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H.Salehi

Analysis of partial safety factor method based on reliability analysis and probabilistic methods

17

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Bauforschung und Baupraxis

From Research to Practice in Construction

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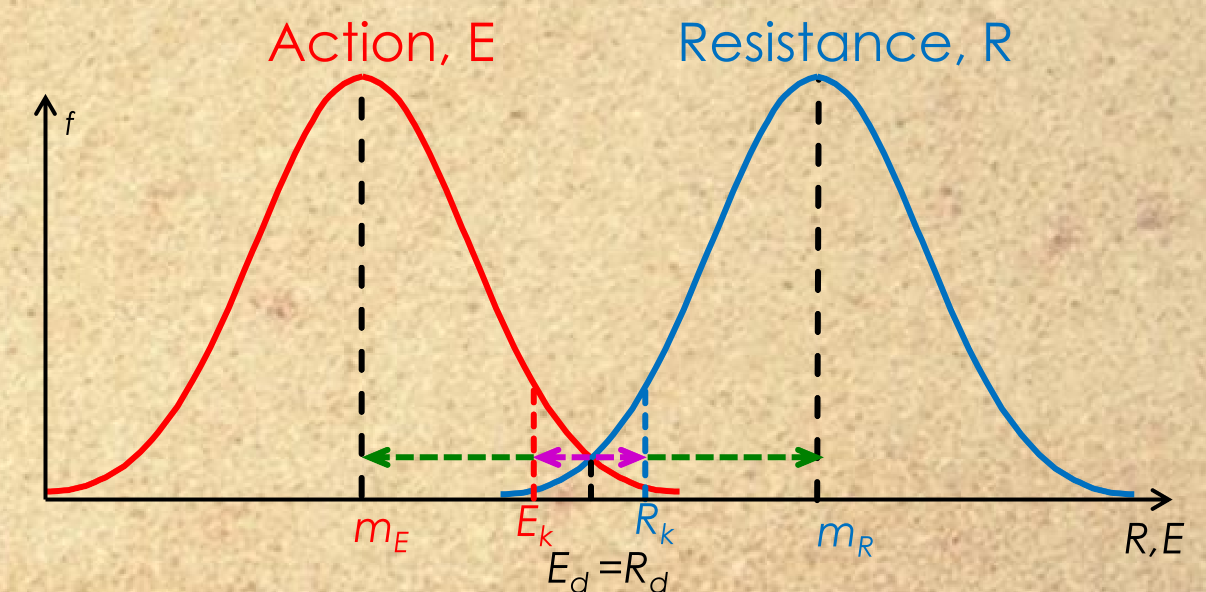
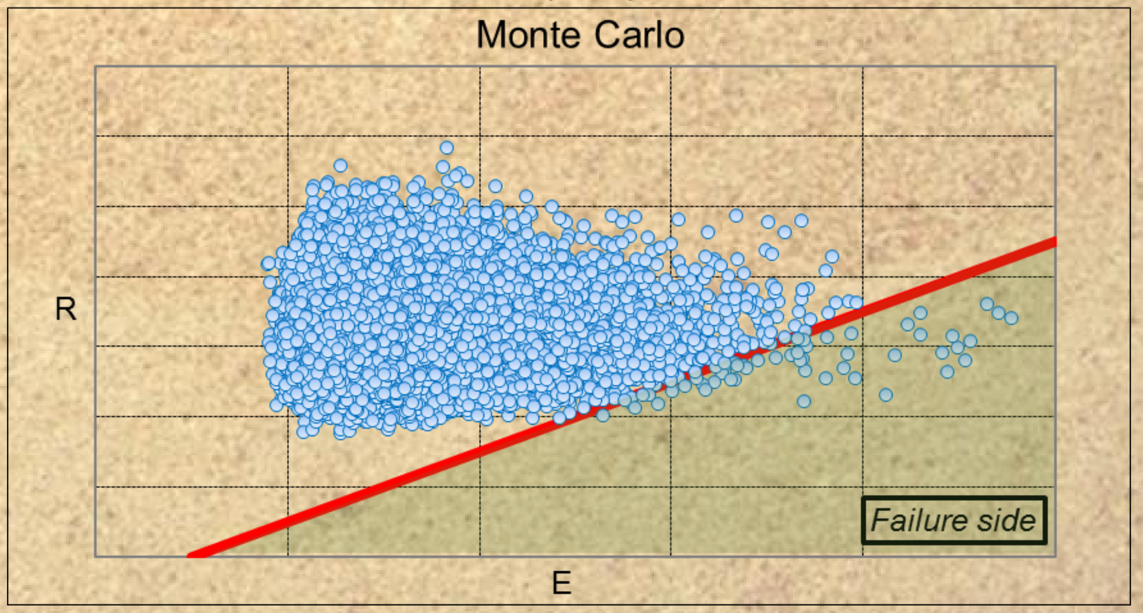
Hamidreza Salehi





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From Research to Practice in Construction

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Analysis of partial safety factor method based on reliability analysis and probabilistic methods

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Faculty of Architecture
Technische Universität Dresden
Germany

For acquiring the academic degree of

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Dissertation by:

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Dedicated
to my father

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Abstract

The partial safety factor method is the main safety concept applied across structural design standards. This method is also presented in EN-1990 as the basis of structural design in Europe. In the review of this code for the new generation of Eurocodes, analysis of the partial safety factor method seems necessary.

The origin of the partial safety factor method is related to probabilistic methods and reliability analysis. Therefore, the latter is selected as tools for the evaluation of the partial safety factor method in the EN-1990 framework. Consequently this research begins with an explanation of the background of partial safety factor methods and reliability analysis.

Different aspects of this safety concept are investigated through this study. The analysis strategy is based on the study of partial safety factor method according to the different part of EN-1990. The research is divided into two main parts, according to the basic components of limit state functions: load and resistance.

Aspects related to loading are investigated first. The available load combinations and the recommended partial factors are investigated based on their reliability levels. The load combinations are compared with each other according to the sustainability of their design. An increased factor for the application of snow load is proposed to overcome safety problems related to snow load on structures. Consequently, a proposal for simplifying these load combinations is offered and verified according to reliability analysis. In the final step, regarding the load’s partial factors, a method of calibration is proposed, based on Monte Carlo reliability analysis.

Afterwards, the aspects related to the resistance are analyzed. Resistances depend mostly on experimental data. Therefore, the relationship between the partial safety factor of resistance and test numbers is investigated. A probabilistic analysis based on Annex D of EN-1990 is then applied to calculate the model uncertainty partial factor and the resistance partial factor for a database from masonry shear walls. A comparison is made to show the influence of different way of partial safety factor utilization in a limit state function.

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1 Introduction

1.1 Motivations and goals

As a matter of engineering, design is crucial. Structural engineering centers on structural design. Since humans construct structures, they try to create safety margin to avoid hazards and major damage to those structures. In ancient civilizations, experience and intuition were the most important considerations in the safety of structures [1],[2]. Today, safety factors are assessed with material strength and stress analysis. This shift from intuition and experience to safety factors and design marks early attempts to formalize structural safety [1],[2].

Structural standards and codes are foundational to acceptable engineering practices and guidelines for the assessment of safety and serviceability issues in structural engineering. The definitions of various components of structural design—such as natural and human-made forces, the magnitudes of these forces for design, and the recommended methods to measure and mitigate these forces—are provided in standards. The goal of safety guidelines is to ensure acceptable levels of safety to prevent structural failure and further consequences. These guidelines attempt to answer a straightforward question: “How safe is safe enough?” [3].

Structural safety originates with the uncertain nature of human products and the randomness of loads and material properties. Moreover, inconsistent structural models are a source of uncertainty in this field: Model predictions of a structure’s behavior do not always accurately predict that behavior in practice. Structural codes are responsible for covering most sources of uncertainty, on the one hand, and providing safe design, on the other hand.

Various design methods have been presented over the last century. In [4], the design methods in modern engineering are categorized as follows: permissible stress, the load factor method and limit state design. Another classification of design methods is offered by [5]: permissible stress, developed permissible stress and limit state design.

In 1960s, engineers began to recognize the weaknesses of previous design strategies, such as allowable stress methods. Given this recognition, the engineers authoring structural codes expressed their intent to implement new approaches in the context of structural design. In the meantime, reliability analysis and probabilistic methods were developed. The probabilistic approaches for structural verification are notable tools due to the statistical nature of the data concerning the strength of a material or the loads to be anticipated.

Freudenthal [6] has reported the early efforts made to define safety factors in structural verification based on probability of failure and on reliability. With these methods, uncertainties in a structural analysis might be modeled based on probabilistic distributions. Representation of loads and resistances by stochastic information provided the ability to assess the risk and safety of design structures. With this ability, codes for probabilistic models were published and applied in restricted fields, such as for steel and concrete structures [7],[8].

In Europe, the first probabilistic based design code was published in 1964 for concrete structures [9], based on the probabilistic method. This design code defined individual safety factors, called “partial safety factors,” for all parameters in the structural design. Thus, this concept was named after these factors, as the partial safety factor method. Other names have also been used for this method, such as load and resistance factor design (LRFD).

The partial factor method is the leading safety concept applied to determine required structural reliability. Most national design standards have implemented the method, which has a reasonable process and is convenient for most engineering applications. The method has been improved and formulated through the last 50 years, in particular by committees in the International Organization for Standardization (ISO), European committee for concrete (Comité Européen du Béton- CEB), and European Convention for Constructional Steelwork (ECCS). The partial factor method is also prescribed in the European standards for structural design of the Eurocodes [10].

The optimizations and improvements of structural design standards are the essential requirements for an excellent practical standard. As the basis of other Eurocodes, EN-1990 is under evaluation as part of its update to a new version. Since its core is the partial safety factor method, it is necessary to investigate the applications of partial safety factor method in various parts of EN-1990. This work aims primarily to evaluate parts of EN-1990 according to the implemented strategy of this research.

The level of safety provided by EN-1990 through its recommendation of partial safety factors has to be assed. Moreover, sustainability is today an essential parameter to consider in constructing structures. Therefore, the economic criteria in the design process must also be considered in creating a sustainable design. Because of the complicated new situation confronting societies due to climate change and lack of resources, developers of safety codes should find the optimum methods to introduce safety measures in structures.

