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Identification of Materials in X-Ray Inspections of Objects by the Dual-Energy Method

S. P. Osipov[a], *, V. A. Udod[a, b], **, and Y. Wang[a], ***

[a] *Tomsk Polytechnic University, Tomsk, 634050 Russia*

[b] *Tomsk State University, Tomsk, 634050 Russia*

**e-mail: osip1809@rambler.ru*

***e-mail: pr.udod@mail.ru*

****e-mail: wyzs007@163.com*

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Abstract—The state-of-the-art, application experience, and trends in further perfection of the dual-energy method as used in digital-radiography and X-ray computed-tomography systems intended for nondestructive testing and screening of objects are reviewed.

Keywords: digital radiography, computed tomography, dual-energy method, material discrimination, effective atomic number

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INTRODUCTION

Modern systems and methods of X-ray testing are widely used in industrial defectoscopy and technical [1–4] and medical [5, 6] diagnostics as well as when inspecting check-in and carry-on luggage, containers, and so on with the aim to ensure transportation safety and prevent illegal trafficking of prohibited items [7–9].

In order to improve the efficiency of testing, many systems (in particular, those for screening) take advantage of the dual-energy method (DEM), in which the test object is examined twice, with two different voltages (that correspond to two effective X-ray energies) across the X-ray tube, while the recorded results are processed by a certain algorithm to estimate the atomic number Z (or the effective atomic number Z_{eff}) of the test-object material [4, 8–10]. There exist different modifications of how this method can be implemented. For example, it can be one-time X-raying of a test object with the radiation being recorded by two detectors placed one after the other in the X-ray propagation direction [11], with X-ray tubes replaced by high-energy sources of ionizing radiation [12, 13]. In practical terms, the application of the DEM makes it possible to recognize (discriminate) the materials of the structural elements of test objects by juxtaposing them (using the calculated estimates of atomic numbers) with one of several classes (groups) of materials, e.g., metal–nonmetal (or organic–inorganic) [14, 15].

Many different DEM modifications that passed practical testing have been developed up till now. This variety gives rise to a natural inclination to describe systematically the above modifications, including their possibilities and limitations as well as trends of their further improvement.

For the sake of convenience of presentation, we will start with setting forth the theoretical foundations of the DEM.

THEORETICAL FOUNDATIONS OF THE DUAL-ENERGY METHOD

The DEM was originally developed by Alvarez and Macovski [16] and (a bit later) Brooks [17]. As developed, it was intended for reducing the effect of X-rays being not energy homogeneous on the quality of tomographic images. Later, the DEM started to be used as an independent technique for discriminating unknown materials (by means of estimating their atomic numbers), in particular, in medical digital-radiography systems [18]. However, as of today, the DEM has become, in fact, the main method for identifying materials in X-ray inspection of a wide nomenclature of objects.

Let us provide, based on [16, 18], basic analytical relationships that express the essence of the DEM when a source of radiation with a continuous energy spectrum is used. For convenience, we will make the following several simplifying assumptions (mainly, for mathematical rigor of design relationships while retaining the physical sense):

- A test object is of uniform and constant thickness;
- A radiation source is not energy-homogeneous and stationary (for example, a continuous X-ray unit);
- Radiation is recorded with a radiometric detector;
- The radiation detector is centered at the radiation-beam axis;
- The radiation source and detector are collimated, thus making it possible to disregard the effect of radiation scattered in the test object;
- The angular and energy distributions of the source are independent of each other;
- The detector aperture (the surface of radiation reception) is much smaller that the source–detector distance;
- The detector aperture is oriented towards the source;
- An analogue (average-current) mode of radiation recording is employed;
- An ideal integrate-and-dump circuit is used as a time filter (integrator) for the output process in the radiation detector.

Taking the above assumptions into account, the total charge $Q(H)$ [to be precise, the mathematical expectation (average value) of the total charge, C] that is registered by the detector in the presence of a test object is described by a relationship of the form

$$Q(H) = \gamma \frac{\psi(\bar{k})}{F^2} ST \int_0^{E_{\max}} g(E, E_{\max}) e^{-\mu(E)H} \varepsilon(E) \overline{E_a(E)} dE, \quad (1)$$

here γ is the coefficient of conversion of the energy of ionizing radiation into electric charge, C/MeV; $Q(H)/\gamma$ is the total amount of energy [to be precise, the mathematical expectation (average value) of the total amount of energy] that is absorbed by the detector in the presence of the test object, MeV; $\psi(\bar{\Omega})$ is a function that describes the distribution of the source over directions $\bar{\Omega}$ of exit of quanta (the angular distribution of the source) and is considered to be normed to 1 quantum into a solid angle $4\pi cp$, that is,

$$\int_{(4\pi)} \psi(\bar{\Omega}) d\Omega = 1,$$

where the symbol (4π) in the left-hand side of the equation indicates that integration is performed over the surface of a unit sphere centered at the coordinate origin [for an isotropic source we have $\psi(\bar{\Omega}) = 1/4\pi$; $\bar{k}$ is a unit vector along the Oz axis (the axis of a radiation beam generated by the source); F is the focal (source–detector) distance; S is the area of the detector aperture; $\frac{\psi(\bar{k})}{F^2} S$ is the likelihood for a quantum to be emitted by the source towards the detector; T is the time of radiation recording (the time constant of the integrate-and-dump circuit); $E_{\max}$ is the maximum energy in the spectrum of X-rays generated by the source; $g(E, E_{\max}) = dN/dE$ is the energy spectrum of X-rays emitted by the source in terms of the number of quanta (not normed), 1/(MeV s); E is the energy of a quantum, MeV; $N_a = \int_0^{E_{\max}} g(E, E_{\max}) dE$ is the total number of quanta emitted by the source per unit time into the entire space, 1/s; $N_F(E) = \frac{\psi(\bar{k})}{F^2} g(E, E_{\max})$ is the energy-differential density of the flux of radiation quanta near the detector surface [located at distance F from the source along the Oz axis (in the direction of the vector $\bar{k}$)], 1/(cm^2 s MeV); $N_0 = \int_0^{E_{\max}} N_F(E) dE = \frac{\psi(\bar{k})}{F^2} \int_0^{E_{\max}} g(E, E_{\max}) dE$ is the density of the flux of radiation quanta near the detector surface [located at distance F from the source along the Oz axis (in the direction of the vector $\bar{k}$)], 1/(cm^2 s); $N_d = N_0 S = \frac{\psi(\bar{k})}{F^2} S \int_0^{E_{\max}} g(E, E_{\max}) dE$ is the flux of radiation quanta incident upon the detector [located at distance F from the source along the Oz axis (in the direction of the vector $\bar{k}$)], 1/s; $\mu(E)$ is

the linear attenuation coefficient (LAC) of radiation for the test-object material, 1/ (cm^{-1}); H is the thickness of the test object, cm; $e^{-\mu(E)H}$ is the transmittance of the test object for radiation quanta with energy E; $\varepsilon(E)$ is the counting efficiency for the registration of radiation in the detector; and $\overline{E_a(E)}$ is the average value (mathematical expectation) of absorbed energy per one quantum with energy E that has experienced interaction with the detector, MeV. Finally, it should be added that the quantity $k_a = \dfrac{\overline{E_a(E)}}{E}$ can be interpreted as the coefficient of absorption of the radiant energy by the detector (the energy efficiency of detection of radiation).

It should be noted that when using the counting mode for recording radiation, the total number of quanta N_{tot} that are registered by the detector within time period T is described by the expression

$$N_{tot} = \frac{\psi(\bar{k})}{F^2} ST \int_0^{E_{max}} g(E, E_{max}) e^{-\mu(E)H} \varepsilon(E) dE.$$

Let us denote

$$C_{det} = \gamma \frac{\psi(\bar{k})}{F^2} ST.$$

Taking this notation into account, relationship (1) can be written in the form

$$Q(H) = C_{det} \int_0^{E_{max}} g(E, E_{max}) e^{-\mu(E)H} \varepsilon(E) \overline{E_a(E)} dE. \quad (2)$$

Using Eq. (2), we, accordingly, derive the following:

$$Q_1(H) = C_{det} \int_0^{E_1} g(E, E_1) e^{-\mu(E)H} \varepsilon(E) \overline{E_a(E)} dE \quad (3)$$

is the total charge registered by the detector in the presence of the test object for $E_{max} = E_1$;

$$Q_1(0) = C_{det} \int_0^{E_1} g(E, E_1) \varepsilon(E) \overline{E_a(E)} dE \quad (4)$$

is the total charge registered by the detector without the test object for $E_{max} = E_1$;

$$Q_2(H) = C_{det} \int_0^{E_2} g(E, E_2) e^{-\mu(E)H} \varepsilon(E) \overline{E_a(E)} dE \quad (5)$$

is the total charge registered by the detector in the presence of the test object for $E_{max} = E_2$;

$$Q_2(0) = C_{det} \int_0^{E_2} g(E, E_2) \varepsilon(E) \overline{E_a(E)} dE \quad (6)$$

is the total charge registered by the detector without the test object for $E_{max} = E_2$.

