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# SURVEY ON MATHEMATICAL MODELS OF QUEUEING SYSTEMS IN RANDOM ENVIRONMENT

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One of the most important point of scientific literature are queueing systems subject with external stochastic influences. For example, Markov modulation and random environment are the part of considerate interest in scientific literature. System breakdowns or arrivals of priority customers (including batch arrivals) are often characterized by such influences. They are the force that provides the behavior of system at a different mode.

In this paper a brief review of papers that are devoted to mathematical models of queueing systems functioning in a random environment is presented.

The infinite-server queue is one of the radical models in queueing theory Its specific property is the existence of an infinite number of servers, so that requests are served right away and there are no waiting times. H.M. Jansen, M.R.H. Mandjes, K. De Turck, S. Wittevrongel in the article [1] studied an infinite-server queue in a random environment and a full large deviation principle (LDP) for the transient number of jobs in the system was proved by the authors. The proof of this LDP includes two substantial aspects, namely the result that the temporary number of jobs in the system has a Poisson distribution with a random parameter and the supposition that the random parameter satisfies an LDP.

In the paper [2] focus is on an extended version of the M/M/1 queue in a n-phase Markovian random environment with disasters that destroy all customers present in the system. The authors applied two methods of analysis, namely (i) probability generating functions and (ii) matrix geometric approach, whereas there was revelation of some relationships between them. Mean queue sizes, mean (conditional and unconditional) waiting times and fraction of customers lost were calculated by the authors, and they investigated two special cases further.

Recently a great deal of attention is paid to studying queueing systems with vacations; for example, the surveys (Doshi) and the monographs (Takagi and Tian and Zhang). Queueing systems with vacations can be characterized by the feature that each time a busy period ends and the systems become vacant, the server starts a vacation of random length of time.

In the paper [3] a discrete-time vacation queue operating in random environment where the time axis is derived into slots with equal length was considered by the J. Li, L. Liu. And the authors showed that it is possible to approximate M/G/1 queue with vacations in random environment by its discrete-time counterpart.

In the article [4] mathematical models of systems and queueing networks in a random environment were considered in queueing theory in connection with the plenty of various applications to transport models and systems with a hysteresis control strategy. However, issues of ergodicity in models of this type have usually been solved in the form of only sufficient, and not necessary and sufficient conditions. The methods of obtaining ergodicity criteria for queueing systems in a random environment proposed in the article can be extended from a single-channel system to other systems.

G.L. Choudhury, A. Mandelbaum, M.I. Reiman, W. Whitt A in their paper [5] noticed that significant feature of many complicated systems is the existence of different behavior on various time scales. In the context of stochastic models, various time scales can be represented by nearly completely-decomposable (NCD) Markov chains. Once by the authors was identified the connection between the fluid models and the associated queueing models, the authors decided that this fact may not actually be needful to apart consider the fluid models. They pointed out that any Markov-modulated fluid model is a limit of MMPP/G/1 queues, where G can be some distribution, including exponential (M). The MMPP/M/1 model is especially amenable because its queue-length process is a quasi-birth-death (QBD) process.

In the article [6] the research was directed on the  $M/G/\infty$  queue, operating in a Markovian random environment. In this way, the authors considered necessarily to analyze the 2-dimensional stochastic process, which is non-Markovian. The authors studied the case when the service-time distribution of customers which are straight away being served does not turn until the customer is totally serviced, even if the environment has jumped to another state. The authors as well deduced the expressions for its parameters — mean and variance.

In the paper [7] an  $M/GI/\infty$  queueing system operating in a semi-Markovian random environment was considered by the authors. Thereby, state transitions are changed by the arrival rate and service-time distribution according to the external semi-Markov process. Using the method of dynamic screening, the authors considered a non-stationary Markov point process  $n(t)$  vice of non-Markovian  $i(t)$  which is the number of customers in the system. In this way, using to the method of supplementary variable the authors defined the residual sojourn time  $z(t)$  in the present state of the environment process  $s(t)$  to be capable to analyze it with theory of Markov processes tools. Using derived system of differential equations in terms of vector characteristic functions of the number of customers in the system, the asymptotic analysis of the system in question was conducted by the authors.

Standard mathematical description of the class of models is time homogeneous Markovian vector processes, where the behaviour of one of the queues is represented by each coordinate. This is stated in the article of R. Krenzler, H. Daduna [8]. Product form networks can be characterized by the fact that in steady state (at any fixed time  $t$ ) the joint distribution of the multi-dimensional (over nodes) queueing process is the product of the stationary marginal distributions of the individual nodes' (non Markovian) queueing processes.

In the paper [9] factors that have influence on the traffic in roadways were considered. In recent years vehicles are the safest while having more power, and the roads have become wider. These and other factors affect on the road traffic, the congestion of vehicles on the road and their speed. Vehicles have higher speed, and this may cause a road section to contain more vehicles every second. Accordingly, a large number of vehicles can provoke incidents on the roadway: they have to minify their speed, or they may not have ability to pass the road section at all. In this research, two kinds of queueing models which describe the traffic flow on a roadway were proposed by the authors. Each model have queue phases, where the various phases represent different situations of the traffic condition and flow. The authors study cases where every phase is characterized either by an  $M/M/\infty$  queue or by an  $M/M/1$  queue, and investigate the corresponding solutions.