1.2 Strategy of the work

The analysis of the partial safety factor method is the main objective of this study. After its brief introduction to the concept, Chapter 2 describes background information concerning the partial safety factor method and its applications in the framework of EN-1990.

Due to the fact that the partial safety factor method is principally the outcome of probabilistic methods, probabilistic methods and reliability analysis are applied in assessing various aspects of partial safety factor methods. Probabilistic methods for reliability analysis are presented and compared with each other in Chapter 3.

The main strategy for the analysis stems from the partial safety factor method in the EN-1990. The analysis compares the reliability levels of the methods applied. The description of limit state functions is based primarily on the definition of resistance and loading in the structure. Thus, the investigation is divided into two main parts: loading, actions and their partial safety factors; and the partial factor resistance and its application.

Different types of loading and different values for relevant partial safety factors appear in the recommendations of EN-1990. In Chapter 4, load parts of the limit state function are investigated. The available load combinations and the recommended partial factors are investigated based on the reliability levels provided by them. The reliability analyses are made for different sets of variable loads and various types of materials.

Load combinations are compared with each other according to the sustainability of their resultant design. An increased factor for the application of snow load is proposed to overcome the associated heightened risks. A proposal to simplify these load combinations is then offered and assessed according to the reliability analysis. In the final step regarding the partial safety factors for load, a calibration method is proposed, based on the Monte Carlo reliability analysis.

The resistances of limit state functions in safety codes are analyzed in Chapter 5. Partial safety factor values for different types of materials are proposed in the relevant Eurocodes rather than in EN-1990. The basics for the application of partial safety factors and the probabilistic determination of these values are presented in EN-1990.

Experimental analyses are conducted primarily to evaluate material properties or resistance model assessments. Resistances are mostly dependent on the experimental data. Therefore, the relationship between the partial safety factor of resistance and test numbers is investigated. A probabilistic analysis based on the Annex D of EN-1990 is then applied to calculate the model uncertainty and the partial safety factor of resistance for database from masonry shear walls. A comparison is made to expose the influence of different ways of partial safety factor utilization in a limit state function.

2 Partial safety factor method and EN-1990

2.1 Introduction

The structural design process has to provide a safe and cost-effective structure. In order to achieve this goal, different standards have been developed. Safety levels can be covered up by increasing expected load while decreasing nominal resistance. Moreover, sustainability must also be considered in the design process.

Through the development of more accurate tools, calculations, and analyses for engineers, various methods of design have been developed over the past century. The safety concept applied in structural design codes were based on allowable stress principles until the 1960s. Structures were designed according to models, and the design process was assessed by considering the elastic behavior. In order to anticipate and mitigate uncertainty, the determined stresses were required not to exceed the values of critical stress divided by a factor of safety. The limit stresses corresponded mostly to yielding, rupture, or instability. These safety factors were chosen individually; one might determine the actions subjected to a structure and assess the structure such that the elastic stresses resulting from the loads stay below 60% of the stress at a critical point or limiting value. Indeed, no overview was offered concerning the amount of risk or safety provided for the designed structure according to this method [3]. Safety factors were mainly determined by an engineer's personal assessment or the practical experience of code writers. As computational methods and facilities improved in the 20th century, the prevalence of the allowable stress method was reduced [11].

To resolve these deficiencies (see [4] and [12]), a new formulation for the safety requirements in the design process was established. This formulation is believed to have first appeared in Russia in the 1930s. Nevertheless, it was developed into its present form at the recommendation of the CEB [9]. This method was the partial safety factor format.

In the current structural standards, a structure's safety is verified according to linear analysis of the structure and the fulfillment of ultimate and serviceability limit states. The verification is done with a semi-probabilistic security format by implementing partial safety factors, as applied for the action values and the characteristic values of the material properties [13].

2.2 EN-1990 basics of design

In 1975 the Commission of the European Communities (CEC) started to develop a new system for construction and structural design for the European structural codes, the so-called Eurocodes. The primary goal of the project was to remove technical problems by unifying technical requirements. Based on this project, the European Commission decided to create a group of unified engineering regulations for structural design and construction projects [14].

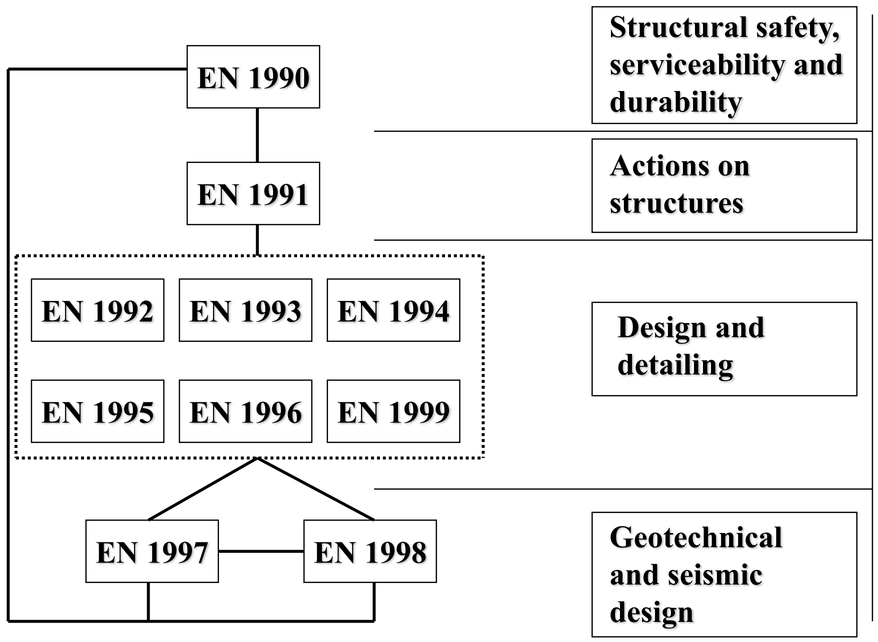


Figure 2.1: Links between the Eurocodes [14]

Since 1989, the European Committee for Standardization (Comité Européen de Normalisation [CEN]) has been in charge of the preparation and publication of the European standards for structural design (Eurocodes-EN) [14]. The full set of CEN Structural Eurocodes, previously known as ENV form, was converted to full EN (Normative) by 2004/5. There are ten Eurocodes, each related to a specific subject in structural engineering [15],[14].

EN-1990 [16] is the fundamental document in the Eurocode standards system, and it provides the requirements and criteria of reliability and safety for all the Structural Eurocodes. Moreover, it gives a general framework for structural design in buildings and constructions. Parameters related to reliability, durability and quality controls are also presented in EN-1990. This information gives engineers an overview of the procedure of the design, construction, and supervision during the construction [17].

2.3 Limit states

The design procedure in EN-1990 is based on the limit state concept, with the partial safety factor method. Based on limit states, the structures may be categorized in two types according to their behavior: acceptable (safe and serviceable) or unacceptable (failed and unserviceable). The criteria that define a condition as acceptable or unacceptable are called limit sates. In other words, limit states represent the unacceptable cases for structure. Generally, the limit states are the boundaries beyond which the structure cannot fulfill the requirements of safety in the code's recommendations. Each performance or characterization of a structure can be represented by one limit state in order to be applied in design procedure [14]. EN-1990 classifies limit states into two types: ultimate limit states and serviceability limit states.

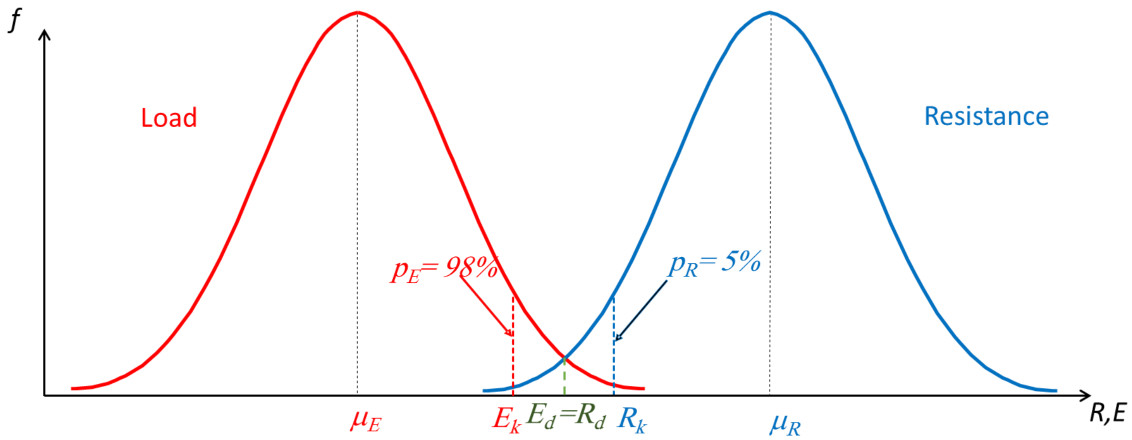


Figure 2.2: Schematic representation of the partial safety factor method

Figure 2.2 offers a basic representation of the partial safety factor method. The parameters of load and resistance are first defined according to their stochastic properties. Then, according to the type of resistance and load, the corresponding characteristic values are determined. Eventually, the design values of each basic variable may be calculated by applying the relevant partial factors.

2.3.1 Ultimate limit state

The ultimate limit state represents a situation in which structural failure occurs in the form of collapse and destruction. It is usually described by the maximum bearing capacity of a structure or structural components. Design that considers the ultimate limit state provides safety for people and structures. In certain cases (e.g., a nuclear power plant, a chemical reservoir, or a museum), however, the limit state concerns the safety of the structural material [14]. EN-1990 defines different categories of ultimate limit states based on failure type:

a) EQU: loss of static equilibrium of the structure or any part of it considered as a rigid body, where
   - minor variations in the value or the spatial distribution of actions from a single source are significant, and
   - the strengths of construction materials or ground are generally not governing;

b) STR: internal failure or excessive deformation of the structure or structural members, including footings, piles, basement walls, etc., where the strength of construction materials of the structure governs;

c) GEO: failure or excessive deformation of the ground where the strength of soil or rock provide significant resistance; and

d) FAT: fatigue failure of the structure or structural members [16].

