magnetic solution of Refs. [4,5] (which is of a dipole shape [5]) produced by an electric charge in a constant magnetic background, because that solution does not belong to the class considered here.

It is well understood that equations of electromagnetism readily accept a magnetic charge, with the "only" reservation that the latter has been never found in Nature [11], except as a quasiparticle in spin ice [12] or a physical imitation [13]. Here we have demonstrated that an electric charge is also a magnetic one if accompanied by (placed into) a combination of constant and homogeneous parallel electric and magnetic fields. Evidently, the restriction on the fields to be parallel is crucial only for the method of derivation, but the condition that they are not mutually perpendicular, $\bar{\mathbf{E}}\cdot\bar{\mathbf{B}} \neq \mathbf{0}$, cannot be circumvented. This means that the charge may move in the Lorentz frame, where the fields $\bar{\mathbf{E}}$ and $\bar{\mathbf{B}}$ are parallel, without stopping being a magnetic monopole.

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