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Application of Wavelet Analysis to Numerical Modeling of Deformations in Multilevel Hierarchical Structures

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Abstract. This paper proposes a wavelet based numerical method for the solution of elastoplastic problem. The method is based on the Lagrange variational equation of elastic static and the Haar wavelet transform of the components of deformation field. Some examples of analysis of multilevel deformation are shown for the demonstration of the method's capabilities. Some differences in the wavelet spectrums of multilevel inhomogeneous media are shown and discussed.

INTRODUCTION

A multi-layered approach considers the evolution of plastic deformation in the entire hierarchy of levels: micro-, meso and macro [1, 2]. Typical dimensions of objects can vary over several orders of magnitude. This can cause some difficulties in the direct numerical simulation of multilevel hierarchical structures. In recent years multi-scale analysis and wavelet analysis are used to study of multi-scale phenomena. Wavelet analysis allows us to obtain new information about signals of a different nature, for example, in spectral analysis, image analysis, etc. In this paper we propose a wavelet based computational approach, which allows us to take into account several structural levels (5 and more), the characteristic dimensions of which may differ by several orders of magnitude. In this paper we use a variational approach [3] with wavelet transform [4] to construct a multi-scale solution to an elastic problem.

WAVELET BASICS

The Haar transform and Haar series are based on the mother wavelet:

$$\psi(x) = \begin{cases} 1, x \in [0, 1/2), \\ -1, x \in [1/2, 1), \\ 0, x \notin [0, 1). \end{cases} \quad (1)$$

Scaling and shift operations make it possible to obtain a set of wavelet functions of the form:

$$\psi^{i,k} = 2^{i/2}\psi(2^i x - k). \quad (2)$$

The corresponding scaling-function is:

$$\varphi(x) = \begin{cases} 1, x \in [0, 1], \\ 0, x \notin [0, 1]. \end{cases} \quad (3)$$

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Likewise (2) the scaling functions with scaling and shift operations lead to a family of functions $\varphi^{i,k}(x) = 2^{i/2}\varphi(2^i x - k)$, where $i = 0, 1, 2, ...$ is a scale parameter and $k = 0, \pm 1, \pm 2, ...$ is a shift parameter, the comma is a separator between scale and shift indexes.

Approximation of function $f(x)$ with wavelet-functions $\varphi^{i,k}$ has a form:

$$P^i(f) = \sum_{k=-\infty}^{\infty} c^{i,k}\varphi^{i,k}(x), \tag{4}$$

where $c^{i,k}$ are approximation coefficients, and $\underset{i\to\infty}{P^i}(x) = f(x)$. Approximation (4) can be written as:

$$P^i(f) = \sum_{k=-\infty}^{\infty} c^{(i-1),k}\varphi^{(i-1),k}(x) + \sum_{k=-\infty}^{\infty} d^{(i-1),k}\psi^{(i-1),k}(x), \tag{5}$$

where $c^{(i-1),k}/2^i$ is a mean value of function on interval $[k/2^i, (k+1)/2^i]$ and $d^{(i-1),k}/2^i$ is a difference of mean values of function on subintervals $[k/2^i, (k+1/2)/2^i]$ and $[(k+1/2)/2^i, (k+1)/2^i]$.

Out of equality $\sum_{k=-\infty}^{\infty} c^{(i-1),k}\varphi^{(i-1),k}(x) = P^{i-1}(f)$ the approximation (5) can be rewritten in *d*-terms only:

$$P^i(f) = P^{i-1}(f) + \sum_{k=-\infty}^{\infty} d^{(i-1),k}\psi^{(i-1),k}(x) = P^0(f) + \sum_{j=0}^{i-1}\sum_{k=-\infty}^{\infty} d^{j,k}\psi^{j,k}(x). \tag{6}$$

Approximation in 2D has the same form with two shift factors: *k* (for *x*-axis) and *m* (for *y*-axis)

$$P^i(f) = P^0(f) + \sum_{j=0}^{i-1}\sum_{k,m=-\infty}^{\infty} d^{j,km}\psi^{j,k}(x)\psi^{j,m}(y). \tag{7}$$

Thus, the approximation of a function is represented as a mean value and a set of detailing coefficients $d^{j,k}$ of different scale factors. The important property of the approximation (7) is that coefficients $d^{j,k}$ decrease rapidly as the scale ratios decrease.