Design strategy may vary in EN-1990 based on each type of ultimate limit state.

2.3.2 Serviceability limit states

The serviceability limit states are associated expected use of a structure. Fulfillment of these conditions covers the requirements needed for particular services in the structure itself or its member. This type of limit state covers a structure's functionality with respect to the demands of the people who use it. According to the time and conditions of the structure's use, serviceability limit states can be divided into two types, as shown in Figure 2.3 [14]:

(1) Irreversible serviceability limit states are those limit states that remain permanently exceeded even when the actions that caused the failure are removed (e.g., permanent local damage or permanent unacceptable deformations) [14].

(2) Reversible serviceability limit states are those limit states that are not exceeded when the actions that caused the failure are removed (e.g., cracks in pre-stressed components, temporary deflections, or excessive vibration) [14].

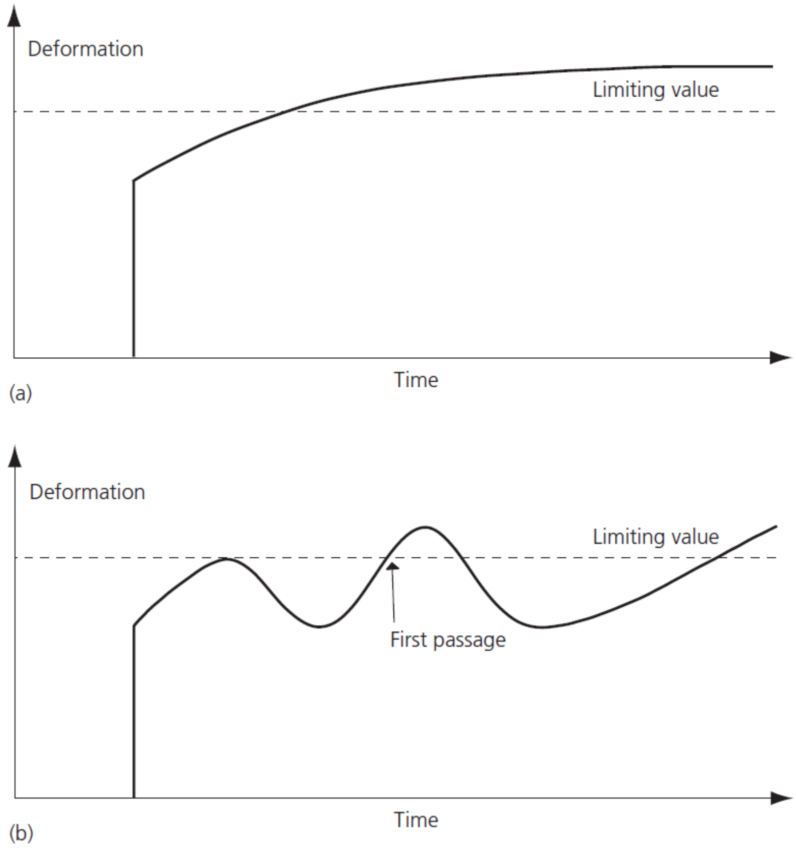


Figure 2.3: (a) Irreversible and (b) reversible limit states [14]

2.4 Design procedure

The design procedure in EN-1990 is done based on a comparison of design values for resistance and action effects. Resistance parameters and actions are defined based on the required limit state. For example, in the case of equilibrium, the stabilizing action or the resistance parameters stabilizing the structure have to be compared with the destabilizer parameter in structure. The verification process in EN-1990 ensures that the relevant limit sate is not exceeded by considering the applicable design values for actions, material properties, and geometrical parameters [14]. The general concept for comparing the design value for action and resistance is presented as (2.1) in EN-1990:

$$E_d \leq R_d \tag{2.1}$$

where

E_d is the design value of the effect of actions such as internal force, moment or a vector representing several internal forces or moments;

R_d is the design value of the corresponding resistance.

Design values for resistance and action effects are determined with partial safety factors and corresponding characteristic values. Characteristic values represent the probabilistic basis of the basic variables. Characteristic values are fractile values of the distribution for each basic variable. In resistance, in most cases, the characteristic value is the lower fractile of distribution. By contrast, in actions, upper fractiles usually represent the characteristic values of actions.

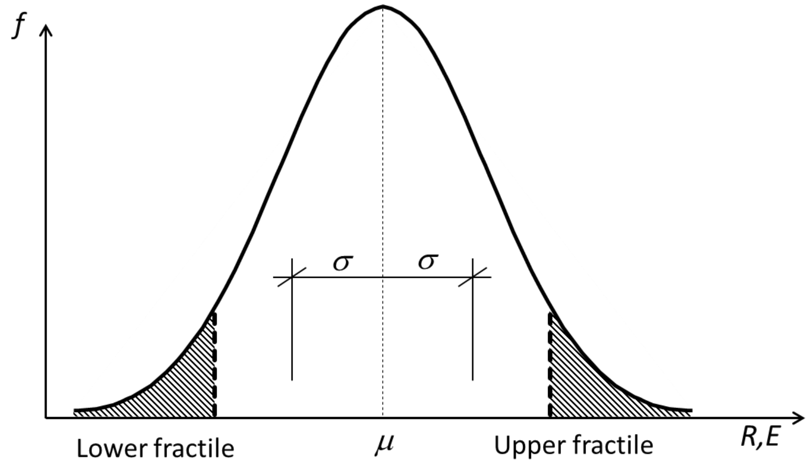


Figure 2.4: Upper and lower fractile in the probability density function (PDF)

In order to determine the critical case, the design values of the effects of actions (E_d) are calculated by considering all existing actions at the same time on structures. Various types of actions may be considered in construction, such as the weight of the structure, live load from vehicles or facilities, wind, snow, temperature, and seismic loading. The combination of all these load types is introduced in EN-1990 for different kinds of limit states and situations. For a given construction, several actions, considered as natural or human-made phenomena, apply permanently. For design, the most critical case through all possible combinations must be considered [16] [14].

The combinations in EN-1990 are recommended for accidental actions, seismic loading, geotechnical cases and transient or persistent situations. The fundamental combination in EN-1990, known as “6.10” in the code text, is the most common combination for design. This combination is shown in (2.2).

$$\sum_{j\geq 1} \gamma_{G,j} G_{k,j} \text{"} + \text{"} \gamma_p P \text{"} + \text{"} \gamma_{Q,1} Q_{k,1} \text{"} + \text{"} \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{2.2}$$

Here, *G* represents the permanent actions, *Q* variable actions, *P* prestress.

Two other combinations in EN-1990, known as equations “6.10a” and “6.10b” in code text, have been proposed for limit states in STR and GEO situations. The less favorable of these two combinations will be used for finding action effects. However, in the German national annex of EN-1990, use of these two combinations [see (2.3) and (2.4)] is not permitted.

$$\sum_{j\geq 1} \gamma_{G,j} G_{k,j} \text{ "+" } \gamma_p P \text{ "+" } \gamma_{Q,1} \psi_{0,i}\, Q_{k,1} \text{ "+" } \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{2.3}$$

$$\sum_{j\geq 1} \xi_j\, \gamma_{G,j} G_{k,j} \text{ "+" } \gamma_p P \text{ "+" } \gamma_{Q,1}\, Q_{k,1} \text{ "+" } \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{2.4}$$

Partial safety factors and combination factors are recommended in Annex A of EN-1990 for different limit state situations. With the application of these values as safety factors in the design process, the required safety level based on EN-1990 recommendations is satisfied. Safety level in EN-1990 is defined according to the reliability index and probability of failure.

2.5 Target reliabilities and consequence classes

In the context of EN-1990, the criteria for safety requirements are provided based on the target values for reliability or failure probability. The recommendation is classified based on the correspondent consequence classes and specific reference periods. In the current version, these values are presented in the context of Annex B along with consequence classes and quality controls (Table 2.1). For each consequence class, a reliability class (RC) is allocated in EN-1990.

Table 2.1: Reliability classes in the current version of EN-1990 [16]

Reliability classes	**Minimum value of $\boldsymbol{\beta}$**	
	1 year reference period	50 years reference period
RC3	5.2	4.3
RC2	4.7	3.8
RC1	4.2	3.3

In the new draft of EN-1990, the target reliabilities are presented only in its Annex C in order to avoid misunderstanding of engineers during the EN-1990 application. Target reliabilities are needed for code calibration and probabilistic design. These are additional approaches for structural design. Subsequently, the reliabilities are mentioned only along with probabilistic methods in the code regarding the concept of ease of use.

Moreover, the reliabilities presented in the new draft represent only a one-year reference period. However, in the current version both one-year and 50-year reference periods are indicated in target reliabilities. The table of target reliabilities in a new draft of EN-1990 is shown in Table 2.2.

Table 2.2: Reliability classes in a new draft of EN-1990 [18]

Consequences of failure	**Consequence class**		
	CC1	CC2	CC3
$\mathbf{p_{f,a}^{tgt}}$	10^{-5}	10^{-6}	10^{-7}
$\boldsymbol{\beta_a^{tgt}}$	4.26	4.75	5.20

In ISO-1394 [19] and the Joint Committee on Structural Safety (JCSS) probabilistic model code [20], the target reliabilities are presented in different forms. In addition to the consequence classes, the cost of safety is also considered for the categorization of target reliabilities in these two documents (Table 2.3). This recommendation is based on the optimization performed in [21]. The cost of reducing risk and increasing structural safety is combined with the consequences of failure to evaluate the optimum economic values for target reliabilities. Structural design is a process of decision making, meaning that various parameters are involved in the final determination. The optimum situation to cover all societal requirements must eventually be selected.

