PHYSICAL REVIEW D **89,** 047504 (2014)

Electric charge is a magnetic dipole when placed in a background magnetic field

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(Received 16 November 2013; revised manuscript received 17 January 2014; published 24 February 2014)

It is demonstrated, owing to the nonlinearity of QED, that a static charge placed in a strong magnetic field B is a magnetic dipole (besides remaining an electric monopole, as well). Its magnetic moment grows linearly with B as long as the latter remains smaller than the characteristic value of 1.2×10^{13} G but tends to a constant as B exceeds that value. The force acting on a densely charged object by the dipole magnetic field of a neutron star is estimated.

DOI: 10.1103/PhysRevD.89.047504 PACS numbers: 04.40.Nr, 04.20.Cv, 04.20.Fy, 11.15.-q

I. INTRODUCTION

In the recent papers [1,2], we started a study of self-interaction of static electric and magnetic fields of moderate strength—in the vacuum [2] or taken against a very strong background formed by a constant and homogeneous magnetic field [1]. By moderate, we mean the fields that are strong enough to make the nonlinearity of QED actual for their self-interaction but still assumed smaller than the characteristic value $B_{\mathrm{Sch}} = m^2/e$, where m and e are electron mass and charge, to enable us to exploit the expansion of the nonlinear Maxwell equations in powers of the fields and confine ourselves to the lowest terms in this expansion. On the contrary, the background field is not limited either from below or from above. It was observed that effects of self-interaction manifest themselves already within the simplest approximation of local action, valid in the infrared region of momenta, i.e., for fields that do not change any essentially at the time or space interval of m^{-1}. Among these effects are nonlinear corrections to the Coulomb field in the vacuum [2], making the field energy of a point charge finite [3], electromagnetic nonlinear renormalization of electric and magnetic dipole moments of mesons and baryons [2], necessary after the latter are calculated following the theory of strong interaction. In the present paper, we continue the work [1], where it was pointed out that the quadratic response of the strong background magnetic field to an applied moderate electric field is purely magnetic, and the nonlinear current induced by an applied electric field was calculated based on the third-rank polarization tensor (a three-photon vertex beyond the mass shell) in the infrared (local) limit. Also, a general expression was given for the magnetic field nonlinearly induced by an electrostatic field. Below we demonstrate that this field is that of a magnetic dipole. The magnetic dipole moment carried by a static spherical charge of finite extension is calculated. We estimate the force acting on this magnetic moment by the inhomogeneous magnetic field of a neutron star and a magnetar. If we admit that in the course of the phase transition of the neutron star to a quark star [4,5] highly charged domains of strange matter [6] may exist, then the observed effect may form a mechanism of charge outflow from the star.

II. INDUCED CURRENT

The nonlinear response of the magnetized vacuum to an applied electrostatic field within the nonlinear Maxwell equations truncated at the third power of the field is purely magnetic [1], with the vector-potential components a^{μ}_{nl} being given as (we keep to the rationalized Heaviside-Lorentz units throughout)

$$a^0_{\mathrm{nl}} = 0, \quad a^i_{\mathrm{nl}}(k) = \sum_{c=1,3} \frac{2\pi\delta(k_0)}{(k^2 - \varkappa_c(k))} \frac{\mathfrak{b}^{(c)}_i}{(\mathfrak{b}^{(c)})^2} (\tilde{j}^{\mathrm{nl}}_j(\mathbf{k}) \mathfrak{b}^{(c)}_j), \quad (1)$$

where $\tilde{j}^{\mathrm{nl}}_j(\mathbf{k})$ is the Fourier-transformed nonlinearly induced current

$$j^{\mathrm{nl}}_{\rho}(x) = -\frac{1}{2} \int d^4x' d^4x'' \Pi_{\nu\sigma\rho}(x', x'', x) a^{\nu}(x') a^{\sigma}(x''), \quad (2)$$

and $\Pi_{\nu\sigma\rho}(x', x'', x)$ is the third-rank polarization tensor in the coordinate representation defined as the third variational derivative of the effective action, with respect to the field potential taken in three space-time points x', x'', x. In QED, it includes all diagrams with three off-shell outer photons in the external magnetic field. In (1), $\mathfrak{b}^{(c)}_i$ are the

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three eigenvectors of the second-rank polarization tensor in the background magnetic field, responsible for the linear response in that field, and $\varkappa_c(k)$ are the corresponding eigenvalues. Out of the three components of the photon propagator that build a field from a (nonlinear) current, only two contribute into the sum in (1). This means that in the static limit only two eigenmodes are responsible for carrying the magnetic field.