Switching to the normed values based on Eqs. (3)–(6), we have

$$d_1 = \frac{Q_1(H)}{Q_1(0)} = \frac{\int_0^{E_1} g(E, E_1) e^{-\mu(E)H} \varepsilon(E) \overline{E_a(E)} dE}{\int_0^{E_1} g(E, E_1) \varepsilon(E) \overline{E_a(E)} dE}; \quad (7)$$

$$d_2 = \frac{Q_2(H)}{Q_2(0)} = \frac{\int_0^{E_2} g(E, E_2) e^{-\mu(E)H} \varepsilon(E) \overline{E_a(E)} dE}{\int_0^{E_2} g(E, E_2) \varepsilon(E) \overline{E_a(E)} dE}. \quad (8)$$

Substantively, the parameters d_1 and d_2 are the "effective coefficients of radiation transmittance" of the test object, i.e., such transmittances $d_1 = e^{-\mu(E_{1\text{eff}})H}$ and $d_2 = e^{-\mu(E_{2\text{eff}})H}$ correspond to the effective energies $E_{1\text{eff}}$ and $E_{2\text{eff}}$. Based on the above, these coefficients can be interpreted as the "radiant transparencies" of the test object for radiation with $E_{\max} = E_1$ and $E_{\max} = E_2$, respectively. It should also be noted that the parameters d_1 and d_2 can be determined experimentally, too.

Relationships (7), (8) comprise the source mathematical model for estimating the atomic number of the test-object material from the results of radiation measurements, i.e., from the quantities $Q_1(H)$, $Q_1(0)$, $Q_2(H)$, and $Q_2(0)$.

The DEM is based on the following assumptions [16, 18]:

1. In the domain of energies that are used in medical diagnostics, the main reasons for X-ray attenuation are photoelectric absorption and Compton scattering, that is,

$$\mu \approx \mu_{\text{ph}} + \mu_{\text{is}}, \quad (9)$$

where μ_{ph} is the LAC component due to photoeffect; μ_{is} is the LAC component due to incoherent scattering (Compton effect);

2. The components μ_{ph} and μ_{is} are approximated analytically by expressions of the form

$$\mu_{\text{ph}} = \rho_e C_p Z^{3.8} f_{\text{ph}}(E); \quad (10)$$

$$\mu_{\text{is}} = \rho_e C_0 f_{\text{KN}}(E). \quad (11)$$

Here,

$$\rho_e = \rho \frac{Z}{A} N_A \quad (12)$$

is the electron density of the test-object material, $1/\text{cm}^3$; ρ, Z, and A are the density, atomic number, and atomic mass of the test-object material, respectively;

$$N_A = 6.022 \times 10^{23}, \text{ Mmol}^{-1} \quad (13)$$

is the Avogadro number;

$$C_p = 9.8 \times 10^{-24}, \text{ cm}^2; \quad (14)$$

$$f_{\text{ph}}(E) = \frac{1}{E^{3.2}}; \quad (15)$$

is a function that describes the energy dependence of the cross section of photoeffect on an atom;

$$C_0 = 2\pi r_0^2 \approx 0.499 \times 10^{-24}, \text{ cm}^2; \quad (16)$$

$$r_0 = 2.818 \times 10^{-13}, \text{ cm} \quad (17)$$

is the classical electron radius;

$$f_{\text{KN}}(E) = \frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha}\right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \quad (18)$$

is a function that describes the energy dependence of the cross section of incoherent scattering (Compton effect) from an atom; $C_0 f_{\text{KN}}(E)$ is the integral Klein–Nishina–Tamm cross section;

$$\alpha = \frac{E}{511 \text{ keV}}, \quad (19)$$

where the energy E is expressed in keV.

With allowance for Eqs. (10) and (11), expression (9) for the LAC μ takes on the form

$$\mu \approx \rho_e C_p Z^{3.8} f_{ph}(E) + \rho_e C_0 f_{KN}(E). \quad (20)$$

The formula in Eq. (20) is a "theoretical foundation" of the DEM. In practice, it is used in the energy range from 20 keV to 1.022 MeV [10].

According to this formula, the LAC μ, being a function of three variables ($\mu = \mu(\rho_e, Z, E)$), can be decomposed into the sum of products of several one-dimensional functions; this simplifies the analysis and processing of measurement results based on this formula.

Substituting Eq. (20) into Eqs. (7), (8) yields the following system of integral parametric equations for estimating the atomic number of the test-object material:

$$d_1 \approx \frac{\int_0^{E_1} g(E, E_1) e^{-Bf_{ph}(E) - Df_{KN}(E)} \varepsilon(E) \overline{E_a(E)} dE}{\int_0^{E_1} g(E, E_1) \varepsilon(E) \overline{E_a(E)} dE}; \quad (21)$$

$$d_2 \approx \frac{\int_0^{E_2} g(E, E_2) e^{-Bf_{ph}(E) - Df_{KN}(E)} \varepsilon(E) \overline{E_a(E)} dE}{\int_0^{E_2} g(E, E_2) \varepsilon(E) \overline{E_a(E)} dE}. \quad (22)$$

Here, d_1 and d_2 are empirical (experimentally determined) "radiant transparencies" of the test object for radiation with $E_{max} = E_1$ and $E_{max} = E_2$, respectively; B and D are the sought-for intermediate parameters

$$B = \rho_e C_p Z^{3.8} H; \quad (23)$$

$$D = \rho_e C_0 H. \quad (24)$$

The right-hand sides in Eqs. (21), (22) can be interpreted as theoretical "radiant transparencies" of the test object".

The system of Eqs. (21), (22) can be used to determine (by one of numerical methods) the parameters B and D. hence, taking Eqs. (23), (24) into account, we ultimately arrive at an estimate $\hat{Z}$ for the atomic number Z of the test-object material, namely,

$$\hat{Z} = \sqrt[3.8]{\frac{C_0 B}{C_p D}} \quad (25)$$

or, with allowance for Eqs. (14) and (16),

$$\hat{Z} = \sqrt[3.8]{0.051 \frac{B}{D}} = 0.457 \sqrt[3.8]{\frac{B}{D}}. \quad (26)$$

Relationships (20)–(26) [combined with Eqs. (9) –(19)] are basic analytical relationships that express the substance of the DEM when sources with continuous energy spectra are used.

To conclude, let us mention that the accuracy of the estimate calculated by the formula in Eq. (26) can be somewhat improved if the totality of objects (the range of the densities, atomic numbers, and thicknesses of objects) to be discriminated is a priori known. In this case, it is expedient to replace the constants 0.457 and 3.8 in formula (26) with parameters A_1 and A_2, respectively, that can be chosen in an optimum manner from a series of preliminary test experiments with some elements of this totality of objects.

THE CASE OF TWO MONOENERGETIC SOURCES

Applying two monoenergetic radiation sources for X-raying a test object is of especial interest as in this case the efficiency of the DEM is manifested the most. To explain, let us provide the relevant analytical expressions

$$g_1(E) = N_1\delta(E - E_{1m}); \tag{27}$$

$$g_2(E) = N_2\delta(E - E_{2m}); \tag{28}$$

i.e., the energy spectra of radiation expressed in terms of the number of quanta (not normed) from the first and second monoenergetic sources, respectively, 1/(MeV s); where N_1 and N_2 are the total numbers of quanta emitted per unit time into the entire space by the first and second sources, respectively, 1/s; E_{1m} and E_{2m} are the energies of quanta emitted by the first and second sources, respectively, MeV; and $\delta(E)$ is the Dirac delta function.

Replacing the energy spectra $g(E, E_1)$ and $g(E, E_2)$ in Eqs. (7) and (8) with the spectra $g_1(E)$ and $g_2(E)$ described by Eqs. (27) and (28), respectively, and assuming, for mathematical rigor (but without losing physical sense), that $0 < E_{1m} < E_1$ and $0 < E_{2m} < E_2$, we have

$$d_1 = e^{-\mu(E_{1m})H}; \tag{29}$$

$$d_2 = e^{-\mu(E_{2m})H}. \tag{30}$$

Finding the logarithms of the equalities in Eqs. (29), (30) and multiplying them by -1 yields

$$-\ln d_1 = \mu(E_{1m})H; \tag{31}$$

$$-\ln d_2 = \mu(E_{2m})H \tag{32}$$

quantities that are "radiant thicknesses" (dimensionless quantities) of the test object for the energies E_{1m} and E_{2m}, respectively.

Now, let us take advantage of the following representation for the LAC μ [19]:

$$\mu(E, \rho, Z) = \rho\mu_m(E, Z), \tag{33}$$

where $\mu_m(E, Z)$ is the mass radiation attenuation coefficient, cm^2/g.

It follows from Eqs. (31), (32) with allowance for Eq. (33) that

$$G(E_{1m}, E_{2m}, z) = \frac{\ln d_2}{\ln d_1} = \frac{\mu_m(E_{2m}, Z)}{\mu_m(E_{1m}, Z)}. \tag{34}$$

The quantity G with unknown energies E_{1m}, E_{2m} is unambiguously related to the atomic number Z.

Thus, using two monoenergetic radiation sources makes it possible, based on the relationship in Eq. (34) and detailed tables for the mass attenuation coefficient (see, for example, [20]), to estimate the unknown atomic number of the test-object material in a relatively simple and reliable manner.

However, monoenergetic sources such as certain radionuclides are hardly used at all in inspections. This is mainly explained by considerable problems in ensuring the radiation safety when operating with radionuclides as well as their relatively high costs and scarcity (and limited availability).

FACTORS DETERMINING THE ACCURACY AND SPEED OF THE DUAL-ENERGY METHOD

It immediately follows from Eqs. (20)–(22) that the accuracy of the DEM is mainly determined by the following factors:

- The accuracy of the approximation formula in Eq. (20) for the LAC μ, in particular, the accuracy of the concrete analytical dependence of the microscopic cross section of interaction for the photoeffect on the atomic number Z of the test-object material and the energy E of radiation quanta;
- The accuracy of measuring the empirical "radiant transparencies" d_1 and d_2 of the test object for different maximum energies in the radiation spectrum; this accuracy is determined, in particular, by the precision (digit capacity) of analog-to-digital transformation of the output processes in the radiation detector and statistical fluctuations in the results of radiation detection due to the quantum nature of radiation;
- The accuracy of estimating (setting) the maximum energy E_{max} in the source radiation spectrum;
- The accuracy of describing the energy spectrum $g(E, E_{max})$ of radiation generated by the source;

- The accuracy of assessing the average value $\overline{E_a(E)}$ of the energy absorbed by the detector;
- The accuracy of assessing (calculating) the efficiency $\varepsilon(E)$ of registration of radiation by the detector;
- The accuracy of solving the system of integral parametric equations (21), (22), a process that is usually done in two stages, viz., numerical integration followed by solution of a system of two nonlinear equations with two unknown (intermediate) parameters B and D.

The efforts of numerous researchers in the field of radiation methods of testing are aimed, in particular, at the search for a more accurate analytical dependence of the photoeffect cross section on energy. For example, Alvarez and Macovski [21] proposed a dependence of the form $f_{ph}(E) = \frac{1}{E^{2.8}}$ instead of Eq. (15) while Ying et al. [22] used $f_{ph}(E) = \frac{1}{E^3}$.

Preference to choosing one or another analytical dependence is usually given empirically for a particular group of materials to be discriminated. It needs to be added that Ying et al. [22] also set forth an approach to solving the system of integral parametric equations of the form in Eqs. (21), (22) based on the least-squares method. According to this approach, a solution to system (21), (22) is such a pair of the parameters B and D that minimizes the special auxiliary function (the sum of squared deviations)

$$\Phi(B, D) = (d_{E_1}(B, D) - d_1)^2 + (d_{E_2}(B, D) - d_2)^2, \quad (35)$$

provided the restrictions $B, D \geq 0$ are observed. Here, $d_{E_1}(B, D)$ and $d_{E_2}(B, D)$ are the right-hand sides of Eqs. (21) and (22), respectively, which can be interpreted as "theoretical" radiant transparencies of the test object for the lower and upper boundary energies, respectively. In this case, the quantities d_1 and d_2 are accordingly treated as "experimental" radiant transparencies of the test object for the lower and upper boundary energies. Taking the above interpretations into account, the function in Eq. (35) will be the sum of squared discrepancies between "theoretical" and "experimental" radiant transparencies of the test object for the lower and upper boundary energies in the radiation spectrum.

It is quite clear that the accuracy of the DEM can be somewhat improved if the formula in Eq. (20) is supplemented with allowance for coherent scattering. For example, based on the data presented in [23], the energy dependence of coherent scattering is satisfactorily analytically approximated by a function of the form $f_{coh}(E) = \frac{1}{E^2}$.

As for the speed of the DEM, it is determined, primarily, by the following stages of the algorithm of calculating the intermediate parameters B and D. At the first stage, functions that describe the energy dependences of cross sections of photoeffect and incoherent scattering are calculated, while at the second stage, the values of a function that approximates the actual energy spectrum of the radiation source are determined. Then, the values of a function that describes the efficiency of registration of radiation by the detector are computed, and the values of a function that describes the mean value of the energy absorbed by the detector are estimated. Further, the integrals in the system of integral parametric equations (21), (22) are assessed, and, ultimately, the parameters B and D are determined. Each of these procedures is characterized by its own number of arithmetical operations; this determines the actual speed of the algorithm of data processing based on the DEM. It should be noted that the overall number of arithmetical operations substantially depends on the selected numerical-integration method and the rate of its convergence.

At the current stage of development of digital-radiography systems that implement the DEM, images containing from 256 × 256 to 1024 × 1024 and more elements are analyzed, i.e., 65536 to 1048576 and more systems of integral parametric equations of the form in Eqs. (21), (22) need to be solved at the stage of preliminary data processing. The total number of computational operations that is required for processing the entire data array (of dual digital radiation images) can be very large. Therefore, minimizing the number of arithmetical operations at each stage of the general algorithm is a very important problem.

The DEM is most effective in the domain of low and medium energies (less than 200 keV), a fact that is accounted for by a strong dependence of the photoeffect cross section on the atomic number of substance [24]. There exists an energy range in which the DEM is physically unrealizable. In this energy range, incoherent scattering (Compton effect), for which dependence on the atomic number is insignificant, prevails [24].

Solving the system of integral parametric equations (21), (22) presents one of the difficulties in the DEM. At the same time, the very system of these equations can be significantly simplified if a full absorption detector is used, as in this case

$$\varepsilon(E) = 1; \quad \overline{E_a(E)} = E. \tag{36}$$

Substituting Eq. (36) into (21), (22) yields

$$d_1 \approx \frac{\int_0^{E_1} \varphi(E, E_1) e^{-Bf_{ph}(E) - Df_{KN}(E)} dE}{\int_0^{E_1} \varphi(E, E_1) dE}; \tag{37}$$

$$d_2 \approx \frac{\int_0^{E_2} \varphi(E, E_2) e^{-Bf_{ph}(E) - Df_{KN}(E)} dE}{\int_0^{E_2} \varphi(E, E_2) dE}. \tag{38}$$

Here,

$$\varphi(E, E_{max}) = Eg(E, E_{max}) = E\frac{dN}{dE} = \frac{dI}{dE} \tag{39}$$

is the energy spectrum of the intensity I of radiation from the source at the maximum energy in the spectrum E_{max}, 1/s.

To calculate the integrals in Eqs. (37), (38), the energy spectrum (39) of the intensity of radiation from the source needs to be set.

According to [25], the energy spectrum of the intensity of bremsstrahlung X-rays is satisfactorily described in many practical applications by Kramers' formula

$$\varphi(E, E_{max}) = \frac{dI}{dE} = \frac{CiZ_a}{h^2}(E_{max} - E), \tag{40}$$

where C is a constant coefficient; i is the anode current; Z_a is the atomic number of the anode material; h is the Planck constant; $E_{max} = eU$ the maximum energy in the spectrum; e is the electron charge; and U is the voltage across the tube.

It follows from Eqs. (37), (38) that, without loss of generality, the energy spectrum $\varphi(E, E_{max})$ can be set except for an arbitrary constant multiplier. Based on this, the following representation for Kramers' spectrum can be used in practical research instead of Eq. (40):

$$\varphi(E, E_{max}) = \frac{dI}{dE} = E_{max} - E. \tag{41}$$

Let us note that the representation in Eq. (41) is rather convenient for mathematical modeling.

It is absolutely clear that the DEM accuracy can be improved if more precise formulae are used instead of Kramers' spectrum. A fairly extensive list of such formulae is available in [26].

MODIFICATIONS OF THE DUAL-ENERGY METHOD AND THEIR APPLICATION EXPERIENCE

Gavrish et al. [27] describe an approach to estimating the atomic number of the test-object material that is based on preliminary compilation of lookup tables for the test-object parameters H_m and Z, where $H_m = \rho H$ is the mass thickness of the test object (g/cm^2), and "radioscopic transparencies" of the test object [in our terms, it is "radiant transparencies" of the test object (the parameters d_1 and d_2)], obtained for different maximum energies in the radiation spectrum. The research was conducted for bremsstrahlung generated by an electron accelerator with maximum energies $E_1 = 4.5$ MeV and $E_2 = 9$ MeV. Identification of the atomic number Z of the test-object material was performed using two mutually comple-

mentary criteria. According to the first criterion, an estimate $\hat{Z}$ of the atomic number Z was taken to be the point of global minimum of the function

$$F(Z) = \frac{\left|\hat{H}_{m2}(Z) - \hat{H}_{m1}(Z)\right|}{\hat{H}_{m2}(Z)}, \tag{42}$$

where $\hat{H}_{m1}(Z)$, $\hat{H}_{m2}(Z)$ are test-object mass thickness estimates that were found for a fixed Z from the lookup tables for the measured values of transparencies for the maximum energies E_1 and E_2, respectively. The second criterion was based on using the ratio of the algorithms of transparencies for the high and low energies

$$R(E_1, E_2, H_m, Z) = \frac{\ln d_2}{\ln d_1}, \tag{43}$$

a value that is the ratio of spectrum-average effective mass radiation-attenuation coefficients for the high and low energies, that is,

$$\frac{\ln d_2}{\ln d_1} = \frac{\mu_m(E_{2eff}(H_m, E_2, Z), Z)}{\mu_m(E_{leff}(H_m, E_1, Z), Z)}. \tag{44}$$

The quantity R for given energies E_1 and E_2 and for a prescribed mass thickness H_m is unambiguously related to the atomic number.

It should be noted that the described approach to estimating the atomic number of the test-object material, apparently, remains valid if mass attenuation coefficients and mass thicknesses in the formulae in Eqs. (42)–(44) are replaced with LAC and test-object thickness, respectively.

This work also mentions that for objects with heterogeneous structure, a situation that is typical of screening, one needs to discriminate the following four groups of elements based on the effective atomic number Z_{eff}:

- light materials ($Z_{eff} = 5$);
- materials with medium atomic number ($Z_{eff} = 13$);
- "inorganic" materials ($Z_{eff} = 26$);
- heavy elements ($Z_{eff} = 82$).

The same authors claimed that combined usage of both criteria [based on the functions $F(Z)$ and $R(E_1, E_2, H_m, Z)$, respectively] made it possible to unambiguously estimate and discriminate the group of elements by the effective atomic number. To enhance the discrimination effect (identification accuracy), the authors proposed to damp the low-energy part of bremsstrahlung spectrum with a preliminary radiation filter.

Gavrish et al. [27] also developed a software facility for processing digital radiation images using the DEM and recommended it to be used when developing inspection systems for large-size objects.

In our opinion, the approach [27] to discriminating materials can be naturally classified as a "reference' technique. This is explained by the fact that it is based on lookup tables between test-object parameters (mass thickness and atomic number) and the output signals of the control system ("radioscopic transparencies" of the test object). Therefore, for this approach to be implemented, a representative (basis) totality of test, i.e., "reference" objects needs to be in place. The "reference" method is superior to others because of its relatively high accuracy. Its drawbacks include the necessity for manufacturing a wide nomenclature of test ("reference") objects and carrying out a large volume of experimental research. In the case where lookup tables are compiled purely theoretically (by means of mathematical modeling with virtual "reference" objects), it is necessary to make sure from the very beginning that the accuracy of the employed mathematical model is acceptable and the model adequately describes the functioning of the actual inspection system, a task that proves fairly hard in many cases.

Results of the research in [27] were used to create a screening radiometric facility for inspecting large-dimensioned vehicles and cargoes for the presence of substances and articles prohibited for transportation [28]. This facility has such specific features as

- two sources of high-energy bremsstrahlung (a linear electron accelerator with an energy of 6 MeV and a betatron with energies of 4.5 and 9 MeV);
- three sets of linear detector arrays [modular linear arrays of detector with high definition (two sets) and high sensitivity (one set)] for registration of bremsstrahlung that passed through the test object.

Lazurik et al. [29] carried out numerical studies of electron converters in the flux of bremsstrahlung and filters for optimum selection of parameters in the systems for shaping low- and high-energy brems-

strahlung beams. The filters were assumed to be made of 1-cm-thick lead plates. Based on the analysis of the results of modeling, it was shown that inspection of large-size objects by the dual-energy method can be carried out based on a single electron accelerator with two spaced-apart converters. In this case, a bremsstrahlung flux with the energy of 8 (or 10) MeV is formed at the exit of the first converter and that at the exit of the second converter has the energy of 3 (or 5) MeV.

According to Spirin et al. [30], electron accelerators with energies of up to 10 MeV are usually used to inspect large-size objects, with the lower bound of the energy range being limited by the penetrating power of bremsstrahlung to no less than 4 MeV for the selected type of objects. The same authors note that the continuous nature of bremsstrahlung spectrum and the inhomogeneity of the inspected object are essential limitations of the accuracy in discriminating materials with this technique. In order to enhance the discrimination effect (improve the accuracy of identifying materials), the authors recommend to perform preliminary filtering of bremsstrahlung with the aim to damp the soft part of the spectrum. They propose to technically realize the damping by equipping the accelerator collimator with a filter in the form of a plate with the large value of the atomic number. In this case, selection of the filter mass thickness is reduced to seeking a balance between the ambiguity in determining the effective atomic number and the filter-induced reduction in the signal-to-noise ratio. It is suggested to diminish the ambiguity in determining the atomic number by an appropriate choice of the lower boundary energy. The authors claim that by varying these two parameters, one can determine their optimum values for which the discrimination effect will be the best for all groups of materials and in the entire range of mass thicknesses. The calculation results have shown that the optimum mass thickness of the lead filter should not be less than 4 g/cm^2, while the dual energy should stay within the limits of 4.5–5 MeV.

Similar to [27], the authors of [31] point out that for the purposes of customs inspection with the use of electron accelerators with energies of up to 10 MeV, it is expedient to conditionally separate discriminated materials into the following four groups: (1) light materials ($Z_{\text{eff}} = 5$), (2) materials with medium atomic numbers ($Z_{\text{eff}} = 13$), (3) inorganic materials ($Z_{\text{eff}} = 26$), and (4) heavy elements ($Z_{\text{eff}} = 82$). In the same work, the authors single out one of the main problems with discriminating materials into groups, viz., only slight distinctions in the degree of absorption of high- and low-energy bremsstrahlung. The authors of [31] conducted research into Z-identification of several test elements such as carbon, aluminum, iron, and lead. The object's mass thickness for different atomic numbers Z of the elements varied from nearly zero to 200 g/cm^2. The research was carried out for the boundary energies of 4.5 and 9.0 MeV. The elements were discriminated by the value of Z in the same way as in [27], based on the measured logarithms of transparencies for both boundary energies, i.e., based on the function $R(E_1, E_2, H_m, Z)$ defined by the relationship in Eq. (43). The research resulted in the conclusion that there is no unambiguous dependence, that is, several elements with different mass thickness may correspond to one and the same value of R. On this basis, it was suggested to minimize this discrimination ambiguity by introducing preliminary filtering of bremsstrahlung spectrum by installing a plate with a large atomic number (e.g., a lead plate) on the accelerator's collimator. Computer simulation with the GEANT4 software confirmed the validity of the statement (which was also made in [30]) on the existence of optimum values of the two parameters (the filter thickness and lower boundary energy) with which the discrimination effect is the best for all groups of materials and in the entire range of mass thicknesses. In this case, a probability measure for the efficiency of discrimination of materials based on the Bayesian approach was used to allow for experimental errors.