Table 2.3: Reliability classes in ISO-2394 [19] and JCSS [20]

Relative cost of safety measures	**Consequences of failure from Table 2.6**		
	Class 2	**Class 3**	**Class 4**
Large	$\beta = 3.1(p_f \approx 10^{-3})$	$\beta = 3.3(p_f \approx 5 \times 10^{-4})$	$\beta = 3.7(p_f \approx 10^{-4})$
Medium	$\beta = 3.7(p_f \approx 10^{-4})$	$\beta = 4.2(p_f \approx 10^{-5})$	$\beta = 4.4(p_f \approx 5 \times 10^{-6})$
Small	$\beta = 4.2(p_f \approx 10^{-5})$	$\beta = 4.4(p_f \approx 5 \times 10^{-6})$	$\beta = 4.7(p_f \approx 10^{-6})$

In the case of structural design, it is possible to robustly consider safety in the design and consume a considerable amount of building material in the construction, yet have an unsustainable structure in the end. This approach will lead to a waste of energy and resources. Contrarily, a design may be economical, but still produce an unsafe structure. Therefore, a balance between these two has been considered in the optimization of target reliability in [21];

this balance is depicted in the Table 2.3. Figure 2.5 presents the relation of costs and risk in the decision-making process. Safety factors can be the decision parameters in the structural design. If the required target reliabilities in the code correspond to the optimum point, the partial factors of this point will provide an optimal design that is sustainable and economical at the same time.

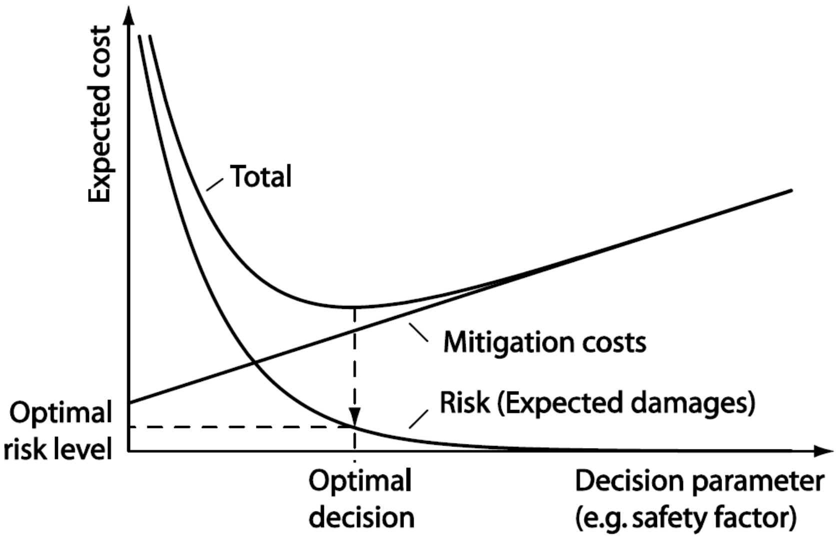


Figure 2.5: Optimization of risk and costs [22]

As mentioned before, each reliability requirement as target reliability is connected with a consequence class. Engineers should decide the consequences of failure and find the correspondent category for a structure to apply the desired level of safety according to the code. The consequences are the parameters for risk evaluation in risk analysis and comprise possible results of an event. In structural engineering, the consequences are defined as outcomes of a structure failure. These consequences have three primary sources: loss of lives, the environmental failure effect and economic damage [23]. However, they are not simple phenomena for evaluation. Quantification of consequences requires multidisciplinary analysis. This kind of analysis requires different experts from various fields to determine the relation between the event and its surroundings [24]. Estimation of consequences is connected with the incident of a hazard, mostly a complicated process that depends on the judgment of experts. These experts must have comprehensive knowledge gained through experiences from similar phenomena. Nevertheless, the determined consequences in this process would include a considerable amount of uncertainty [25].

The current version of EN-1990 [16] categorizes the consequences in three different classes. The representations of the number of consequences are mentioned together in each class. The table in Annex B of EN-1990 is shown in Table 2.4.

Table 2.4: Consequence classes in EN-1990 [16]

Consequence classes	Description	Examples of buildings and civil engineering works
CC3	**High** consequence for loss of human life, or economic, social or environmental consequences **very great**	Grandstands public buildings where consequences of failure are high (e.g. a concert hall)
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequences **considerable**	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	**Low** consequence for loss of human life,and economjc, social or environmental consequences **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

In the final draft of a new version of EN-1990, consequence classes are described more clearly. The different classes regard loss of human life, along with economic, social and environmental consequences. For loss of human life, there are three levels: high, medium and low. For economic, social and environmental consequences as well, there are three levels: very great, considerable and small or negligible. These two levels are presented in the form of a sentence in the current version, but in the new draft they are separated in two different columns, and the more severe of these two columns has to be considered in selecting consequence classes for the relevant structure or structural component.

Table 2.5: Consequence classes in the final draft of new version EN-1990 [18]

Consequence classes	Description	Loss of human life	Economical social or environmental consequences
CC4	Highest	Extreme	Huge
CC3	Higher	High	Very great
CC2	Normal	Medium	Considerable
CC1	Lower	Low	Small
CC0	Lowest	Very low	Insignificant

This classification aligns with the concept of ease of use for the code. Table 2.5 presents the classification of consequences in the new draft. There are also two extra classes in the new draft, namely the highest and lowest consequence classes. These are related to exceptional constructions and based on the description of EN-1990, and they are not covered through the Eurocode system. These kinds of structures need individual structural design and analysis with particular considerations.

The classification of consequences in ISO-2394 [19] has five different categories. In this document, the explanation of each category is more comprehensive, complete with the details of each failure consequence. This clarification helps engineers to choose the corresponding consequence class conveniently, according to the structure's characteristics. The consequences in ISO-2394 are presented in Table 2.6.

Table 2.6: Consequence classes in ISO-2394 [19]

Consequences class	**Description of expected consequences**	**Examples of structures**
Class 1	Predominantly insignificant material damages	Low-rise buildings where only a few people are present, minor wind turbines, stables, etc.
Class 2	Material damages and functionality losses of significance for owners and operators but with little or no societal impact Damages to the qualities of the environment of an order which can be restored completely in a matter of weeks Expected number of fatalities fewer than 5	Smaller buildings and industrial facilities, minor bridges, major wind turbines, smaller or unmanned offshore facilities, etc.
Class 3	Material losses and functionality losses of societal significance, causing regional disruptions and delays in important societal services over several weeks. Damages to the qualities of the environment limited to the surroundings of the failure event and which can be restored in a matter of weeks. Expected number of fatalities fewer than 50.	Most residential buildings, typical bridges and tunnels, typical offshore facilities, larger and or hazardous industrial facilities
Class 4	Disastrous events causing severe losses of societal services and disruptions and delays at national scale over periods in the order of months. Significant damages to the qualities of the environment contained at national scale but spreading significantly beyond the surroundings of the failure event and which can only be partially restored in a matter of months. Expected number of fatalities fewer than 500.	High-rise buildings, grandstands, major bridges and tunnels, dikes, dams, smaller offshore facilities, pipelines, refineries, chemical plants, etc.
Class 5	Catastrophic events causing losses of societal services and disruptions and delays beyond	Buildings of national significance, major

	national scale over periods in the order of years. Significant damages to the qualities of the environment spreading significantly beyond national scale and which can only be partially restored in a matter of years to decades. Expected number of fatalities larger than 500.	containments and storages of toxic materials, major offshore facilities, major dams, and dikes, etc.

In JCSS probabilistic model code, a quantity is defined for classification of failure consequences [20], and ρ is the ratio between total costs (i.e., construction costs Ck plus direct failure costs H) and construction costs [20]. Parameter ρ is determined based on (2.5) [26].

$$\rho = \frac{H + Ck}{Ck} \tag{2.5}$$

The cost of failure H is calculated based on the cost of fatalities, and it is represented as (2.6):

$$H = n \cdot k \cdot SLSC \tag{2.6}$$

where *n* is the number of people in the building at the time of failure and where *k* is the parameter related to the proportion of fatalities-per-person based on Table 2.7. Social life-saving cost "SLSC" is a social indicator representing the implied cost of averting fatalities, and it depends on gross domestic product per capita (*g*), life expectancy (*e*) and the ratio of life to earn a living (*w*) [27]. Some selected values of SLSC, *g*, and *e* are presented in Table 2.8.

Table 2.7: Parameter *k* for determining failure cost [28]

Types and cause of failure	***k***
Earthquake	0.01–1.0
Avalanches, rock fall, explosions, impact etc.	0.01–1.0
Floods and storms	0.0001–0.01
Sudden structural failure in places of public entertainment	0.1–0.5
Fire in buildings	0.0005–0.002
Fire in road tunnels	0.01–1.0

Table 2.8: Some SLSC and social indicators *g* and *e* [26, 28]

Country	***g* [US$]**	***e* [year]**	***w***	**SLSC**
Canada	27330.16	78.84	0.13	$1.3 \cdot 10^6$
USA	34260.22	77.86	0.15	$1.6 \cdot 10^6$
Germany	25010.15	78.87	0.12	$1.1 \cdot 10^6$
Czech Rep.	12900.67	73.77	0.17	$4.6 \cdot 10^5$

Quantification of consequences with (2.5) and (2.6) helps engineers contextualize failures. The number of people and fatalities plays an important role in this formula, allowing the opportunity to distinguish between the loss of one human life or more. According to the parameter ρ and (2.5), the classification of consequences in JCSS is presented as in Table 2.9.

Table 2.9: Consequence classes JCSS [20]

Consequences class	**ρ**	**Description**
Class 1 Minor Consequences	ρ is less than approximately 2	The risk to life, given a failure, is small to negligible and economic consequences are small or negligible (e.g., agricultural structures, silos, masts)
Class 2 Moderate Consequences	ρ is between 2 and 5.	Risk to life, given a failure, is medium or economic consequences are considerable (e.g., office buildings, industrial buildings, apartment buildings)
Class 3 Large Consequences	ρ is between 5 and 10	Risk to life, given a failure, is high, or economic consequences are significant (e.g. main bridges, theaters, hospitals, high rise buildings)

JCSS also introduces the classification of consequences, which depends on failure type. In this classification, ductility is the criteria for different categories, and three different classes are defined [20]:

a) ductile failure with reserve strength capacity resulting from strain hardening,
b) ductile failure with no reserve capacity, and
c) brittle failure.

