

## 2D MODELLING OF A SENSOR FOR ELECTRICAL CAPACITANCE TOMOGRAPHY IN ECTSIM TOOLBOX

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**Abstract.** Electrical capacitance tomography is used to visualize a spatial distribution of dielectrical permittivity of materials placed in a tomographic sensor. An image is reconstructed from measurements of mutual capacitances of electrodes placed around the examined volume. This technique is characterized by very high temporal resolution – it is possible to achieve even few thousands of images per second. One of drawbacks of the method is low spatial resolution. Electrical capacitance tomography is mainly used in industry, e.g. for multiphase flow visualization. One of important elements of a tomographic system is a sensor which parameters influence quality of measurements and therefore affects quality of reconstructed images. In the Division of Nuclear and Medical Electronics a Matlab toolbox called ECTsim was developed. It is used for modelling of sensors, simulations of electrical field and image reconstruction. In this article we present the latest improvement which is modelling of a sensor using algebra of sets. Using primitive elements like rectangle and sector of a ring it is possible to perform operations like union, intersection and difference of two elements with a designed language. With such tools it is easy to prepare complex models of tomographic sensors which have different geometries. In this paper we show two models of sensors with different geometry in order to show how ECTsim solves forward problem.

**Keywords:** electrical capacitance tomography, 2D modelling, algebra of sets

### DWUWYMIAROWE MODELOWANIE SONDY DO ELEKTRYCZNEJ TOMOGRAFII POJEMNOŚCIOWEJ W PAKIECIE ECTSIM

**Streszczenie.** Elektryczna tomografia pojemnościowa służy do obrazowania rozkładu przenikalności elektrycznej materiałów w sondzie. Obraz rekonstruowany jest dzięki pomiarom pojemności wzajemnych elektrod umieszczonych wokół badanego obszaru. Ta technika obrazowa charakteryzuje się dużą rozdzielczością czasową – możliwe jest obrazowanie nawet kilku tysięcy przekrojów na minutę. Wadą jest niska przestrzenna zdolność rozdzielcza. Elektryczną tomografię pojemnościową stosuje się w przemyśle, między innymi do obrazowania przepływów wielofazowych. Istotnym elementem systemu tomograficznego jest sonda, której parametry wpływają na jakość pomiaru, a tym samym na jakość rekonstruowanego obrazu. W Zakładzie Elektroniki Jądrowej i Medycznej stworzono pakiet ECTsim uruchamiany w środowisku Matlab, który służy do modelowania sond tomograficznych, symulacji pola elektrycznego oraz rekonstrukcji obrazów. W niniejszym artykule opisano najnowszą modyfikację pakietu polegającą na modelowaniu sondy przy pomocy algebry zbiorów. Wprowadzono podstawowe kształty geometryczne, takie jak prostokąt i wycinek pierścienia, oraz zaproponowano język opisujący operacje sumowania, odejmowania i iloczynu elementów, co pozwala na proste tworzenie złożonych modeli sond tomograficznych o różnych geometriach. W artykule pokazujemy modele dwóch różnych sond tomograficznych i prezentujemy, jak ECTsim rozwiązuje problem prosty.

**Słowa kluczowe:** elektryczna tomografia pojemnościowa, modelowanie dwuwymiarowe, algebra zbiorów

### Introduction

Electrical capacitance tomography is an imaging technique used to visualize a spatial distribution of electrical permittivity of materials placed in a tomographic sensor. An image is reconstructed from measurements of mutual capacitances of electrodes placed around the examined volume [1]. This technique is characterized by very high temporal resolution – it is possible to achieve even few thousands of images per second. One of drawbacks of the method is low spatial resolution which comes from small number of electrodes used for measurements. It is not possible to place many electrodes because this means decrease in size of electrodes and therefore decrease of measured capacitances which are already low – from tens of femtofarads to single picofarads [6]. Electrical capacitance tomography is mainly used in industry, e.g. for multiphase flow visualization or monitoring of processes which involve combustion [2][4]. One of important elements of a tomographic system is a sensor which parameters (geometric dimensions, number of electrodes) influence quality of measurements and therefore affects quality of reconstructed images. In the Division of Nuclear and Medical Electronics a Matlab toolbox called ECTsim was developed. It is used for modelling of sensors, simulations of electrical field and image reconstruction. In this article we present the latest improvement which is modelling of a sensor using algebra of sets. Using primitive elements like rectangle and circle it is possible to perform operations like union, intersection and difference of two elements with a designed language. With such tools it is easy to prepare complex models of tomographic sensors which have different geometries.

