

# Adaptive fuzzy neural production network with MIMO-structure for the evaluation of technology efficiency

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**Abstract.** The paper presents an example of modeling the algorithm operation. The paper analyses the modified Wang and Mendel MIMO-architecture of the adaptive fuzzy neural production network with a logical conclusion. It is distinguished by the automatic generation of a set of fuzzy rules based on a fuzzy decision tree and a hybrid algorithm for parameter adaptation by the neural network (centers, widths of membership functions, and conclusions) starting from the leaf nodes to the root nodes of the tree.

## 1. Introduction

When solving problems of evaluating the effectiveness of technologies, traditional statistical methods have some drawbacks, especially when two or more classes are connected by a chain of internally connected sampling objects, have a non-spherical form, are linearly inseparable sets, or when the density or volume of classes is different [1]. In the case of class crossing, contact or chaining, the classification model is blurred; therefore, fuzzy procedures are preferable when choosing mathematical methods for solving the problem of automatic classification [2]. The theory of fuzzy sets operates with such concepts as the weighted membership of the object under investigation to the set and offers a rather flexible apparatus for a formal description of similar situations. Therefore, the target setting moves from the question "Does the object belong to the class?" to determine the degree of membership of the object to a particular class. Summarizing, we can say that the fuzzy-multiple approach to the solution of classification problems in certain cases allows separating clusters of complex shape providing new opportunities for interpreting classification results.

Comparing the reliability of classification results obtained by different authors, we can identify the most effective classifiers, which include neural networks [1,3], fuzzy logic, and the methods of cluster analysis [4,5]. The best results were obtained using three-layer perceptron, probabilistic neural network, and Kohonen self-organizing maps. During testing, Takagi-Sugeno-Kang fuzzy network showed good results of complex functions approximation [6]. At the same time, studies that we have analyzed, did not consider the classification of the effectiveness of scientific and technological solutions and technologies, and did not use neural-fuzzy networks for technology classification.

## 2. The structure of Wang and Mendel fuzzy neural production network

From the functional point of view, Wang and Mendel fuzzy neural production network (L.-X. Wang and J.M. Mendel) is similar to the zero-order Takagi-Sugeno-Kang network (TSK) [7]. The components of the expressions that determine the prerequisites of the production rules are identical for both networks. Differences are observed in presenting conclusions. In the Takagi-Sugeno-Kang network, the result is polynomial:



$$r_{i0} + \sum_{j=1}^m r_{ij}x_j,$$

whereas in the Wang and Mendel network, the output variable in the rules is represented by the constant  $r_i$ , which can be considered as a zero-order polynomial that determines the center of the membership function of the conclusion of the corresponding rule.

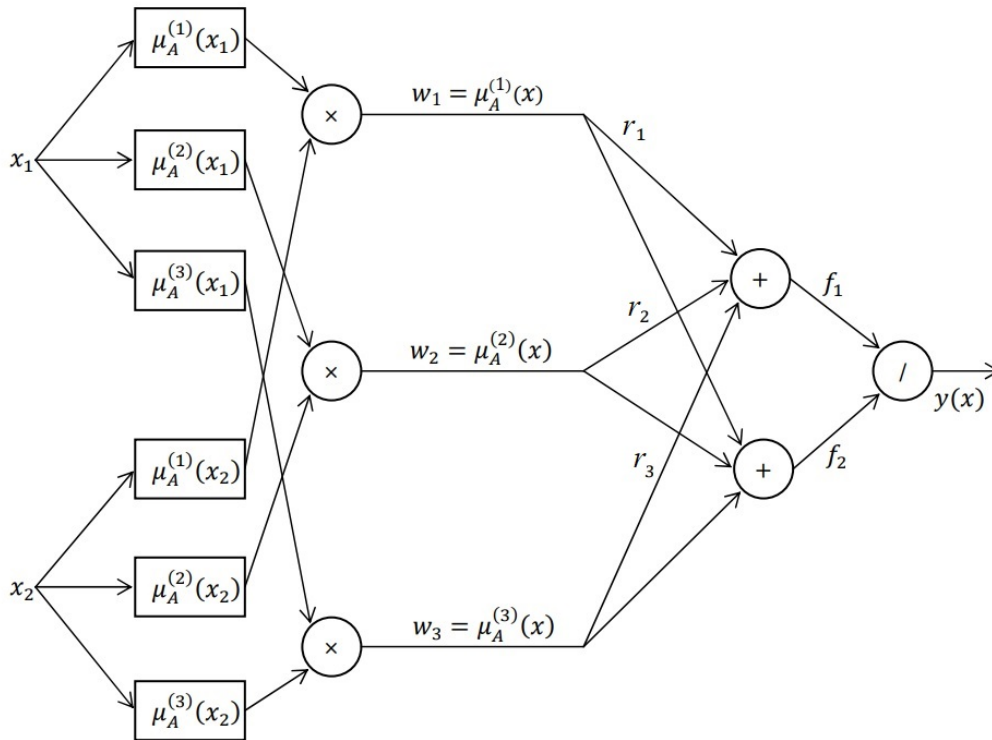
Thus, Wang and Mendel fuzzy neural production network implements a fuzzy product model based on the following rules:

$$\text{Rule } i: \text{ IF } x_l \text{ is } A_{il} \text{ AND } \dots \text{ AND } x_j \text{ is } A_{ij} \text{ AND } \dots \text{ AND } x_m \text{ is } A_{im}, \text{ THEN } y \text{ is } r_i, \\ i = 1, \dots, n,$$

where  $n$  is a number of rules;

$m$  is a number of input variables.

Figure 1 shows an example of a Wang and Mendel network with a MISO- structure (Multi Inputs Single Output) containing two inputs, three rules, and one output.



**Figure 1.** MISO-structure of Wang and Mendel fuzzy neural production network with two inputs and one output.

This network consists of five layers.

*The first layer (input).* Inputs can be both numerical and linguistic variables.

*The second layer (non-linear parametric)* performs the fuzzification of the input variables  $x_k$ ,  $k = 1, \dots, m$ . Elements of this layer calculate the values of the membership functions  $\mu_{A_{ki}}(x_{kj})$  in the  $j$ -th example ( $i$  is the rule number) given, for example, by Gaussian functions with the parameters  $a_{ij}$  and  $b_{ij}$  to be adapted in the learning process:

$$\mu_{A_{ki}}(x_{kj}) = \exp \left[ - \left( \frac{x_{kj} - a_{ki}}{b_{ki}} \right)^2 \right].$$

This layer contains  $m \times n \times 2$  parameters of the Gaussian function.

The third layer (*nonparametric*) contains the number of elements equal to the number of rules in the database; it aggregates the degrees of truth of the conditions of the corresponding rules.

The fourth layer (*linear parametric*) consists of two neurons. The first neuron serves to activate the conclusions of the rules  $r_i$  in accordance with the values of the preconditions of rules aggregated in the previous layer. The second neuron generates a normalizing signal for subsequent defuzzification of the result. This is the parametric layer, in which linear weights  $r_i$  interpreted as centers of membership functions of the conclusion of the  $i$ -th fuzzy inference rule are the subject to adaptation.

The stage of accumulation of conclusions of fuzzy production rules in this algorithm is absent because of clear values of output variables.

The fifth layer (*nonparametric*) consists of one output neuron and performs defuzzification forming an output signal. The output variables are single-point fuzzy sets, therefore, their values are calculated using the modified center-of-gravity method (center-of-gravity for singletons), and another name for this method is fuzzy mean [7]. It allows clarifying the output variable without accumulating the activated conclusions of the individual rules as follows:

$$y_j(x) = \frac{\sum_{i=1}^n w_{ij} r_i}{\sum_{i=1}^n w_{ij}}, \quad (1)$$

where  $w_{ij}$  is an aggregated degree of truth for all conditions of the  $i$ -th rule in the  $j$ -th example;  $r_i$  is a value of the output variable of the  $i$ -th rule ( $y_i = r_i$ );  $n$  is a number of rules in the database.