In other words, a structure whose collapse occurs with some warning ensures certain precautions can be taken to avoid severe consequences, so it may be designed a lower level of reliability. On the other hand, a structure whose collapse would be sudden must be designed with greater reliability [20].

In the safety concept of limit state design, the criteria of safety are defined for each structure based on the consequence classes and their correspondent target reliability. In next chapters, the safety requirements of EN-1990 are implemented as target reliabilities.

3 Reliability analysis

3.1 Introduction

This chapter presents an overview of the reliability analysis and probabilistic methods. Reliability analysis is the main tool in this research work for evaluation of the analysis. The results are interpreted based on reliability analysis. Therefore, different components involved in reliability analysis must be described, along with various methods for further applications.

3.2 Random variable

As defined by [29], a random variable is a means for representation of an incident in analytical format. Random variable definition is based on mathematics. In contrast to a deterministic variable that can be considered as a certain value, the value of a random variable may be defined within a range of possible values. The event A may be mapped through the random variable X, and thus can be identified as indicated in (3.1) [30]:

$$A = a < X \leq b \tag{3.1}$$

If X is a random variable, its probability distribution can always be described by its cumulative distribution function (CDF), as in (3.2):

$$F_X \equiv P(X \leq x) \qquad for\ all\ x \tag{3.2}$$

For a continuous random variable, the probability law is described in terms of the probability density function (PDF) denoted as $f_x(x)$ such that the probability of X in the interval $(a, b]$ is

$$P(a < X \leq b) = \int_a^b f_X(x)dx = \int_{-\infty}^b f_X(x)dx - \int_{-\infty}^a f_X(x)dx \tag{3.3}$$

3.3 Failure probability and reliability

A limit state is a mathematical of a structure. Beyond the limit state, the model no longer fulfills the relevant design criteria (ultimate or serviceability) and as a result, failure occurs (virtually). A failure event can be defined with a so-called limit state function, as

$$g(x) \leq 0 \tag{3.4}$$

where the vector x includes the realization of basic random variable X.

Based on (3.4), the failure of the limit state is defined as the set of realizations for function $g(\boldsymbol{x})$ with zero and negative values [31].

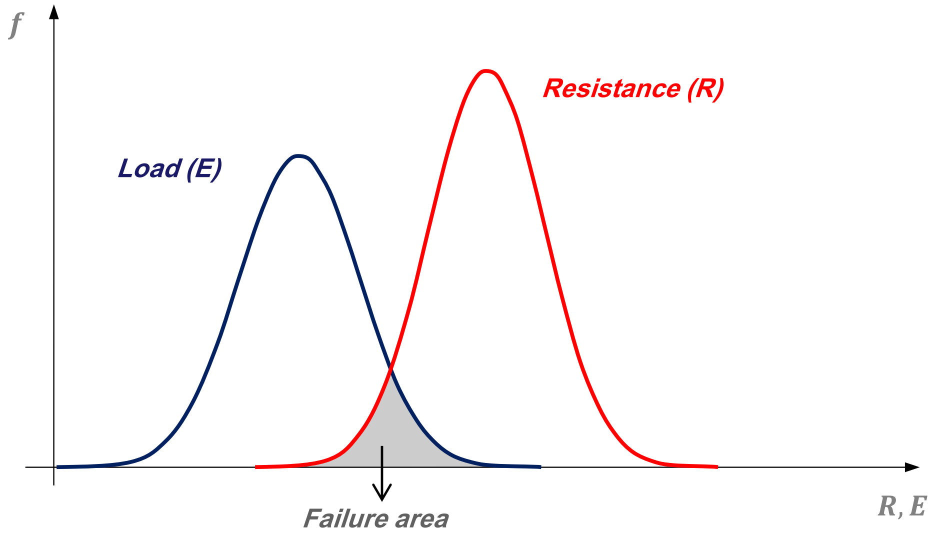


Figure 3.1: Schematic representation of failure with probability density function (PDF) of load and resistance

A general mathematical relation similar to (3.5) could be assumed for the structural model, which consists of two independent linear random variables: resistance and load variables. In [29], this model is called the idealized case and is defined by

$$R - E > 0 \tag{3.5}$$

In this case, the failure event is $R - E \leq 0$, and the function $g = R - E = 0$ is known as the limit state function. In general by neglecting time effects, any failure criterion of a particular design situation containing finite variables can be written in multi-dimensional limit state form:

$$g(x_1, x_2, x_3, \dots x_n) = R(x_1, x_2, x_3, \dots x_m) - E(x_{m+1}, x_{m+2}, \dots x_n) \tag{3.6}$$

Huntington [32] has established a theorem conveniently used to represent the probability of failure regarding distribution functions of resistance and action. For the limit state considered to contain only two independent variables (*R*, *S*), the probability of failure is computed as follows:

$$P_f = P(R - E \leq 0) = \int_0^\infty F_R(x) \cdot f_E(x) d_x \tag{3.7}$$

in which

F_R is the cumulative probability distribution function (CDF) in *R*, and

f_E is the probability density function (PDF) for *E*.

In the structural calculation, the probability of failure is very low (in the range of 10^{-4} to 10^{-7}). For the sake of convenience, the probability of failure is transferred to another mathematical parameter, called the "reliability index," as defined in (3.8).

$$\beta = -\Phi^{-1}(P_f) \tag{3.8}$$

Here, Φ^{-1} represents the inverse cumulative of standard normal distribution. The values of different reliability indexes relative to various probabilities of failure are shown in Figure 3.2.

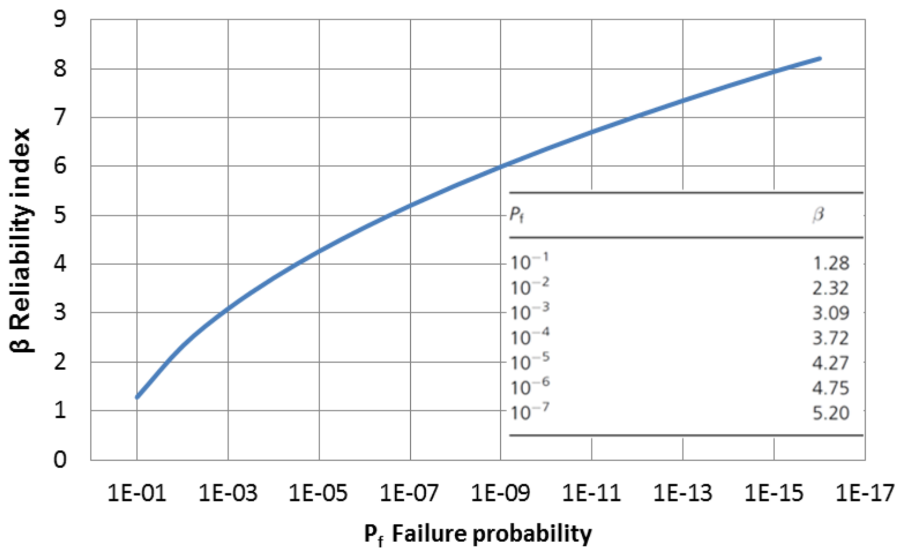


Figure 3.2: Logarithmic plot of failure probability and reliability index

A useful comparative measure of reliability is β, which can be used to evaluate the relative safety of various design alternatives. However, an assessment of reliability may be provided by solving (3.7). An explicit analytical solution exists for very simple models where resistance and load are independent (both having Gaussian distribution functions).

$$\beta = \frac{(\mu_R - \mu_E)}{\sqrt{\sigma_R^2 + \sigma_E^2}} \tag{3.9}$$

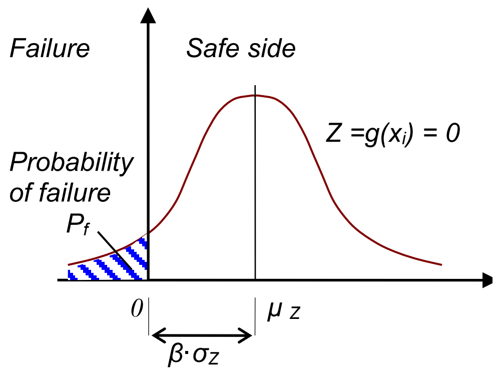


Figure 3.3: Illustration of a reliability index [11]

However, most practical engineering problems are very complicated, and non-normal distributions are involved. Hence other numerical methods need to be examined to solve (3.7). Some of the reliability methods for solving the integral of (3.7) are classified in Table 3.1.

Table 3.1: Overview of some reliability methods [33]

Category	**Method or technique**
Level III	Analytical or numerical integration
	Monte Carlo
	Importance sampling
Level II	First Order Second Moment Method (FOSM)
	First Order Reliability Method (FORM)
	Second Order Reliability Method (SORM)

3.4 Linear First Order Second Moment method (FOSM)

Certain methods are generally referred to as “first-order” methods because they require a first-order (linear) approximation of the failure criteria (limit state) in terms of the design variables [30]. The linearization is done by the first two terms of Taylor expansion at the design point used for first and second moments of the random variables. Therefore, the term “second-moment” is included. This method is also referred to as the “mean value FOSM” (MNFOSM) [34].