If the effective action $\Gamma = \int \mathcal{L}(x) d^4x$ is assumed to be a local functional of the scalar ($\mathfrak{F}$) and pseudoscalar ($\mathfrak{G}$) field invariants, like what the Euler-Heisenberg in QED or Born-Infeld actions are, the current (2) has the form [1]

$$j^0_{\rm nl}(\mathbf{x}) = 0, \qquad \mathbf{j}_{\rm nl}(\mathbf{x}) = \nabla \times \mathfrak{h}(\mathbf{x}), \tag{3}$$

$$\mathfrak{h}_i(\mathbf{x}) = \frac{B_i}{2}\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\mathbf{E}^2 - \frac{B_i}{2}\mathcal{L}_{\mathfrak{F}\mathfrak{G}\mathfrak{G}}(\mathbf{B}\cdot\mathbf{E})^2 - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}(\mathbf{B}\cdot\mathbf{E})E_i. \tag{4}$$

Here $\mathbf{E}$ is the static applied field contained in the vector potential in the right-hand side of (2), and $\mathbf{B}$ is the external constant and homogeneous magnetic field contained in $\Pi_{\nu\sigma\rho}(x', x'', x)$. The scalar coefficients in the auxiliary field (4) are the derivatives of the effective Lagrangian $\mathcal{L}(x)$ with respect to the field invariants taken at $\mathfrak{G} = 0$, $2\mathfrak{F} = B^2 = \text{const}$. They depend only on $B = |\mathbf{B}|$.

III. INDUCED MAGNETIC FIELD AND MAGNETIC MOMENT

The magnetic field strength $\boldsymbol{h}(\mathbf{x})$ generated by the current (3) according to the Maxwell equation $\nabla \times \boldsymbol{h}(\mathbf{x}) = \boldsymbol{j}^{\rm nl}(\mathbf{x})$ is

$$h_i(\mathbf{x}) = \mathfrak{h}_i(\mathbf{x}) + \nabla_i\Omega, \tag{5}$$

because $\nabla \times \nabla\Omega \equiv 0$. To find the scalar function Ω, we should exploit the other Maxwell equation $(\nabla\cdot\mathbf{b}) = 0$, where the magnetic induction $\mathbf{b}$ is related to the magnetic field as $h_i(\mathbf{x}) = \mu^{-1}_{ij} b_j$ through the (inverse) magnetic permeability tensor of the magnetized vacuum, in the present local approximation given as [7] $\mu^{-1}_{ij} = (1 - \mathfrak{L}_{\mathfrak{F}})\delta_{ij} - \mathfrak{L}_{\mathfrak{F}\mathfrak{F}} B_i B_j$. Its eigenvalues are $\mu_\perp^{-1} = 1 - \mathfrak{L}_{\mathfrak{F}}$, $\mu_\parallel^{-1} = 1 - \mathfrak{L}_{\mathfrak{F}} - \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}B^2$. Then

$$\Omega = -\frac{\nabla_i \mu_{ij}}{\nabla_m \mu_{mn} \nabla_n}\mathfrak{h}_j(\mathbf{x});$$

hence (the tilde designates again the Fourier transform),

$$h_i(\mathbf{x}) = \left(\delta_{ij} - \nabla_i \frac{\nabla_j + \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\mu_\parallel(\mathbf{B}\cdot\nabla)B_j}{\nabla^2 + \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\mu_\parallel(\mathbf{B}\cdot\nabla)^2}\right)\mathfrak{h}_j(\mathbf{x}) \tag{6}$$

$$= \mathfrak{h}_i(\mathbf{x}) + \nabla_i \frac{\nabla_j + \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\mu_\parallel(\mathbf{B}\cdot\nabla)B_j}{(2\pi)^3} \times \int \frac{\tilde{\mathfrak{h}}_j(\boldsymbol{k}) e^{i\mathbf{k}\cdot\mathbf{x}}}{\boldsymbol{k}^2 + \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\mu_\parallel(\mathbf{B}\cdot\mathbf{k})^2} d^3k, \tag{7}$$

