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Bayesian Peer Calibration with Application to Alcohol Use

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Abstract

Peers are often able to provide important additional information to supplement self-reported behavioral measures. The study motivating this work collected data on alcohol in a social network formed by college students living in a freshman dormitory. By using two imperfect sources of information (self-reported and peer-reported alcohol consumption), rather than solely self-reports or peer-reports, we are able to gain insight into alcohol consumption on both the population and the individual level, as well as information on the discrepancy of individual peer-reports. We develop a novel Bayesian comparative calibration model for continuous, count and binary outcomes that uses covariate information to characterize the joint distribution of both self and peer-reports on the network for estimating peer-reporting discrepancies in network surveys, and apply this to the data for fully Bayesian inference. We use this model to understand the effects of covariates on both drinking behavior and peer-reporting discrepancies.

Keywords

Alcohol Use; Bayesian Comparative Calibration; Multiple Sources; Peer-Reports; Self-Reports

1 Introduction

Gold-standard measurements of individual-level alcohol consumption are rarely collected [1; 2; 3]. In the absence of a gold standard, self-reports or peer-reports are utilized as proxies [3]. The utility of peer-reports, termed collateral reports in the alcohol literature, has been extensively studied and reviewed in both non-college settings [4] and in the college setting [5]. In the college setting, while self-reports and peer-reports are often in agreement [4; 5],

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peer-reports of drinking tend to be higher than self-reports [6]. There is general agreement in the alcohol literature that college students' peer-reports of alcohol consumption are biased towards over-reporting [7; 8]. Hence both self-reports and peer-reports contain information about the true level of alcohol consumption, particularly if the discrepancies (i.e., the amount of over or under-reporting) contained within the peer-reports can be accounted for.

Measurement error due to self-reports of health behaviors, such as physical activity [9], food consumption [10], and alcohol consumption [11], is an important issue because ignoring measurement error may lead to incorrect inference [12; 13]. Existing measurement error models typically assume self-reports are unbiased [10] or contain systematic biases [14; 9; 15; 16]. In the presence of self-report error, another type of data on the health-related behavior is collected on each individual. Using both types of data, a method of estimation (i.e. method of moments) is employed to estimate all parameters in the measurement error models as well as the quantity of interest.

Another way of dealing with measurement error is absolute calibration [17; 18; 19], in which both a gold standard and error-prone measures are collected on a limited number of data points in order to infer the relationship between the gold standard and the error-prone measures. This relationship is then exploited to allow for measurement with the error-prone measure. Comparative calibration, which uses information from two or more error-prone measurement instruments, is applied when a gold standard is not available but there is prior information about the structure of the measurement error of each of the instruments [20; 18]. Comparative calibration has been implemented in both the frequentist [20; 21; 22; 23] and the Bayesian setting [24]. [18] has an extensive review of both absolute and comparative calibration approaches.

Both measurement error and calibration models deal with the situation where we are interested in variable Y, but instead we observe a proxy variable X that is related to Y [25], and both types of models allow that X may be biased for Y, prone to error, or both. One contrast between the two methods is that calibration models require more than one proxy variable for Y, say X and W, whereas measurement error models do not require this [26]. Further, measurement error models seek to identify and decompose the different sources of bias and variation that are included in the proxy measurements [27], whereas calibration models aim to learn about the measurement errors, but do not decompose these different sources of bias and variation included in the proxy measurements. Further, measurement error models may investigate measurement error by incorporating covariates into the estimation of the underlying values of interest in a linear mixed effects model [28], akin to a calibration model with a zero-bias assumption. Alternately, when multiple sources of information are available, measurement error models may allow for the measurement error to be modeled with covariates [29]. However, these measurement error models do not jointly estimate the relations between covariates and the underlying values of interest and the unknown bias, as we do in our Bayesian peer calibration with covariates model which we present below. Further, among measurement error models that make use of multiple sources, none are situated in a social network context which necessarily implies that individuals only report on their associates (those with whom they share a connection in the network, here representing a friendship) rather than all members of the network.

Social networks provide a setting in which there are multiple sources of information which are not gold-standards. Consider a social network in which individuals report on both themselves and their peers in the network. Neither self nor peer-reports are ideal measurements, but both types of reports contain information about the quantity of interest. In this paper we present a novel Bayesian comparative calibration model framework: Bayesian Peer Calibration (BPC) which utilizes both self-reports and peer-reports of alcohol consumption. We assume self-reports are unbiased but measured with error. Peer reports may be biased and are also assumed to be measured with error.

1.1 Motivating Study

The data motivating this analysis are drawn from a study of alcohol and drug use in a social network formed by college students residing in a freshman dormitory [30]. The primary objective of our analysis is to characterize the alcohol consumption of the participants through the use of self and peer-reports of the number of alcoholic drinks consumed on drinking days. In particular, this study seeks to identify characteristics that are associated with alcohol consumption. Though previous studies have collected both self and peer-reports of alcohol consumption, few have used both sources of information to estimate alcohol consumption. This paper is concerned with how to use both information sources which may provide contradictory information on alcohol consumption. In order to address this question we use a Bayesian Peer Calibration (BPC) model on the social network from the motivating study to generate the posterior distributions of individual-level alcohol consumption while using both sources of information. In Section 2 we develop the BPC models (both with and without covariates) for binary, count, and continuous data, in Section 3 we apply these models to the data, in Section 4 we perform a simulation to investigate the model's sensitivity to assumptions, and in Section 5 we present a brief discussion.

2 Bayesian Peer Calibration Models

2.1 Overview

We consider data drawn from a network of n nodes (representing members of the social network) with e directed edges (representing the presence or absence of peer nominations), represented by an $n \times n$ adjacency matrix A, where $A_{ij} \in \{0, 1\}$ and equals 1 if node i reports on node j (denoted as $i \to j$). For each of the n nodes in the network we have a self-report on a quantity of interest, denoted by z_i. Furthermore, for each node, there are reports on the same quantity of interest of their nominated alters, denoted by y_{ij} which is the peer-report of node j on node i. The observed data at each node i is $(z_i, y_{ij} : j \to i)$. We assume self-reports are made with mean-zero error and peer-reports are potentially subject to some systematic bias which we call a discrepancy. We assume that the true underlying quantity of interest is θ_i. The self-report z_i is an unbiased but possibly error-prone proxy for θ_i, such that $E(z_i) = \theta_i$. The peer-reports y_{ij} are subject to a systematic discrepancy that can be additive, multiplicative, or exponential depending on the type of outcome. Specifically, if we denote the discrepancy by γ_{ij}, the peer-reports can follow $E(y_{ij}) = \theta_i + \gamma_{ij}$, $E(y_{ij}) = \theta_i \gamma_{ij}$, or $E(y_{ij}) = \theta_i^{\gamma_{ij}}$.

We now introduce a three-level model which may include covariates, which we call the Bayesian Peer Calibration with Covariates(BPCC) model, or not include covariates, which we call the Bayesian Peer Calibration model (BPC). The first level of the model specifies the joint distribution of the observed data at each node, conditional on parameters θ_i and γ_j that respectively capture the node-specific mean and the alter-specific discrepancy. The second level characterizes variation in θ_i and γ_j, potentially allowing both to be influenced by covariates. At the third level, priors are introduced as needed.

We provide specific model formulations for continuous, binary, and count versions of z and y. In our application to the available data, we have interval-censored count data; hence we focus on posterior inference for Poisson distributed data. The appendices (Section 6) contain information about posterior inference for binary and continuous data.

2.2 Bayesian Peer Calibration Model

The general form of the first level of the BPC model is

$$z_i \sim F(\theta_i) \quad (1)$$

$$y_{ij} \sim G(\theta_i, \gamma_{ij}), \quad (2)$$

where the distributions F, G and the support of θ_i and γ_{ij} depend on the choice of the model. We treat three parametric formulations here. For Normal y and z, we assume

$$z_i \sim N(\theta_i, {\sigma_z}^2) \quad (3)$$

$$y_{ij} \sim N(\theta_i + \gamma_{ij}, {\sigma_y}^2), \quad (4)$$

where $\sigma_y^2 > 0$ and $\sigma_z^2 > 0$ are the variance parameters, and θ_i and γ_{ij} are real numbers. If y and z are Bernoulli distributed,

$$z_i \sim \mathrm{Ber}(\theta_i) \quad (5)$$

$$y_{ij} \sim \mathrm{Ber}(\theta_i^{\gamma_{ij}}), \quad (6)$$

where θ_i is between 0 and 1, and γ_{ij} is a positive real number. Lastly, if y and z are Poisson distributed,

$$z_i \sim \mathrm{Poi}(\theta_i) \quad (7)$$

$$y_{ij} \sim \mathrm{Poi}(\theta_i \gamma_{ij}), \quad (8)$$

and θ_i and γ_{ij} are positive real numbers. In each of these model formulations, the self-reports are assumed to be unbiased.

Level 1 can be parameterized in terms of generalized linear models for z and y, where

$$g\{\mathrm{E}(z_i)\} = \alpha_i \quad (9)$$

$$g\{\mathrm{E}(y_{ij})\} = \alpha_i + \phi_{ij}, \quad (10)$$

where $g(\cdot)$ is an appropriate link function. The GLM parameters map directly to the parameters in the cases above. For the normal case and identity link $g(x) = x$, $\alpha_i = \theta_i$ and $\phi_{ij} = \gamma_{ij}$. For binary data and log-log link $g(x) = \log\{-\log(x)\}$, we have $\alpha_i = \log\{-\log(\theta_i)\}$ and $\phi_{ij} = \log(\gamma_{ij})$. Finally, in the case of count data, using the link $g(x) = \log(x)$ yields $\alpha_i = \log(\theta_i)$, and $\phi_{ij} = \log(\gamma_{ij})$. These relationships are summarized in Table 1.