Equation (3.10) describes a linearization of the limit state, which apparently is the approximation by the first two terms of Taylor expansion at the design point $P^* = (X_1^*, X_2^*, \dots, X_n^*)$.

$$z = g(X_1^*, X_2^*, \ldots, X_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} \cdot (x_i - x_i^*) = 0 \tag{3.10}$$

The point P^* is called a checking point. Multi-failure criteria and time variation for random variables are neglected as well. Most of the early approaches selected P^* to equal the mean of basic variables. The distribution of Z has a mean value of

$$\mu_z \cong g\left(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right) \tag{3.11}$$

Assuming the random variables to be statistically uncorrelated, the standard deviation in Z can be approximated by

$$\sigma_z \cong \left[\sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i} \cdot \sigma(x_i)\right)^2\right]^{\frac{1}{2}} \tag{3.12}$$

By using these equations, the reliability index β is defined by (3.13), which is a geometric measure. It gives the distance from the mean of limit state to the origin. This method is also known as the “mean-value method.”

$$\beta = \frac{\mu_z}{\sigma_z} \tag{3.13}$$

Based on Ravindra’s linearization [35], it is convenient to express σ_Z as a linear combination of σ_i. A useful and symmetrical expression of this value is

$$\sigma_z = \sum_{i=1}^{n} \alpha_i \cdot \frac{\partial g}{\partial x_i} \cdot \sigma_i \tag{3.14}$$

with,

$$\alpha_i = \frac{\frac{\partial g}{\partial x_i} \cdot \sigma_i}{\sqrt{\sum_{j=1}^{n} \left(\frac{\partial g}{\partial x_j} \sigma_j\right)^2}} \tag{3.15}$$

Equation (3.15) allows the separation of the contribution of variables and enables the development of simple partial safety factor code formats. Note that the statistical distributions of random variables are not regarded in these methods. Although the theory does not give a complete description of uncertainty for any particular variable, the extension of this idea has encouraged many researchers to develop probability-based structural codes [30].

3.5 First-Order Reliability Method (FORM)

3.5.1 Considering normal distribution

Mean value methods have two fundamental shortcomings. First, the limit state function is linearized at the mean values of the random variables. Using only two terms form the Taylor series may cause significant errors for some nonlinear limit states. Secondly, the mean value methods fail to be invariant to different mechanically equivalent formulations of the same problem. This problem arises not only for nonlinear forms of limit states but also even in certain linear forms: for example, when the loads (or load effects) counteract one another [36]. Ellingwood recognized that the linear expansion of limit state should take place not about the means but as a point on the failure surface $g(x) = 0$—that is, in the upper tail of load parameter distributions and in the lower tail of resistance parameter distributions [11]. Therefore, the main result of the recent efforts is that safety checking can be considered to measure the (random) distance from the mean to any point in the sample space of the structural variables on the surface, representing the failure criterion. If the distance is measured towards the failure side, a positive distance implies a safe outcome [37].

This improvement was made also by [38], in a way which the expansion point is changed from the mean value to most-probably point (MPP). It also represents the minimum distance between origin point and the MPP.

The procedure of calculation can be started by a transformation of a random variable to the standard normal space [34].

$$u_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \quad , \quad x_i = u_i\,\sigma_{x_i} + \mu_{x_i} \tag{3.16}$$

The procedure is illustrated in Figure 3.4; in this normalized space, the new variables have unit standard deviation and zero mean, and therefore this space is occasionally called the reduced space.

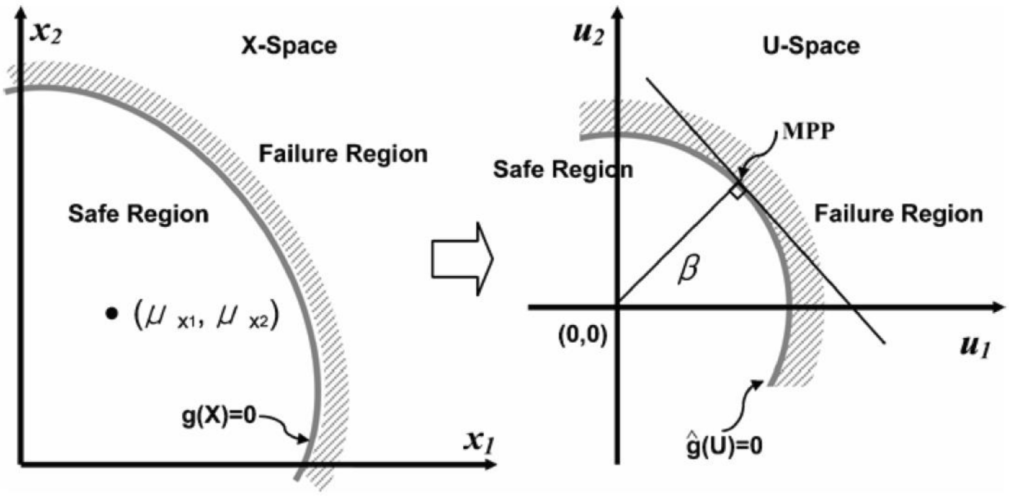


Figure 3.4: Mapping to the standard space (U) [34]

In the reduced coordinate space, the new limit state is

$$g_1(u_1, u_2, u_3, \dots u_n) = 0 \tag{3.17}$$

Failure occurs when $g_1 < 0$, and reliability is defined as the shortest distance between the surface $g_1(x) = 0$ and the origin.

$$\beta = \frac{g(X) - \sum_{i=1}^{n} \frac{\partial g(X)}{\partial x_i} \cdot \sigma_{x_i} \cdot u_i^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g(X)}{\partial x_i} \cdot \sigma_{x_i}\right)^2}} \tag{3.18}$$

The design point or MPP, $P^* = (x_1^*, x_2^*, \dots, x_n^*)$ on $g_1(x) = 0$, must be determined through finding the corresponding coordinate of MPP by means of sensitivity factors.

$$\alpha_i = -\frac{\frac{\partial g(X)}{\partial x_i} \cdot \sigma_{x_i}}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g(X)}{\partial x_i} \cdot \sigma_{x_i}\right)^2}} \tag{3.19}$$

Parameter α_i illustrates the relative effects of each individual random variable on the total variation and reliability index. Based on these parameter, the coordinate of MPP can be calculated as follows:

$$u_i^* = \frac{x_i^* - \mu_{x_i}}{\sigma_{x_i}} = \beta \cdot \alpha_i \quad (3.20)$$

$$x_i^* = \beta \cdot \alpha_i \cdot \sigma_{x_i} + \mu_{x_i} \quad (3.21)$$

3.5.2 Considering various distribution types

The mentioned methods are set to determine the reliability of random variables with a Gaussian distribution. It is recognized in [39] that the approximation caused by non-normal distribution in the algorithm may become more and more inaccurate if the original distribution becomes increasingly skewed. In contrast, many structural problems involve random variables that are non-normal.

The solution is to transform the non-normal variables into equivalent normal variables prior to the calculation. This transformation should be applied such that both distributions match as closely as possible in the range of the design point. Therefore, this method is also referred to as "normal-tail approximation." This transformation may be accomplished by approximating the exact distribution of random variable X by a normal distribution at the value X^* corresponding to a point on the failure surface. In order to determine the mean and standard deviation of the equivalent normal variable, the following equations for approximating normal distribution are suggested in [39]:

$$\sigma' = \frac{\varphi\{\Phi^{-1}[F(x^*)]\}}{f(x^*)} \quad (3.22)$$

$$\mu' = x^* - \sigma' \cdot \Phi^{-1}[F(x^*)] \quad (3.23)$$

where

x^*	is the approximation point,
$F(x^*)$	is non-normal cumulative probability distribution function (CDF) in x^*,
$f(x^*)$	is non-normal density function (PDF) in x^*,
Φ^{-1}	is inverse cumulative for standard normal distribution (CDF),
φ	is probability density function (PDF) for the standard normal distribution.

For the sake of completeness, a summary of Rackwitz and Fiessler's algorithm, according to [11] and [34], is given in Figure 3.5.

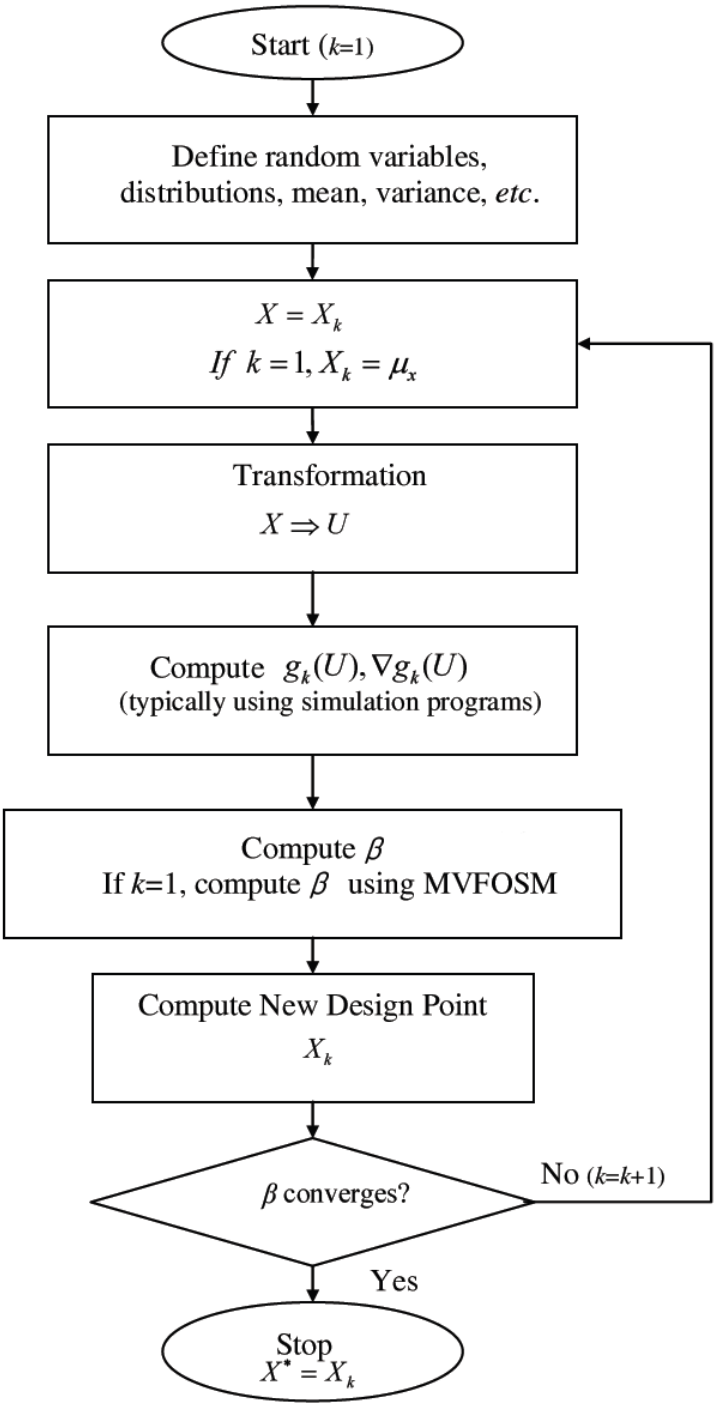


Figure 3.5: Rackwitz Fiessler algorithm [34]

3.6 Monte Carlo method

3.6.1 General

The current version of the Monte Carlo method was established in the 1940s. However, some restricted activities are similar to the Monte Carlo simulation in earlier studies [40]. It is also mentioned by [41] that the accepted origin of the Monte Carlo method was in 1949 when Metropolis and Ulam published their work on the Monte Carlo method [42, 43].