and

$$h_i(\mathbf{x}) = \mathfrak{h}_i(\mathbf{x}) + \nabla_i\left[\nabla_k + \left(\frac{\mu_\parallel}{\mu_\perp} - 1\right)\frac{(\mathbf{B}\cdot\nabla)B_k}{B^2}\right]\mathfrak{T}_k(\mathbf{x}), \tag{8}$$

$$\mathfrak{T}_k(\mathbf{x}) = \frac{1}{4\pi}\int d^3y \frac{\mu_\perp^{1/2}\mathfrak{h}_k(\mathbf{y})}{(\mu_\parallel(\mathbf{x}_\perp - \mathbf{y}_\perp)^2 + \mu_\perp(\mathbf{x}_\parallel - \mathbf{y}_\parallel)^2)^{1/2}}. \tag{9}$$

The indices $\parallel$ and $\perp$ mark the radius-vector components, parallel and orthogonal to the background magnetic field $\mathbf{B}$. We are interested in asymptotic behavior of (8) in the far-off region $x_i \to \infty$. The contribution of (9) is

$$\mathfrak{T}_k(\mathbf{x}) \sim \frac{1}{4\pi}\frac{\mu_\perp^{1/2}\int d^3y \mathfrak{h}_k(\mathbf{y})}{(\mu_\parallel \mathbf{x}_\perp^2 + \mu_\perp \mathbf{x}_\parallel^2)^{1/2}},$$

provided that the integral here converges. This is guaranteed assuming that the applied electric field E_i in (4) decreases fast enough at $y_i \to \infty$. If its source is concentrated in a finite space domain, E decreases as fast as the square of the distance from this domain and the auxiliary field (4) as its fourth power. This decrease is faster than that of the second term in (8). Therefore, only the latter should be kept in the remote region

$$h_i(\mathbf{x}) \sim \frac{\mu_\perp^{1/2}}{4\pi}\int d^3y \mathfrak{h}_k(\mathbf{y})\nabla_i\left(\nabla_k + \left(\frac{\mu_\parallel}{\mu_\perp} - 1\right)\frac{(\mathbf{B}\cdot\nabla)B_k}{B^2}\right)\frac{1}{(\mu_\parallel \mathbf{x}_\perp^2 + \mu_\perp \mathbf{x}_\parallel^2)^{1/2}}$$
$$= \frac{\mu_\perp^{1/2}\mu_\parallel}{4\pi}\int d^3y \mathfrak{h}_k(\mathbf{y})\frac{1}{\check{r}^3}\left(-\delta_{ki} + \frac{3x_k x_i \mu_\parallel}{\check{r}^2} + \frac{3x_k B_i(\mathbf{B}\cdot\mathbf{x})(\mu_\perp - \mu_\parallel)}{B^2 \check{r}^2}\right), \tag{10}$$

where we have denoted the *effective* distance as $\check{r} = (\mu_\parallel \mathbf{x}_\perp^2 + \mu_\perp \mathbf{x}_\parallel^2)^{1/2}$. This magnetic field satisfies both sourceless Maxwell equations $\nabla \times \boldsymbol{h}(\mathbf{x}) = 0$, $(\nabla\cdot\mathbf{b}(\mathbf{x})) = 0$ in the whole space domain (not only in the remote region), except the point $r' = 0$. It is the field of a pointlike magnetic dipole in anisotropic medium, with the magnetic moment being $\mathbf{M} = \frac{\mu_\perp^{1/2}\mu_\parallel}{4\pi}\int d^3y \mathfrak{h}(\mathbf{y})$. If we also assume that the electric field is no less than cylindrically symmetric with the symmetry axis coinciding with the direction of $\mathbf{B}$ the magnetic moment becomes parallel with $\mathbf{B}$. In the limit of isotropic medium $\mu_\perp = \mu_\parallel = \mu$, the last term in (10) disappears, and $\check{r}$ becomes $\mu^{1/2}|\mathbf{x}|$; hence, μ cancels out from Eq. (10), and the latter acquires the standard form of a magnetic dipole in the vacuum

$$\mathbf{h}(\mathbf{x}) = \frac{1}{r^3}\left(-\mathbf{M} + \frac{3\mathbf{x}(\mathbf{x}\cdot\mathbf{M})}{r^2}\right), \quad \mathbf{M} = \frac{1}{4\pi}\int d^3y\mathfrak{h}(\mathbf{y}), \quad (11)$$