### 1. ECTsim

ECTsim is a Matlab toolbox which purpose is to aid studies on electrical capacitance tomography. It allows to perform modelling and simulations in two dimensions. A square mesh is used to discretize modelled sensor. A cell method is used to establish equations which describe distribution of electrical field in the modelled sensor [3]. Linear equations system is solved using specialized algorithm of Gaussian elimination for band diagonal matrix. The simulations are performed three times: for the empty sensor, the sensor filled with a phantom and the sensor fully filled with a material with maximum permittivity used in an experiment. Those simulations are performed in order to allow image reconstruction using normalized values of mutual capacitances. Sensitivity matrices for all cases are calculated as well as simulated capacitances. ECTsim allows to reconstruct images from simulated capacitances as well as measurements made with tomographs made in the Division of Nuclear and Medical Electronics. LBP, Landweber and Levenberg-Marquardt algorithms are used.

#### 1.1. Algebra of sets

Until now ECTsim allowed to create only cylindrical sensors. Only several parameters like diameter of the sensor, number of electrodes and their axial width could be set. To extend capabilities of the toolbox and give more flexibility to users a new way of defining sensors was implemented. Two primitive shapes were introduced:

- rectangle (Fig. 1a),
- sector of a ring (Fig. 1b).

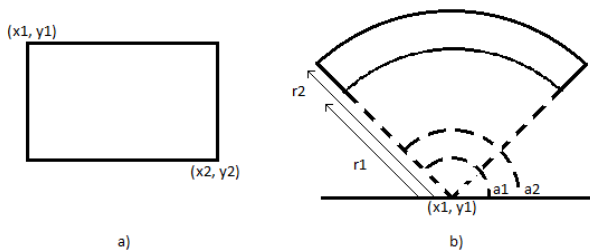


Fig. 1. Primitive shapes: a) rectangle; b) sector of a ring

Rectangle is defined by coordinates of the left upper corner  $(x1, y1)$  and the right bottom corner  $(x2, y2)$ . Sector of a ring is defined by inner radius  $r1$ , outer radius  $r2$ , angles  $a1$  and  $a2$  between left and right edge and a line  $x = 0$  and coordinates of the point from which both radii start  $(x1, y1)$ . Every shape has constant permittivity value inside. Having those two primitive shapes it is possible to create complex sensors by defining regions consisting of one or more shapes connected by operations known from algebra of sets like union, intersection and difference. Each region has a unique name which can be used to perform further operations on selected regions and is described by an equation which use unique names of other regions or primitive shapes. This way it is possible to define more complex regions which interact with another. Each model has to have required regions:

- boundary – a region in which boundary conditions are set (value of electric potential is known);
- calc\_potential – a region in which a distribution of electric field is calculated;
- calc\_sensitivity – a region in which a sensitivity matrix is calculated;
- model\_points – all points of model.

Following is an excerpt from a script which defines a sensor to show how to define a sensor using developed language:

```
'boundary', '+electrodes +screen', ...
'calc_potential', '+fov +insulators', ...
'calc_sensitivity', '+fov +insulators', ...
'model_points', '+fov +insulators +boundary'
```

An exemplary model is shown in Fig 2. A model consists of following elements: a field of view (FOV), a sensor wall (insulator1), electrodes (16 electrode elements), an insulator between electrodes (insulator2), an insulator which wraps the whole sensor (insulator3) and separates electrodes from a metal screen (screen) used to limit an influence of external electric fields on measurements.