The approaches described in previous sections are all approximation methods. The linearization of limit state function is applied in these methods by implementing the first order or second order of the Taylor series. Therefore, in complex limit state functions, such as a highly nonlinear failure function, multiple failure points or a combination of failure functions (serial and parallel systems), this simplification by the Taylor series will conduct an error calculation to determine failure probability [44]. In such complex cases, it is very difficult to determine whether the result is conservative or the failure probability is underestimated. Hence, neither FORM nor SORM can offer accurate results [45]. These methods were proposed in the 1960s and 1970s because their application for the different limit states was a simple way to calculate failure probability. These methods give an overall view on the probability failure problem. By simulation methods or the Monte Carlo method, the result of the failure probability calculation is the exact solution, but in the 1970s there were no powerful computational instruments to produce large samples for a component of limit states or random variables. At the same time, it was not normal practice to use simulation methods to evaluate failure probability. Therefore, the application of the Monte Carlo method has recently gained popularity due to developments in computing abilities and skills [34]. Calculation now takes much less time, using new random generating algorithms. Hence, implementing simulations or the Monte Carlo method with accurate results could be the best option for reliability analysis.

The Monte Carlo is a technique that takes a finite random sample of the basic variables with their statistical properties and calculates the related limit state. The ratio of the number of simulations that exceed the limit state to the total number of trails is taken as the probability of failure.

Therefore, the computation approach in the Monte Carlo method can be represented in three steps:

1. choosing an individual distribution for each random variable,
2. creating a sampling for each distribution based on random numbers, and
3. applying the simulation and finding the probability of failure [34].

As a consequence of implementing high-performance computers, physical model simulations have become routine, and the Monte Carlo method seems to be a powerful tool in dealing with complicated statistical processes that could not otherwise be handled [34].

The implementation of the Monte Carlo method is examined in [46] for the evaluation of distribution functions for load and resistance. These functions are the results of complicated functions from various random variables in practice. This technique is also recommended in [6].

For the first step of calculation in selecting a suitable distribution, various methods are proposed in literature. In principle, two methods are available to estimate the distribution for a database, namely the method of point estimates and the method of interval estimates ([31]). Maximum likelihood and method of moments as point estimates methods are widely used in recent probabilistic analyses. These are explained in detail in [47], [48] and [49].

3.6.2 Random numbers generation

In order to implement the selected distribution of random variables in a Monte Carlo simulation, input data should be generated as random numbers. The core of any Monte Carlo method is a random number generator [50]. This procedure generates independent random values that follow the same distribution. When the corresponding distribution of random numbers is a uniform distribution on the interval (0,1) then this process will be called “Uniform random number generator” ([50], [51]).

Based on the handbook for the Monte Carlo method [50], two algorithms have the most effective performance in generating random numbers:

1. “Combined multiple recursive generators: some of which have excellent statistical properties, are simple, have a large period, and are relatively fast” [50].
2. “Twisted general feedback shift register generators: some of which have very good equidistributional properties, are among the fastest generators available (due to their essentially binary implementation), and can have extremely long periods which is currently the default generator in MATLAB” [50].

3.6.3 Random variable generation

Using the generated random numbers in the interval of (0 1), the random variable can be generated based on the corresponding distribution of basic variables. Each random value represents the probability of one random variable realization. Eventually, by means of the CDF of basic variables, the corresponding value for the realization can be calculated. The main concept of generating random variable is represented in Figure 3.6.

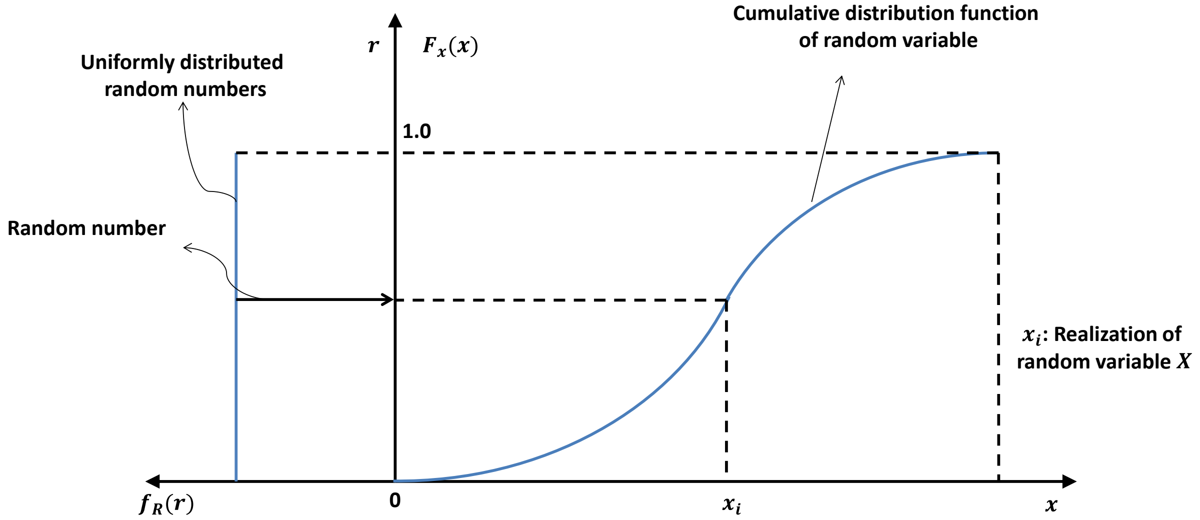


Figure 3.6: Generating a random variable

3.6.4 Crude Monte Carlo

The principles of the crude version of these kinds of simulation techniques will also be applied in this study. However, there are different ways to improve the efficiency of the Monte Carlo method, such as important sampling or variance reduction (see [52]). The crude Monte Carlo method is so far the most uncomplicated method for simulation of structural reliability [31].

The simulation method can be proposed by reformatting the probability integral in the form of (3.24), using an indicator function $I[\,]$.

$$\mathrm{P}_f = \int_{g(x)\leq 0} f_x(x)\,dx = \int I[g(x) \leq 0]\, f(x) dx \tag{3.24}$$

In this integral, the integration domain is changed from a part of X $(x_1, x_2 \dots x_n)$ where $g(x) \leq 0$ for the whole domain of X. In this domain, the indicator function $I[g(x) \leq 0]$ is equal to 1, where $g(x) \leq 0$ is otherwise equal to zero [31]. In Figure 3.7 a schematic illustration of a crude Monte Carlo sample domain is represented.

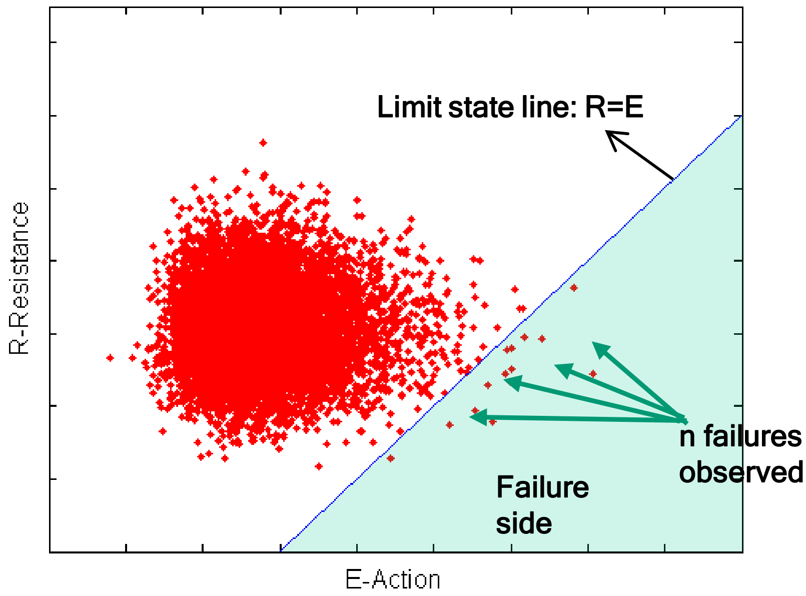


Figure 3.7: Schematic representation of crude Monte Carlo simulation

Considering the first moment that represents the expected value of a random variable and comparing it with (3.24), the failure probability can be represented as a mean value of indicator function $I[g(x) \leq 0]$. If *N* realizations of the vector $\boldsymbol{X}$ are sampled, the failure probability can be calculated as follows:

$$P_f = E[I(X)] = \frac{1}{N}\sum_{j=1}^{N} I[x_j] \tag{3.25}$$

Or based on the number of failure points in simulation, the probability of failure can be formulated as in (3.26).

$$P_f = \frac{n, failure\ realisation}{N, total\ realisation} \tag{3.26}$$

The statistical error corresponding to the estimation of failure probability by the Monte Carlo method is proportional to $\frac{1}{\sqrt{N}}$. The probability of failure has been calculated based on the mean value of the indicator function. The variance of the indicator function can also be described based on the failure probability, following the procedure of [51].

$$Var[I(X)] = E[I(X)^2] - E[I(X)]^2 = E[I(X)] - \{E[I(X)]\}^2 = E[I(X)] - \{1 - E[I(X)]\}$$

$$Var[I(X)] = \sigma_I^2 = P_f(1 - P_f) \tag{3.27}$$

Eventually, based on the indicator function variance and the number of trials in the Monte Carlo method, the standard deviation or variance of the final result of Monte Carlo estimation can be represented, as in (3.28).

$$S_I^2 = \frac{1}{N-1}\left\{\sum_{j=1}^{N} I[X]^2 - N\left(\frac{1}{N}\sum_{j=1}^{N} I[X]\right)^2\right\} \tag{3.28}$$

The term $N-1$ can be changed to N in case of a large N. The (3.28) will be reformed in this case and can be represented by the failure probability.

$$S_I^2 = \frac{P_f(1-P_f)}{N} \tag{3.29}$$

This value represents the estimation error for failure probability based on the trial numbers and estimated failure probability. Clearly, the required number of total trials is related to the desired accuracy for failure probability. In [33], it is reported that some attempts have been made to evaluate the minimum required number, such as (3.30).

$$N \geq \frac{1}{[Var(P_f)]^2}\left|1-\frac{1}{P_f}\right| \tag{3.30}$$

3.7 Importance sampling method

Efficiency and accuracy generally contrast one another. More efficiency leads to less accuracy and vice versa. The variance of an estimated probability in the Monte Carlo method represents the accuracy of the result. To produce better or more accurate results, the variance should be decreased by increasing the number of sample points [53].