The GLM formulation easily accommodates covariate information, and allows the analyst to put separate priors on the mean and variance parameters. For example we would recommend a diffuse Normal prior on the α and ϕ parameters, and a flat uniform prior on the variance parameters. However, except for the normal case, the scaling of the GLM parameters may not always be natural for the application at hand. The parameterizations given above allows for more natural interpretation of model parameters and use conjugate priors for easier computation. Additionally, the posteriors for the model parameters have intuitive forms that transparently show from where the information is derived. Incorporation of covariates needs to be done carefully, but we provide full details for each formulation of the model.

In addition to assuming that self-reports are unbiased, we also assume that for all i, j $\gamma_{1j} = \gamma_{2j} = \cdots = \gamma_{nj} = \gamma_j$, where $ij, 2j, \ldots, nj$ are the indices for the nodes on which j provided a peer-report, which implies that each individual j has a constant discrepancy γ_j when reporting on peers. Finally we assume $\theta_i \perp\!\!\!\perp \gamma_j$, $\theta_i \perp\!\!\!\perp \theta_j$ and that $\gamma_i \perp\!\!\!\perp \gamma_j$. These constraints impose structure that is most easily understood in the continuous-data normal distribution case. Specifically, we have $E(z_i / \theta_i) = \theta_i$, $\mathrm{Var}(z_i|\theta_i) = \sigma_z^2$, $\mathrm{Cov}(y_{ij}, y_{ik} \mid \theta_i) = \mathrm{Var}(\theta_i)$, $\mathrm{Cov}(y_{ij}, z_i \mid \theta_i) = \mathrm{Var}(\theta_i)$, $\mathrm{Cov}(y_{ij}, z_i \mid \theta_i, \gamma_j) = 0$, and $\mathrm{Cov}(y_{ij}, y_{ik} \mid \theta_i, \gamma_j, \gamma_k) = 0$.

We proceed by specifying level two of the BPC model. We focus on the formulation for count data (which we assume to be Poisson distributed) due to our application. We assume that both θ_i and γ_j are each independent and Gamma distributed with shape parameters τ_θ, τ_γ and rate parameters κ_θ, κ_γ such that $E(\theta_i) = \tau_\theta / \kappa_\theta$, $E(\gamma_j) = \tau_\gamma / \kappa_\gamma$; this is denoted by

$$\theta_i \sim \text{Gam}(\tau_\theta, \kappa_\theta) \quad (11)$$

$$\gamma_j \sim \text{Gam}(\tau_\gamma, \kappa_\gamma). \quad (12)$$

The third level of the model specifies the priors. In our application, we use diffuse Gamma priors with hyperparameters a, b on τ_θ, κ_θ, τ_γ, κ_γ where a is the shape parameter and b is the rate parameter. Details regarding the full conditionals are in the appendices (Section 6).

The BPCC model is elaborated to include covariate effects in the components characterizing both the quantity of interest θ and the peer-reporting discrepancy γ. Carrying forward the Poisson formulation, we again assume that both θ and γ are Gamma distributed, the difference being that the expected value of both θ and γ are now functions of the covariates:

$$\theta_i | X_i, \beta, \omega_\theta \sim \text{Gam}(\omega_\theta e^{X_i \beta}, \omega_\theta) \quad (13)$$

$$\gamma_j | X_j, \alpha, \omega_\alpha \sim \text{Gam}(\omega_\gamma e^{X_i \alpha}, \omega_\gamma). \quad (14)$$

Hence, $\log E(\theta_i) = X_i \beta$ and $\log E(\gamma_j) = X_j \alpha$. The ω_θ and ω_γ parameters help determine the variance of θ and γ respectively such that the $\text{Var}(\theta_i) = e^{X_i \beta} / \omega_\theta$. For instance, if ω_θ is large relative to $e^{X_i \beta}$ then $\text{Var}(\theta_i)$ is small. This BPCC model specification also allows us to gain insight into the relationship between the covariates and θ_i and γ_j

In the third level of the BPCC model, we specify diffuse zero-mean Normal priors for β and α with a large variance σ^2, and diffuse Gamma priors for rate parameters ω_θ and ω_α with hyperparameters f and g. The full conditionals are detailed in the appendix (Section 6).

2.3 Posterior inference for count data

Under the BPC model framework (which does not include covariates), for each iteration of the MCMC we draw from the posterior distribution using full conditionals which have the following Gamma distributions:

$$P(\theta_i | \gamma, \theta_{-i}, \tau_\theta, \kappa_\theta, \tau_\gamma, \kappa_\gamma, a, b, z, y) \sim \text{Gam}(z_i + \sum_{j:j \rightarrow i} y_{ij} + \tau_\theta, 1 + \sum_{j:j \rightarrow i} \gamma_j + \kappa_\theta) \quad (15)$$

$$P(\gamma_j|\theta,\gamma_{-j},\tau_\theta,\kappa_\theta,\tau_\gamma,\kappa_\gamma,a,b,z,y)\sim\text{Gam}(\sum_{i:j\to i}y_{ij}+\tau_\gamma,\sum_{i:j\to i}\theta_i+\kappa_\gamma). \quad (16)$$

Full conditionals of the other parameters in this model are detailed in the appendices (Section 6. The expected value of the full conditionals of θ_i can be instructive:

$$\frac{z_i+\sum_{j:j\to i}y_{ij}+\tau_\theta}{1+\sum_{j:j\to i}\gamma_j+\kappa_\theta}=\frac{z_i}{1+\sum_{j:j\to i}\gamma_j+\kappa_\theta}+\frac{\sum_{j:j\to i}y_{ij}}{\sum_{j:j\to i}\gamma_j}\frac{\sum_{j:j\to i}\gamma_j}{1+\sum_{j:j\to i}\gamma_j+\kappa_\theta}+\frac{\tau_\theta}{\kappa_\theta}\frac{\kappa_\theta}{1+\sum_{j:j\to i}\gamma_j+\kappa_\theta}, \quad (17)$$

which is a weighted average of the self-report of person i (z_i), the discrepancy corrected peer-reports on person i, $\frac{\sum_{j:j\to i}y_{ij}}{\sum_{j:j\to i}\gamma_j}$, and the prior information about the mean value of the θ parameters, $\tau_\theta/\kappa_\theta$. We also obtain full conditionals of γ_j which have the expected value:

$$\frac{\sum_{i:j\to i}y_{ij}+\tau_\gamma}{\sum_{i:j\to i}\theta_i+\kappa_\gamma}=\frac{\sum_{i:j\to i}y_{ij}}{\sum_{i:j\to i}\theta_i}\frac{\sum_{i:j\to i}\theta_i}{\sum_{i:j\to i}\theta_i+\kappa_\gamma}+\frac{\tau_\gamma}{\kappa_\gamma}\frac{\kappa_\gamma}{\sum_{i:j\to i}\theta_i+\kappa_\gamma},$$

which is a weighted average of $\frac{\sum_{i:j\to i}y_{ij}}{\sum_{i:j\to i}\theta_i}$ and $\tau_\gamma/\kappa_\gamma$, the prior information about the mean value of the γ parameters.

We note that when an individual i has multiple peer-reports, and those peers have a high-level of agreement on the quantity of interest *theta*$_i$, then this high agreement would impact both the posterior distribution of θ_i, as well as the posterior distribution for each of the discrepancy parameters γ_j. First, the posterior distribution of θ_i will be pulled from the self-report value to the peer-reports, and should the peer-reports and self-reports be in high agreement, then the posterior distribution of θ_i will be quite narrow. Secondly, the discrepancy parameters for those reporting on individual i will be shrunk closer to zero.

In the BPCC model, which includes covariates, for each MCMC step we draw from the posterior distribution using full conditionals having the following Gamma distributions:

$$\begin{aligned}P(\theta_i|\theta_{-\mathrm{i}},\gamma,\alpha,\beta,\omega_\theta,\omega_\gamma,\mathbf{x},\mathbf{y},\mathbf{z}) &\sim \text{Gam}(z_i+\omega_\theta e^{X_i\beta}+\sum_{j:j\to i}y_{ij},1+\omega_\theta+\sum_{j:j\to i}\gamma_j)\\ P(\gamma_j|\theta,\gamma_{-\mathbf{j}},\alpha,\beta,\omega_\theta,\omega_\gamma,\mathbf{x},\mathbf{y},\mathbf{z}) &\sim \text{Gam}(\omega_\gamma e^{X_j\alpha}+\sum_{i:j\to i}y_{ij},\omega_\gamma+\sum_{i:j\to i}\theta_i).\end{aligned}$$

In the BPCC model, we obtain posterior draws of θ_i which are weighted averages of the self-report of person i (z_i), the discrepancy corrected peer-reports on person i $\frac{\sum_{j:j\to i}y_{ij}}{\sum_{j:j\to i}\gamma_j}$, and the information about the mean value of the θ parameter for those with covariate profile X_i,

$e^{X_i\beta}$. We also obtain posterior draws of γ_j which are weighted averages of the peer-reports' discrepancy for person j, $\frac{\sum_{i:j\to i} y_{ij}}{\sum_{i:j\to i} \theta_i}$, with $e^{X_j\alpha}$, the posterior the mean value of the peer-reporting discrepancy for those with covariate profile X_j. Similar to the BPC, the posterior distributions of the θ parameters are less diffuse when there is more precise information about the γ parameters, and vice-versa.

With the use of covariates in the BPCC model, we can gain insight into the relationships between the covariates with the peer-reporting discrepancies γ and the quantity of interest θ. Additionally, by including covariates to model θ and γ we are relaxing the independence assumption in the BPC model by only imposing conditional independence between θ and γ such that $\theta_i \perp \gamma_j | X_i, X_j$.