The DEM is based on using two bremsstrahlung spectra for identification of chemical elements in inspected objects, and due to the continuity of these spectra, the method has low resolving ability when detecting certain materials [32]. As a result, Afanas'ev et al. [32] proposed a version of the DEM aimed at improving the reliability of discriminating materials with large atomic numbers (Pb, U) against masking backgrounds. The method essentially consists in special formatting of bremsstrahlung spectra by setting an upper bound for the low-energy beam and a lower bound for the high-energy one, with filters being used for setting the lower energy bound. The notion of a "discrimination effect" that is defined (in terms of our symbols) as a relationship of the form

$$\text{DE} = \frac{d_2 - d_1}{d_1}.$$

It is noted that changes in DE give a qualitative idea about the presence of enclosures with heavy elements. To estimate the efficiency of the proposed DEM version, numerical experiments were carried out in the same work using the PENELOPE software package. When performing calculations for the low-energy beam, it was assumed that the initial electron energy is 3.5 MeV; the converter is made of 0.3-mm-thick tungsten; and the filter is 1-cm-thick lead. For the high-energy beam, it was assumed that the initial electron energy is 10 MeV; the converter is made of 0.5-mm-thick tungsten; and the filter consists of

10-mm-thick carbon (graphite) and 2-mm-thick lead. Test-object mockups were used in the form of various combinations of several elements such as aluminum, carbon, vinyl plastic, cellulose, water, nylon, and polyethylene. Iron and lead were used as enclosures with large atomic numbers into the test objects. The results proved the possibility for revealing enclosures of heavy metals with a mass thickness from 10 g/cm^2 against masking backgrounds.

Svistunov et al. [33] presented a review of research conducted at D.V. Efremov NIIEFA-Energo on the development of linac-based inspection facilities using both X-ray and nuclear-physics methods. In the former case, we are speaking of discriminating materials with the DEM, while in the latter case it is elemental analysis of the contents of containers for the presence of explosive and fissile substances. Explosives and fissiles are identified in a container by the presence of nitrogen, oxygen, and carbon nuclei in a certain proportion. The very problem of identification of materials by their atomic number in large-dimensioned cargo containers using the dual-energy method started to be studied at D.V. Efremov NIIEFA-Energo around the year 2000. One of the essential hindrances to its solution is said to be substantial quantum noise in the images. At present, D.V. Efremov NIIEFA-Energo is the manufacturer of X-ray facilities intended for high-performance nonintrusive inspection of vehicles and large-dimensioned containers. These facilities allow inspectors to compare the actual content of the test object to that declared in freight documents and reveal illegal enclosures and hiding places with weapons, ammunition, drugs, jewelry, alcohol, etc. Three facility types were developed: EFASKAN facility for customs inspection of large-size cargoes at customs checkpoints at airports, seaports, etc.; EFASKAN-2 facility that is intended for inspection of vehicles on highways (the accelerator, detector array, and system for shaping the bremsstrahlung field are mounted in this facility on a transportation device and move with respect to the vehicle); and EFASKAN-3 facility that is intended for nonintrusive inspection of freight trains consisting of freight cars, cisterns, containers (as X-rayed, the freight train moves at a speed of 18 km/h under a bremsstrahlung beam). Record-breaking parameters have been obtained in these systems with respect to the spatial (1 mm) and density (1%) resolutions. Monoenergies ranging from 6 to 15 MeV obtained at the exit of linear HF electron accelerators were used for X-raying test objects. In particular, further research is indicated to be toward developing an accelerator that shapes "duplex" pulse pairs—alternating high-energy [9(6) MeV] and low-energy [5(3) MeV] current pulses. In this case, the current in the low-energy pulse should exceed that in the high-energy one by several times. At the same time, one more accelerator (with two targets and with scanning of the electron beam on each target by a control magnetic field) is planned for development.

Ogorodnikov [34] notes that at the moment, there are two main types of X-ray systems that use the DEM to discriminate materials by atomic number. In the systems of the first type, energy modulation is achieved by spectral filtering of X-ray radiation. The filters cut out the low-energy part of the radiation spectrum as compared with the unfiltered beam. Transmitted radiation is registered by two linear detector arrays placed one after the other and separated by an absorber; this ensures formation of two images with different spectral compositions. A drawback of these systems is the weak modulation of the median part of the spectrum and, hence, a low signal-to-noise ratio. The systems of the second type use two different working voltages across the X-ray tube. In this case, the energy modulation depth can be made arbitrarily high. In commercial installations, the tube voltage is approximately 150 kV at the nominal level and 75 kV for the dual mode. These systems usually use two spaced-apart beams and two linear detector arrays (to register radiation), respectively. The two gathered images are combined and computer processed in the online mode. The resultant image is displayed on a monitor at the inspector's workstation in a special color palette in which different color shades are ascribed to different materials. The facilities of both types commonly use X-rays with a boundary energy of up to 200 keV. However, the penetrating power of low-energy X-rays is limited to several centimeters of steel equivalent; this rules out the possibility for using them to inspect large-size containers and vehicles. It is mentioned that high-quality shadow radioscopic images of these objects can only be obtained by using high-energy bremsstrahlung with a boundary energy of up to 10 MeV produced by electron accelerators. Ogorodnikov [34] carried out numerous theoretical and experimental studies on material identification with the dual-energy method based on a linear electron accelerator with the nominal energy of 8 MeV and the dual energy of 4 MeV. The implementation of the DEM was based on the notion of "radioscopic transparency" of a barrier defined, similar to [27], as the ratio of intensities before and after the barrier, that is,

$$T(E_0, H_\mathrm{m}, Z) = \frac{\int\limits_0^{E_0} \frac{dP}{dE}(E, E_0) e^{-\mu_\mathrm{m}(E,Z)H_\mathrm{m}} dE}{\int\limits_0^{E_0} \frac{dP}{dE}(E, E_0) dE}, \quad (45)$$

where E_0 is the boundary (maximum) energy of bremsstrahlung quanta (the energy of accelerated electrons); $H_m = \rho H$ is the mass thickness of the test object in the beam propagation direction, g/(cm^2);

$$\frac{dP}{dE}(E, E_0) = \frac{dI}{dE}(E, E_0)\varepsilon(E)\frac{\mu_{det}^{en}(E)}{\mu_{det}(E)}; \quad (46)$$

$dI/dE(E, E_0)$is the spectral distribution of bremsstrahlung intensity described by the Schiff formula; $\varepsilon(E)\frac{\mu_{det}^{en}(E)}{\mu_{det}(E)}$ is detector's response function; $\mu_{det}(E)$ is the mass coefficient of damping of radiation quanta with energy E in the material of the detector crystal, (cm^2)/g; $\mu_{det}^{en}(E)$ is the mass coefficient of absorption of radiation quanta with energy E in material of the detector crystal, (cm^2)/g; and $\mu_m(E)$ is the mass coefficient of damping of radiation quanta with energy E in the test-object material. It was suggested to seek two unknown quantities (the atomic number of the test-object material and its mass thickness) based on solution of the system of two integral equations [similar to Eqs. (7), (8)] that relate the "theoretical" $T(E_1, H_m, Z)$, $T(E_2, H_m, Z))$ [defined as per Eqs. (45), (46)] and "experimental" T_{1exp}, T_{2exp} "radioscopic transparencies" of the test object for two boundary energies E_1 and E_2, namely,

$$T_{1exp} = T(E_1, H_m, Z) = \frac{\int_0^{E_1} \frac{dP}{dE}(E, E_1)e^{-\mu_m(E,Z)H_m}dE}{\int_0^{E_1} \frac{dP}{dE}(E, E_1)dE}; \quad (47)$$

$$T_{2exp} = T(E_2, H_m, Z) = \frac{\int_0^{E_2} \frac{dP}{dE}(E, E_2)e^{-\mu_m(E,Z)H_m}dE}{\int_0^{E_2} \frac{dP}{dE}(E, E_2)dE}. \quad (48)$$

Minimization of a function defined by the formula

$$L(H_m, Z) = \sqrt{(T(E_1, H_m, Z) - T_{1exp})^2 + T(E_2, H_m, Z) - T_{2exp})^2}. \quad (49)$$

was used as a method for solving the system of Eqs. (47), (48).

