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HOMING SEQUENCES FOR NONDETERMINISTIC FINITE STATE MACHINES: ON THE EXPONENTIAL UPPER BOUND REACHABILITY

We discuss the problem of homing experiment derivation for nondeterministic finite state machine. In general case, the complexity of such experiment is exponential. In this paper, we show that this upper bound is reachable.

Ключевые слова: *nondeterministic finite state machine, homing sequence.*

Finite state machines are widely used in many application areas, such as analysis and synthesis of digital circuits, telecommunication protocols, software testing and verification etc. A Finite State Machine is a state transition model with finite non-empty sets of inputs, outputs, states, and transitions that moves from state to state producing an output when an input is applied. When testing a system without reset there is a problem of detecting the initial state of a system under test (SUT) and usually this is done using so-called homing experiments [1]. During the experiment a sequence of inputs (input sequence) is applied to a SUT, output responses are observed and the conclusion is drawn what is the final state of the SUT. An input sequence that allows such an experiment is a homing sequence. For a deterministic reduced FSM a homing sequence always exists and a shortest homing sequence has length $n(n-1)/2$ that is polynomial w.r.t. the number of states of the FSM [2]. However, nowadays the behavior of many systems is described by a nondeterministic FSM. Nondeterminism occurs due to various reasons such as performance, flexibility, limited controllability, and abstraction etc, see, for example [3].

In [4], an algorithm for homing sequence derivation for nondeterministic FSM has been proposed and length of a shortest homing sequence for NFSM with n states has been shown to be at most of the order 2^{n^2}. Moreover, it has been also shown that exponential upper bound is reachable for nondeterministic FSM when the number of inputs is exponential w.r.t. to the number of states. In this paper, we show that exponential bound of a shortest homing sequence is still reachable for an FSM of polynomial size. Namely, we show that for every $n > 1$ there exists a nondeterministic FSM with n states, $(n-1)$ inputs and $n(n-1)/2$ outputs that has a shortest homing sequence of length $2^{n-1} - 1$.

A *finite state machine* (*FSM*), or simply a *machine* throughout this paper, is a 5-tuple $\mathrm{S} = \langle S, I, O, h_S, S' \rangle$, where S is a finite nonempty set of states with a non-empty subset S' of initial states; I and O are finite input and output alphabets; and $h_S \subseteq S \times I \times O \times S$ is a *behavior relation*. A machine is *deterministic* if for each pair $(s, i) \in S \times I$ there exists at most one pair $(o, s') \in O \times S$ such that $(s, i, o, s') \in h_S$; otherwise, the machine is *nondeterministic*. If for each pair $(s, i) \in S \times I$ there exists $(o, s') \in O \times S$ such that (s, i, o, s') $\in h_S$ then FSM S is *complete*; otherwise, the machine is *partial*. A machine is *observable* if for each triple $(s, i, o) \in S \times I \times O$ there exists at most one state $s' \in S$ such that $(s, i, o, s') \in h_S$; otherwise, the machine is *nonobservable*. In this paper, we consider complete and observable nondeterministic machines, hereafter denoted as NFSMs. In a usual way, the behavior relation is extended to input and output sequences.

Given NFSM $\mathrm{S} = \langle S, I, O, h_S, S' \rangle$, $S' \subseteq S$, $|S'| = m \geq 2$, a sequence $\alpha \in I^*$ is a *homing sequence* (HS)[1] for S if for each subset $\{s_{i_1}, \ldots, s_{i_j}\} \subseteq S'$, the following holds:

$$\forall \beta \in out(s_{i_1}, \alpha) \cap out(s_{i_2}, \alpha) \cap \ldots \cap out(s_{i_j}, \alpha)\ [next\_state(s_{i_1}, \alpha/\beta) =$$

$$= next\_state(s_{i_2}, \alpha/\beta) = \ldots = next\_state(s_{i_j}, \alpha/\beta)],\ j \in \{1, \ldots, m\}, \quad (1)$$