Next the designed sensor is discretized with a square mesh. The index of mesh points is built for all regions.

Using designed sensor a forward problem could be solved. Assuming permittivity distribution in a sensor capacitance measurements may be simulated. The calculated sensitivity matrix of the tomographic sensor may be exported to a tomograph software which performs data acquisition and image reconstruction.

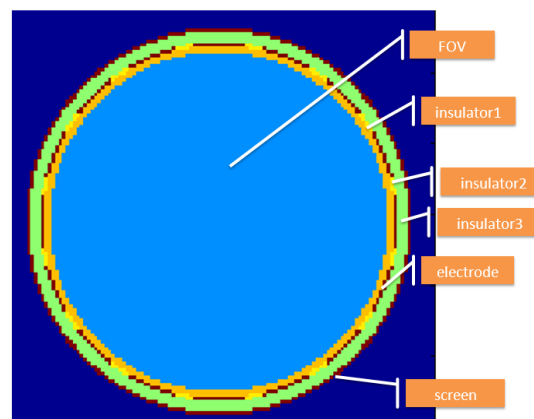


Fig. 2. A model of a cylindrical sensor consisting of a field of view (FOV), a sensor wall (insulator 1), electrodes (16 electrode elements), insulator between electrodes (insulator 2), an insulator which wraps the whole sensor (insulator 3) and a metal screen (screen)

## 2. Results

Two different types of sensors were modelled and simulated in order to show how ECTsim solves a forward problem.

### 2.1. Cylindrical sensor

A cylindrical sensor was modeled. It consisted of:

- field of view (diameter = 156 mm),
- wall ( $\epsilon = 2$ , thickness = 3),
- 16 electrodes (height = 80, angular width = 16 degrees,  $\epsilon = 10000$ , thickness = 0.1 mm),
- Insulator between electrodes and the screen ( $\epsilon = 2$ , thickness = 5 mm),
- screen ( $\epsilon = 10000$ , thickness = 0.1 mm).

A phantom consisting of 6 rods having electric permittivity equal to 3 was modeled.

The model was meshed using square elements having side length equal to 2.54 mm.

Permittivity in the model is shown in Fig 3. For clarity all high permittivity values (electrodes and the screen) were changed from 10000 to 4. Using cell method and the given distribution of electric permittivity it is possible to create a linear system matrix which describes electric potential in the model. The sparsity pattern of this matrix is shown in Fig 4. ECTsim calculates potential only in pixels which are inside the screen in order to decrease number of equations. In case of 2D forward problem matrices are small enough to be calculated using methods like Gauss elimination. This method was used in the described study to calculate potential. ECTsim is also able to use Krylov methods along with sparse matrices which decreases time needed for calculations.

The distribution of electric potential when the first electrode was the excitation electrode and other electrodes were the measurement electrodes is shown on Fig. 5. The sensitivity matrix is calculated using simulated electric fields using reciprocity rule. The sensitivity map for electrodes 1 and 9 is shown on Fig. 6.

Capacitance between electrodes can be numerically calculated using linear model  $C = S\epsilon$ . Simulated capacitances for the empty sensor, the sensor filled with the phantom and the sensor fully filled with the maximum value of permittivity are shown on Fig. 7. Logarithmic scale was used to better visualize small values. All values were scaled to the minimum value. It is noticeable that the phantom which was used for simulations doesn't change much capacitances between electrodes. The difference between neighbor and opposite electrodes in the modeled sensor is of two orders of magnitude. This shows that the range of capacitances which have to be measured is very wide. Fig. 8. shows the capacitance change between the empty and fully filled sensor.

