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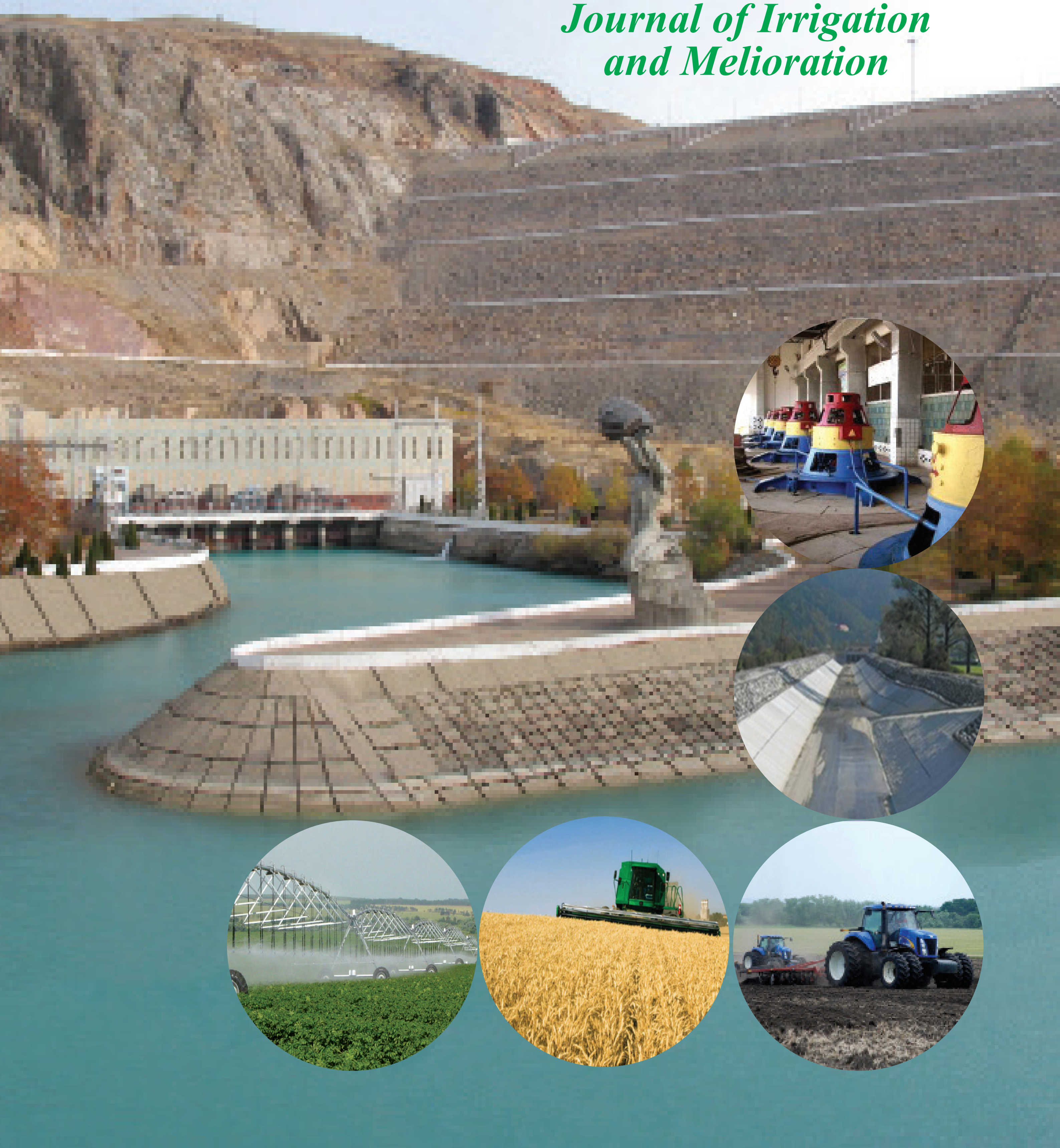

IRRIGATION AND MELIORATION

HYDRAULIC ENGINEERING STRUCTURES AND PUMPING STATIONS

ELECTRIFICATION AND AUTOMATION OF AGRICULTURE AND WATER RESOURCES MANAGEMENT

ECONOMICS OF WATER MANAGEMENT AND USE OF LAND RESOURCES

RESULTS OF NUMERICAL RESEARCH OF DISCHARGE CAPACITY OF A SPILLWAY WITH A WIDE THRESHOLD

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Abstract

The problem of dramatically changing river flow in the reaches of hydro-technical structures is considered in the paper. The discharge capacity of these structures is determined by the well-known formula of a spillway with a wide threshold based on the Saint-Venant system of equations. It was stated that the Froude number in an undisturbed flow increases at the narrowing of the channel and decreases with its widening, and in a turbulent flow the Froude number decreases with the narrowing of the channel, and increases with its widening. Proceeding from this, the transition of a flow from a calm regime to a turbulent one can occur only when the form of the channel transfers from a narrow form to a wide one; in the narrowest section of the channel flow, the Froude number and the kinetic parameter is Fr=Pk=1,0. The characteristic features of the flow coefficient of a spillway with a wide threshold are identified and a recommendation is given for its determination. The main factor that affects the discharge capacity of a spillway with a wide threshold is analyzed. It is determined by the product of the values of the hydraulic resistance and compression coefficients in the inlet of the structure.

Key words: spillway with a wide threshold, channel, dramatically changing flow, Froude number, Reynolds number, curve of water free surface, hydrodynamic equation, average rate, calm mode, turbulent mode, flow rate, coefficient.

The purpose of research: Spillways with a wide threshold are widely used in the practice of hydro-technical engineering, for example, in the inlet of spillway structures at water flow, in water intake, etc. One of the main hydrodynamic parameters of a spillway with a wide threshold is its discharge capacity, determined by the well-known formula of a spillway with a wide threshold in a narrowing channel, in weirs, in the sites of jet-directing dams, narrowing flood-lands, approaches to bridges, and in dams with partial discharge of high floods on the flooded plains. Since the above calculation formula is used in various problems of computational fluid dynamics, its study is the aim of this work

The methods of research: In conditions of sharp variability of hydrological processes in rivers, the channel capacity increases in a relatively short period of time, which is accompanied by an abrupt change in water flow. This type of flow can occur in riverbeds and be of a natural origin (floods and high waters); in hydro-systems and reservoirs they can be the flood release waves from an overlying hydro-system and the breaking waves caused by hydrodynamic accidents. In this case, the role of the spillway is played by the gash in the dam body [1, 2,3,4].

In numerical studies of dramatically changing flows, the inclusion into the calculation program of an internal boundary condition approximating the spillway formula greatly complicates it. It should be noted that the calculation formula of general type of a spillway with a wide threshold can be obtained from the equation of the curve of water free surface in the channel, which is a partial case of the Saint-Venant equation of motion for a steady flow [5,6,7,8,9]. From the properties of the equation of free surface curve it is easy to show that the transition of a steady river flow from a calm flow regime (the Froude number and the kinetic parameter are less than unity (Fr <1 or Pk <1)) to a turbulent one (Fr> 1 or Pk> 1) is possible in the narrowing/widening sections of the river. This property of channel flows is similar to the property of pressure gas flows in Laval nozzles; it is a striking example of the hydro-gas-dynamic analogy discovered by the classical scholars of the hydrodynamics N.E. Zhukovsky and D. P. Ryabushinsky [10, 11].

To obtain an improved calculation formula for impounded spillway with a wide threshold using the curve of free surface of the water flow directly, without the Belange hypothesis, the following hydrodynamic equations of motion of water flow are used [12,13,14,15]:

$$\begin{cases} \dfrac{\partial \omega}{\partial t}+\dfrac{\partial Q}{\partial x}=0, \\ \dfrac{\partial Q}{\partial t}+\dfrac{\partial \alpha Q^2/\omega+gS}{\partial x}-g\left.\dfrac{\partial S}{\partial x}\right|_{Z_{fs}=const}+\int_B \tau/\rho dy=0, \end{cases} \qquad (1)$$

where: t – is the time, x, y are the length of flow along the channel and the width of flow across the channel, respectively, a is the momentum correction considering the form of velocity diagram, g is the acceleration of gravity, Q is the water flow through the gate, ω is the cross-sectional area, S is the static moment of the cross sections of the relatively free surface, $S=\omega h_{цm}$, $h_{цm}$ - is the depth of the center of gravity of the flow cross section, Z_{fs} is the mark of the free surface, B is the channel width along the top, ρ is the density of water, and τ is the friction stress along the gate width. For water flow, the value of $\int_B \tau/\rho dy$ is expressed using Chezi or Darcy-Weisbach formulas generally accepted in the hydraulics; Darcy-Weisbach formula is

$$\int_B \tau/\rho dy=\lambda\frac{V^2}{2g}\chi \qquad (2)$$

where: $V=Q/\omega$ –is the average water rate, χ - wetted perimeter, λ – the Darcy-Weisbach hydraulic friction coefficient, using Manning formula [16.17]

$$\lambda = \frac{2gn^2}{R_h^{1/3}} \tag{3}$$

where: n is the bottom roughness, $R_h^{1/3} = \omega/\chi$ - the hydraulic radius, .