The results of the research with different materials of different mass thicknesses enabled the conclusion on the possibility of compiling, based on the system of Eqs. (47), (48), lookup tables for direct calculation of the atomic number of the test-object material from experimentally measured transparencies to high- and low-energy boundary energies in the bremsstrahlung spectrum. However, it is pointed out that practical implementation of this approach requires that the transparencies be measured with an accuracy of, at least, three significant figures. For customs inspection purposes, it is desirable to be able to identify four main types of materials: light or "organic" materials with low atomic numbers ($1 < Z < 10$); materials with medium atomic numbers ($10 < Z < 20$); "inorganic" materials ($20 < Z < 50$); and heavy metals with high atomic numbers ($Z > 50$). To assess quantitatively the discrimination effect in a pair of materials with atomic numbers Z_1 and Z_2 in per cent, the following quantity was introduced:

$$D(Z_1, Z_2, H_m) = \left(1 - \frac{R(H_m, Z_2)}{R(H_m, Z_1)}\right) \times 100\%, \quad (50)$$

where

$$R(E_1, E_2, H_m, Z) = \frac{\ln T(E_1, H_m, Z)}{\ln T(E_2, H_m, Z)} \quad (51)$$

is the ratio of logarithmic transparencies for the nominal E_1 and dual E_2 boundary bremsstrahlung energies. To improve the discrimination effect [assessed according to Eq. (50)], it was proposed to pre-filter the bremsstrahlung spectrum in order to suppress the contribution of its soft part. Lead (which has a high photoabsorption cross section) was recommended as a material for such a filter.

Table 1. Interpolation nodes for material–color transformation interpretation [35]

Substance	Reference materials	Z	Shade	Color
Organics	Polyethylene (CH_2)	5.3	0	Red
Organics–inorganics	Aluminum (Al)	13	$2\pi/3$	Green
Inorganics	Iron (Fe)	26	$4\pi/3$	Blue
Heavy metals	Lead (Pb)	82	$5\pi/3$	Lilac

Proper selection of the filter thickness consists in searching for a balance between the degree of ambiguity in determining Z and the extent of degradation of the signal-to-noise ratio. The latter is accounted for by a decrease in the bremsstrahlung dose rate. Some researchers suggest to decompose the spectrum of bremsstrahlung that passed through the test object into components that are responsible for pair formation and Compton effect.

Petrunin [35] comments that due to health and safety standards, the energy of the accelerator used in a customs-inspection system must not exceed 10 MeV, while its base configuration usually incorporates a unit for spectral filtering of bremsstrahlung field based on a lead filter with a thickness of 3–5–10 mm at the accelerator exit. The design of the control system in [35] provides for the use of 24-digit analog-to-digital converters. The construction and circuit-design principles made it possible to achieve a dynamic range of 300000 in this system. Radiation is detected in this system by a linear array of detectors of the scintillator–photodiode type, with cadmium tungstate being used as the material for the scintillators. The system was optimized by the Monte Carlo method. Two main types of noises in the digital radiation images were considered, viz., noise from electronic devices and quantum noise. The efficiency of the DEM was investigated in a range of 4–10 MeV. Similar to [34], the notion of barrier transparency [the expression in Eq. (45)] and the ratio of logarithmic transparencies for nominal and dual boundary bremsstrahlung energies [the expression in Eq. (51)] were used. It was proposed to use an approach to material discrimination that is based on applying the effective mass radiation-attenuation coefficient

$$\mu_{\text{meff}} = -\frac{\ln T}{H_{\text{m}}}, \tag{52}$$

where T is the transparency of the test object, while H_{m} is its mass thickness, g/(cm^2). The substance of this approach is to construct, using Eq. (52), the dependence of the effective mass coefficient μ_{dual} of radiation damping for the dual energy on the effective mass coefficient μ_{nom} of radiation damping for the nominal energy. However, the author of this approach himself points out that it is only operable in the ideal case, which, in particular, assumes an unbounded accuracy of measuring the test-object transparency. The knowledge of the effective atomic number and density of the contents of a container makes it possible to discriminate effectively drugs, explosives, and other contraband substances. A customs-inspection system is described that makes use of an idea about estimating the effective atomic number by using a dual-energy bremsstrahlung beam with the concurrent measurement of the cargo density by scanning the cargo in two directions. The same author carried out experiments related to the imaging of radiation patterns of discriminated materials based on the information provided in Table 1.

Image segmenting was employed to enhance the synthesized images (suppress noises) and reduce the number of statistically independent image elements.

Ishkhanov et al. [36] studied the system of two integral equations [similar to that of Eqs. (47), (48)] that relate the "theoretical" and "experimental" transparencies of the test object that correspond to test-object X-raying for the two boundary energies of 5 and 9 MeV. It was shown that this system is uniquely solvable with respect to Z in the Z range of 1 to 44, while in the range of 44 to 97, two essentially different solutions will correspond to a pair of "theoretical" and "experimental" transparencies. In order to overcome this ambiguity, it was proposed to X-ray the test object for several (three and more) boundary energies. Numerical experiments confirmed the efficiency of this approach. In particular, it made it possible to determine unequivocally that the test object with dimensions of $2.5 \times 2.5 \times 2.5$ cm^3 was made of uranium ($Z = 92$).

Gorshkov [37] used the DEM to develop an upgraded technique for estimating the mass absorption coefficient and effective atomic number of a multicomponent object in the case of a continuous radiation spectrum.

The same author proposed [38] a dual-energy technique for estimating the average density of multicomponent objects that uses a continuous spectrum and allows one to reduce considerably the measurement error.

Linacs and betatrons with a maximum energy of 1–10 MeV are used as X-ray sources in the high-energy DEM [39]. Chakhlov and Osipov [39] presented an algorithm for discriminating the test-object material by the high-energy DEM that is based on decomposing the LAC into Compton-effect and pair-formation components with subsequent solution of the relevant system of integral parametric equations [similar to the system of Eqs. (21), (22)]. They analyzed factors that determine the accuracy of the algorithm and aspects of its practical usage. It was noted that in a number of screening tasks, there is no need to estimate the exact value of the effective atomic number. The actual requirement is to ascribe the test-object material to one of several rather wide classes of substances such as organic substances; metals with low atomic numbers (aluminum, magnesium); metals with medium atomic numbers (iron, copper); and metals with large atomic numbers (lead, silver). In other words, it is necessary to juxtapose the test-object substance with one of the four classes (substances that have effective atomic numbers close to 6, 13, 26, and 40).

Klimenov et al. [10] estimated contributions from various components to the total error of DEM measurements of the effective atomic number of the substance of a homogeneous test object in the energy range of up to 200 keV. They showed that the greatest contribution to the error in measuring the effective atomic number is rendered by deviations of the maximum energies in X-ray spectra.

In [13], a series of calculations was performed for the commercially produced small-size MIB 4.5/7.5 betatron in order to estimate the effect that various physical factors have on the quality of identification of the test-object material by the high-energy DEM. The following factors were considered: deviations of the maximum energies of high-energy X-rays from their nominal values; radiation scattered in the test object; and radiation scattered in the body of the inspection facility. The research proves that these factors need to be taken into account when designing inspection facilities that allow discrimination of the test-object material by the high-energy DEM.