CALCULATION METHOD

The method uses a variational equation of the elastic-plastic theory, which has the form:

$$\iiint_V (\sigma_{ij}^E + \Delta^*\sigma_{ij})\delta(\Delta^* e_{ij})\mathrm{d}V^{(n)} - \iiint_V (\bar{P}_i + \Delta\bar{P}_i)\delta(\Delta u_i)\mathrm{d}V^{(n)} - \iint_{S_\sigma} (\bar{R}_i + \Delta\bar{R}_i)\delta(\Delta u_i)\mathrm{d}S^{(n)} = 0, \tag{8}$$

where $\bar{P}_i$ and $\Delta\bar{P}_i$ are surface forces and their change, $\bar{R}_i$, $\Delta\bar{R}_i$ are volume forces and their change, Δu_i, $\delta(\Delta u_i)$ are displacements and their variations, $\Delta\sigma_{ij}^E + \Delta^*\sigma_{ij}$ is the stress tensor and $\Delta^* e_{ij}$ is the deformation tensor.

The variational equation is supplemented by consideration of a deformation field as a multi-level hierarchical system of strains of various overlapping scales. We apply the Haar transform to components of a deformation tensor to obtain a multilevel representation of deformations and displacements of material points.

In the incremental theory of plasticity, the process of a quasi-static deformation of a solid is considered as a series of transitions between near equilibrium states [3] $\Theta^0 \to \Theta^1 \to ... \to \Theta^n \to \Theta^{n+1}$. The Lagrange variational equation for the state Θ^{n+1} has the form:

$$\delta E^{n+1} = \int_V (\sigma_{ij}^{(n)} + \Delta\sigma_{ij}^{(n+1)})\delta\varepsilon_{ij}\mathrm{d}V + \int_S P_{ij}^{(n+1)}\delta u_i\,\mathrm{d}S_j^{n+1} = 0, \tag{9}$$

where $\sigma_{ij}^{(n)}$, $\Delta\sigma_{ij}^{(n+1)}$ are the stress tensor and its change, $\varepsilon_{ij}^{(n)}$ and $\delta\varepsilon_{ij}$ are deformations and their variations, $P_{ij}^{(n+1)}$ is the surface force, δu_i are variations of components of displacement vector and $\mathrm{d}S_j^{n+1}$ are components of a surface vector for the part of surface with boundary condition in stresses (hereinafter the summation rule is applied on a recurring lower index).

After linearization of (9) via small deformations, one can obtain [3]:

$$\delta E^{n+1} = \int_V (\sigma_{ij}^{(n)} + C_{ijkl}\Delta\varepsilon_{kl}^{(n+1)})\delta\varepsilon_{ij}\mathrm{d}V + \int_S P_{ij}^{(n+1)}\delta R_i\,\mathrm{d}S_j^{n+1} = 0, \tag{10}$$

where C_{ijkl} —elastic constants tensor.

Integration of the mother wavelet (2) in a two-dimension case gives us a multi-scale shape function in form:

$$\Phi^{i,km}(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}\psi^{i,k}(x)\psi^{i,m}(y)\mathrm{d}x\mathrm{d}y=\sup[(1-|x-k|/2^{i})(1-|y-m|/2^{i}),0]. \qquad (11)$$

If the Haar transform is applied to components of the deformation tensor, it is found that the displacement vector can be defined via the same coefficients $d^{i,km}$, but with the use of a shape function $\Phi^{i,km}$ instead of the 2D Haar wavelet $\psi^{i,k}\psi^{i,m}$. Each scale i can be associated with rectangular mesh and each shape-function of the scale i can be associated with the corresponding node of this mesh.

