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ABSTRACTS

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On the FS-property of homeomorphisms of continuous function spaces on countable ordinals

Mathematics Subject Classification (MSC): 54C35

Abstract. For given a Tykhonoff space X the space of functionals with a finite support was defined in [2] as follows. A subset $A \subset \mathbb{R}^X$ is called *zero-sufficient family* if A contains zero-function 0^X and for each point $x \in X$, each its neighborhood U, each real interval I, the set A contains a function φ with $\varphi(x) \in I$ and φ is equal to zero out of U. We say that a function $f : A \to \mathbb{R}$ with $f(0^X) = 0$ is a *functional with a finite support* (or *FS-functional*) if there exists a finite subset $K \subset X$, such that the following two conditions hold:

(FS 1) For any positive ε, any $\varphi \in A$, there exists a positive δ such that if $\psi \in A$ and $|\varphi(x) - \psi(x)| < \delta$ for all $x \in K$, then $|f(\varphi) - f(\psi)| < \varepsilon$;

(FS 2) There exists a positive ε, such that for each $x \in K$, each its neighborhood U one can find a function $\varphi \in A$ which coincides with 0^X out of U, but $|f(\varphi)| \geq \varepsilon$.

The space of all FS-functionals on A is dense in the subspace

$$\{f : A \to \mathbb{R}; f(0^X) = 0\} \subset \mathbb{R}^A.$$

In the case $A = C_p(X)$ the FS-functional may be considered as a natural wide generalization of the notion of continuous linear functional.

In [1] S. Gul'ko described a uniform homeomorphism $h : C_p[1, \alpha] \to C_p[1, \omega]$ for an arbitrary countable ordinal number α. Here we are investigating the images of $[1, \omega]$ and $[1, \alpha]$ under h's dual h^* and invers dual mapping $(h^*)^{-1}$ respectively. We establish, that such a homeomorphism h has the *FS-property*, namely, $h^*([1, \omega])$ and $(h^*)^{-1}([1, \alpha])$ consist of FS-functionals on $C_p[1, \alpha]$, $C_p[1, \omega]$ respectively. Since it is impossible to replace the Gul'ko's homeomorphism by any linear one provided by $\alpha \geq \omega^\omega$, then

we conclude that the class of homeomorphisms with the FS-property is irreducible to the class of linear homeomorphisms. This result is the positive answer on the question 5.3 (first part) from [2].

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Quotient images of linearly ordered topological spaces

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Abstract. We prove that every continuous image of a Hausdorff topological space X is a generalized ordered space iff X is homeomorphic to a countable successor ordinal (with the order topology).

Then we report on the recent new results towards the solution of the following problem: characterize Hausdorff spaces X such that every Hausdorff *quotient* image of X (in particular, X itself) is a linearly ordered topological space.