$$\{s_{i_1}, s_{i_2}, \ldots, s_{i_j}\} \subseteq S'.$$

In [4], it is shown that given an observable NFSM $\mathrm{S} = \langle S, I, O, h_S, S' \rangle$ with an input sequence $\alpha \in I^*$ such that for each pair $\{s_{i_1}, s_{i_2}\} \subseteq S'$ it holds that $\forall \beta \in out(s_{i_1}, \alpha) \cap out(s_{i_2}, \alpha)$ $[next\_state(s_{i_1}, \alpha/\beta) = next\_state(s_{i_2}, \alpha/\beta)]$, the sequence α is an HS for NFSM S. The latter means that a homing sequence of an observable FSM is an input sequence that homes each pair of initial states of the FSM. Based on this definition an algorithm for HS derivation for NFSM with m initial states has been proposed.

[1] In [4], such a sequence is called a preset homing sequence (PHS).

Differently from reduced deterministic FSMs for which there always exists an HS of the polynomial length [2], there exist reduced NFSMs for which an HS does not exist as well as for each *n* there exists an FSM with *n* states and $2^{n^2/4}$ inputs such that a shortest homing sequence has length $2^{n^2/4}$ [4]. However, the number of inputs of the above FSM is also exponential w.r.t. the number of states. In this paper, we show that for each *n* there exists an NFSM with *n* states, $n-1$ inputs and $n(n-1)/2$ outputs such that its shortest homing sequence has length $2^{n-1}-1$. We now show that the same result holds for nondeterministic FSMs of polynomial size.

Consider NFSM `Sn`, $n > 1$, with the set $S = \{0, 1, 2, \ldots, n-1\}$ of states, set $I = \{i_0, i_1, \ldots, i_{n-2}\}$ of inputs, and set $O = \{(i, j)\colon i, j = 0, \ldots, n-1 \text{ and } i < j\}$ of outputs.

We define the transition relation of FSM `Sn` in the following way. Given input i_0, there is a single transition at state 0 under this input, namely, the transition to state $(n - 1)$ with the output $(0, n-1)$. Moreover, at each state j, $0 < j \le n-1$, there is a loop labeled with the input/output pair $i_0/(0, n - 1)$.

For each input i_k, $0 < k < n-1$, we define transitions under this input as follows. Given state *k*, FSM `Sn` moves from state *k* to state $q \in \{0, 1, \ldots, k-1\}$ under input i_k producing outputs (q, l) where $l \in \{0, 1, \ldots, k-1, k+1 \ldots, n-1\}$ and $l > q$, i.e., there exist transitions $(k, i_k, (q, l), q)$.

Thus, given FSM S_4 and $k = 2$, at state 2, there exist the following transitions:

$$2 - i_2/(0, 1) \to 0;$$
$$2 - i_2/(0, 3) \to 0;$$
$$2 - i_2/(1, 3) \to 1.$$

Define now transitions of FSM S_n under input i_k, $0 < k < n-1$, at state $p > k$.

At each state *p* where $p > k$ there are loops labeled with $i_k/(l, p)$ for all $l < p$ and with $i_k/(p, l)$ for all $p < l < n$. Moreover, at state *p* there are transitions to each state $q, \in \{0, 1, \ldots, k-1\}$ labeled with $i_k/(l, q)$ for all $l < q$.

At each state $p < k$ there are a transitions to state *k* under input i_k with the output (p, l) for $p, l \ne k$ и $0 \le p < l \le n-1$.

The following statement can be proved.

Theorem 1. The length of a shortest homing sequence for NFSM `Sn` is $2^{n-1} - 1$.

In this paper, we have shown that the exponential upper bound of homing sequence is reachable for NFSM of polynomial size. As future work, we are going to estimate the reachability of exponential upper bound for adaptive homing experiments.

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