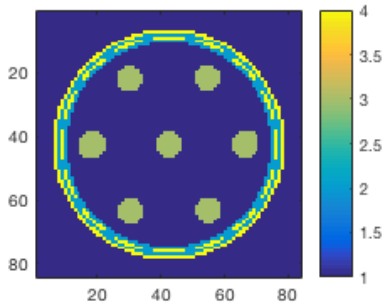


Fig. 3. Permittivity distribution in a model of a cylindrical sensor with 16 external electrodes filled with a phantom consisting of 6 rods made from a material with electric permittivity equal to 3

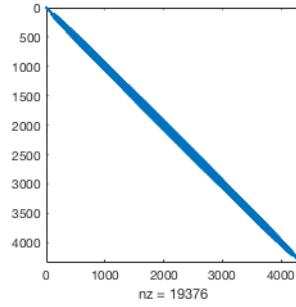


Fig. 4. Sparsity pattern of linear system matrix which describes electric potential in the model. Number of non-zero elements is equal to 19376

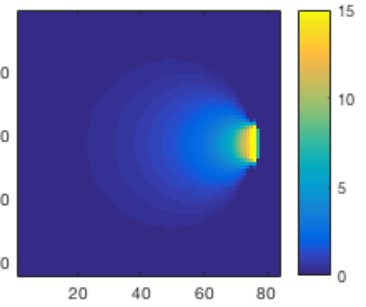


Fig. 5. Distribution of electric potential when the first electrode is the excitation electrode and other electrodes are measurement electrode

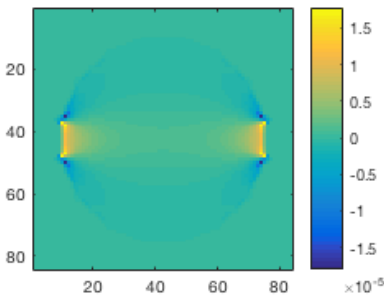


Fig. 6. Sensitivity map calculated from distributions of electric potential when first and ninth electrodes where excitation electrodes

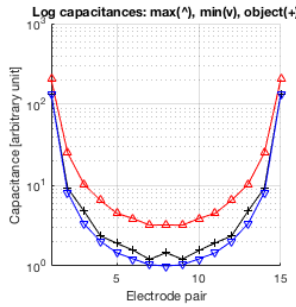


Fig. 7. Simulated capacitance measurements for the empty sensor, the sensor filled with the phantom and the sensor fully filled with the maximum permittivity

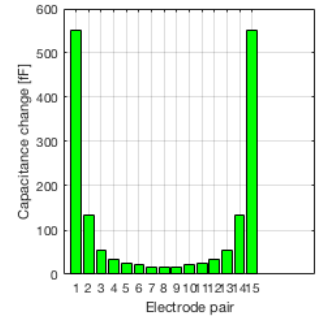


Fig. 8. Differences between simulated capacitances for the empty and fully filled sensor

## 2.2. Square sensor

A square sensor was modeled. It consisted of:

- field of view (diameter = 160 mm),
- wall ( $\epsilon = 2$ , thickness = 4),
- 16 electrodes (height = 160, width = 26 mm,  $\epsilon = 10000$ , thickness = 0.1 mm),
- Insulator between electrodes and the screen ( $\epsilon = 2$ , thickness = 5 mm),
- screen ( $\epsilon = 10000$ , thickness = 0.1 mm).

A phantom consisting of 6 rods having electric permittivity equal to 3 was modeled.

The model was meshed using square elements having side length equal to 2.54 mm.

Permittivity in the model is shown in Fig. 9. As previously, all high permittivity values (electrodes and the screen) were changed from 10000 to 4 for clarity of the figure. The sparsity pattern of this linear equation matrix prepared using the given distribution of permittivity is shown in Fig. 10.

The distribution of electric potential when the first electrode was the excitation electrode and other electrodes were the measurement electrodes is shown on Fig. 11. The sensitivity map for electrodes 1 and 9 is shown on Fig. 12.

Simulated capacitances for the empty sensor, the sensor filled with the phantom and the sensor fully filled with the maximum value of permittivity are shown on Fig. 13. Logarithmic scale was used to better visualize small values. All values were scaled to the minimum value. The difference between neighbor and opposite electrodes in the modeled sensor is of two orders of magnitude, just like in the cylindrical sensor. Fig. 14. shows the capacitance change between the empty and fully filled sensor.