It is important to note that these models are subject to several assumptions. The assumption that the θ and γ parameters are independent (in the BPC model) or conditionally independent (in the BPCC model), may not hold. Additionally, the assumption that all self-reports have no systematic bias may not be true. We proceed with these assumptions and will address them in Section 4 and Section 5.

3 Application

The objective of this analysis is to learn about the unknown average number of alcoholic drinks consumed on drinking days, and the association between certain personal characteristics and alcohol consumption among study participants. To do so, we formulate Bayesian comparative calibration models (BPC, BPCC) that allow for drawing from the posterior distribution of peer-reporting discrepancies, in reports of alcohol consumption, and use the peer-reports corrected for the posterior draws of the discrepancies along with self-reports to learn about the posterior distribution of the number of alcoholic drinks consumed on drinking days for each participant in the study. We wish to learn how covariates are associated with the number of alcoholic drinks consumed on drinking days as well as which covariates are associated with over or under reporting the alcohol consumption of peers. All simulations and analyses are performed in the R statistical programming language [31]. Please contact the corresponding author if you are interested in the R code.

3.1 Data

In the social network study, data are collected from residents of a freshman dormitory. Each participant is 18 years old or older when the survey is administered and is asked to report the number of alcoholic drinks they consume on drinking days (self-reports) as either 0, 1–2, 3–4, 5–6, or 7 or more drinks. Further, participants are asked to provide demographic information, including their age, sex, race, sexual orientation, and whether or not they intend to join a fraternity or sorority. Additionally, each participant is asked to nominate which of the other participants are important to them. If participant A nominates participant B as important to them, then B is an alter of A. Each participant can nominate alters and serve as an alter for other participants. Lastly, each participant is asked to report the number of alcoholic drinks each of their alters consume on drinking days (0, 1–2, 3–4, 5–6, or 7 or more drinks), which we refer to as peer-reports. By using nominations to connect participants, we construct a network of social relationships.

Of the 129 participants included in the sample, 4 did not nominate alters and were not nominated, and are excluded from the present analysis. This leaves 125 participants who are included in the analysis, with 507 nominations. Of these 507 nominations, there are 366 peer-reports on alcohol consumption on 108 different dorm residents. In the sample of 125 participants, 47.2% are male, 24% report considering joining a fraternity or sorority, 90.4% identify as heterosexual, and 59.2% identify as white. Among the 108 participants whose alters report on their alcohol consumption, 48.1% are male, 23.1% report considering joining a fraternity or sorority, 88.9% identify as heterosexual, and 62.0% identify as as white. Self-reports and peer-reports of alcohol consumption are presented in Figure 1, and peer-reports stratified by self-reports are presented in Figure 2. The most common self-report of alcohol consumption was 3–4 drinks on drinking days, with 47 out of 129 providing this self-report (Figure 1). Further information about the UrWeb Study can be found in [30].

We treat the number of drinks as a count, and as such will apply the Poisson formulations of the BPC and the BPCC. However, because the number of drinks are reported in intervals (0, 1–2, 3–4, 5–6, 7+) we are not certain of the exact number of drinks. In order to incorporate this loss of information in the data collection in the model, we perform a data augmentation step for each of the 35,000 iterations which allows us to apply the Poisson models. The data augmentation proceeds as follows. For iteration t, the self-report for i is generated from a truncated Poisson distribution $z_{it} \sim \text{Poi}(\theta_{it-1})$ such that each z_{it} value is within the specified interval of the self-report. Similarly, for iteration t, the alter report made by j on i is generated $y_{ijt} \sim \text{Poi}(\theta_{it-1}\gamma_{jt-1})$. Once we generate the count from the truncated Poisson distribution, we are able to apply the Poisson formulation.

3.2 Model Specification Without Covariates

In the BPC model θ_i and γ_j are the parameters of primary interest. The distribution of posterior means for these parameters is shown in Figure 3. The posterior draws of each θ_i stratified by self-reported drinks on drinking days are presented in the first column of Figure 4; we see that while the posterior distributions of the θ_i parameters are related to the self-reported number of drinks on drinking days, there is variation within strata which indicates that the information from the peer-reports and the self-reports has been combined to give us a different result than if we had ignored the peer-report information. For the BPC model, we carry out 35,000 iterations of Gibbs Sampling, and of these, discard the first 10,000 as burn-in and use every fifth of the final 25,000 simulations, leaving us with 5,000 simulations to calculate our posterior values of parameters. We assumed diffuse priors by specifying the hyper-parameters $a = 0.02$ and $b = 0.02$.

3.3 Bayesian Peer Calibration with Covariates

We specify diffuse priors on ω_θ, ω_γ, β, and α by specifying the hyper-parameters $f = 0.2$ and $g = 0.2$ and $\sigma^2 = 1000$. The f and g values specify a prior mean of $0.2/0.2 = 1$ and a prior variance of $0.2/0.2^2 = 5$ for both $\omega_\theta, \omega_\gamma$. As with the BPC model, for the BPCC model we have 35,000 iterations of Gibbs Sampling, discard the first 10,000 as burn in, and use every fifth of the remaining simulations, leaving us with 5,000 simulations to calculate our posterior values of parameters.

The parameters of most interest in the BPCC model are β and α which indicate the extent to which being male, having an interest in joining a fraternity or sorority, having a heterosexual sexual orientation, being white, and self-reporting zero drinks(used for α only) are associated with alcohol consumption and peer-report discrepancies respectively. We present the posterior values of β and α in Table 2. As the posterior distributions for β and α have no closed form, we used MCMC to draw the posterior values. For the posterior means of the β parameters, we can see that being male, interest in joining a fraternity or sorority, heterosexual sexual orientation and white race are associated with higher number of drinks on drinking days, though only interest in joining a fraternity or sorority and white race have 95% credible intervals that does not contain zero. For example, average consumption for a white male who is heterosexual and is interested in joining a fraternity is exp(0.44 + 0.26 + 0.36 + 0.00 + 0.55) = 5.00 drinks on drinking days, whereas we expect a *non-white* male who is heterosexual and is interested in joining a fraternity or sorority to drink exp(0.44 + 0.26 + 0.36 + 0.00) = 2.89 drinks on drinking days. For the posterior means of the α parameters, we can see that interest in joining a fraternity or sorority and self-reporting zero drinks on drinking days are both associated with having discrepancies that could represent under-reporting, assuming that self reports are unbiased. For example, we expect that a white male who is heterosexual, is interested in joining a fraternity, and reported drinking zero drinks would have multiplicative peer-reporting discrepancies of exp(0.52 + 0.24 − 0.30 − 0.25 − 0.02 − 0.81) = 0.54, or under-reporting peer-alcohol consumption by about half.

Next we turn our attention to the distribution of the posterior values of the θ and γ parameters which represent the number of drinks consumed on drinking days, and discrepancies in reporting on an alter's drinks on drinking days respectively. These are displayed in Figure 5. The posterior distributions of θ stratified by self-reported drinks on drinking days are presented in the second column of Figure 4. Similar to the BPC model, we can see that the comparative calibration has utilized information from both the self and peer-reports since dorm-residents with the same self-reports have varying posterior predictive distributions of the θ which reflect the influence of the peer-reports.

3.4 Role of Peer-Reports

To understand the role of peer-reports we look at a model with only self-reports (naive model). We present results from the BPCC where we ignore the peer-report information, so that we can learn about the effect of peer reports on both covariates as well as on the model fit. Since only self-reports are considered, this Naive model is not a calibration model. In this model we are primarily interested in the β parameters, and present the posterior means and credible intervals for these parameters in the third column of Table 2. For the posterior means of the β parameters, we can see that male sex, interest in joining a fraternity or sorority, heterosexual sexual orientation and white race are associated with higher number of drinks on drinking days, though only white race has a 95% credible interval that does not contain zero. Comparing the posterior distributions for the β parameters from the model without peer-reports to the β parameters from the BPCC model (Table 2), we see that there are some differences. Most notably, the 95% credible interval for the β corresponding to interest in joining a fraternity or sorority does not contain zero in the posterior distribution from the BPCC model, but does contain zero in the posterior distribution from the model without peer-reports. In other words, if we only used self-reports we expect that those interested in joining a fraternity or sorority drink $e^{0.12} = 1.13$ times the number of drinks than someone who is not considering joining a fraternity or sorority with a 95% credible interval that includes 1: (0.81, 1.57), whereas if we consider both self-reports and peer reports, we expect that those joining a fraternity or sorority have a $e^{0.36} = 1.43$ times the number of drinks as someone not interested in joining a fraternity or sorority with a 95% credible interval that does not include 1: (1.02, 2.01). This demonstrates that the inclusion of the peer-reports may result in different inferences than had we not included peer-reports in our models.

3.4.1 Comparing Model Fit—We utilized posterior predictive draws to assess the fit of these models [32]. At each step of the Gibbs sampler, we use the relevant parameters to “predict” self-reports z according to our model, which are posterior predictive draws. For example, if at the t^{th} step of the Gibbs sampler, we have θ_{it} would generate predictive values for the self-reports as $z_i \sim Poisson(\theta_{it})$ for each self-report observed in the data. A well-fitted model will produce posterior predictive draws that closely approximate the data that were used to fit the model, while a poorly-fitted model will produce posterior predictive draws that diverge from the data used to fit the model. By comparing the posterior predictive distribution of different models, we are able to compare the fit of different models. Specifically, we generated 5,000 posterior predictive draws of the self-reports z. In order to compare the fit of the BPC and BPCC models, we found the proportion of posterior predictive z values that were within the interval observed in the data (0, 1–2, 3–4, 5–6, 7+). These are presented in the first two columns of Table 3. We also found the proportion of posterior predictive z and values that were within the interval observed in the data for the model without peer-reports, and present these in the third column of Table 3.

By comparing the posterior predictive z’s to the observed z’s, we can determine whether the fit of the model seems reasonable, and compare the model fits for the models, keeping in mind that the BPCC model requires estimation of additional parameters. Here we can see that the BPC and BPCC models have a comparable fit to the data. For example, about 46% of the posterior predictive z values for participants reporting 1–2 drinks were 1 or 2 for both the BPC and BPCC models. While the use of covariates does not seem to markedly change the model fit, the use of peer-reports seems to improve the model fit, particularly for participants who reported zero drinks.

4 Simulations Allowing for Self-Report Bias

We carried out simulations to investigate how the BPCC model performs when the assumption of unbiased self-reports is violated. We base these simulations on the UrWeb data (n=125 individuals), and we only use the Fraternity/Sorority covariate. We then generated simulated data under three conditions: 1.) self-reports are unbiased, 2.) self-reports are positively biased, with bias unrelated to the covariates in the BPCC model, and

3.) self-report bias is a function of the covariates included in the BPCC model. In each simulation we set $\alpha_1 = 0.52$, $\alpha_2 = -0.30$, $\beta_1 = 0.44$, $\beta_2 = 0.35$, which are the posterior means for the intercepts and the Fraternity/Sorority parameters. We set both ω_θ and ω_γ equal to 2.

In each simulation we first generate θ and γ values from $\theta \sim \text{Gamma}(\omega_\theta e^{X\beta}, \omega_\theta)$ and $\gamma \sim \text{Gamma}(\omega_\gamma e^{X\beta}, \omega_\gamma)$. Next we generate the self reports (z) and the peer-reports (y) from $z_i \sim \text{Pois}(\delta_i \theta_i)$ and $y_{ij} \sim \text{Pois}(\theta_i \gamma_j)$. In the case of the simulations where self-reports are unbiased, we set $\delta_i = 1$. When self-reports are positively biased but that bias is unrelated to the covariates in the model, we set $\delta_i \sim \text{Unif}(1, 1.5)$. Finally, where self-report bias is a function of the covariate δ_i, we have $\sim (1 - X_i) + X_i\text{Unif}(1, 1.5)$. One hundred simulated datasets were generated under each of the 3 different settings of self-report bias.

For each simulated dataset we carried out 5,200 iterations of Gibbs sampling, and discarded the first 200 of these, leaving 5,000 simulations to calculate posterior mean values of the α and β parameters. We present the empirical mean of the posterior means, the bias, and the empirical standard deviation of the posterior means in Table 4.

We can see that in the simulations in which self-reports are generated without bias, the BPCC performs well with relatively small biases, and standard deviations. In the second set of simulations, where self-reports are generated with a bias that is unrelated to the covariates, we find that the posterior means of the intercept terms are biased but the posterior means of α_2 and β_2 seem to have very small biases, indicating that covariate effects are accurately captured. Finally, in the set of simulations where self-report bias is a function of the covariate in the model, we see that the intercept terms seem to have very small biases, but the posterior means of α_2 and β_2 seem to be affected by the self-report bias. This simulation suggests that BPCC properly captures the relationship between covariates and the quantity of interest when there is no self-reporting bias, and when self-reporting bias is not a function of the covariates of interest, but that regression coefficients may be biased when self-report bias is related to the covariates.

5 Discussion

We show how to use self and peer-reports to learn about both individual level alcohol consumption and peer-reporting discrepancies. Further, we show that the use of peer-reports improves model fit. Among college students, peer-reports of alcohol consumption are often thought to be overestimates [6]. Under the assumptions of the BPC model framework, we have found that a vast majority of the peer-reports indeed had positive discrepancies. Males tended to have larger discrepancies than females. Those intending to join a fraternity or sorority, and those who self-reported zero drinks on drinking days tended to have more negative discrepancies than those who do not intend to joining a fraternity or sorority or reported at least one drink on drinking days, respectively. We also found that people who identified as white had higher levels of alcohol consumption than those who did not, and that those intending to join a fraternity or sorority have a higher level of alcohol consumption than those not intending to join.

This method may be applied to any situation where members of a network report measurements on the same quantities of interest for peers as well as themselves. Any network study that asks for self and peer-reports, and has a similar error structure, could make use of this method. Consider, for example, an online rating system wherein users provide ratings of different products or services. By applying a BPC model, the network of users and ratings could be leveraged to learn about both raters' reporting biases as well as an underlying "true" rating. By applying this method to an online-rating system, products or services that were rated by those with a negative bias could have their over-all rating corrected, thereby reflecting its true quality rather than the biases of the raters.

With the increase of available network data [33], there is a growing need to develop methods that allow for the integration of information from different sources within the network. Bayesian comparative calibration offers a coherent way to utilize data on the same quantity which come from many sources. Prior comparative calibration formulations assumed that there are $p > 1$ imperfect measurement instruments, and that each of these instruments are used to measure the same quantity from n objects, resulting in $n \times p$ measurements [22; 24]. In our models, we generalize prior work in Bayesian comparative calibration in three ways 1.) we do not require that each measurement instrument(either a self-report or a peer-report) is applied to measure the quantity of interest from each object (the members of the social network) 2.) we have formulated models for continuous, count, and binary data 3.) we jointly model the covariate effects between both discrepancies and the quantity of interest.

We plan to extend these models in order to relax some of the assumptions that are made. In this paper, we assume that self-reports are unbiased on average. In future work we will allow for self-reports to contain a systematic bias by placing a prior on a self-report discrepancy. Our simulations of data that contain both peer-report and self-report bias show that the BPCC model performs well when self-reporting bias is independent of the covariates included in the models. In the BPCC model, we make the assumption that alcohol consumption and peer-reporting discrepancies are independent given covariate values. Following [34], we could allow for marginal correlations, and since we know the network structure of this data, we could allow for there to be correlation of these quantities between individuals who are connected in the network.

References

1. Stahre M, Naimi T, Brewer R, Holt J. Measuring average alcohol consumption: the impact of including binge drinks in quantity-frequency calculations. Addiction. 2006; 101:1711–1718. [PubMed: 17156170]
2. Snell LD, Ramchandani VA, Saba L, Herion D, Heilig M, George DT, Pridzun L, Helander A, Schwandt ML, Philips MJ, et al. The biometric measurement of alcohol consumption. Alcoholism: Clinical and Experimental Research. 2011 Epub ahead of print.
3. Gmel G, Graham K, Kuendig H, Kuntsche S. Measuring alcohol consumption- should the graduated frequency approcah become the norm in survey research? Addiction. 2006; 101:16–30. [PubMed: 16393189]
4. Connors GJ, Maisto SA. Drinking reports from collateral individuals. Addiction. 2003; 14(Suppl 2): 21–29.
5. Borsari B, Muellerleile P. Collateral reports in the college setting: A meta- analytic integration. Alcoholism: Clinical and Experimental Research. 2009; 33:826–838.

6. Hagman BT, Patrick CR, Noel NE, Davis CM, Cramond AJ. The utility of collateral informants in substance use research involving college students. Addictive Behaviors. 2007; 32:2317–2323. [PubMed: 17329032]
7. Baer JS, Carney MM. Biases in the perceptions of the consequences of alcohol use among college students. Journal of Studies on Alcohol. 1993; 54:54–60. [PubMed: 8355500]
8. Borsari B, Carey K. Descriptive and injunctive norms in college drinking a meta- analytic integration. Journal of Studies on Alcohol. 2003; 64:331–341. [PubMed: 12817821]
9. Nusser SM, Beyler NK, JWG, Carriquiry AL, Fuller WA, King BM. Modeling errors in physical activity recall data. Journal of Physical Activity and Health. 2012; 9(Suppl 1):S56–S67. [PubMed: 22287449]
10. Dodd KW, Guenther PM, Laurence Freedman S, Subar AF, Kipnis V, Midthune D, Tooze JA, Krebs-Smith SM. Statistical methods for estimating usual intake of nutrients and foods: A review of the theory. Journal of the American Dietetic Association. 2006; 10:1640–1650.
11. Kraus L, Augustin R. Measuring alcohol consumption and alcohol-related problems: comparison of responses from self-administred questionnaires and telephone interviews. Addiction. 2001; 96:459–471. [PubMed: 11255585]
12. Buonaccorsi J, Lin CD. Berkson measurement error in designed repeated measures studies with random coefficients. Journal of Statistical Planning and Inference. 2002; 104:53–72.
13. Richardson S, Gilks WR. A Bayesian aprroach to measurement error problems in epidemiology using conditional independence models. American Journal of Epidemiology. 1993; 138:430–442. [PubMed: 8213748]
14. Beyler, NK. Unpublished Dissertation. 2010. Statistical methods for analyzing physical activity data.
15. Spiegelman D, Schneeweiss S, McDermott A. Measurement error correction for logistic regression models with an "alloyed gold standard". American Journal of Epidemiology. 1997; 145:184–196. [PubMed: 9006315]
16. Ferrari P, Friedenreich C, Matthews CE. The role of measurement error in estimating levels of physical activity. American Journal of Epidemiology. 2007; 166:832–840. [PubMed: 17670910]
17. Brown P. Multivariate calibration. Journal of the Royal Statistical Society Series B (Statistical Methodology). 1982; 44:287–321.
18. Osborne C. Statistical calibration: A review. International Statistical Review. 1991; 59:309–336.
19. Barnett VD. Simultaneous pairwise linear structural relationships. Biometrics. 1969; 25:129–142. [PubMed: 5786303]
20. Theobald CM, Mallinson JR. Comparative calibration, linear structural relation- ships and congeneric measurements. Biometrics. 1978; 34:39–45.
21. Gimenez P, Patat M. Estimation in comparative calibration models with replicate measurement. Statistics and Probability Letters. 2005; 71:155–164.
22. Kimura D. Functional comparative calibration using an EM algorithm. Biometrics. 1992; 48:1263–1271.
23. Lu Y, Ye K, Mathur AK, Hui S, Fuerst T, Genant HK. Comparative calibration without a gold standard. Statistics in Medicine. 1997; 16:1889–905. [PubMed: 9280040]
24. Blangiardo M, Richardson S. A Bayesian calibration model for combining different pre-processing methods in affymetrix chips. BMC Bioinformatics. 2008; 9:512. [PubMed: 19046434]
25. Stefanski L. Measurement error models. Journal of the American Statistical Association. 2000; 95:1353–1358.
26. Nusser SM, Carriquiry A, Dodd KW, Fuller W. A semiparametric transformation approach to estimating usual daily intake distributions. Journal of the American Statistical Association. 1996; 91:1440–1449.
27. Fuller WA. Estimation in the presence of measurement error. International Statistical Review. 1995; 63:121–141.
28. Tooze JA, Kipnis V, Buckman DW, Carroll RJ, Freedman LS, Guenther PM, Krebs-Smith SM, Subar AF, Dodd KW. A mixed-effects model approach for estimating the distribution of usual

intake of nutrients: The NCI method. Statistics in Medicine. 2010; 29:2857–2868. [PubMed: 20862656]

29. McIntyre J, Stefanski LA. Regression-assisted deconvolution. Statistics in Medicine. 2011; 30:1722–1734. [PubMed: 21284016]
30. Barnett N, Ott MQ, Rogers M, Loxley M, Linkletter C, Clark M. Peer associations for substance use and exercise in a college student social network. Health Psychology. 2013 In Press.
31. R Core Team. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing; Vienna, Austria: 2012. http://www.R-project.org/
32. Gelman A, Meng XL, Stern H. Posterior predictive assessment of model fitness via realized discrepancies. Statistica sinica. 1996; 6(4):733–760.
33. Knoke, D., Yang, S. Quantitative Applications in the Social Sciences. 2. Vol. 154. Sage Publichations; 2008. Social Network Analysis.
34. Vangeneugden T, Molenberghs G, Verbeke G, Demtrio CG. Marginal correlation from an extended random-effects model for repeated and overdispersed counts. Journal of Applied Statistics. 2011; 38(2):215–232. http://EconPapers.repec.org/RePEc:taf:japsta:v:38:y:2011:i:2:p:215-232.

6 Appendix

6.1 Model Specification and Full Conditions for Normal Data Bayesian Peer Calibration

We assume that θ and γ are independent and Normally distributed, $\theta \sim N(\mu_\theta, \sigma^2_\theta), \gamma \sim N(\mu_\gamma, \sigma^2_\gamma)$ that the self-reports are on average error-free, and that the peer-reports are subject to an additive systematic discrepancy γ_j.

We assume conjugate prior distributions for μ_θ, μ_γ, $\sigma^2_\theta, \sigma^2_\gamma, \sigma^2_y$ and σ^2_z specifying diffuse prior distributions. Normal zero-centered priors are placed on μ_θ and μ_γ, with variance term ϕ. The variance terms $\sigma^2_\theta, \sigma^2_\gamma, \sigma^2_y$ and σ^2_z are given Inverse-Gamma prior with shape a and rate s. These prior specifications result in the following full conditionals, where $t_i = \frac{\sum_j y_{ij} - \gamma_j}{p_i}$, $w_j = \frac{\sum_i y_{ij} - \theta_i}{r_j}$, p_i = number of peer-reports on person i, r_j = number of reports that person j gave, $\overline{\theta} = \frac{\sum_i \theta_i}{n}$, $\overline{\gamma} = \frac{\sum_j \gamma_j}{g}$, and g = the number of people who made reports. This yields full conditionals:

$$
\begin{aligned}
P(\theta_i|\gamma,\sigma_y^2,\sigma_z^2,\sigma_\theta^2,\mu_\theta,y_{i.},z_i) &\sim \mathrm{N}\left[\frac{t_i\sigma_z^2\sigma_\theta^2+z_i\sigma_\theta^2\sigma_y^2/p_i+\mu_\theta\sigma_y^2\sigma_z^2/p_i}{\sigma_z^2\sigma_\theta^2+\sigma_\theta^2\sigma_y^2/p_i+\sigma_z^2\sigma_y^2/p_i},\frac{\sigma_z^2\sigma_\theta^2\sigma_y^2/p_i}{\sigma_z^2\sigma_\theta^2+\sigma_\theta^2\sigma_y^2/p_i+\sigma_z^2\sigma_y^2/p_i}\right]\\
P(\gamma_j|\boldsymbol{\theta},\sigma_z^2,\sigma_\gamma^2,\mu_\gamma,y_{.j}) &\sim \mathrm{N}\left[\frac{w_j\sigma_\gamma^2+\mu_\gamma\sigma_y^2/r_j}{\sigma_\gamma^2+\sigma_y^2/r_j},\frac{\sigma_\gamma^2\sigma_y^2/r_j}{\sigma_\gamma^2+\sigma_y^2/r_j}\right]\\
P(\mu_\theta|\phi,\overline{\theta},\sigma_\theta^2) &\sim \mathrm{N}\left[\frac{\overline{\theta}\phi}{\phi+\sigma_\theta^2/n},\frac{\phi\sigma_\theta^2/n}{\phi+\sigma_\theta^2/n}\right]\\
P(\mu_\gamma|\phi,\overline{\gamma},\sigma_\gamma^2,) &\sim \mathrm{N}\left[\frac{\overline{\gamma}\phi}{\phi+\sigma_\gamma^2/g},\frac{\phi\sigma_\gamma^2/g}{\phi+\sigma_\gamma^2/g}\right]\\
P(\sigma_z^2|\boldsymbol{\theta},\mathbf{y},\boldsymbol{\gamma}) &\sim \mathrm{Inv{-}Gam}\left[a+e/2,\left(\frac{1}{s}+\frac{1}{2}\sum_i(y_{i,j}-\theta_i-\gamma_j)^2\right)^{-1}\right]\\
P(\sigma_\theta^2|\boldsymbol{\theta},\mathbf{z}) &\sim \mathrm{Inv{-}Gam}\left[a+n/2,\left(\frac{1}{s}+\frac{1}{2}\sum_i(\theta_i-\mu_\theta)^2\right)^{-1}\right]\\
P(\sigma_\gamma^2|\boldsymbol{\gamma},\mathbf{z}) &\sim \mathrm{Inv{-}Gam}\left[a+g/2,\left(\frac{1}{s}+\frac{1}{2}\sum_j(\gamma_j-\mu_\gamma)^2\right)^{-1}\right].
\end{aligned}
$$

6.2 Model Specification and Full Conditionals for Normal Data Bayesian Peer Calibration with Covariates

Here, we specify θ_i and γ_j as being Normally distributed with means and variances $X_i\beta_\theta$, σ_β^2 and $X_j\alpha_\gamma$, σ_α^2 respectively. Assuming diffuse prior distributions such that: $\mathrm{P}(\beta_\theta,\sigma_\beta^2)\propto\sigma_\beta^{-2},\mathrm{P}(\alpha_\gamma,\sigma_\alpha^2)\propto\sigma_\alpha^{-2},\mathrm{P}(\sigma_y^2)\sim\mathrm{Inv{-}Gam}(a,s)$, and $\mathrm{P}(\sigma_z^2)\sim\mathrm{Inv{-}Gam}(a,s)$ we get the following full conditionals:

$$
\begin{aligned}
P(\gamma_j|\boldsymbol{\theta},\sigma_z^2,\sigma_\alpha^2,y_{.j},\alpha_\gamma,X_j) &\sim \mathrm{N}\left[\frac{w_j\sigma_\alpha^2+X_j\alpha_\gamma\sigma_y^2/r_j}{\sigma_\alpha^2+\sigma_y^2/r_j},\frac{\sigma_\alpha^2\sigma_y^2/r_j}{\sigma_\alpha^2+\sigma_y^2/r_j}\right]\\
P(\theta_i|\boldsymbol{\gamma},\sigma_z^2,\sigma_y^2,\sigma_\beta^2,\beta_\theta,y_{i.},z_i,X_i) &\sim \mathrm{N}\left[\frac{t_i\sigma_z^2\sigma_\beta^2+z_i\sigma_\beta^2\sigma_y^2/p_i+X_i\beta_\theta\sigma_z^2\sigma_y^2/p_i}{\sigma_z^2\sigma_\beta^2+\sigma_\beta^2\sigma_y^2/p_i+\sigma_z^2\tau_y^2/p_i},\frac{\sigma_z^2\sigma_\beta^2\sigma_y^2/p_i}{\sigma_z^2\sigma_\beta^2+\sigma_\beta^2\sigma_y^2/p_i+\sigma_z^2\sigma_y^2/p_i}\right]\\
P(\beta_\theta|\boldsymbol{\theta},\sigma_\beta^2,\mathbf{X}) &\sim \mathrm{MVN}\left[\hat{\beta}_\theta,(X'X)^{-1}\sigma_\beta^2\right]\\
P(\alpha_\gamma|\boldsymbol{\gamma},\sigma_\alpha^2,\mathbf{X}) &\sim \mathrm{MVN}\left[\widehat{\alpha_\gamma},(X'X)^{-1}\sigma_\gamma^2\right]\\
P(\sigma_z^2|\boldsymbol{\theta},\mathbf{z}) &\sim \mathrm{Inv{-}Gam}\left[a+n/2,\left(\frac{1}{s}+\frac{1}{2}\sum_i(z_i-\theta_i)^2\right)^{-1}\right]\\
P(\sigma_y^2|\boldsymbol{\theta},\mathbf{y},\boldsymbol{\gamma}) &\sim \mathrm{Inv{-}Gam}\left[a+e/2,\left(\frac{1}{s}+\frac{1}{2}\sum_i(y_{i,j}-\theta_i-\gamma_j)^2\right)^{-1}\right]\\
P(\sigma_\beta^2|\boldsymbol{\theta},\mathbf{X}) &\sim \mathrm{Inv{-}}\chi^2\left[n-k,s_\beta^2\right]\\
P(\sigma_\alpha^2|\boldsymbol{\gamma},\mathbf{X}) &\sim \mathrm{Inv{-}}\chi^2\left[g-k,s_\alpha^2\right],
\end{aligned}
$$

where $\hat{\beta}_\theta=(X'X)^{-1}X'\boldsymbol{\theta}$, $\widehat{\alpha_\gamma}=(X'X)^{-1}X'\boldsymbol{\gamma}$, k is the number of parameters used in the model, $s_\beta^2=\frac{1}{n-k}(\boldsymbol{\theta}-\mathbf{X}\hat{\beta}_{\boldsymbol{\theta}})'(\boldsymbol{\theta}-\mathbf{X}\hat{\beta}_{\boldsymbol{\theta}})$, and $s_\alpha^2=\frac{1}{g-k}(\boldsymbol{\gamma}-\mathbf{X}\widehat{\alpha_\gamma})'(\boldsymbol{\gamma}-\mathbf{X}\widehat{\alpha_\gamma})$.