where now r is just $|\mathbf{x}|$. Earlier [8], we reported this result under the neglect of the linear response of the vacuum by setting $\mu_\perp = \mu_\parallel = \mu = 1$. Now we see that it is sufficient to admit only the absence of anisotropy of this response. In the magnetized vacuum in QED, the isotropization occurs [1] for the very large background field $B \gg 2.7 \cdot B_{\text{Sch}}$, where $\mathfrak{L}_\mathfrak{F} \gg 2\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}B^2$. However, if QED is treated perturbatively, the whole linear response of the background magnetic field introduces only a higher order correction into (10): for small B, the coefficients $\mathcal{L}_{\mathfrak{F}\mathfrak{F}}$, $\mathcal{L}_{\mathfrak{F}\mathfrak{G}\mathfrak{G}}$, $\mathcal{L}_{\mathfrak{G}\mathfrak{G}}$ are proportional to the fine-structure constant $\alpha = e^2/4\pi$ squared [see Eq. (18) below], as well as $\mathcal{L}_\mathfrak{F}$, whereas for large B, they all are proportional to α [consult Eq. (15) below and the relation $\mathcal{L}_\mathfrak{F} = \frac{\alpha}{3\pi}\ln\frac{eB}{m^2}$, valid in the large-field regime].

IV. A BASIC EXAMPLE

For demonstrative purposes, we accept this approximation in considering the simplest example of the magnetic response to a spherically symmetric electric field given by the potential

$$a_0(r) = a_0^{\text{in}}(r)\theta(R-r) + a_0^{\text{out}}(r)\theta(r-R), \qquad r = |\mathbf{x}|$$
$$a_0^{\text{in}}(r) = -\frac{Ze}{8\pi R^3}r^2 + \frac{3}{8\pi}\frac{Ze}{R}, \qquad a_0^{\text{out}}(r) = \frac{Ze}{4\pi r}, \qquad (12)$$

where $\theta(z)$ is the step function

$$\theta(z) = \begin{cases} 1, & z > 0, \\ 0, & z < 0. \end{cases}$$

Once the applied electric field is spherically symmetric, $\mathbf{E} = \mathbf{x}\mathcal{E}(r)$, the current $\mathbf{j}_{\text{nl}}(\mathbf{x})$ (3), (4) is proportional to the vector product $\mathbf{B}\times\mathbf{x}$; hence, it is circular in the plane orthogonal to the background magnetic field. Simultaneously, the contribution of the first mode $c = 1$ disappears from (1).

If not for the linear electric polarization, the potential (12) would be the field of extended spherically symmetric charge distributed with the constant density $\rho(r)$ inside a sphere $r \le R$ with the radius R:

$$\rho(r) = \left(\frac{3}{4\pi}\frac{Ze}{R^3}\right)\theta(R-r). \qquad (13)$$

With the account of the linear vacuum polarization, the potential distribution (12) cannot be supported by any spherically symmetric charge, strictly localized in a finite space domain. The genuine source of the field (12) is $\rho_{\text{lin}}(\mathbf{x}) = (\nabla\cdot\mathbf{d})$, where the electric induction is defined as $d_i = -\varepsilon_{ij}\nabla_j a_0$ in terms of the potential (12) and of the dielectric tensor ε_{ij} of the vacuum, which in the constant magnetic background has, in the present local approximation, the form [7] $\varepsilon_{ij} = (1-\mathfrak{L}_\mathfrak{F})\delta_{ij} + \mathfrak{L}_{\mathfrak{G}\mathfrak{G}}B_iB_j$. We find

$$\rho_{\text{lin}}(\mathbf{x}) = \rho(r)(1-\mathcal{L}_\mathfrak{F}) + 2\mathfrak{F}\mathcal{L}_{\mathfrak{G}\mathfrak{G}}\left(1 + \frac{(\mathbf{B}\cdot\mathbf{x})^2}{B^2}\frac{d}{rdr}\right)\frac{d}{rdr}a_0(r).$$