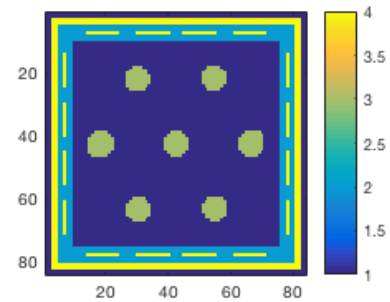


Fig. 9. Permittivity distribution in a model of a square sensor with 16 external electrodes filled with a phantom consisting of 6 rods made from a material with electric permittivity equal to 3

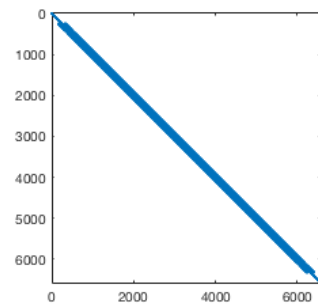


Fig. 10. Sparsity pattern of linear system matrix which describes electric potential in the model. Number of non-zero elements is equal to 28968

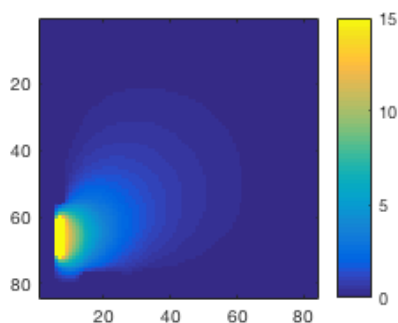


Fig. 11. Distribution of electric potential when the first electrode is the excitation electrode and other electrodes are measurement electrodes.

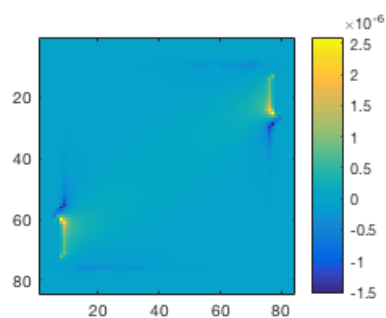


Fig. 12. Sensitivity map calculated from distributions of electric potential when first and ninth electrodes where excitation electrodes

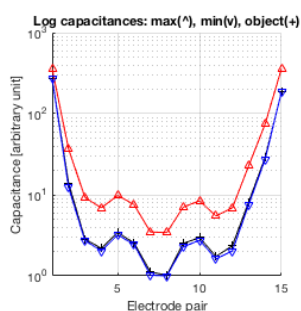


Fig. 13. Simulated capacitance measurements for the empty sensor, the sensor filled with the phantom and the sensor fully filled with the maximum permittivity. Fig 8. Differences between simulated measurements for the empty and fully filled sensor

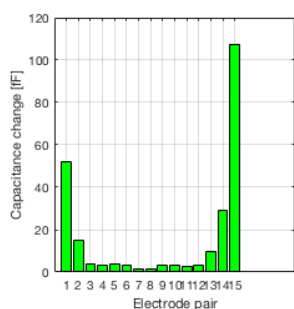


Fig. 14. Differences between simulated capacitances for the empty and fully filled sensor

### 3. Conclusions

ECTsim is a robust toolbox for two dimensional modeling and simulations in electrical capacitance tomography. It allows to model a sensor and evaluate its parameters before construction of a real equivalent of the model. In order to increase usefulness of the toolbox a new way of sensor definition was implemented. Using an intuitive language which allows to perform algebraic operations on primitive shapes it is possible to design a sensor with expected geometry. This feature extends possible fields in which ECTsim can be applied not only to cylindrical sensors but also cubical or even not regular geometries.

In this paper we showed simulations of models with different geometry: cylindrical and square. Distributions of electric potential for each model were calculated as well as sensitivity matrices. Numerically simulated capacitances were shown. This knowledge can be used to establish if hardware used for measurements would be able to measure such values.

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