The propagation velocity of waves of small amplitude along the channel, corresponding to equation (1), is determined by the formulas:

$$C = \sqrt{g\frac{\omega}{B}} \tag{4}$$

$$\omega\frac{\partial V}{\partial t} + V\frac{\partial \omega}{\partial t} + V\frac{\partial \omega V}{\partial x} + V\omega\frac{\partial V}{\partial x} + {} + g\frac{\partial S}{\partial x} - g\frac{\partial S}{\partial x}\Big|Z_{fs} = const + \frac{\lambda}{2}V^2\chi = 0 \tag{5}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{g}{\omega}\left(\frac{\partial S}{\partial x} - \frac{\partial S}{\partial x}\Big|Z_{fs} = const\right) + \frac{\lambda}{2}\frac{V^2}{R} = 0 \tag{6}$$

$$S = \int_{Z_{rb}}^{Z_{fs}} B(Z_{fs} - z)dz \tag{7}$$

To differentiate the static moment of the cross section, the formula known from mathematical analysis [18] is used, where the function is:

$$F(x) = \int_{\alpha(x)}^{\beta(x)} \Phi(x,y)dy \tag{8}$$

$$\frac{dF}{dx} = \int_{\alpha}^{\beta}\frac{\partial \Phi}{\partial x}dy + \Phi_{|y=\beta}\frac{d\beta}{dx} - \Phi_{|y=\alpha}\frac{d\alpha}{dx} \tag{9}$$

$$\begin{aligned}\frac{\partial S}{\partial x} &= \frac{\partial}{\partial x}\int_{Z_{rb}}^{Z_{fs}}(Z_{fs} - z)Bdz = \int_{Z_{rb}}^{Z_{fs}}\frac{\partial (Z_{fs} - z)B}{\partial x}dz + {}\\ &+ (Z_{fs} - z)B\Big|z = Z_{fs}\frac{\partial Z_{fs}}{\partial x} - (Z_{fs} - z)B\Big|z = Z_{Z_{rb}}\frac{\partial Z_{rb}}{\partial x} = \\ &= \int_{Z_{rb}}^{Z_{fs}}\frac{\partial Z_{fs}}{\partial x}Bdz + \int_{Z_{rb}}^{Z_{fs}} Z_{fs}\frac{\partial B}{\partial x}dz - HB\frac{\partial Z_{rb}}{\partial x} = \\ &= \frac{\partial Z_{fs}}{\partial x}\int_{Z_{rb}}^{Z_{fs}}Bdz + Z_{fs}\int_{Z_{rb}}^{Z_{fs}}\frac{\partial B}{\partial x}dz - HB\frac{\partial Z_{rb}}{\partial x} = \\ &= \omega\frac{\partial Z_{fs}}{\partial x} + Z_{fs}\int_{Z_{rb}}^{Z_{fs}}\frac{\partial B}{\partial x}dz - HB\frac{\partial Z_{rb}}{\partial x}\end{aligned} \tag{10}$$

$$\frac{\partial S}{\partial x}\Big|Z_{fs} = const = Z_{fs}\int_{Z_{rb}}^{Z_{fs}}\frac{\partial B}{\partial x}dz - HB\frac{\partial Z_{rb}}{\partial x} \tag{11}$$

So,

$$\frac{\partial S}{\partial x} - \frac{\partial S}{\partial x}\Big|Z_{fs} = const = \omega\frac{\partial Z_{fs}}{\partial x} \tag{12}$$

and

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial Z_{fs}}{\partial x} + \frac{\lambda}{2}\frac{V^2}{R} = 0 \tag{13}$$

hence:

$$\frac{\partial V}{\partial t} + \frac{\partial \left(V^2/2 + gZ_{fs}\right)}{\partial x} + \frac{\lambda}{2}\frac{V^2}{R} = 0 \tag{14}$$

Equation (14) with a steady flow regime, at $\frac{\partial V}{\partial t} = 0$, is the equation of the free surface curve in the channel. At *λ = 0*, it turns into the Bernoulli equation:

$$\frac{d\left(V^2/2 + gZ_{fs}\right)}{dx} + \frac{\lambda}{2}\frac{V^2}{R} = 0 \tag{15}$$

Given the stationary nature of the flow in the computational domain, we get:

$$\frac{d\left(Q^2/2\omega^2 + gh\right)}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{R} \tag{16}$$

If to assume a channel of rectangular cross section, with varying width and bottom mark along the flow, then in each gate we get *ω=Bh* where *B* and *h* can vary along x. Instead of (16), we get

$$\left(g - \frac{Q^2}{\omega^3}\frac{\partial \omega}{\partial h}\right)\frac{dh}{dx} - \frac{Q^2}{\omega^3}\frac{\partial \omega}{\partial B}\frac{dB}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{R} \tag{17}$$

B case of arbitrary form of the channel, we get

$$\frac{\partial \omega}{\partial h} = B \tag{18}$$

$$\begin{aligned}&\left(g - \frac{V^2B}{Bh}\right)\frac{dh}{dx} - \frac{V^2h}{Bh}\frac{dB}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{R}\\ &\left(1 - \frac{V^2}{gh}\right)\frac{dh}{dx} - \frac{V^2h}{Bgh}\frac{dB}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{hR}\end{aligned} \tag{19}$$

$$R = \frac{Bh}{(B+2h)} \tag{20}$$

The following formula is introduced,

$$Fr = \frac{V^2}{gh} \tag{21}$$

The Froude number is defined by the formula

$$(1 - Fr)\frac{dh}{dx} = I + Fr\frac{h}{B}\frac{dB}{dx} - \frac{\lambda}{2}\frac{V^2}{gR} \tag{22}$$

$$\frac{dh}{dx} = \frac{I + Fr\frac{h}{B}\frac{dB}{dx} - \frac{\lambda}{2}\frac{V^2}{gR}}{1 - Fr} \tag{23}$$

In cases where the friction effect is small, it follows from (23) that

at narrowing of channel cross section (*I<0*, $\frac{dB}{dx}$*<0*):

- in a calm state of the flow (*Fr <1, Pk <1*) the depth decreases, an increase in the average flow rate is observed;

- in the turbulent state of the flow (*Fr> 1 Pk> 1*), the depth of the flow increases, and the average rate decreases;

at widening of channel cross section (*I>0*, $\frac{dB}{dx}$*>0*):

- in a calm flow (*Fr <1*), the depth increases, and the flow rate decreases,

- in a turbulent flow (*Fr> 1*), the depth decreases and the rate increases.

Research results and discussion: According to the above statements, it was found that the Froude number in a calm stream increases with the narrowing of the channel and decreases with its widening; and in a turbulent flow, the Froude number decreases with the narrowing of the channel, and increases with its widening. Proceeding from this, the transition of the flow from a calm flow regime to a turbulent one can occur only when the channel form transfers from a narrow form to a wide one, and in the narrowest section of channel flow the Froude number and the kinetic parameter are Fr = Pk = 1.0.

According to the N.E. Zhukovsky and D.P. Ryabushinsky theory of the hydrodynamics, the depth of the flow is similar to the gas density, the pressure forces of the channel flow are similar to gas flow in the pipe, the propagation velocity of small-amplitude waves in the channel is similar to the velocity of sound in a gas, the squared Mach number is analogous to the Froude number. A detailed description of the hydro-gas-dynamic analogy and its application in technology are given in the studies of many authors [8, 9, 10].

Consider the gate of the minimum area, in which the Froude number Fr = 1 and the flow parameters are critical, and mark it with an asterisk. Of course, the transition from a calm flow regime of a channel to a turbulent one is possible provided that the critical depth of the flow is not flooded from the downstream side. So,

$$Fr = \frac{V_*^2}{gh_*} = 1 \qquad (24)$$

These considerations were carried out with a slight effect of hydraulic friction along the channel, where the basic conditions of the Bernoulli equation are satisfied.

Suppose that the channel is wide in the upstream site (a reservoir), the head velocity in this zone can be neglected. Let the depth of the water in the reservoir above the bottom of critical section of the channel be equal to H. Then the Bernoulli equation will have the form:

$$H = h_* + \frac{V_*^2}{2g} \qquad (25)$$

Considering (24), $V_*^2 = gh_*$

$$H = \frac{3}{2}h_* \qquad (26)$$

From here follows the widely known in hydraulics formula of a spillway with a wide threshold

$$Q = B_* h_* V_* = B_* h_* \sqrt{gh_*} = B_* \sqrt{g} h_*^{3/2} = \\ = B_* \sqrt{g}\left(\frac{2}{3}H\right)^{3/2} = mB_* \sqrt{2g} H^{3/2} \qquad (27)$$

where – *m* is the coefficient of water flow through an impounded spillway with a wide threshold (27) obtained by some mathematical transformations of equation (14) directly, without using the Belange hypothesis.

$$m = \left(\frac{2}{3}\right)^{3/2} = 0{,}385 \qquad (28)$$

In the general case $\omega = \omega(h, x)$, and

$$\frac{d\omega}{dx} = \frac{\partial\omega}{\partial h}\frac{dh}{dx} + \frac{\partial\omega}{\partial x} = B\frac{dh}{dx} + \frac{\partial\omega}{\partial x} \qquad (29)$$

$$\left(g - \frac{Q^2}{\omega^3}B\right)\frac{dh}{dx} - \frac{Q^2}{\omega^3}\frac{\partial\omega}{\partial x} = gI - \frac{\lambda}{2}\frac{V^2}{R} \qquad (30)$$

$$\left(g - \frac{V^2}{\omega}B\right)\frac{dh}{dx} - \frac{V^2}{\omega}\frac{\partial\omega}{\partial x} = gI - \frac{\lambda}{2}\frac{V^2}{R} \\ (1 - Fr)\frac{dh}{dx} - \frac{V^2}{g\omega}\frac{\partial\omega}{\partial x} = I - \frac{\lambda}{2}\frac{V^2}{Rg} \qquad (31)$$

$$\frac{dh}{dx} = \frac{I + \frac{V^2}{g\omega}\frac{\partial\omega}{\partial x} - \frac{\lambda}{2}\frac{V^2}{Rg}}{1 - Fr} \qquad (32)$$

In cases where the effect of friction is small, it follows from (32) that at section narrowing

$$(I < 0, \quad \frac{\partial\omega}{\partial x} < 0): \qquad (33)$$

- in a calm flow (*Fr <1*), the depth decreases, and the flow rate increases,

- in a turbulent flow (*Fr> 1*), the depth increases, and the flow rate decreases,

at section widening (*I> 0,> 0*):

- in a calm flow (*Fr <1*), the depth increases, and the flow rate decreases,

- in a turbulent flow (*Fr> 1*), the depth decreases, and the flow rate increases.

As in the case of a rectangular section, the gate (with the Froude number $Fr=V^2/gh=1$) is critical with a minimum area; water consumption is determined by formula Q=V* ω*, where V* and ω* are the water rate and the area of critical section. To determine the flow rate from a large reservoir through an impounded spillway with a wide threshold of non-rectangular shape, the critical depth of the flow is determined by the Bernoulli equation

$$H = \frac{Q^2}{2g\omega^2(h_*)} + h_* \qquad (34)$$

In the general case, the critical depth h* is determined from (30) approximately, although in some cases, for example, for a triangular channel, it can be found analytically.

Thus, the formula for determining the water flow rate through an impounded spillway with a wide threshold (27) is found directly from the formula for the curve of water free surface (5), without using the Belange hypothesis. Note that an analogue of formula (14) for pressure flows is the well-known Saint-Venant-Vancel formula [5], which allows us to determine the gas flow rate from the pressure tank through the nozzles (assuming the process is adiabatic); the method is widely used in engineering practice.

Since the equation of the curve of water free surface is a consequence of the Saint-Venant equations, in numerical calculations of flows over spillways with a wide threshold (and on polygonal spillways of flat profile) it is possible to use numerical methods for solving the Saint-Venant equations directly, by the "through" method, without introducing the spillway formula as internal boundary condition. Of course, this does not apply to the flow calculation in spillway sites of practical profile or with a thin rib, with large jet curvature, due to which the pressure distribution over the depth differs from hydrostatic one; this eliminates the application of the Saint-Venant equations.

For flows with a small jet curvature, acceptable for the Saint-Venant equations, the flow coefficient $m=2/3^{3/2}$ is maximal and achievable only in the absence of any hydraulic losses in the inlet section.