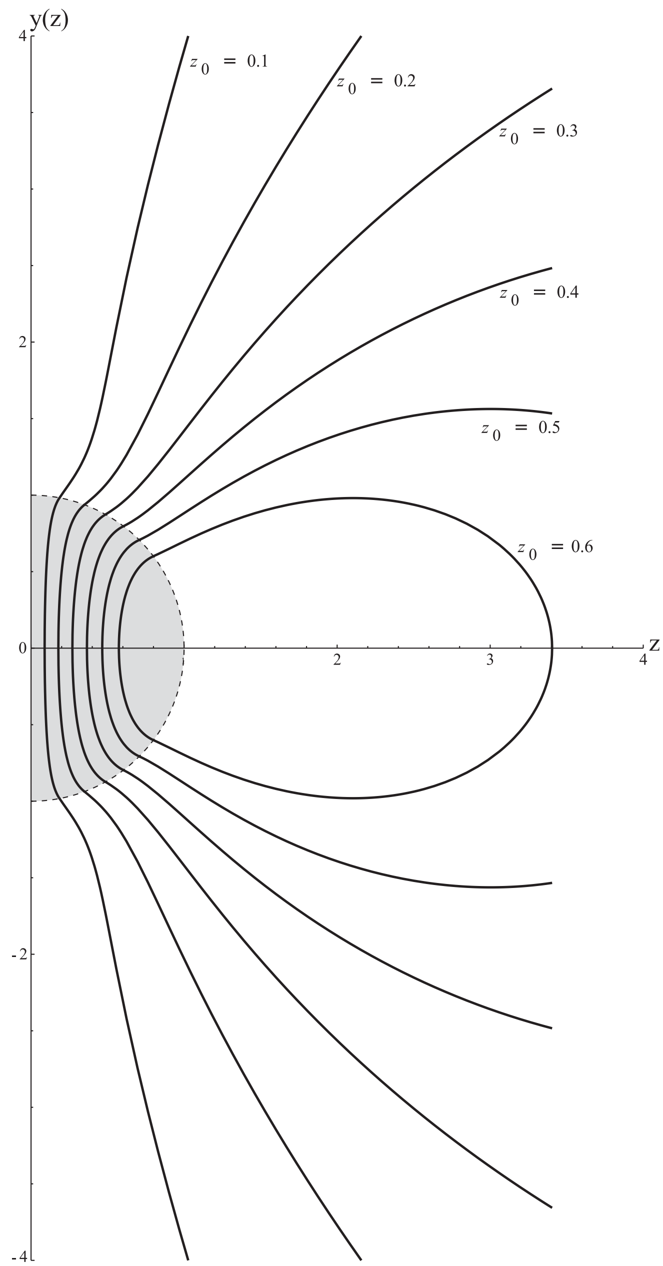

FIG. 1. Magnetic dipole lines of a static charge in an external magnetic field exampled with $B = \infty$. Shaded is the area of the charge.

This charge density is cylindrically symmetric and extends beyond the sphere, $r > R$, decreasing as $1/r^3$, or $1/x_3^3$ far from it, depending on the direction. However, the same argument as above allows us to neglect the linear polarization in the electric sector by setting $\varepsilon_{ij} = \delta_{ij}$, as well as we did in the magnetic. Then $\rho_{\rm lin}(\mathbf{x}) = \rho(r)$, and therefore, we refer to the magnetic field nonlinearly produced by the electric field (12) as the nonlinear response to the field of a homogeneously charged sphere (13). It is calculated following Eqs. (8) and (9) with $\mathbf{E} = -\nabla a_0$ used in (4) and with $\mu_\parallel = \mu_\perp = 1$ to be (the details of this calculation can be found in [8])

$$h_i(\mathbf{x}) = h_i^{\rm in}(\mathbf{x})\theta(R-r) + h_i^{\rm out}(\mathbf{x})\theta(r-R), \qquad (14)$$