Thus, for the  $j$ -th example, the neural network implements the approximation function, which can be written as follows:

$$y_j(x) = \frac{\sum_{i=1}^n \left( r_i \prod_{k=1}^m \mu_{ki}(x_{kj}) \right)}{\sum_{i=1}^n \prod_{k=1}^m \mu_{ki}(x_{kj})} = \frac{\sum_{i=1}^n \left( r_i \prod_{k=1}^m \exp \left[ - \left( \frac{x_{kj} - a_{ki}}{b_{ki}} \right)^2 \right] \right)}{\sum_{i=1}^n \prod_{k=1}^m \exp \left[ - \left( \frac{x_{kj} - a_{ki}}{b_{ki}} \right)^2 \right]}. \quad (2)$$

Expression (2) defines a continuous function that can be used to approximate an arbitrarily given continuous function  $g(x)$  from the many variables  $x_i$ , that form the vector  $x$ . With the appropriate selection of the parameters of the condition and conclusion, the function (2) can approximate the given function  $g(x)$  with arbitrary accuracy  $\varepsilon$ .

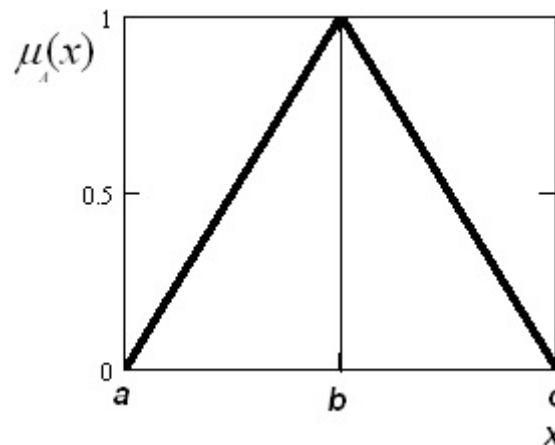
### 3. Development of the modified MIMO-structure of Wang and Mendel fuzzy neural production network

#### 3.1 Selection of the type of membership function

Fuzzification of the input space of attributes is associated with the formalization of input parameters in the form of a vector of interval values (fuzzy interval); the hit in each interval is characterized by a certain degree of uncertainty. We assume that based on primary data, experience, and intuition, experts usually do not have difficulties during quantifying the boundaries (intervals) of possible (admissible) values of the parameters. For example, experts can characterize the primary data with triangular fuzzy numbers with the following membership function  $\mu_A(x)$  of the fuzzy set  $A$ :

$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{x-b}, & b \leq x \leq c \\ 0, & c \leq a \end{cases}$$

This expression models the following expert position: "parameter A is approximately equal to b and uniquely enters the segment [a, c]" (Figure 2).



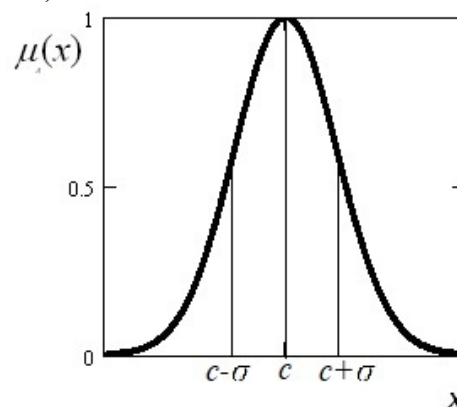
**Figure 2.** Triangular membership function.

One of the main advantages of triangular membership functions is their simplicity.

In practice, an analytical representation of the membership function is often used, for example, the Gaussian membership function has the form shown in Figure 3:

$$\mu(x) = \exp\left[-\left(\frac{x-c}{\sigma}\right)^2\right],$$

where  $c$  is a center,  
 $\sigma$  is a width coefficient (variation).

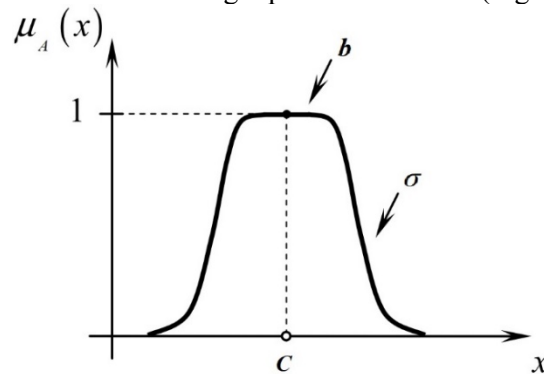


**Figure 3.** Gaussian membership function.

To develop the modified MIMO-structure (Multi Inputs Multi Output) of the Wang and Mendel fuzzy neural production network, we will use a symmetrical bell-shaped two-sided membership function controlled by three parameters and described by an analytical expression:

$$\mu(x) = \frac{1}{1 + \left| \frac{x - c}{\sigma} \right|^{2b}}$$

where  $c$  determines the location of the center (the coordinate of the displacement point of the curve symmetry center along the X axis);  
 $\sigma$  is the parameter that determines the width of the upper part of the curve;  
 $b$  is the coefficient of flatness of the left and right parts of the curve (Figure 4).



**Figure 4.** Generalized bell-shaped membership function.

The difference between the generalized bell-shaped membership function and the above-considered membership functions is addition of the third parameter, which allows implementing a smooth transition between fuzzy sets.

### 3.2 Generating a knowledge base using a fuzzy decision tree

To form the base of fuzzy rules, we propose to use a fuzzy decision tree [8, 9], which can be regarded as a hierarchical network with direct signal propagation. Therefore, in order to adapt its parameters (centers and width of membership functions) and the parameters of rule conclusions in tree leaves, we propose to use a hybrid algorithm for back propagation of the error.

To isolate a single layer in the network where the neurons correspond to the leaves of the tree, in the formula (1) we select the normalized coefficients  $\overline{w_{ij}}$  of the truth degree of the conditions for each rule in the  $j$ -th example. In this case, the expression (1) for the output  $y_j$  of Wang and Mendel fuzzy neural production network in the  $j$ -th example takes the following form:

$$y_j = \frac{\sum_{i=1}^n w_{ij} r_i}{\sum_{i=1}^n w_{ij}} = \frac{w_{1j} r_1 + \dots + w_{nj} r_n}{\sum_{i=1}^n w_{ij}} = \frac{w_{1j} r_1}{\sum_{i=1}^n w_{ij}} + \dots + \frac{w_{nj} r_n}{\sum_{i=1}^n w_{ij}} = \overline{w_{1j} r_1} + \dots + \overline{w_{nj} r_n} = \sum_{i=1}^n \overline{w_{ij} r_i},$$

where  $\overline{w_{ij}} = \frac{w_{ij}}{\sum_{i=1}^n w_{ij}}$  is the normalized degree of truth conditions on the  $i$ -th rule in the  $j$ -th example.

Thus, in addition to formula (1), the last layer in the Wang and Mendel fuzzy neural production network (Figure 1) functionally realizes a weighted sum of the normalized degrees of truth conditions for each rule with weight coefficients  $r_i$ . Therefore, in accordance with the expression obtained, we replace the 4-th and 5-th layers of the network in a modified structure isolating a separate layer, the neurons of which correspond to the leaves of the tree [10].

Layer 4 (nonparametric) performs normalization of the truth condition degrees for each of the rules:

$$\overline{w}_{ij} = \frac{w_{ij}}{\sum_{i=1}^n w_{ij}}.$$

Layer 5 (*linear parametric*) consists of the leaves of the tree, each of which activates the corresponding rule conclusions  $r_i$  in accordance with the values of the rule conditions normalized in the previous layer:

$$y_{ij} = \overline{w}_{ij} r_i.$$

Layer 6 (*nonparametric*) performs defuzzification summing up the rule conditions for each class that were activated in the previous layer:

$$y_j = \sum_{i=1}^n y_{ij}.$$

The number of neurons in this layer is also determined by the number of classes used in the task, at the same time the output signal of each neuron is the degree of truth to which the example agrees with the corresponding class.

#### 4. Conclusion

Using the structural-functional diagram of the modified Wang and Mendel fuzzy neural production MIMO-network, we have implemented the adaptive neuro-fuzzy classifier with fuzzy output based on a combination of “hierarchical techniques (decision trees) + fuzzy logic + neural networks”. This successful combination of intelligent techniques provides a synergistic effect allowing complex usage of the strengths of decision trees (simplicity, high speed, the use of any type of variables, and generation of rules in areas where experts have difficulties in formalizing their knowledge), fuzzy systems (interpretability of knowledge), and neural networks (the ability to learn on the data). The results of the study are listed below.

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