The field of displacements of material points can be expressed as a sum of displacements of different scales, defined via scaled shape functions and detailing coefficients:

$$u_{\gamma}(\mathbf{r})=\sum_{i=0}^{N}\sum_{k,m}d_{\gamma}^{i,km}\Phi^{i,km}(\mathbf{r}), \qquad (12)$$

where i is the scale factor, k and m are shift factors in 2D, $\mathbf{r}=(x,y)$ is the position vector. The displacement field $u_{\gamma}(\mathbf{r})$ corresponds to the strain field of the form:

$$\varepsilon_{\gamma\beta}(\mathbf{r})=\sum_{i=0}^{N}\sum_{km}(d_{\gamma}^{i,km}\,\partial\Phi^{i,km}(\mathbf{r})/\partial r_{\beta}+d_{\beta}^{i,km}\,\partial\Phi^{i,km}(\mathbf{r})/\partial r_{\gamma})/2. \qquad (13)$$

Let

$$G_{s\gamma\beta}^{i,km}=\sum_{i=0}^{N}\sum_{km}(I_{s\gamma}\,\partial\Phi^{i,km}(\mathbf{r})/\partial r_{\beta}+I_{s\beta}\,\partial\Phi^{i,km}(\mathbf{r})/\partial r_{\gamma})/2, \qquad (14)$$

where I is a unity matrix. Substitution (13), (14) to (10) gives us:

$$\delta E=\sum\left\{\delta d_{s}^{I,KM}\left[\int_{V}\left(\sigma_{\alpha\beta}+C_{\alpha\beta pq}\sum_{ikm}d_{s}^{i,km}G_{spq}^{i,km}\right)G_{s\alpha\gamma}^{I,KM}\,\mathrm{d}V+\int_{S}P_{\alpha\beta}^{(n+1)}\frac{\partial R_{\alpha}}{\partial d_{s}^{I,KM}}\mathrm{d}S_{\beta}^{n+1}\right]\right\}=0. \qquad (15)$$

Variation of internal energy is defined by displacement variations, and the displacement field is fully defined by $d_{\gamma}^{i,km}$ coefficients. Because of the independence of variations $\delta d_{\gamma}^{i,km}$, equation (15) must be satisfied for any $\delta d_{\gamma}^{i,km}$ [3], leading to a linear equation:

$$\delta d_{s}^{I,KM}\left[\int_{V}\left(\sigma_{\alpha\beta}+C_{\alpha\beta pq}\sum_{ikm}d_{\gamma}^{i,km}G_{\gamma pq}^{i,km}\right)G_{s\alpha\gamma}^{I,KM}\,\mathrm{d}V+\int_{S}P_{\alpha\beta}^{(n+1)}\frac{\partial R_{\alpha}}{\partial d_{s}^{I,KM}}\mathrm{d}S_{\beta}^{n+1}\right]=0. \qquad (16)$$

Hence the system of equations for the unknown coefficients $d_{\gamma}^{i,km}$ takes the form:

$$\left[\int_{V}\left(\sigma_{\alpha\beta}+C_{\alpha\beta pq}\sum_{ikm}d_{\gamma}^{i,km}G_{\gamma pq}^{i,km}\right)G_{s\alpha\beta}^{I,KM}\,\mathrm{d}V+\int_{S}P_{\alpha\beta}^{(n+1)}\frac{\partial R_{\alpha}}{\partial d_{s}^{I,KM}}\mathrm{d}S_{\beta}^{n+1}\right]=0,\ \forall I,K,M. \qquad (17)$$

Integrals $\int_{V}\sigma_{\alpha\beta}G_{s\alpha\beta}^{I,KM}\,\mathrm{d}V=F_{s}^{I,KM}$ and $\int_{V}C_{\alpha\beta pq}\sum_{ikm}G_{\gamma pq}^{i,km}G_{s\alpha\beta}^{I,KM}\,\mathrm{d}V=\partial F_{S}^{I,KM}/\partial d_{\gamma}^{i,km}$ are the force for node (I, KM) and derivation of this force with respect to a displacement of the near node (i, km). Surface integrals in (17) correspond to the force that acts on the surface if boundary conditions in stresses are used.