$$\begin{aligned} h_i^{\rm in}(\mathbf{x}) = &-\left(\frac{Ze}{4\pi R^2}\right)^2\left\{\left[\frac{1}{2}\left(1-\frac{4r^2}{5R^2}\right)\mathcal{L}_{\mathfrak{GG}} + \frac{r^2}{10R^2}\mathcal{L}_{\mathfrak{FF}} + \mathcal{L}_{\mathfrak{FGG}}\left(\frac{1}{10}\left(1-\frac{4r^2}{7R^2}\right)B^2 + \frac{2r^2}{7R^2}\left(\frac{\mathbf{B}\cdot\mathbf{x}}{r}\right)^2\right)\right]B_i\right. \\ &\left.+\frac{1}{5}\left(\frac{1}{2}\mathcal{L}_{\mathfrak{FF}} + \frac{1}{2}\mathcal{L}_{\mathfrak{GG}} - \frac{2}{7}B^2\mathcal{L}_{\mathfrak{FGG}}\right)\frac{(\mathbf{B}\cdot\mathbf{x})x_i}{r^2}\right\}, \\ h_i^{\rm out}(\mathbf{x}) = &\left(\frac{Ze}{4\pi r^2}\right)^2\left\{\left[\frac{1}{2}\left(1-\frac{6r}{5R}\right)\mathcal{L}_{\mathfrak{FF}} - \frac{1}{2}\left(1-\frac{4r}{5R}\right)\mathcal{L}_{\mathfrak{GG}} - \left(\frac{1}{2}\left(1-\frac{2r}{5R}-\frac{18R}{35r}\right)B^2 - \left(1-\frac{9R}{7r}\right)\left(\frac{\mathbf{B}\cdot\mathbf{x}}{r}\right)^2\right)\mathcal{L}_{\mathfrak{FGG}}\right]B_i\right. \\ &-2\left(\frac{Ze}{4\pi r^2}\right)^2\left\{\left(1-\frac{9r}{10R}\right)\mathcal{L}_{\mathfrak{FF}} - \frac{1}{2}\left(1-\frac{6r}{5R}\right)\mathcal{L}_{\mathfrak{GG}}\right. \\ &\left.+\left[\left(-1+\frac{3r}{10R}+\frac{9R}{14r}\right)B^2 + \frac{3}{2}\left(1-\frac{R}{r}\right)\left(\frac{\mathbf{B}\cdot\mathbf{x}}{r}\right)^2\right]\mathcal{L}_{\mathfrak{FGG}}\right\}\frac{(\mathbf{B}\cdot\mathbf{x})x_i}{r^2}. \end{aligned}$$

which agrees with (11), once the auxiliary field (4) is taken on the Coulomb field (12).

The shape of a family of magnetic lines of force is depicted in Fig. 1, drawn, for definiteness, for asymptotically large values of the background field $B \gg m^2/e$, in which domain it holds, using the Euler-Heisenberg effective Lagrangian (see, e.g., [9]), that

$$\mathcal{L}_{\mathfrak{FF}} = \frac{\alpha}{3\pi}\frac{1}{B^2}, \qquad \mathcal{L}_{\mathfrak{GG}} = \frac{\alpha}{3\pi}\left(\frac{e}{m^2}\right)\frac{1}{B},$$
$$B^2\mathcal{L}_{\mathfrak{FGG}} = B^2\frac{d\mathcal{L}_{\mathfrak{GG}}}{d\mathfrak{F}} = -\mathcal{L}_{\mathfrak{GG}}. \qquad (15)$$

Inside the charged sphere R, the curves follow the formula

$$y(z) = \sqrt{\frac{14}{11} - \frac{6}{11}z^2 - \left(\frac{z_0}{z}\right)^2}, \qquad (16)$$

being labeled by positive values of the integration constant z_0 in the interval $0 < z_0 < \frac{7}{\sqrt{66}}$, whereas outside the sphere they have resulted from the computer solution of the corresponding first-order differential equation $\frac{\mathrm{d}z}{\mathrm{d}y} = \frac{h_z}{h_y}$ (we have directed axis z along the background magnetic field and axis y along any direction in the orthogonal plane). It is seen that in the long-range domain the pattern of the lines of force is that of a magnetic dipole. Indeed, the long-range contribution of (14) does behave like a magnetic field of a solenoid (11) with the equivalent magnetic moment M given by

$$M_i = \left(\frac{Ze}{4\pi}\right)^2\frac{1}{5R}(3\mathcal{L}_{\mathfrak{FF}} - 2\mathcal{L}_{\mathfrak{GG}} - B^2\mathcal{L}_{\mathfrak{FGG}})B_i, \qquad (17)$$

V. ESTIMATES

To estimate the effect of nonlinear magnetization numerically, note that, within the Euler-Heisenberg Lagrangian, for the background field $B \ll \frac{m^2}{e}$, much smaller than Schwinger's optional value $B_{\rm Sch} = \frac{m^2}{e} = 1.2 \times 10^{13}$ G it holds

$$\mathcal{L}_{\mathfrak{FF}} = \frac{16\alpha^2}{45m^4}, \qquad \mathcal{L}_{\mathfrak{GG}} = \frac{28\alpha^2}{45m^4}, \qquad (18)$$

while the contribution of the term $B^2\mathcal{L}_{\mathfrak{FGG}}$ in (17) and in (14) is negligible in this order as being proportional to an extra power of B^2. Then in this field domain, after the substitution $E = -\frac{Ze}{4\pi R^2}$, we have from (14) for the component orthogonal to the background field, taken approximately at $r = R$,

$$\begin{aligned} h_\perp &= \frac{-11\alpha}{225}\left(\frac{eE}{m^2}\right)^2 B\frac{\sin 2\theta}{2} \\ &= 3.5\times 10^{-4}\left(\frac{E}{E_{\rm Sch}}\right)^2 B\frac{\sin 2\theta}{2}, \end{aligned} \qquad (19)$$

where θ is the angle between $\mathbf{B}$ and the radius-vector $\mathbf{x}$, directed toward the points where the induced magnetic field is orthogonal to $\mathbf{B}$ (see Fig. 1), and $E_{\rm Sch} = \frac{m^2}{e} = 1.3 \times 10^{16}$ V/cm. Our results are not expected to be applicable to charged microscopic objects, since their electric field close to themselves is too large to be treatable using only the second-power nonlinearity (2) and varies too fast at the distance of the electron Compton length m^{-1} to be treatable within the approximation of the local action. For this reason, we consider a macroscopic device at extremum laboratory conditions. Let the potential of accelerator scale, $(1 \div 25)$ MV, be applied to a point ball with the curvature radius $R = (1 \div 0.1) \times 10^{-5}$ cm, like the one in an ion projector, to produce electric field $E = (1 \div 250) \times 10^{11}$ V/cm $= (0.77 \div 200) \times 10^{-5} E_{\rm Sch}$. Then, according to (19), $h_\perp \simeq (2 \times 10^{-5} \div 1.4) \times 10^{-9} B$. This quantity may reach maximum values for the background field that may achieve $B = 10^6$ G in a laboratory. It is hard to say whether they can be registered against the strong background field, although directed orthogonally to $h_\perp$. As Eq. (19) remains valid up to the values of pulsar scale $B \simeq 0.1 B_{\rm Sch} = 1.2 \times 10^{12}$ G, the field of the above device would make $h_\perp \simeq (2.2 \times 10^{-5} \div 1.7) \times 10^3$ G if placed into such pulsar. The magnetic moment (17) in the small-field domain follows from (18) to be

$$M \simeq -\frac{Z^2\alpha^2}{(Rm)}\frac{1}{225\pi^2}\mu_e \frac{eB}{m^2} \simeq -\left(\frac{E}{E_{\rm Sch}}\right)^2 \frac{2\alpha}{225\pi} B R^3, \quad (20)$$

where $\mu_e = \frac{e}{2m} = 9.27 \times 10^{-21}$ G · cm^3 is the Bohr magneton. The force acting to it by the inhomogeneous (dipole) magnetic field B near a pulsar surface is $F \simeq M \frac{dB}{dr} \simeq M \frac{d}{dr} \frac{M_{\rm pl}}{r^3} \simeq -3M \frac{M_{\rm pl}}{r^4} \simeq -4\pi M \frac{B}{r}$, where $M_{\rm pl} = B \frac{4}{3}\pi r^3$ is the magnetic moment of the neutron star and r is its radius. If we imagine a macroscopic ball of the radius $R = 4 \times 10^9$ m$^{-1} \simeq 1$ cm carrying the charge $Ze = 1$ C, i.e., $Z = 0.6 \times 10^{19}$ and place it in the magnetic field of a pulsar, taking $r = 10^6$ cm for its radius and $B \simeq 0.1 B_{\rm Sch}$ for its field at the surface, we see that its magnetic moment is 0.2 G · cm^3 and it is subject to a force of the order of $\simeq 3$ kG. Note that since $r \gg R$, the magnetic moment has enough space to be formed before it would sense the inhomogenuity of the background field and undergo its forcing influence.