In [10] the author studied a special case where  $\frac{\lambda_i}{\mu_i} = c$  for every phase  $i$ , author identifies

a full reconcilability which appears between-phase  $M/M/\infty$  model and the regular  $M/M/\infty$  queue. In this work proposals were presented of application a general approach for cases with  $n \geq 2$  phases. Considering road traffic, the author precisely revealed the dependence which appears between the transition rates between phases and the number of vehicles in the system. Respectively, considering safety requirements of distance between vehicles on the road have dependence of their speed, one can find the optimum driving speed for a road section, considering incidents which might happen while the vehicles are driving along a given road section.

In the paper [11] written by O.J. Boxma studied an  $M/G/1$  queue in a multi-phase random environment with disasters. Using the supplementary variable technique, the author identified a Markov process and gained the probability generating function of the stationary queue size at an arbitrary epoch. Performance measures such as the mean system length, the mean length of a cycle and the probability that the system in phase also were obtained by the author. Moreover, the author derived the Laplace – Stieltjes transform (LST) of the stationary sojourn time distribution of an arbitrary customer and the server's working time in a cycle, relatively.

Lastly, some numerical examples whose purpose is to show the effect of parameters on the mean system length were given by the authors.

Subject of the article of S. Sophia, B. Praba [12] is an M/M/1 queue with the special feature that the speed of the server alternates between two constant values  $s_L$  and  $s_H$ . The high-speed periods have exponential distribution, and the low-speed periods have a regularly varying distribution. By the authors was obtained evident asymptotics for the tail of the workload distribution. The two cases in which the offered traffic load is smaller respectively and larger than the low service speed are shown to result in completely different asymptotics.

In this article [13], an M/M/1 queue with vacation in a multi-phase random environment was studied. For this model, by the author was obtained probability generation function (PGF) of the steady-state queue length at arbitrary epoch. Performance measures such as the average queue length, the average length of a cycle, and the probability that the system is in phase that was also presented by the author. Further, the author gained the LST of the steady-state sojourn time distribution of an arbitrary customer and the server's working time distribution in a cycle, respectively. The authors investigated two special cases of that model. Finally, they gained some numerical examples to demonstrate the impact of parameters on the average queue length and the average sojourn time, accordingly.

In the paper of B. D'Auria it's shown that using a matrix-geometric approach it is possible to resolve the problem of detection the factorial moments of the random number of customers in an M/M/ $\infty$  system when its parameters are modulated by a quasi-markovian random environment. By the authors was showed that this is possible by looking at this random variable as the random measure of a bidimensional random set by a modulated Poisson random measure. Like so, the case when the environment includes only 2 states is more deeply investigated and it is shown that explicit formulas are obtainable given that one state has exponential sojourn times. It is then colorable to believe that for this last case it would be possible to get an explicit expression for the complete characteristic function.

The authors in [14] considered an M/G/ $\infty$  queue where customers arrive according to a Poisson process with constant rate  $\lambda$  and require independent service,  $\sigma$ , that is distributed according to a general probability function  $G(d\sigma)$ . The queue is supposed to operate in a random environment  $\Gamma(t)$  that is a non-negative stochastic process independent of the arrival process. They were noticed the space of the sample paths of  $\Gamma(t)$  by  $G$ . The authors assumed that at time  $t$  all servers operate at the same speed,  $v(\Gamma(t))$ , that is a function of the random environment. In particular they supposed that  $v: [0, \infty] \rightarrow [0, 1]$ . Before proving the stochastic decomposition formula for the M/G/ $\infty$  queue, the authors first proved the stability of the queue in a more general setting. Also they supposed that the queue is of G/G/ $\infty$  kind, whose input is a stationary marked point process with intensity  $\lambda$ , and whose service speeds depend on the stationary and ergodic process  $\Gamma(t)$ .

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## **SIMPLE MODEL – A WAY TO INTEGRATE SOFTWARE DEVELOPMENT PROCESSES AND PROJECT MANAGEMENT SOFTWARE**

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### **Introduction**

In the present condition of the industrial software development project managers have a special role. The manager's role consists in managing a project, which implies monitoring its progress, risk analysis and identifying the key stages of development, building and managing a team, documentation and other activities.

To make the work of the manager easier there are many manuals, reference books, knowledge bases, that describe the project activities in quite different ways. The manager can approach studying those sources more competently to solve problems more efficiently. Many of the difficulties that project managers face are not exclusive to their subject area. For example, a skyscraper architect, a head of a department developing an operating system or even a student studying project management can face similar problems of motivating the team members [6]. But, naturally, there is a great influence of the subject area on project management. In software development this influence resulted in so-called software development process (SDP).

An SDP is effectively a set of instructions that should help organize teamwork in the most efficient way to reach the goals of a project. These instructions can range from a set of abstract principles describing the main work stages and correlation with the tasks to strict rules that specify the order of artefacts that should be added at each step of the working process of the project team. Practical application of the SDP has shown that there is no ideal and universal process. Every one of them has its advantages and disadvantages and is applicable for a certain group of projects. A project manager in software development uses either an existing SDP (it can be adapted for a certain project team) or tries to manage work relying on personal experience (but usually his experience is based on methods from different SDPs).

There are different project management software solutions (PM) created to aid managers. Such systems allow managers and their teams to formalize the work on the projects. At present there are many examples of software solutions the most popular of which are Microsoft Project [4], Team Foundation Server [5], Redmine [2] and Jira [3]. Some of the PM are targeted for specific development processes which means that their features focus on the specific aspects of a certain development process (for example, digital Kanban board for marking the progress of tasks). Nonetheless it is evident that regardless of the system (even if it is created for specific purposes) the observance of the development process in the software development project is the professional responsibility of the manager. That leads to the following problems: