

# Method of gas flows calculation in solid propellant rocket engines taking into account the combustion of solid fuel charge

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**Abstract.** The paper presents a method for calculating the local and integral characteristics of the flow in the axisymmetric gas-dynamic paths of solid propellant rocket motors, taking into account the combustion of a charge of solid fuel. The numerical method of calculation is based on the use of the Godunov scheme, formulated for moving computational grids. The speed of movement of the combustion surface is defined locally on the edge of each calculation boundary cell. This approach allows us to take into account the uneven distribution of the pressure of the combustion products in the free volume of the combustion chamber. In test calculations, the power law of burning rate is used. Calculations of the gas flow in the solid propellant combustion chamber with cylindrical charge of solid fuel are carried out. Unsteady pressure curve in the combustion chamber is obtained. The method allows to determine all integral characteristics of the developed solid propellant rocket motors as a function of the engine running time.

## 1. Introduction

Currently in Russia and in the world there is the task of developing heavy launch vehicles. One of the concepts is to equip the launch vehicle with solid-fuel launching booster. Solid propellant rocket engines (SPRE) have a number of advantages: simple design, reliability and a quick preparation for launch. The control of engine thrust in SPRE is usually carried out in the “passive” mode and is determined by the form of solid fuel charge and fuel characteristics. The form of fuel charge in this case should provide the necessary burning law of the combustion surface depending on the pressure in the combustion chamber. The transient pressure curve in the general case determines the behavior of propulsive power. The law of change of the burning surface as a function of time is, as a rule, unregulated and is designed for a specific cycle of engine operation and task. As a rule, in modern SPRE use charges of a complex spatial or axisymmetric form such as the “cocoon”, “umbrella”, “star”, “slot”. Structural design of such engines is made using physical and mathematical modeling. Mathematical modeling is used to create a general layout with the aim of reducing costs at conducting field experiments [1-3].

In engineering practice, to solve such the problems, approximate approaches based on averaging the pressure over the chamber volume and changing the burning surface in parallel layers are often used. It is known that the operation process in a combustion chamber under actual usage conditions is characterized by the spatial distribution of the combustion products pressure in the free volume of combustion chamber. This leads to distortion changes of a charge burning surface of solid fuel during engine operation. Therefore, when calculating non-stationary performance characteristics, it is necessary



to take into account the change in the filler geometry according to the local pressure at the combustion surface, and not the average pressure in the combustion chamber volume. Minkov et al. ([4, 5]) were developed a calculation method taking into account non-stationary changes in the combustion surface based on the use of ghost point and the Lax-Wendroff method. This approach allows calculations to be carried out without rebuilding the grid even for complex combustion surfaces. However, the assumption that the combustion of propellant occurs in a quasi-stationary mode, and the pressure field has time to adjust to the change in the burning surface does not allow for the effects of chamber relaxation [6]. This feature makes it impossible to simulate unstable modes of solid propellant in SPRE.

The aim of this work is to develop an algorithm and methodology for calculating the local and integral characteristics of a solid propellant rocket motor, taking into account the change in the geometry of the flow path of the combustion chamber depending on the local pressure distribution at the combustion surface.

## 2. Calculation Methodology

The Euler equations, which govern the compressible, inviscid gas flow in three-dimensions, are [7]:

$$\frac{d}{dt} \left( \iiint_G \rho dG \right) + \oint_S \rho \mathbf{v} \cdot d\mathbf{S} = 0, \quad (1)$$

$$\frac{d}{dt} \left( \iiint_G \rho \mathbf{v} dG \right) + \oint_S (\rho \mathbf{v} \mathbf{v} + p \hat{\mathbf{I}}) \cdot d\mathbf{S} = 0, \quad (2)$$

$$\frac{d}{dt} \left( \iiint_G e dG \right) + \oint_S (e + p) \mathbf{v} \cdot d\mathbf{S} = 0, \quad (3)$$

where:  $\rho$  – density;  $t$  – time;  $p$  – pressure;  $\mathbf{v} = [u, v, w]^T$  – gas velocity;  $\hat{\mathbf{I}}$  –  $3 \times 3$  identity tensor;  $\varepsilon$  – specific internal energy;  $e = \rho\varepsilon + \rho(u^2 + v^2 + w^2)/2$  – total energy per unit volume;  $G$  – final volume in three-dimensional space;  $dG = dx dy dz$  – element of volume;  $S$  – surface limiting the volume  $G$ ;  $d\mathbf{S} = \mathbf{n} dS$  – orientable surface element  $S$ , where  $\mathbf{n}$  – external normal unit vector to the surface  $S$ ;  $dS$  – differential surface element.