In the opposite asymptotic regime $B \gg \frac{m^2}{e}$, characteristic of some magnetars [10], it follows from (15) that the second and the third terms in (17) dominate to provide the saturation of the magnetic moment in the limit $B = \infty$ at the level of

$$M = -\left(\frac{Ze}{4\pi}\right)^2 \frac{\alpha}{15R\pi}\frac{e}{m^2} = -\frac{Z^2\alpha^2}{(Rm)}\frac{1}{30\pi^2}\mu_e$$
$$= -\left(\frac{E}{E_{\rm Sch}}\right)^2 \frac{\alpha}{15\pi} B_{\rm Sch} R^3, \quad (21)$$

where $\mu_e = \frac{e}{2m} = 9.27 \times 10^{-21}$ G · cm^3 is the Bohr magneton. Once $\frac{\alpha}{15\pi} = 1.5 \times 10^{-4}$, the upper bound on (21) admitted by the quadratic approximation (2), which requires that $E < E_{\rm Sch}$, is $|M| < 1.5 \times 10^{-4} B_{\rm Sch} R^3 = 1.8 \times 10^9$ GR^3. This means that the average magnetic field at the surface of a sphere, charged so high that its surface electric field approaches 1.3×10^{16} V/cm (we may think of strangelets [6] and quark stars [5]), approaches 10^9 G. The magnetic moment (21) of the ball of $R = 1$ cm and $Ze = 1$ C is now $M \simeq 1.6 \times 10^{21} \mu_e \simeq 15$ G · cm^3. It is subject to a force of 2.3×10^4 kG if placed in a magnetar field $B \simeq 10 B_{\rm Sch}$.

ACKNOWLEDGMENTS

T. C. A. acknowledges the support of FAPESP under the Contracts No. 2013/00840-9 and No. 2013/16592-4. He is grateful to the Department of Physics of the University of Florida for the kind hospitality and to Professor John Klauder for his warm reception there. D. G. thanks CNPq and FAPESP for permanent support. A. S. acknowledges the support of FAPESP, Process No. 2011/51867-9, and of RFBR under Project No. 14-02-01171. He also thanks USP for kind hospitality in Sao Paulo, Brazil, where this work was partially fulfilled. The authors are thankful to C. Costa for discussions.

[1] D. M. Gitman and A. E. Shabad, Phys. Rev. D **86**, 125028 (2012).

[2] C. V. Costa, D. M. Gitman, and A. E. Shabad, Phys. Rev. D **88**, 085026 (2013).

[3] C. V. Costa, D. M. Gitman, and A. E. Shabad, arXiv:1312.0447.

[4] R. P. Negreiros, F. Weber, M. Malheiro, and V. Usov, Phys. Rev. D **80**, 083006 (2009); R. P. Negreiros, I. N. Mishustin, S. Schramm, and F. Weber, Phys. Rev. D **82**, 103010 (2010).

[5] N. K. Glendenning, in *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity*, edited by J. R. Oppenheimer and G. Volkoff (Springer-Verlag, New York, 2000), 2nd ed.; A. Sedrakian, Prog. Part. Nucl. Phys. **58**, 168 (2007); B. Golf, J. Hellmers, and F. Weber, Phys. Rev. C **80**, 015804 (2009).

[6] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984); M. S. Berger and R. L. Jaffe, Phys. Rev. C **35**, 213 (1987); Y. Zhang and R.-K. Su, Phys. Rev. C **67**, 015202 (2003); M. G. Alford and D. A. Eby, Phys. Rev. C **78**, 045802 (2008).

[7] S. Villalba-Chávez and A. E. Shabad, Phys. Rev. D **86**, 105040 (2012).

[8] T. C. Adorno, D. M. Gitman, and A. E. Shabad, arXiv:1311.4081.

[9] A. E. Shabad and V. V. Usov, Phys. Rev. D **83**, 105006 (2011).

[10] R. Turolla, S. Zane, J. A. Pons, P. Esposito, and N. Rea, Astrophys. J. **740**, 105 (2011); C. Kouveliotou *et al.*, Nature (London) **393**, 235 (1998); S. Mereghetti, Astron. Astrophys. Rev. **15**, 225 (2008); V. V. Usov, Nature (London) **357**, 472 (1992).