6.3 Model Specification and Full Conditionals for Bayesian Peer Calibration with Bernoulli data

We assume that θ and γ are independent and Bernoulli distributed, that the self-reports are on average error-free, and that the peer-reports are subject to an exponential systematic discrepancy given by γ_j such that: $z_i \sim \text{Bern}(\theta_i)$ and $y_{ij} \sim \text{Bern}(\theta_i^{\gamma_j})$. Further, $\theta_i \sim \text{Beta}(a, b)$ and $\gamma_j \sim \text{Gam}(c, d)$. We specify diffuse Gamma priors for a, b, c, d with shape and rate parameters ω and ψ. This results in the following full conditionals:

$$\begin{aligned}
P(\theta_i|\boldsymbol{\gamma}, a, b, c, d, y_{i.}, z_i) &\propto \theta_i^{a+z_i+\sum_{j:j\to i}\gamma_j y_{ij}-1}(1-\theta_i)^{b-z_i}\prod_{j:j\to i}(1-\theta_i^{\gamma_j})^{1-\gamma_{ij}} \\
P(\gamma_j|\boldsymbol{\theta}, a, b, c, d, y, z) &\propto \gamma_j^{c-1}e^{-d\gamma_j}\prod_{i:j\to i}\theta_i^{\gamma_j y_{ij}}(1-\theta_i^{\gamma_j})^{1-y_{ij}} \\
P(a|\boldsymbol{\gamma}, \boldsymbol{\theta}, z, y, b, c, d, \omega, \psi) &\propto a^{\omega-1}e^{-\psi a}\prod_i \frac{\theta_i^a}{\text{Beta}(a,b)} \\
P(b|\boldsymbol{\gamma}, \boldsymbol{\theta}, z, y, a, c, d, \omega, \psi) &\propto b^{\omega-1}e^{-\psi b}\prod_i \frac{(1-\theta_i)^b}{\text{Beta}(a,b)} \\
P(c|\boldsymbol{\gamma}, \boldsymbol{\theta}, z, y, a, b, d, \omega, \psi) &\propto c^{\omega-1}e^{-\psi c}\prod_j \frac{(d\gamma_j)^c}{\Gamma(c)} \\
P(d|\boldsymbol{\gamma}, \boldsymbol{\theta}, z, y, a, b, c, \omega, \psi) &\sim \text{Gam}(\omega+\sum_j c, \psi+\sum_j \gamma_j).
\end{aligned}$$

6.4 Model Specification and Full Conditionals for Bayesian Peer Calibration with Covariates with Bernoulli data

We assume that θ and γ are independent and Bernoulli distributed, that the self-reports are on average error-free, and that the peer-reports are subject to an exponential systematic discrepancy given by γ_j, as with the BPC model. We specify distributions on θ_i and γ_j that make use of covariate information: $\theta_i \sim \text{Beta}(a, \frac{a}{e^{X\beta}})$, $\gamma_j \sim \text{Gam}(\omega_\gamma e^{X\alpha}, \omega_\gamma)$, such that the expected value of θ_i equals $\frac{e^{X_i\beta}}{1+e^{X_i\beta}}$. We specify diffuse Gamma priors for a, ω_γ with shape and rate parameters, and diffuse Normal priors on a, β which result in the following full conditionals:

$$\begin{aligned}
P(\theta_i|\boldsymbol{\gamma}, a, \beta, x_i, \omega_\gamma, y_{i.}, z_i) &\propto \theta_i^{a+z_i+\sum_{j:j\to i}\gamma_j y_{ij}-1}(1-\theta_i)^{\frac{a}{e^{X_i\beta}}-z_i}\prod_{j:j\to i}(1-\theta_i^{\gamma_j})^{1-y_{ij}} \\
P(\gamma_j|\boldsymbol{\theta}, a, \beta, \omega_\gamma, \alpha, y, z, x) &\propto \gamma_j^{\omega_\gamma e^{X-j\alpha}-1}e^{-\omega_\gamma\gamma_j}\prod_{i:j\to i}\theta_i^{\gamma_j y_{ij}}(1-\theta_i^{\gamma_j})^{1-y_{ij}}.
\end{aligned}$$

6.5 Full Conditionals of Bayesian Peer Calibration for Count Data

$$\begin{aligned}
P(\theta_i|\gamma, \theta_{-i}, \tau_\theta, \kappa_\theta, \tau_\gamma, \kappa_\gamma, a, b) &\sim \text{Gam}(z_i+\sum_{j:j\to i}y_{ij}+\tau_\theta, 1+\sum_{j:j\to i}\gamma_j+\kappa_\theta) \\
P(\gamma_j|\theta, \gamma_{-j}, \tau_\theta, \kappa_\theta, \tau_\gamma, \kappa_\gamma, a, b) &\sim \text{Gam}(\sum_{i:j\to i}y_{ij}+\tau_\gamma, \sum_{i:j\to i}\theta_i\kappa_\gamma) \\
P(\kappa_\theta|\theta, \gamma, \tau_\theta, \tau_\gamma, \kappa_\gamma, a, b) &\sim \text{Gam}(\tau_\theta n+b, \sum_i \theta_i+a) \\
P(\kappa_\gamma|\theta, \gamma, \tau_\theta, \kappa_\theta, \tau_\gamma, a, b) &\sim \text{Gam}(\tau_\gamma n_e+b, \sum_j \gamma_j+a) \\
P(\tau_\theta|\theta, \gamma, \kappa_\theta, \tau_\gamma, \kappa_\gamma, a, b) &\propto \tau_\theta^{b-1}e^{-a\tau_\theta}\prod_i \frac{(\kappa_\theta\theta_i)^{\tau_\theta}}{\Gamma(\tau_\theta)} \\
P(\tau_\gamma|\theta, \gamma, \tau_\theta, \kappa_\theta, \kappa_\gamma a, b) &\propto \tau_\gamma^{b-1}e^{-a\tau_\gamma}\prod_j \frac{(\kappa_\gamma\gamma_j)^{\tau_\gamma}}{\Gamma(\tau_\gamma)}
\end{aligned}$$

6.6 Full Conditionals of Bayesian Peer Calibration with Co-variates for Count Data

$$\begin{aligned}
P(\theta_i|\theta_{-\mathrm{i}},\gamma,\alpha,\beta,\omega_\theta,\omega_\gamma,\mathbf{X},\mathbf{y},\mathbf{z},f,g) &\sim \mathrm{Gam}(z_i+\omega_\theta e^{X_i\beta}+\sum_{j:j\to i} y_{ij}, 1+\omega_\theta+\sum_{j:j\to i}\gamma_j) \\
P(\gamma_j|\theta,\gamma_{-\mathrm{j}},\alpha,\beta,\omega_\theta,\omega_\gamma,\mathbf{X},\mathbf{y},\mathbf{z}f,g) &\sim \mathrm{Gam}(\omega_\gamma e^{X_j\alpha}+\sum_{i:j\to i} y_{ij}, \omega_\gamma+\sum_{i:j\to i}\theta_i) \\
P(\beta_k|\theta,\gamma,\alpha,\beta_{-\mathbf{k}},\omega_\theta,\omega_\gamma,\mathbf{X},\mathbf{y},\mathbf{z},f,g) &\propto e^{-\frac{\beta^2 k}{2\sigma^2}}\prod_i \frac{{\omega_\theta}^{\omega_\theta e^{X_i\beta}}{\theta_i}^{\omega_\theta e^{X_i\beta}}}{\Gamma\omega_\theta e^{X_i\beta}} \\
P(\alpha_l|\theta,\gamma,\alpha_{-\mathbf{l}},\beta,\omega_\theta,\omega_\gamma,\mathbf{X},\mathbf{y},\mathbf{z},f,g) &\propto e^{-\frac{\alpha^2 l}{2\sigma^2}}\prod_j \frac{{\omega_\gamma}^{\omega_\gamma e^{X_i\alpha}}{\gamma_j}^{\omega_\gamma e^{X_j\alpha}}}{\Gamma\omega_\gamma e^{X_j\alpha}} \\
P(\omega_\theta|\theta,\gamma,\alpha,\beta,\omega_\gamma,\mathbf{X},\mathbf{y},\mathbf{z},f,g) &\propto {\omega_\theta}^{f-1}e^{-g\omega_\theta}\prod_i \frac{{\omega_\theta}^{\omega_\theta e^{X_i\beta}}}{\Gamma\omega_\theta e^{X_i\beta}}{\theta_i}^{\omega_\theta e^{X_i\beta}}e^{-\omega_\theta\theta_i} \\
P(\omega_\gamma|\theta,\gamma,\alpha,\beta,\omega_\theta,\mathbf{X},\mathbf{y},\mathbf{z},f,g) &\propto {\omega_\gamma}^{f-1}e^{-g\omega_\gamma}\prod_j \frac{{\omega_\gamma}^{\omega_\gamma e^{X_j\alpha}}}{\Gamma\omega_\gamma e^{X_j\alpha}}{\gamma_j}^{\omega_\gamma e^{X_j\alpha}}e^{-\omega_\gamma\gamma_j}
\end{aligned}$$