The final equation system for coefficients of wavelet-transform of displacement field has the form:

$$F_{S}^{I,KM}+\sum_{i=0}^{N}\sum_{km}d_{\gamma}^{i,km}(\partial F_{S}^{I,KM}/\partial d_{\gamma}^{i,km})+P_{S}^{I,KM}=0,\ \forall I,K,M. \qquad (18)$$

This SLAE (18) can be efficiently solved using iterative methods. Fast convergence of a solution of the system (18) makes it possible to solve an elastic problem on detailed grids, taking into account several scales.

FIGURE 1. Test structures: St40-TiC (A, B, C), Aluminum foam (D), St40 with cracks (E)

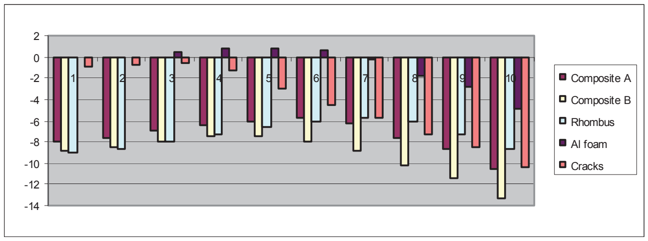

FIGURE 2. M^j for different structures: level *1* corresponds to cells of size 19.5 μm, level *10*—10 mm

Let ε^j be the strain of a large-scale level of the index *j*, then a measure of heterogeneity strain in a volume V^* can be defined as a natural logarithm of the ratio of deformations of smaller scale to the current:

$$M^j = \ln\left[\left(\sum_{i>j}\iiint_{V^*}\left|\varepsilon^i\right|\mathrm{d}V\right)\bigg/\iiint_{V^*}\left|\varepsilon^j\right|\mathrm{d}V\right]. \quad (19)$$

This allows us to evaluate how significant the contribution of deformations of fine levels in the resulting strain field is. Therefore, a set of M^j can be treated as heterogeneity parameters—small values corresponding to a relatively homogeneous stain field on the j-scale, and vice versa.

RESULTS

The main result of this work is the Eq. (18), which is well defined SLAE that can be effectively solved by iterative methods and provide a solution directly in terms of wavelet transform of a field of deformations. As an application of the method we modeled several different structures (Fig. 1). Each structure had a size of 20×20 mm. Structures *A*, *B*, *C* had a matrix of steel ST40 with inclusions consisting of titanium carbide, structure D is aluminum foam and structure E is steel with cavities. All calculations were performed on a 2D-grid of 1024×1024 cells (11 scale levels). The problem of uniform uniaxial loading with pressure of 20 MPa was solved (a linear problem was chosen in order to avoid the influence of plastic deformations and corresponding inhomogeneous effects).

The upper level of the scale corresponds to the entire sample, the finest level of the scale corresponds to a cell with a side 19.5 μm. The condition of convergence was the fall of the maximum residual 17×10^5 times.

As shown in Fig. 2, cracked material and aluminum foam have different distributions of deformations over scales in comparison with solid composites: they have an expressed influence of intermediate (meso) levels and the influence of fine levels (micro) is much greater then for St4-TiC composites. This result shows some differences in the meso-behavior of materials and allows one to connect micro-, meso- and macrolevels in a single picture. It is also interesting that large inclusions in the structure *C* have a greater impact at the meso and micro levels, rather than a large number of small inclusions in the structures *A* and *B*.

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