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ASSESSMENT OF STRESS-STRAIN STATE OF EARTH DAMS WITH ACCOUNT FOR GEOMETRICAL NONLINEARITY UNDER STATIC LOADS

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Abstract

Mathematical statement, methods and algorithms for assessing the stress-strain state (SSS) of earth structures are given in this paper taking into account geometric nonlinear soil strain under static influences. In the course of research, it was found that an account for geometrically nonlinear strain in soil leads to a noticeable change in the stress-strain state in high-rise structures and to an increase in all stress components. High concentration of stresses, which are the cause of strength damage in the zones of earth dams, is observed in the in the upper part of the core-transition zone contact point and in the upstream side.

Key words: earth dam, stress-strain state, nonlinearity, inhomogeneity, stress, strain, strength.

ntroduction. World experience shows that timely prevention is much more economical and effective than eliminating the consequences associated with flood events and accidents at hydrotechnical structures (HTS). Therefore, the monitoring and forecasting organization of possible emergencies, the implementation of protective engineering and technical measures to increase the stability of hydrotechnical structures are highlighted [1].

Today in the republic there are more than 270 large and especially important hydro-technical structures. The guaranteed water supply in agriculture of the Republic largely depends on their reliability. Therefore, the issue of reliable and safe operation of the HTS is of particular relevance.

The safety of hydro-technical structures, first of all, is determined by its reliability. The term "reliability" means the failure-free operation of the structure as a whole and its individual elements throughout the entire service life [2, 3]. The seriousness and relevance of ensuring the safety and reliability of dams during their operation is confirmed in [1-4].

The reliability and safety of dams depend on the stress-strain state of the structures under various loads. The forecast of the change in the magnitude of the stress-strain state components of the dam allows us to get complete information about the structure strength.

In the study of the stress-strain state and dynamics of specific structures in order to ensure their strength, a number of questions arise related to the real geometry, inhomogeneity and design features of the structures, the real properties of their material. An account for these aspects allows us to more accurately predict the state of structures under various influences. Along with this, the accuracy of determining the SSS (stressstrain state) depends on the selected design scheme, mathematical models used, methods of solution and equations of state of the materials [5-14].

In connection with the construction of high-rise earth dams, the relevance of accounting for nonlinear soil properties is increasing. One of the nonlinear models that satisfactorily describe the properties of earth materials is the model of an elastoplastic body [8,9,15-17,27]. The model [17] is based on the assumption that, until the state of ultimate equilibrium is reached, the material behaves according to the model of the linear theory of elasticity. If the stresses exceed the tensile strength or shear, the material strength is violated according to the Coulomb-Mohr theory of strength.

The main principles of the methods to assess the earthquake resistance of dams considering plastic properties of soil and the effect of mineral skeleton and pore fluid interaction, are presented in [8,9,18,20,28]. The problems considered there are solved in a plane statement for a specific dam.

The studies in [11,19,22] are also devoted to the development (in a plane statement) of theoretical conditions and methods for assessing the dynamics, stress-strain state, strength and seismic resistance of earth dams, considering their design features, real conditions of structure operation and various linear and non-linear elastic, viscoelastic, elastoplastic and moisture properties of soil under various influences.

To date, not all issues of soil behavior under load have been completely clarified and one of the important problems here is a nonlinear strain of soil, in particular, geometrical nonlinear strain.

The need to take into account geometrical nonlinear strain of earth structures is increasing in connection with the construction of high-rise earth structures. The question of how this factor affects the SSS, the strength of structures and the stability of slopes, remains insufficiently studied and requires extensive research.

The investigation presented here is devoted to the development of a mathematical model, methods and algorithms for assessing the stress-strain state of earth structures considering geometric non-linearity of soil and to the study of the stress-strain state of specific earth dams under various static influences.

1. Models, methods and algorithms for assessing the stress-strain state of structures

1.1. Statement of the problem

To forecast the SSS and the dynamics of earth dams in a three-dimensional statement, we consider a spatial model of the structure, presented as an inhomogeneous system (Fig. 1). The surfaces of the base and side slopes $\Sigma^{\alpha} \Sigma^{\alpha} \Sigma^{\alpha}$ are rigidly fixed, the surface of the downstream slope Σ_3 is stress-free, water pressure acts on the surface Sp (part of the upstream slope below the line of the normal water level (NWL)), and an external load is

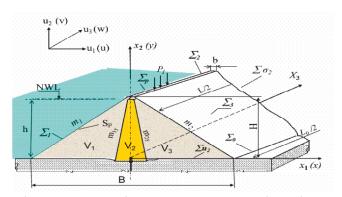


Fig. 1 Model of an inhomogeneous three-dimensional system

applied to the surface Σ_p of the crest surfaces Σ_2 .

Here: $V = V_1 + V_2 + V_3$ is the volume of the dam body (V_1, V_3 are the volumes of the upper and lower retaining prisms, V_2 is the core volume); $\sum_{\sigma_1} \sum_{\sigma_2} \sum_{\sigma_2}$ are the surfaces of the shorefaces, \sum_{o} is the surface of the base along the bottom, and $\sum_{1}, \sum_{2}, \sum_{3}$ are the surfaces of retaining prisms and crest.

To simulate the strain process and the dynamics of dams (Fig. 1) in a spatial statement, the Lagrange variation equation is used based on the D'Alembert principle for inhomogeneous deformable threedimensional bodies:

$$-\int_{V_{1}} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{V_{2}} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{V_{3}} \sigma_{ij} \delta \varepsilon_{ij} dV -$$

+
$$\int_{V} \vec{f} \delta \vec{u} dV + \int_{S_{p}} \vec{p} \delta \vec{u} dS + \int_{\Sigma_{p}} \vec{P}_{1} \delta \vec{u} d\Sigma = 0, \quad i, j = 1, 2, 3.$$
(1)

Here, \vec{u} , ε_{ij} , σ_{ij} , $\overline{}$ are the displacement vector, strain and stress tensors, respectively; $\delta \vec{u}$, $\delta \varepsilon_{ij}$ are the isochronous variations of displacements and strains; ρ_n is the density of the elements material of the system in question (index n = 1,2,3 means the part of the system to which this quantity belongs); \vec{f} is the vector of mass forces; \vec{P}_i is the vector of external forces applied to Σ_p ; \vec{p} is the water pressure (the sum of hydrodynamic and hydrostatic pressures) resulting from the structure-water interaction and determined at the point (x_1, x_2) [23].

Not all issues of soil behavior under load are clarified to the end. In this regard, there are many different theories that are more or less difficult to implement when solving specific problems. One of the important problems here is the consideration of non-linear strain of structures, in particular, geometrically nonlinear strain, i.e. finite strain.

In theoretical terms, geometrical nonlinear strain when solving specific problems consists in holding in the components of the strain tensor ϵ_{ij} not only linear, but also quadratic terms from the derivatives of the displacements along the coordinates, i.e.:

$$\varepsilon_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} + u_{\ell,i} * u_{\ell,j} \right); \ i, j, \ell = 1, 2, 3$$
(2)

In all the problems under consideration, the displacement vector in the spatial coordinate system $\vec{x} = \{x_1, x_2, x_3\} = \{x, y, z\}$ has three components $\vec{u} = \{u_1, u_2, u_3\} = \{u, v, w\}$.

When creating mathematical models, the following boundary conditions are taken into account

$$\vec{x} \in \sum_{O} : \vec{u} = 0 \tag{3}$$

Now, the general variation problem can be formulated as follows: it is necessary to determine the fields of displacements \vec{u} , strains ε_{ij} and stresses σ_{ij} in a nonlinear inhomogeneous spatial system (Fig. 2.7), arising under the action of mass (\vec{f}), external forces (\vec{P}_i), and water pressures \vec{p} that satisfy equations (1), (2) and correspond to the boundary conditions (3) for any virtual displacement $\delta \vec{u}$.

1.2. Method and algorithm for solving the problem 1.2.1. For an inhomogeneous linear elastic system (Fig. 1) under static load, variation equation (1) using the procedure of the finite element method is reduced to solving a system of linear algebraic equations of the *N*-th order

$$[K]{u} = {F}$$

$$\tag{4}$$

here the elements of the system stiffness matrix [K] are constant and depend only on elastic physicalmechanical parameters of the structure material.

1.2.2. For geometrically nonlinear inhomogeneous elastic systems (Fig. 1) under static load, variation equation (1) using the finite element method procedure [24,25] is reduced to solving a system of nonlinear algebraic equations of the *N*-th order

$$[K(u)]{u} = {F}$$
(5)

here, the elements of the stiffness matrix [K(u)] are the variables and depend not only on the geometrical and physical parameters of the structure, but also on the nodal displacements (u); $\{F\}$ is the vector of external load from mass forces, hydrostatic pressure of water.

Then, the system of nonlinear algebraic equations (5) is replaced by an equivalent system of the following form [11, 26]:

$$[K]\{u\} = \{F\} - [K_n(u)]\{u\}$$
(6)

where [K] is the stiffness matrix of the linear-elastic problem; $[K_n(u)]$ is the nonlinear part of the stiffness matrix, depending on the movement of the system nodes, obtained by extracting from the matrix $[K\{u\}]$ its linear component [K].

To solve equation (6), the progressive approximation method [26] is used; its convergence is determined by the choice of the initial estimate. As an initial estimate, the solution of the linear-elastic problem is used:

$$[K](u_0) = \{F\}$$

$$\tag{7}$$

Further approximations are found by the formula: $[K]\{u_{s+1}\} = \{F\} - [K_n\{u_s\}\{u_s\}] \quad s = 0,...,n \quad (8)$

The criterion for the iteration end is the fulfillment of the condition:

$$\left|u_{s+1}-u_{s}\right|\leq\varepsilon\tag{9}$$

Where ϵ is the given accuracy.

2. Results of the stress-strain state assessment

In this section, using the developed methods, algorithm and computer program, the stress-strain state of the following earth dams is studied under various static loads taking into account geometrical nonlinearity: 1) Gissarak dam: height H = 138.5 m, crest width $b_g = 16.0$ m, slope steepness $-m_r=2.2$ and $m_2=1.9$; 2) Pachkamar dam: height H = 70.0 m, crest width $b_g = 8.0$ m, slope steepness $m_r=2.0$ and $m_2=2.0$. Inhomogeneity, design features, real geometry and elastic characteristics of the material for each individual section of the structure were taken into account in concrete calculations.

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2.1. The stress component was determined for the models of homogeneous earth dams of various heights (H = 25m; H = 50m; H = 70m). Other geometrical and physico-mechanical parameters of the models remained unchanged: m₁=2.0; m₂=2.0; E=83500 tf/m²; γ =1.9 tf/m³; μ =0.3. After calculations, the results obtained with and without geometrical nonlinearity were compared.

Figure 2 shows the relative difference (in %) of the results of linear and nonlinear calculation of stress components in the central section of dam under own weight of the structure. The difference was determined by the formula $((\sigma_{ij}^{gnl} - \sigma_{ij}^{l})^{*}100\%)/\sigma_{ij}^{l}$. Here: σ_{ij}^{l} is the stress component of linear calculation, σ_{ij}^{gnl} is the stress component with

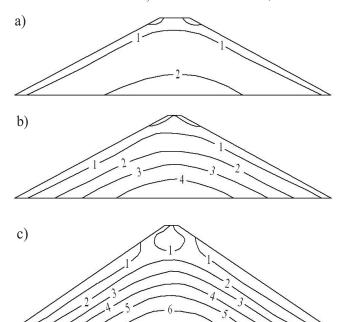


Fig. 2. Isolines of the difference (in %) of the linear and non-linear calculation of stresses in the cross section of homogeneous dams of different heights: H = 25m - (a); H = 50m - (b); H = 70m - (c)

account for geometrical nonlinearity.

The results (Fig. 2) obtained for homogeneous dams of different heights show that an account for geometrical nonlinearity increases by approximately 2% the stresses with every twenty meters in height of the structure as compared to a linear calculation.

2.2. The SSS of a homogeneous Gissarak earth dam under its own weight, with account for geometrical nonlinearity, is considered.

Figure 3 shows the stress isolines σ_1^{out} in a homogeneous Gissarak dam under own weight of the structure, with account for geometrical nonlinearity (Fig. 3 a) and isolines of the difference (in %) between the results of linear and geometrical nonlinear calculations (Fig. 3b).

Figure 4 shows the stress isolines σ_{22}^{gnl} for the dam under consideration, with account for geometrical nonlinearity (Fig. 4a) and the difference (in %) between linear and geometrical nonlinear calculations (Fig. 4b).

An analysis of the results in Figs. 3 and 4, shows that in high-rise structures the non-linear components of strain are observed. An account for nonlinear components in calculation gives a difference of 15-20%, compared to a linear calculation; this confirms the conclusions made earlier based on the results in Fig. 2 for a homogeneous

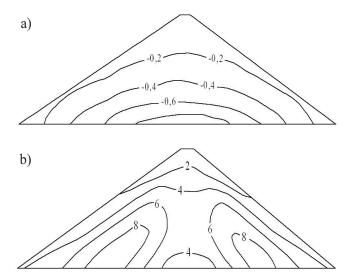


Fig. 3. Isolines of horizontal stresses σ_{11} (MPa) in the cross section of a homogeneous dam under its own weight, with account for geometrical nonlinearity (a) and the difference (in %) between linear and nonlinear calculations (b).

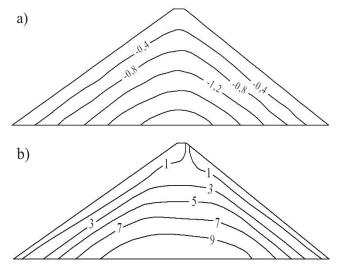


Fig. 4. Isolines of horizontal stresses (MPa) in the cross section of a homogeneous dam taking into account geometrical nonlinearity (a) and the difference (in %) between linear and nonlinear calculations (b)

structure: every 20 m increase in height with account for geometrical nonlinearity increases the values of stress components by approximately 2%.

2.3. The assessment of the SSS of an inhomogeneous Gissarak earth dam with account for geometrical nonlinear strains is carried out. The heterogeneity of the structure is associated with the presence of a core in the dam center.

Figure 5 shows the stress isolines for an inhomogeneous Gissarak dam under its own weight, with account for geometrical nonlinearity (a), and the difference in stresses (in %) in linear and nonlinear calculations (b).

An analysis of the results in Fig. 5 shows that for an inhomogeneous dam, an account for geometrical nonlinearity leads to slight change in the horizontal stresses σ_{11}^{ged} in the slopes, in the upstream slope and in the core. In the lower part of the upstream slope there is a greater probability of the soil riser occurrence, in comparison with

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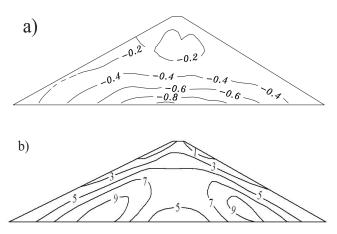


Fig. 5. Isolines of horizontal stresses (MPa) in the cross section of an inhomogeneous Gissarak dam with account for geometrical nonlinearity (a) and the difference between linear and nonlinear calculations (in %) (b).

the elastic case, and at the boundary with the core there is a high probability of cracking.

The isolines of the principal stress in the central section of the inhomogeneous dam, with account for geometrical nonlinearity, and the isolines of the difference between the

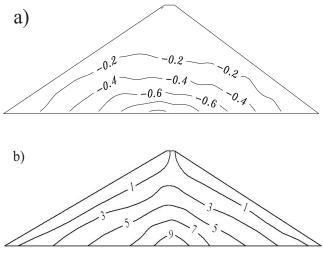


Fig. 6. Isolines of the principal stress (MPa) in the cross section of an inhomogeneous dam with account for geometrical nonlinearity (a) and the isolines of the difference (in %) between linear and nonlinear calculations (b).

stresses σ_1^{2n} and σ_1^{n} (in %), are shown in Fig. 6.

Here (Fig. 6 b), a difference (up to 10%) in the principal stresses obtained by nonlinear (σ_1^{gnl}) and linear (σ_1^{l}) calculations is observed. This confirms that an account for the structure inhomogeneity leads to an increase in compressive stresses, compared with a homogeneous structure throughout the dam body, and more so in the central part of the core.

Figure 7 shows the isolines of the stress intensity σ_i in the central section of the inhomogeneous Gissarak dam under its own weight: geometrical nonlinear calculation σ_i^{gnl} (Fig.6a) and the difference (in %) between (σ_i^l) and (σ_i^{gnl}) (Fig.7 c), and between the maximum tangential stresses r_{max} in linear and nonlinear calculations.

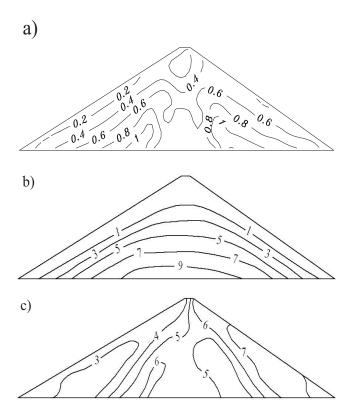


Fig. 7. Isolines of stress intensity σ_i^{2u} (MPa) in the cross section of an inhomogeneous Gissarak dam with account for geometrical nonlinearity (a) and isolines of the difference (in %) σ_i - (b) and τ_{max} - (c)

Figure 7b also shows a significant (up to 10%) difference in the intensity of stresses σ_1^{gnl} for inhomogeneous dams with account for geometrical nonlinearity compared with the stresses σ_1^{I} obtained for the linear problem (for an inhomogeneous structure) and a difference in the values of τ_{max} , the maximum of which occurs at the core–slope contact point; this can also lead to the crack formation in the near-crest zone and at the joints of dam sections of different soil materials.

The noted effect occurs in highly inhomogeneous dams. Therefore, when assessing the strength of high-rise structures, it is necessary to take into account geometrically nonlinear strain and real structural features.

Conclusions

1. Computer methods, algorithms and programs were developed to assess the stress-strain state of inhomogeneous earth dams, taking into account geometrical nonlinearity under static loads.

2. Studies of the SSS of earth dams under static loads with account for geometrical nonlinear strain showed that the consideration of finite strain leads to a noticeable change in the SSS in high-rise structures and leads to approximately 2% increase in all stress components (compared with the linear case) with every 20 m increase in structure height.

3. An account for the structural inhomogeneity and geometric nonlinearity of the structure under static influences leads to a change in stress fields, compared with a homogeneous structure. High concentration of stresses is observed in the upper part of the core-transition zone contact point and upstream slope, which is the reason for the strength violation in these zones.

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