In [40], the authors proposed a method for solving the system of integral parametric equations that relate the parameters (atomic number and mass thickness) of the test object to the measured data, that is, a system that is similar to that of Eqs. (21) and (22), based on using the notion of the level lines of a two-dimensional function. Carbon $Z = 6$ (organics) and iron $Z = 26$ (inorganics) are used [41] as the basis reference materials in technical applications when discriminating materials with the dual-energy radiography method. If a more accurate three-energy radiography is used, the following three ranges are singled out: substances with low ($1 < Z < 10$), medium ($10 < Z < 18$), and high ($18 < Z < 40$) atomic numbers.

The major part of hazardous and prohibited luggage enclosures (drugs, explosives, medication, etc.) belong to "light" substances ($Z_{\text{eff}} < 10$) and any improvement in the accuracy of determining Z_{eff} gives better chances for their detection [42]. On this basis, Ryzhikov et al. [42] made an attempt to improve the level of material discrimination by using a three-energy emission–detection tract in the inspection system. The posed problem was solved by using a collimator that formed three fan-shaped beams. One could obtain shadow X-ray images of test objects in different spectral ranges for each of the beams by adjusting the anode voltage of the X-ray source and selecting filters made of different materials (copper, aluminum, etc.) Three linear detector arrays were used accordingly to register radiation. The resultant three-energy test-object image was processed by a special algorithm that made it possible to separate, relatively reliably, imitators of explosives; this confirms the prospects of the approach proposed in [42]. However, the following relationship that relates the effective and maximum radiation energies was used in this work as one of the fundamental assumptions for developing the recognition algorithm:

$$E_{\text{eff}} \approx \frac{2}{3} E_{\max}. \tag{53}$$

In our opinion, the validity of the approximate equality in Eq. (53) raises some doubts in the general case, as the effective energy depends not only on the maximum radiation energy but also on the material and thickness of the test object, which are a priori unknown during inspections.

As a rule, medical dual-energy systems and X-ray scanners for luggage screening operate in an energy range of 80–160 keV [43]. At the same time, inspection of airfreighted cargo containers requires the use of X-ray sources that possess high penetrating power. These requirements are met, for example, by a betatron with boundary energies of 3 and 7.5 MeV. This source generates radiation with a penetrating power of up to 350 mm of steel equivalent or 1.5 m of dissimilar concrete structures. In addition, due to a small size of the betatron focal spot (1.5×3 mm^2), the quality of synthesized images can be substantially improved. In the research with the dual and nominal energies of 3 and 7.5 MeV, a pre-filter of radiation was used in the form of a copper plate with a thickness of 4 and 10 mm, respectively. The research was based on analyzing the ratio

of effective radiation attenuation coefficients that correspond to the nominal E_1 and dual E_2 boundary energies. That is to say, a relationship of the form in Eq. (51) was analyzed that allows unambiguous betatron discrimination of materials in the Z-range of 10–46, while for lower ($Z < 10$) and higher ($Z > 46$) values of Z the discrimination remains equivocal. The physical experiments conducted by the same authors with a number of phantom samples confirmed the results of theoretical calculations.

Firsching et al. [44] describe the usage of the DEM for detection of natural diamonds enclosed in pieces of kimberlite (magmatic rock). An X-ray source with the boundary energies of 80 and 120 keV was used in the experiments. The results make this method a promising technology in diamond-mining industry for screening crushed rocks moving on a conveyor belt.

Mazoochi et al. [45] theoretically studied a DEM-assisted approach to material recognition based on using the ratio R of the logarithmic test-object transparencies that correspond to two different boundary energies in the X-ray spectrum as an identification (recognition) parameter. This parameter is significantly affected by the test-object thickness (which is, as a rule, unknown); this deteriorates considerably the accuracy of recognition. Aiming at reducing this negative effect, the authors proposed a novel algorithm for processing detector signals that correspond to a double X-raying of the test object with two different maximum energies in the X-ray spectrum. The algorithm is based on Simpson's rule for numerical integration and a set of iterative relationships for selecting intermediate parameters that relate the test-object parameters and the characteristics of the radiation energy spectrum and detector sensitivity. At the formal level, this algorithm makes it possible to reduce the problem of material recognition when using two beams of radiation with wide energy spectra to the problem of recognition with two quasi-monochromatic radiation beams. Numerical experiments performed by the same authors using the Monte Carlo method and the MATLAB software package illustrated the effectiveness of this algorithm when applied to the example of discriminating two materials (aluminum and plastic).

In our view, the shortcomings of the above-described algorithm include a rather cumbersome system of iterative relationships for selecting intermediate parameters, the assumption of ideal separability of the low- and high-energy parts of bremsstrahlung X-ray spectrum as well as lack of physical experiments that would have confirmed practically the algorithm efficiency.

Rebuffel and Dinten [46] study the DEM based on an assumption of the form in Eq. (16, that is, based on decomposition of the X-ray LAC into two effects, viz., photoelectric absorption and Compton (incoherent) scattering. The authors describe different approaches to the technical implementation of the DEM in inspection systems. The first approach assumes the same detectors (detector) being exposed twice with different voltages across the X-ray tube. The second approach uses one exposure of two-layer ("sandwich"-type) detectors that are placed one after the other in the ray propagation direction and are separated by an intermediate filter (a copper plate), with the detectors that appear first along the X-ray propagation direction registering predominantly the low-energy quanta and those that appear second detecting the high-energy ones. However, the poorly expressed separation of the low- and high-energy parts of the X-ray spectrum is pointed out to be a drawback of these approaches, deteriorating material recognition. At the same time, these approaches are quoted as promising in the systems of spectrometric detectors. They ensure a narrow (several keV) energy resolution. In addition to that, the authors describe a method for solving the system of integral parametric equations [similar to that of Eqs. (21), (22)] that consists in modeling an inverse relationship (inverse functions) using second-order polynomial functions (second-degree polynomials of two variables), that is,

$$\begin{cases} B = \alpha_0 + \alpha_1 x + \alpha_2 y + \alpha_3 xy + \alpha_4 x^2 + \alpha_5 y^2; \\ D = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 xy + \beta_4 x^2 + \beta_5 y^2, \end{cases} \tag{54}$$

here α_i, β_i ($i = 0, 1, \ldots, 5$) are undefined coefficients that are determined experimentally using reference test objects;

$$x = -\ln d_1; \quad y = -\ln d_2. \tag{55}$$

The same authors comment on a considerable effect of noise (photons and detection-electronics induced noise) on the accuracy of solving a system of the form in Eqs. (21), (22) and, hence, on the accuracy of material discrimination by the DEM.

In our opinion, the efficiency of modeling inverse functions based on relationships (54), (55) should be assessed separately in each particular case and as compared to other techniques for estimating the atomic number of the test-object material.

Chang et al. [47] presented several versions of DEM implementation that differ in the form of decomposition (expansion) of the LAC $\mu(E)$, in particular,

$$\mu(E) = a_1 f_{PE}(E) + a_2 f_{KN}(E), \quad (56)$$

where a_1 and a_2 are energy-independent expansion coefficients; $f_{PE}(E)$ and $f_{KN}(E)$ are functions that describe the energy dependence of the photoeffect and incoherent-scattering (Compton effect) cross sections, respectively;

$$\mu(E) = b_1\mu_1(E) + b_2\mu_2(E), \quad (57)$$

where b_1 and b_2 are energy-independent expansion coefficients; and $\mu_1(E)$, $\mu_2(E)$ are the mass radiation-attenuation coefficients for two selected (basis) materials. The authors also performed theoretical numerical research into the efficiency of DEM application in the systems of computed X-ray tomography. The research was based on approximating a polychromatic (not energy-homogeneous) X-ray radiation with a monochromatic radiation (a monoline). The monoline was chosen to be the average energy in the X-ray spectrum or an equivalent energy (as determined from the condition of the equality of the half-value layer of a radiation with this equivalent energy and the half-value layer for the given polychromatic radiation). The effective atomic number and electron density of the test-object material served as informative parameters "extracted" from the reconstructed tomographic images of model test objects. Comparative analysis proved the advantage of the average energy over the effective energy being used as a monoline.

The approach [47] to approximating a polychromatic X-ray radiation with a monoline produces an explicit positive effect. However, such approximation is only possible in practical terms when the test-object material and thickness are known, with this kind of information being, as a rule, unavailable during single-scan X-ray inspections. Therefore, additional investigation is needed on adequate approximation of X-ray radiation with a monoline in X-ray screening systems with single scanning of test objects.