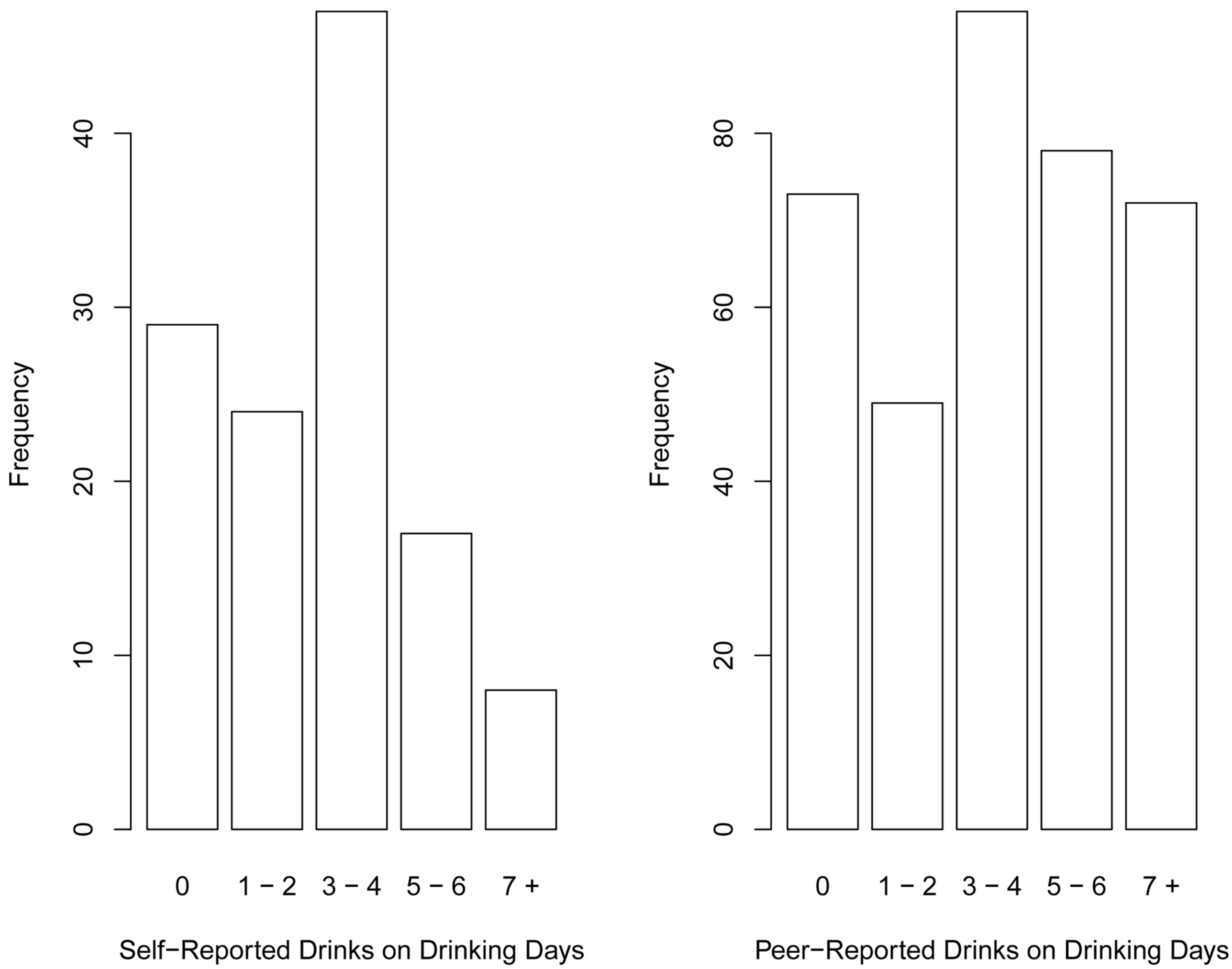

Figure 1.
Self-reports and peer-reports of number of drinks on drinking days

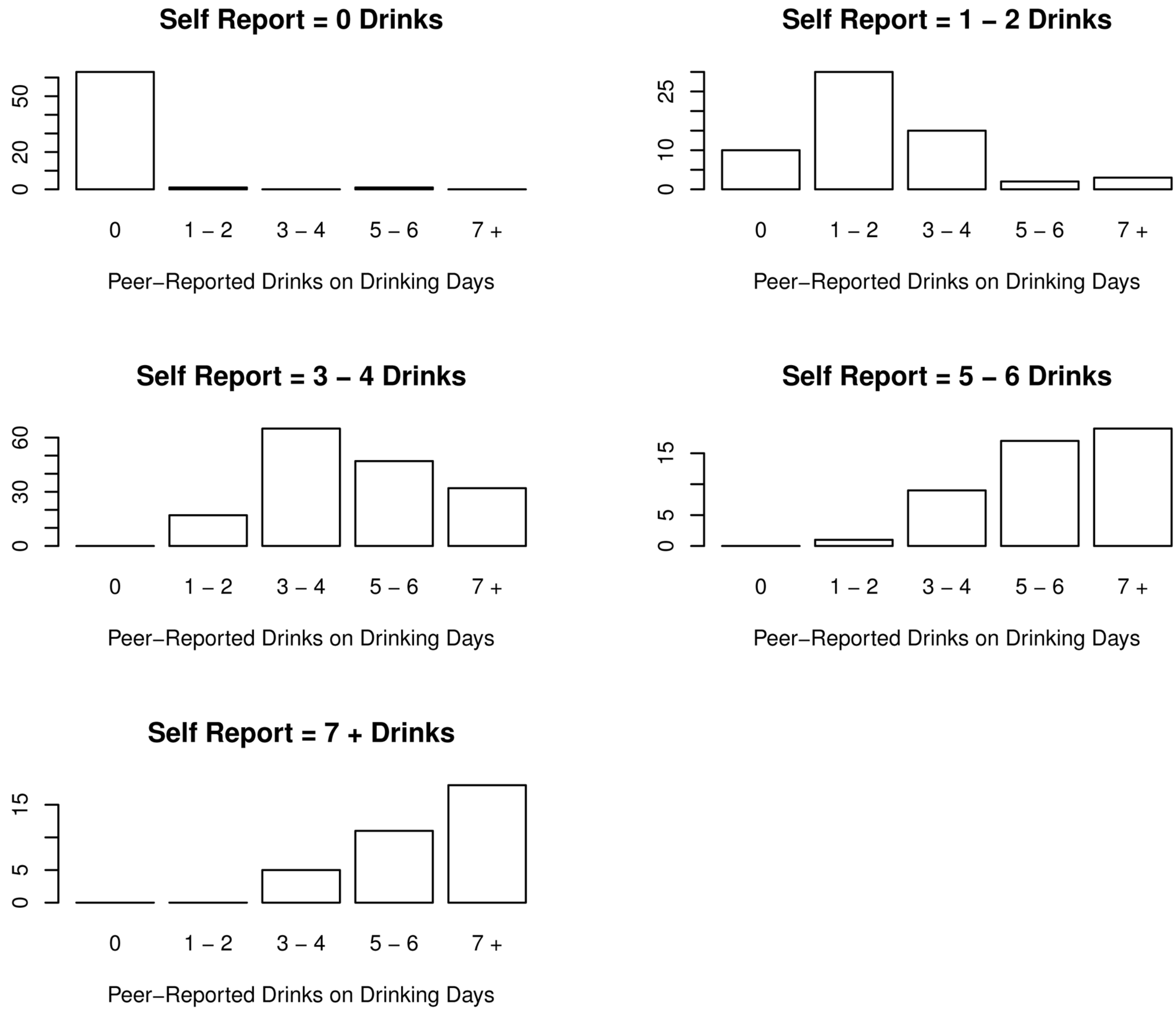

Figure 2.
Peer-reports on number of drinks on drinking days by self-reported number of drinks on drinking days