System (1) – (3) is closed by equation of state:

$$\varepsilon = \frac{p}{(k-1)\rho}. \quad (4)$$

In order to build difference scheme the finite volume method is applied. Therefore, we will cover all computational domain with discrete cells consisting of arbitrary convex polyhedrons with volume  $G_i$ , where  $i = 1, 2, \dots$  – volume number, and  $m(i)$  – face number of  $i$  finite volume. Each face of  $i$  finite volume has surface  $S_j$ , where  $j = 1, 2, \dots, m(i)$ . Approximation of integral equations in each of polyhedrons obtains in the following way:

$$G_i \frac{\rho_i^{k+1} - \rho_i^k}{\Delta t} + \sum_{j=1}^{m(i)} R_j (\mathbf{V}_j \cdot \mathbf{S}_j) = 0, \quad (5)$$

$$G_i \frac{(\rho \mathbf{v})_i^{k+1} - (\rho \mathbf{v})_i^k}{\Delta t} + \sum_{j=1}^{m(i)} (R_j \mathbf{V}_j) (\mathbf{V}_j \cdot \mathbf{S}_j) + \sum_{j=1}^{m(i)} P_j \mathbf{S}_j = 0, \quad (6)$$

$$G_i \frac{e_i^{k+1} - e_i^k}{\Delta t} + \sum_{j=1}^{m(i)} (E_j + P_j) (\mathbf{V}_j \cdot \mathbf{S}_j) = 0. \quad (7)$$

As a result we will get Godunov's scheme for custom computational mesh. Here  $\mathbf{S}_j = \mathbf{n}_j S_j$ , and  $\Delta t$  – time step. Lower index  $i$  signifies value of functions, which are referred to mass center of  $i$  polyhedrons, and lower index  $j$  signifies values referred to center of  $j$  face of discrete cell. Upper index  $k$  signifies the number of time step. Flux values of  $R$ ,  $\mathbf{V}$ ,  $P$ ,  $E$  signify correspondingly density, speed, pressure and total energy at faces of control volume. These values are defined from the solution of Riemann problem [14] in the direction of outward normal. Also for the flow calculations can be used other methods, based on approximate solutions of Riemann problem: Roe, Osher, HLL, HLLC, HLLC and others [8].

For moving grids, the Godunov scheme looks like:

$$\frac{(\rho G)_i^{k+1} - (\rho G)_i^k}{\Delta t} + \sum_{j=1}^{m(i)} R_j \left( [\mathbf{V} - \mathbf{D}] \cdot \mathbf{S}^{k+1/2} \right)_j = 0, \quad (8)$$

$$\frac{(\rho \mathbf{v} G)_i^{k+1} - (\rho \mathbf{v} G)_i^k}{\Delta t} + \sum_{j=1}^{m(i)} (R \mathbf{V})_j \left( [\mathbf{V} - \mathbf{D}] \cdot \mathbf{S}^{k+1/2} \right)_j + \sum_{j=1}^{m(i)} P_j \mathbf{S}_j^{k+1/2} = 0, \quad (9)$$

$$\frac{(e G)_i^{k+1} - (e G)_i^k}{\Delta t} + \sum_{j=1}^{m(i)} E_j \left( [\mathbf{V} - \mathbf{D}] \cdot \mathbf{S}^{k+1/2} \right)_j + \sum_{j=1}^{m(i)} P_j \mathbf{S}_j^{k+1/2} = 0, \quad (10)$$

$$\frac{G_i^{k+1} - G_i^k}{\Delta t} - \sum_{j=1}^{m(i)} (\mathbf{D} \cdot \mathbf{S}^{k+1/2})_j = 0. \quad (11)$$

Here  $\mathbf{D}_j$  – velocity of the center of the  $j$  cell face. Top index  $k + 1/2$  signifies values at time  $t + \Delta t / 2$ . Equation (11) describes changes in the volume of  $G_i$  discrete cell. To determine the flow through the moving faces of a discrete cell, a modified solution of the Riemann problem is used [7]. Scheme (8) – (11) is first-order accurate in space and time.

The geometrical parameters of the cells on the burning surface and the local mass flow from the edge of the computational cell at the current time step are calculated using the power law of the burning rate:

$$u_L = u_0 p_L^\nu, \quad (12)$$

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \mathbf{n}_x u_L \Delta t, \quad (13)$$

$$m = \rho_T u_L, \quad (14)$$

where:  $u_L$  – local burning rate at the edge of the cell;  $u_0$ ,  $\nu$  – constants in the law of burning rate;  $p_L$  – local pressure in the neighborhood of the vertex of moving face;  $\mathbf{x}$  – the coordinate of the vertex of the movable edge;  $\mathbf{n}_x$  – external normal unit vector to the burning surface at  $\mathbf{x}$  point;  $m$  – local mass flow;  $\rho_T$  – solid fuel density.

The solution algorithm at the current time step on a moving computational grid is constructed as follows:

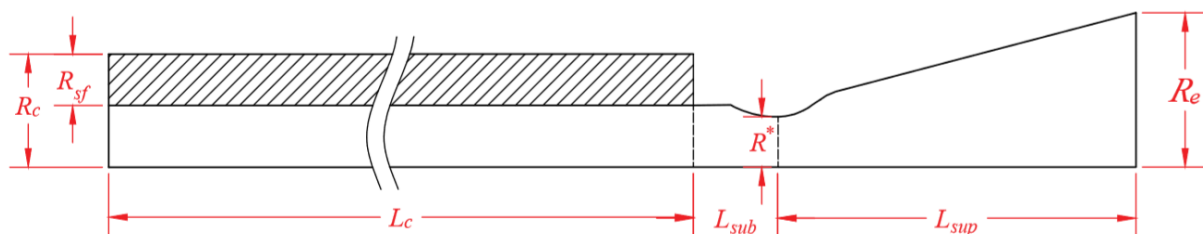
1. Using relations (12) – (14), new coordinates of the vertices of the moving faces of the cells are determined.
2. For each moving cell, geometric parameters (cell volume and face areas) are recalculated.
3. Using (14), the local mass flow for each face of the moving computational grid is determined.

### 3. Numerical results

Based on the above calculation method, a software package for calculating flow characteristics in the combustion chambers and nozzles was developed. To verify the methodology and program for calculating, numerical studies in a combustion chamber with a cylindrical charge of solid fuel are carried

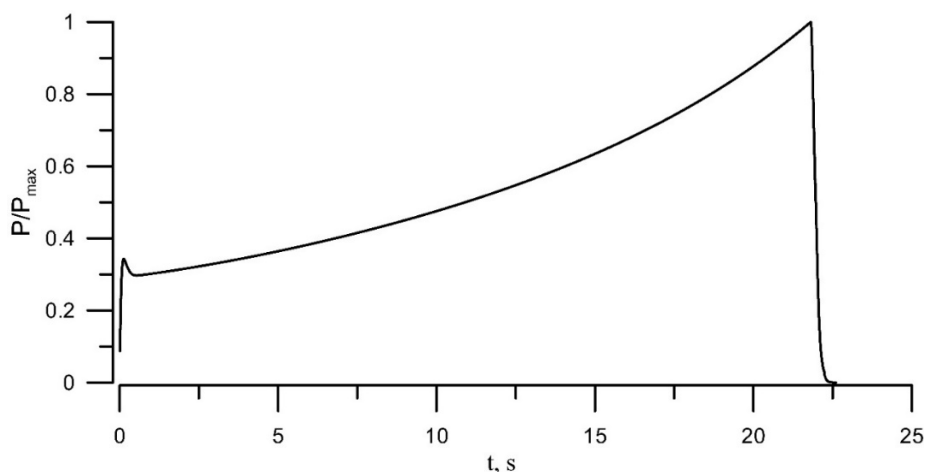
out. The scheme of the engine is shown in Figure 1. The linear dimensions are related to the radius of the critical section. Here  $R^*=1$ ,  $L_c=28.93$ ,  $R_c=2.24$ ,  $R_{sf}=1.02$ ,  $R_e=3$ ,  $L_{sub}=1.6$ ,  $L_{sup}=7.1$ . The computational domain was divided into 2 subdomain: the subdomain of the combustion chamber, which includes the combustion surface; subdomain of the nozzle block. For each subregion, a structured computational grid was constructed, in accordance with the methodology [9]. At the initial time, in the first subregion, the size of the computational grid consist of  $4 \times 50$  cells, and in the subregion of the nozzle unit -  $10 \times 20$  cells. During the calculation the number of cells remains unchanged.

On all impermeable boundaries the solid wall boundary condition was specified, and in the exit section of the nozzle, a supersonic output. The boundary conditions on the burning surface were determined through a local gas input through the edge of the computational cell according to scheme (12) - (14). The coefficients in the law of burning rate of the model fuel is:  $u_0 = 0.006$  m/s,  $\nu = 0.3$ . The heat capacity ratio of combustion products  $k = 1.19$ . The combustion temperature of the combustion products is 3400 K. In the combustion chamber at the initial time, the pump pressure was set to 5 atm.



**Figure 1.** SPRE scheme

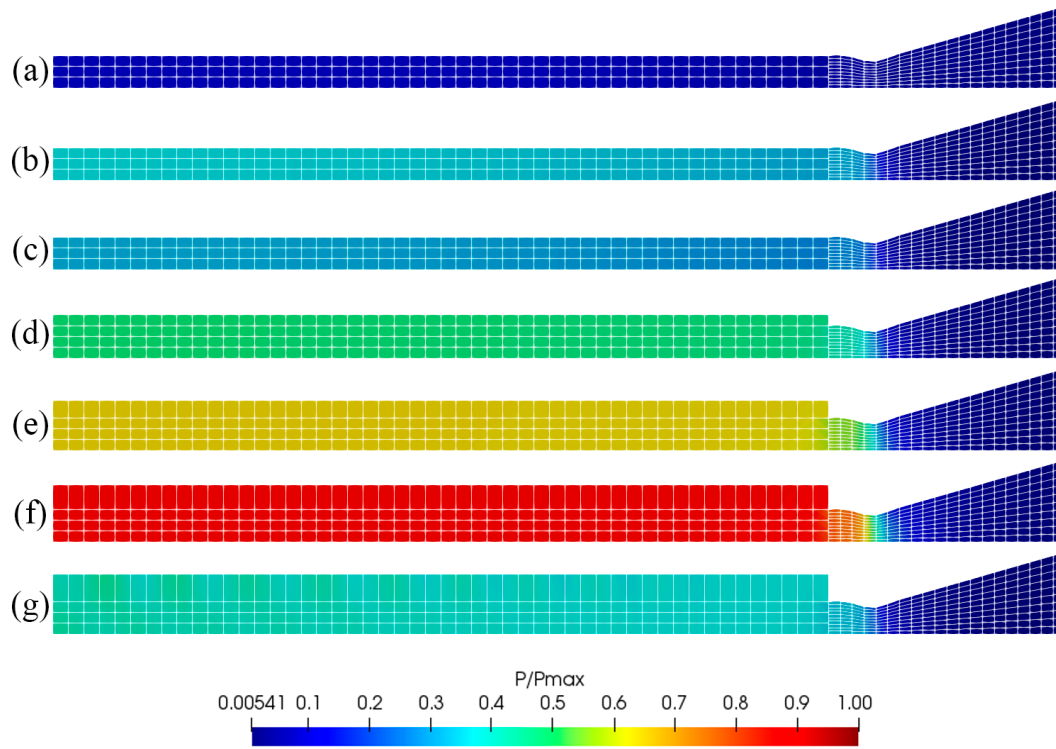
Figure 2 shows the complete pressure curve in the combustion chamber with a cylindrical charge from the start of engine operation to the moment of shutdown. We can see the sections of the starting operation, the main mode of engine operation and pressure drop. The pressure peak at 0.12 sec is caused by the effect of chamber relaxation. Figure 2 a) - e) shows the change in the distribution of the pressure field and the evolution of the computational grid at 0 s, 0.12 s, 0.54 s, 10.0 s, 15.0 s, 20.0 s, 22.5 s.



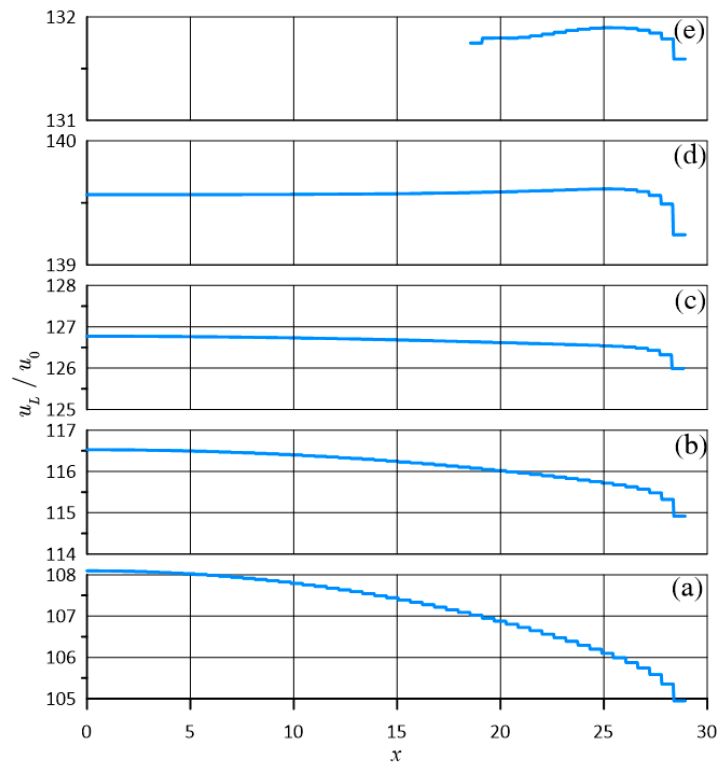
**Figure 2.** Pressure curve of engine operation process

Figure 4 shows the distribution of local values of the burning rate along the burning surface for time  $t=5$  s (a), 10 s (b), 15 s (c), 20 s (d), 21.9 s (e). It can be seen that the rate of combustion along the surface of the charge of solid fuel is substantially uneven, higher at the front bottom, and lower at the entrance to the nozzle. This correlates with the pressure distribution along the axis of the combustion chamber. Figure 4 (d) shows an uncharacteristic increase of burning speed at the front of the chamber. This is due to a local increase of pressure during the flow around a ledge at the entrance to the nozzle, which is the

element of the combustion chamber armor. The time  $t = 21.9$  s. (Figure 4 (e)) is characterized by a partial burnout of the charge of solid fuel in the region starting from the front bottom to  $L = 18.5$ .



**Figure 3.** Pressure fields distribution and calculation grid change at various time a) 0 s, b) 0.12 s, c) 0.54 s, d) 10.0 s, e) 15.0 s, f) 20.0 s, g) 22.5 s



**Figure 4.** Local burning rate at time a) 5 s, b) 10 s, c) 15 s, d) 20 s, e) 21.9 s.

#### 4. Conclusion

The method for calculating the local and integral characteristics of axisymmetric solid propellant rocket motors, taking into account the combustion of a charge of solid fuel, has been developed. The calculation method is based on the application of the Godunov scheme, formulated for moving computational grids. The speed of movement of the burning surface is determined by the local pressure values at the edges of each boundary computational cell according to the empirical law of the burning rate. Such an approach makes it possible to determine the combustion surface and obtain a pressure curve of the combustion chamber, taking into account the uneven distribution of pressure in the free volume. The technique allows to determine the intraballistic and integral characteristics of the developed solid propellant rocket motors as a function of the engine running time, for example, launching booster with cylindrical sectioned charges.

#### Acknowledgments

The work executed at financial support of the Ministry of education and science of the Russian Federation in the framework of the state assignment, project No 9.9063.2017/8.9.

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