Ryzhikov et al. [48] studied experimentally the possibilities offered by digital roentgenography systems based on "scintillator–photodiode" linear detector arrays. One of the research areas was to discriminate materials according to their atomic numbers using the DEM. Experiments were conducted using an 11-mm-thick graphite ($Z = 6$) plate and five aluminum ($Z = 13$) plates, each 1.5 mm thick. The combined samples (the graphite plate combined with different sets of the aluminum plates) were X-rayed for two different anode voltages (70 and 140 kV) with additional radiation filtering (a 0.75-mm-thick copper filter for the anode voltage of 140 kV). The source data were LED and HED detector signals when registering radiation with the minimum (70 keV) and maximum (140 keV) boundary energies, respectively. LED + HED (it characterizes the overall level of X-ray attenuation by the test-object material) and HED/LED (it characterizes the effective atomic number of the test-object material) pairs were used instead of the pair of LED and HED for material discrimination. Processing and subsequent analysis of the data were performed using a dedicated piece of software. The experiments demonstrated the possibility for discrimination of substances by the effective atomic number for "light" elements (with atomic numbers ranging from 6 to 13). In particular, the authors managed to reliable distinguish between water ($Z_{eff} \approx 7.43$) and glycerine ($Z_{eff} \approx 6.87$). A higher accuracy in discrimination of substances by their atomic number can be achieved by using quasi-monochromatic X-ray radiation or a spectrometric radiation detection mode.

Unfortunately, the method [48] for determining the effective atomic number of the test-object material with the DEM has not been set forth in enough detail. For example, the algorithm for determining Z_{eff} given the pair of values LED + HED and HED/ LED is missing. In addition, the very quantities LED and HED have not been analytically expressed in any shape or form (no formulae are provided whatsoever).

Alvarez [49] presented an extensive cycle of research on the effect of polychromaticity (non-energy-homogeneity) of X-rays on the quality of reconstructed images in computed tomographs that are mainly intended for medical purposes. Various versions of decomposition (expansion) of the LAC μ are considered for different problems. In particular, when decomposing into two effects of interaction between X-rays and substance [photoelectric absorption (photoeffect) and incoherent scattering], it is suggested to use the energy dependence of the photoeffect cross section in the form $f_{ph}(E) = \frac{1}{E^3}$. Comparative analysis of different modes of radiation detection—counting, analog, and spectrometric—was carried out, with the signal-to-noise ratio used as a quantitative efficiency indicator for the above modes. Given the zero dead time, the spectrometric mode was ranked the highest, followed by counting and, finally, analog radiation detection modes. A lot of research was performed using the Monte Carlo method and the MATLAB software package.

Iovea et al. [50] described an X-ray tomograph intended to be used on board of an oceanographic research vessel. The tomograph is equipped with a single 160-kV X-ray tube and two linear detector arrays, 240 detectors each, placed one after the other in the X-ray propagation direction and separated by copper

filters ("sandwich"-detectors). The spatial resolution of reconstructed images of the linear attenuation coefficient was 0.5 mm in the tomographic and roentgenographic modes, while in the dual-energy mode—where reconstructed images reflect the distribution of densities and effective atomic numbers and additional filtering is required—the resolution was approximately 1 mm. With this tomograph, the accuracy of calculating the density and effective atomic number of test samples was better than 8%.

Du Plessis et al. [51] report an X-ray computed tomograph that is used for controlling the density of homogeneous polymeric materials. The control is effected with the DEM, while the range of densities of studied materials is 0.9 to 2.2 g/cm^3.

With two or more materials present along the X-ray propagation path, the efficiency of their discrimination by the DEM considerably reduces. To overcome this problem, Liang et al. [52] developed a novel dynamic material-discrimination algorithm that includes the following three stages: preliminary classification of materials based on recorded data; decomposition of dual X-ray patterns of overlapping materials into "basis" materials; and final recognition of the materials. The algorithm is based on creating a database of dual X-ray patterns of "pure basis" materials and their pairwise combinations.

Alves et al. [53] describe the use of the DEM in a computed microtomograph intended for estimating the density and effective atomic number of heterogeneous geological samples.

Rudychev et al. [54] studied the efficiency of DEM application in a digital radiography system intended for revealing unauthorized enclosures containing heavy elements in cargo containers. They developed a technique for optimizing the characteristics of the bremsstrahlung beams—the low-energy one that contains the maximum number of photons in the energy range dominated by the Compton effect and the high-energy one with the maximum number of photons in the energy range where the effect of formation of electron–positron pairs prevails. The effect of the thickness and material of the converters on the spectral characteristics of photon beams was investigated; versions of efficient three-layer converters were proposed. The Monte Carlo method and a quasi-analytical technique were used to model a numerical experiment on the detection of enclosures by the digital radiography method. It was shown that using optimally shaped radiation beams makes it possible to reliably extract lead enclosures against the masking background of steel constructions of up to 25 cm.

CONCLUSIONS

1. The main DEM application areas are medical and technical diagnostics and, increasingly, inspection of check-in and carry-on luggage, etc. performed at airports, customs checkpoints, and so on with the aim of ensuring transportation safety and preventing illegal trafficking of prohibited items, with linacs and betatrons with the maximum energy of 10 MeV (defined by health and safety standards) being used as ionizing radiation sources when inspecting large-size objects (cargo containers, etc.)

2. Discrimination of the following four groups of materials is desirable for customs-inspection (screening) purposes: light or "organic" materials with low ($1 < Z < 10$) atomic numbers; materials with medium ($10 < Z < 20$) atomic numbers; "inorganic" materials ($20 < Z < 50$); and heavy metals with high ($Z > 50$) atomic numbers.

3. The following main methods of physico-technical implementation of the DEM are currently used in the cases where X-ray units are employed as ionizing-radiation sources: X-raying the test object twice with two different voltages across the X-ray tube and with the same detector array; single X-raying of the test object followed by radiation detection by an array of "sandwich"-detectors (two-layer detectors separated by an intermediate filter); and single X-raying of the test object with radiation detected by an array of spectrometric sensors. In terms of the achievable accuracy (the information value), the third scheme (with spectrometric sensors) is preferred the most. However, the question of the optimality of selection of one or another scheme should be answered in each particular case separately.

4. Mathematical (informational) implementation of the DEM is performed, mainly, by one of the following three methods:

- solving integral parametric equations of the form in Eqs. (21), (22);
- compiling lookup tables for the values of test-object parameters [the mass thickness and atomic number of reference (basis) test objects] and the measured values of the radiant transparencies (or logarithmic radiant transparencies) of the test objects;
- constructing second-order polynomial functions of the form in Eq. (54) to estimate the intermediate parameters B and D that are further used to determine ultimately the atomic number of the test-object material.

The first method can be essentially classified as analytical; the second method can be described as reference; and the third one can be treated as a certain intermediate one between the former two and interpreted as reference-analytical or design-experimental. Giving preference to one or another method of mathematical implementation of the DEM should be decided, similar to physical implementation, in each particular case separately.

5. The accuracy of estimating the (effective) atomic number of the test-object material using the DEM depends on many factors such as the accuracy of solving integral parametric equations that relate "theoretical" and "experimental" radiant transparencies of test objects; the accuracy of estimating (determining) the maximum radiation energies; quantum noise (statistical fluctuations of the results of radiation detection); and the accuracy of analytical description of the actual energy spectrum of radiation.

6. In order to enhance the discrimination effect (to improve the accuracy of material discrimination), it is expedient to perform pre-filtering of bremsstrahlung with the aim to suppress the soft part of its spectrum. This can technically be realized by means of installing a filter in the form of a plate with a large atomic number at the source exit. The mass thickness and material of the filter are selected based on a balance between the ambiguity in determining the effective atomic number of the test-object material and the filter-induced reduction in the signal-to-noise ratio.

7. The following can be named as trends in further perfection of the DEM: development of sources that form "duplex" pulse pairs, with alternating high and low-energy pulses; creating a single bremsstrahlung source with two targets and electron-beam scanning on each target by means of a control magnetic field; formation of quasi-monochromatic X-ray radiation; and creation of highly efficient and fast algorithms of automatic segmentation of dual digital radiation images of test objects.

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