A numerical experiment was conducted to study the flow passing over an impounded spillway with a wide threshold by A.N. Militeev explicit finite difference scheme adapted for the channels of arbitrary form [6,7,8]. In the experiment, the channel of rectangular section without inclination and friction was abruptly compressed 100 times, and then abruptly expanded 100 times.

On the finite-difference grid, the spillway was simulated by two narrowed gates, the flow rate was set as the boundary condition at the input to the computational domain, and the Froude number Fr was set at the output. Generally speaking, it could be set to be greater than 1, but the Froude number was Fr = 0.12. As the initial condition, the water flow rate over the entire section was taken equal to 0, a large depth was set above the narrowed site, and a depth corresponding to the inlet flow rate and the Froude number at the outlet of the domain was set below the narrowed site.

In a numerical experiment, after a certain time, a flow regime was established in the channel, in which the depth in the upstream substantially exceeded the depth in the downstream; below the narrowed zone there appeared a section of turbulent flow, a hydraulic jump, and a section of calm flow. In the absence of bottom friction, the solution to the problem of numerical simulation of the driving away section of hydraulic jump is impossible, but this is not the subject of this study. Figure 1 shows a graph of the value of $m=\frac{Q}{B_*\sqrt{2g}H^{3/2}}$ obtained in a numerical experiment, which in a steady-state regime is the flow coefficient of the spillway. In the experiment, the flow coefficient of the spillway is equal to m≈0.394, which practically does not differ from the theoretical value of m≈0.385.

When setting the hydraulic friction in the narrowing section with a step of a finite difference grid of 20 H, the flow coefficient of the spillway decreases (Fig. 2).

Conclusions and recommendations: The equation of

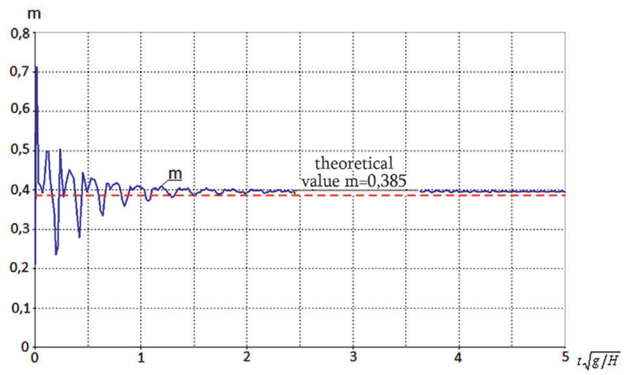

Fig. 1. The change of the value of $m=\frac{Q}{B_*\sqrt{2g}H^{3/2}}$ ***in time***

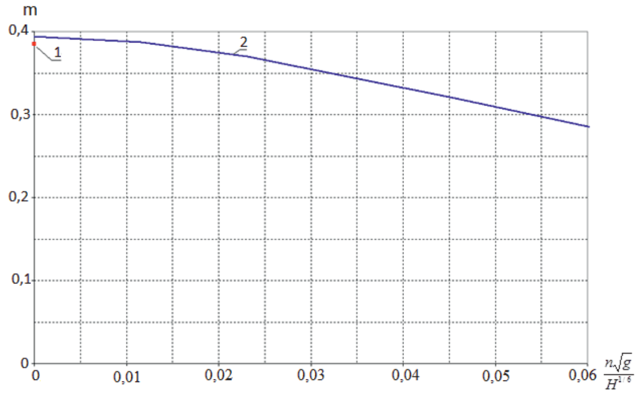

Fig. 2. The effect of bottom roughness n on the value of the flow coefficient of the spillway m. Notations: 1 - theoretical value of m at n = 0; 2 – values of m obtained in numerical experiments.

the curve of water free surface is a consequence of the Saint-Venant equations and numerical calculations of flows over spillways with a wide threshold (and polygonal spillways of flat profile). It is concluded that it is possible to use numerical methods for solving the Saint-Venant equations directly, by the “through” method, without introducing the spillway formula as an internal boundary condition. This does not apply to calculations of spillways of practical profile or with a thin wall, with large jet curvature, due to which the pressure distribution over the depth differs from hydrostatic one; this eliminates the application of the Saint-Venant equations. For flows with a small jet curvature, acceptable for the Saint-Venant equations, the flow coefficient $m=\left(\frac{2}{3}\right)^{3/2}=0.385$ is maximal and is used in the absence of any additional hydraulic losses at the inlet section.

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