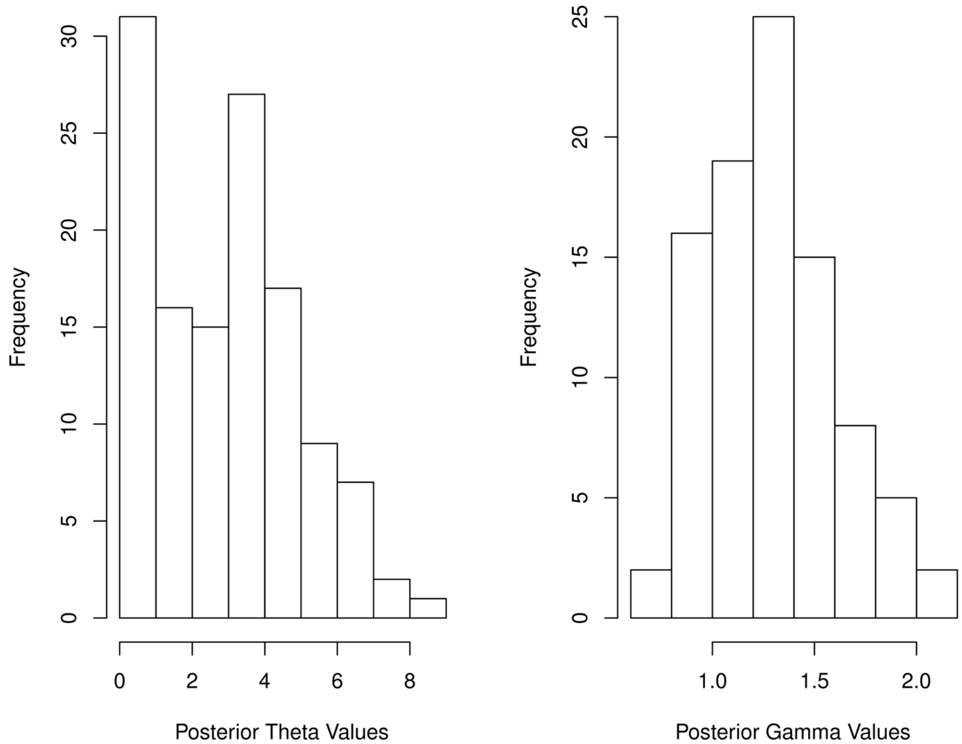

Figure 3.
Posterior θs and posterior γs from BPC model

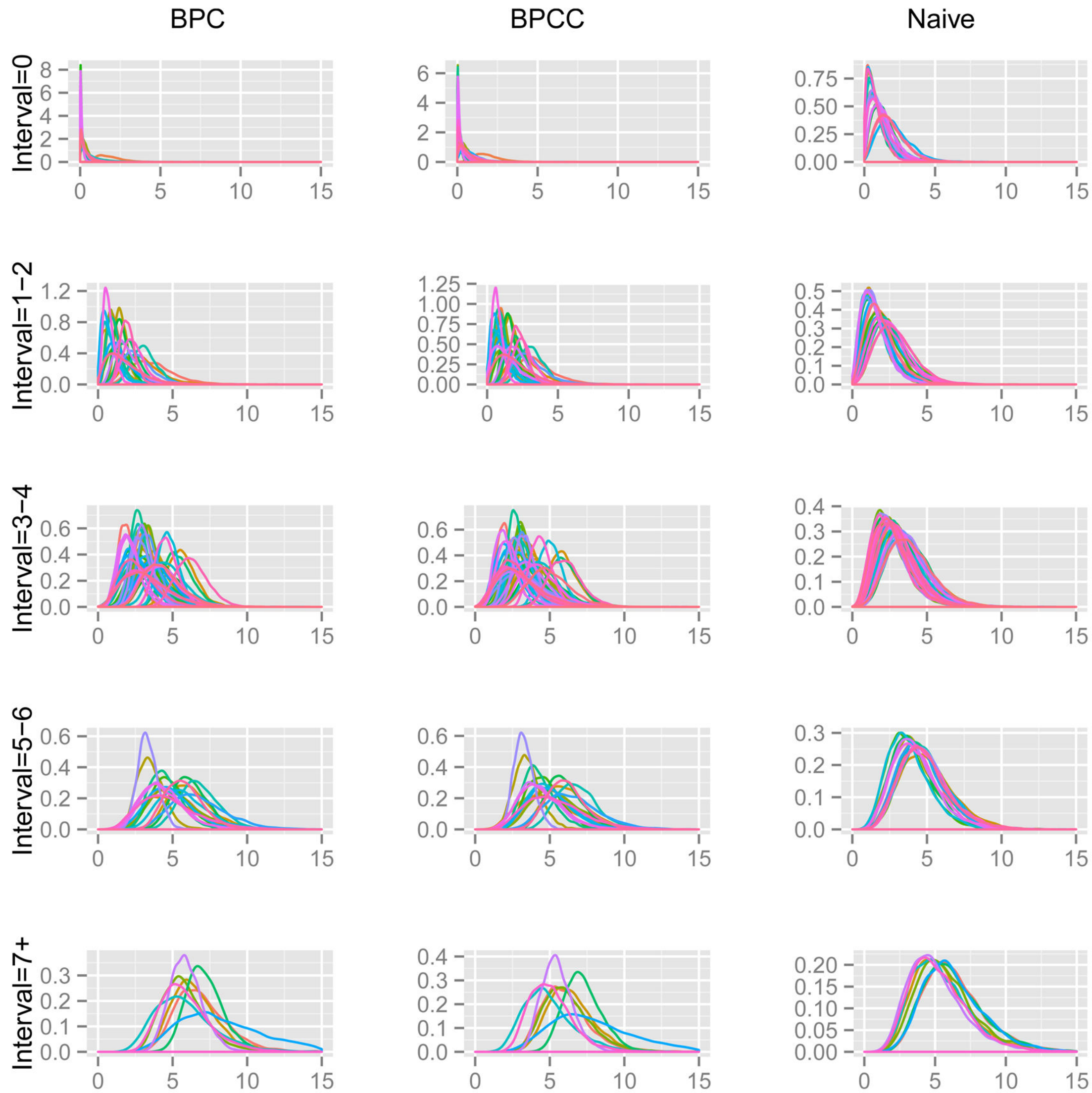

Figure 4.
Posterior densities of θs stratified by observed self-reports

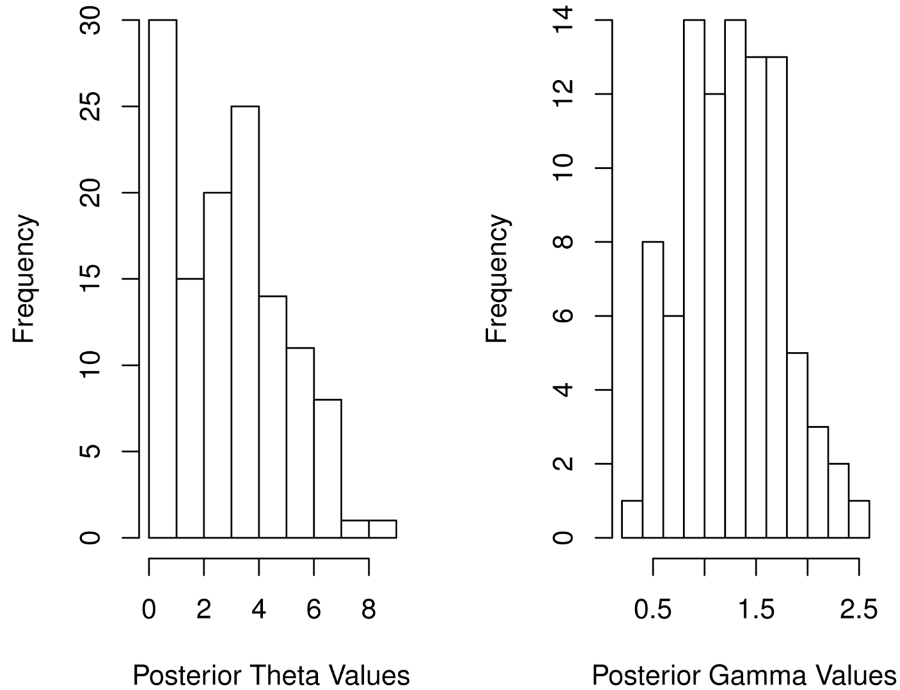

Figure 5.
Posterior θs and Posterior γs from BPCC Model

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Table 1

Relationships between θ, γ and GLM parameterizations

	Normal	**Bernoulli**	**Poisson**
Link	identity	log − log	log
$\alpha_i =$	θ_i	$\log\{-\log(\theta_i)\}$	$\log(\theta_i)$
$\phi_{ij} =$	γ_{ij}	$\log(\gamma_{ij})$	$\log(\gamma_{ij})$

Table 2

Posterior Mean Values and 95% CI of β and α

Effect	Discrepancy BPCC α	Drinks Per Day BPCC β	Drinks Per Day Naive β
Intercept	0.52 (0.09, 0.97)	0.44 (−0.15, 1.01)	0.53 (−0.10, 1.10)
Male	0.24 (−0.01, 0.50)	0.26 (−0.06, 0.57)	0.28 (−0.03, 0.58)
Frat/Sorority	−0.30 (−0.59, −0.02)	0.36 (0.02, 0.70)	0.12 (−0.21, 0.45)
Heterosexual	−0.25 (−0.64, 0.10)	0.00 (−0.47, 0.52)	−0.06 (−0.55, 0.48)
White Race	−0.02 (−0.29, 0.25)	0.55 (0.22, 0.90)	0.57 (0.25, 0.93)
Self-Reported 0	−0.81 (−1.46, −0.28)		

Table 3

Posterior Predictive $\boldsymbol{z}$, generated by Posterior θ

	% of posterior *z* in interval		
Reported Interval	**Without Covariates**	**With Covariates**	**Without Peer-Reports**
0	75	71	40
1 – 2	46	46	46
3 – 4	33	32	32
5 – 6	25	26	24
7+	45	41	36

Table 4

Effect of self-report bias on posterior means from BPCC with 100 simulated datasets

Simulation	Empirical mean of posterior means	Absolute Bias	Empirical SD of posterior means
Unbiased self-reports			
α_1	0.521	0.001	0.101
α_2	−0.296	0.004	0.166
β_1	0.433	−0.007	0.097
β_2	0.350	−0.009	0.141
Self-report bias $\perp X$			
α_1	0.728	0.208	0.085
α_2	−0.306	−0.006	0.161
β_1	0.652	0.212	0.085
β_2	0.371	0.011	0.112
Self-report bias function of X			
α_1	0.520	<0.001	0.166
α_2	−0.079	0.221	0.097
β_1	0.432	−0.008	0.093
β_2	0.570	0.210	0.117