Otherwise, if the efficiency of Monte Carlo is considered, fewer random points should be obtained for decreasing the calculation time to reach the same level of variance. These two scenarios are so-called variance-reduction techniques. In these techniques, the variance is decreased with the same number of random points, or the variance level remains constant but with fewer random realizations [54].

As mentioned, the problem in applying the crude Monte Carlo is that the joint density function of basic variable is located in an area far from the limit state failure side. For example, to reach a reliability level of 3.8 with 1 million sample points, 7 points should be located in the failure side. Therefore, a method has been proposed to increase the efficiency of the sampling method.

An importance sampling method was first introduced by Harbitz [55]. Importance sampling is the most effective reduction technique [56]. The most important characteristic of this method is the process of producing samples. The sample points are mostly distributed in the failure domain, which helps speed convergence on the final probability value [45].

In order to apply the importance sampling method, the integral in (3.24) can be rewritten as follows:

$$\mathrm{P}_f{}^* = \int I[g(x) \leq 0]\, f(x) dx = \int \frac{I[g(x) \leq 0]}{h_v(x)}\, f(x)\, h_v(x)\, dx \tag{3.31}$$

where h_v is the importance sampling density function.

Again here by comparing the integral in (3.31) and the first-moment function, it can be concluded that P_f is the expected value of $\frac{I[g(x)\leq 0]\, f(x)}{h_v(x)}$, and component x is distributed based on importance sampling distribution function $h_v(x)$ ([31]). Comparing (3.31) to the (3.26) shows that $I[\,]\frac{f}{h}$ is applied instead of $I[\,]$. Therefore, the unbiased estimation of failure probability can be represented as (3.32).

$$\mathrm{P}_f{}^* = E[I(X)] = \frac{1}{N}\sum_{j=1}^{N} \frac{I\left[g\left(x_j\right) \leq 0\right]}{h_v(x_j)}\, f(x_j) \tag{3.32}$$

The variance of estimated probability with importance sampling is formulated in (3.33) [52].

$$Var[\mathrm{P}_f{}^*] = \frac{Var\left(\frac{If}{h}\right)}{N} \tag{3.33}$$

$$Var\left(\frac{If}{h}\right) = \int \dots \int \left\{I[\,]\frac{f(x)}{h_v(x)}\right\}^2 h_v(x) dx - \mu_{p_f}^2 \tag{3.34}$$

Based on these equations, the optimum choice of h_v can be easily found and represented, as in (3.35) [34].

$$h_v = \frac{|I[\,]\, f(x)|}{\int \dots \int\, |I[\,]\, f(x)|\, dx} \tag{3.35}$$

Then by substituting in (3.34),

$$Var\left(\frac{If}{h}\right) = \left\{\int \dots \int\, |I[\,]\, f(x)|\, dx\right\}^2 - \mu_{p_f}^2 \tag{3.36}$$

If $|\mathrm{I}[\,]\, \mathrm{f(x)}|$ remains positive everywhere, the integral will be identical with $\mu_{\mathrm{p_f}}$, and subsequently $\mathrm{Var}[\mathrm{P_f}^*]$ will be zero. So the optimal choice of h_v is equal to (3.37).

$$h_v = \frac{I[g(x) \leq 0]\, f(x)}{\mu_{p_f}} \tag{3.37}$$

At first, it seems that this equation is not advantageous, because for evaluating h_v, μ_{p_f}, probability of failure is needed, which is impossible to find. However, progress can be made by an initial estimation of failure probability μ_{p_f}, which is close to the final value. It should be taken into account that a poor choice of importance sampling function will lead to increase of failure probability variance. Therefore, importance sampling should be used with caution ([52], [34]). The critical point for the importance sampling technique is to produce a positive sampling located near the most probable failure point (maximum likelihood or design point) ([45], [57]).

Generally, finding the optimal h_v is a difficult task, typically requiring appropriate prior information such as design point [52]. Usually, this point is unknown, though, and must be evaluated based on other approaches, such as FORM or FOSM initially; the importance sampling function is then applied on this point.

In order to skip this prior analysis alternative approach, the so-called adaptive method can be applied. By means of this method and its algorithm, the sample domain will be guided in the direction of the design point. Thus, it is not necessary to find the design point before applying the method and nor to apply importance sampling afterwards. Another advantage is that the importance sampling function can be modified through each iteration of the adaptive importance sampling algorithm.

In the application of the adaptive importance sampling method, an arbitrary point in the failure side of the limit state would be the initial sampling point (x^*) for the algorithm. By processing the algorithm of the adaptive method, x^* will be the design point in the final iteration. The importance sampling function h_v for a problem with *n* basic variables, could be an *n*-dimensional normal joint density function with mean values based on the initial selected point on the failure side of the first iteration and with a standard deviation for each random variable ([31], [52]).

In each iteration, the sample domain is reproduced based on the h_v, which is modified according to the sample domain in the final iteration. The standard deviation for the function h_v can be considered constant in all iterations, but the mean value is changed in order to guide the sampling domain toward the design point [58].

The next point among sample points can be selected based on the likelihood of sample realization. The point with the maximum likelihood ($f(x)$) is the point needed for the next step of the calculation. This selected point will be considered the mean value of the h_v function in next iteration. This loop will be continued until the convergence error is reached. Considering the first iteration, the estimated probability of failure can be calculated in the first iteration from (3.38).

$$\mathrm{P}_f{}^{[1]} = \frac{1}{N^{[1]}} \sum_{j=1}^{N} \frac{I\left[g\left(x_j{}^{[1]}\right) \leq 0\right]}{h_v{}^{[1]}(x_j{}^{[1]})} \; f(x_j{}^{[1]}) \tag{3.38}$$

And after *s*-th iteration, the probability will be estimated by (3.39) [52].

$$\mathrm{P}_f{}^{[s]} = \frac{1}{N_{all}} \sum_{u=1}^{s} \sum_{j=1}^{N^{[u]}} \frac{I\left[g\left(x_j{}^{[u]}\right) \leq 0\right]}{h_v{}^{[u]}(x_j{}^{[u]})} \; f(x_j{}^{[u]}) \tag{3.39}$$

Then the termination condition can be defined with a convergence error value ε.

$$\varepsilon < \left| \frac{\mathrm{P}_f{}^{[s]} - \mathrm{P}_f{}^{[s-1]}}{\mathrm{P}_f{}^{[s-1]}} \right| \tag{3.40}$$

3.8 Comparing reliability methods

In the first step, a generic model for the structure resistance is used to apply the methods introduced in previous sections. A conventional model, so-called generic, is considered in this part for the resistance probabilistic model. The loading includes two mutually independent actions: a permanent load *G* and leading imposed load *Q*. Resistance of generic member (which covers all types uncertainty related to material and resistance modeling) is defined by a lognormal distribution with a coefficient of variation COV = 0.15 and the mean μ_R as follows [15]:

$$\mu_R = R_k \; \mathrm{e}^{1.65\, cov} \tag{3.41}$$

In the case of a generic structural member, it is assumed that the characteristic value R_k of the resistance R may be calculated as the 5% fractile of *R* and the design value R_d as follows:

$$R_d = R_k / \gamma_R \tag{3.42}$$

where γ_R is a resistance partial factor considered for generic members to be $\gamma_R = 1.1$.

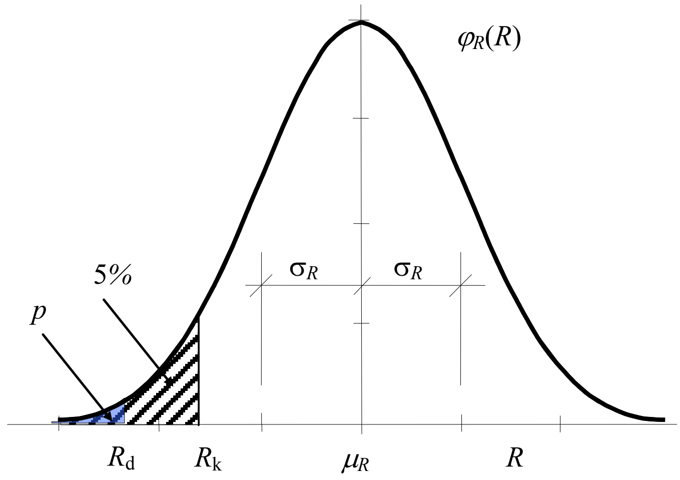


Figure 3.8: Random variable R, the characteristic value R_k and design value R_d [15]

The analysis of generic resistance assumes the linear behavior of structural members: namely actions and their characteristic values. The action part includes permanent actions G and variable actions Q. Variable actions may comprise two actions at the same time in the limit state, leading variable Q_1 and accompanying variable action Q_2. Additionally, by considering the uncertainty in a loads model with θ, the limit state can be defined by (3.43) as a limit state for a generic resistance model.

$$g = R - \theta(G + Q) \tag{3.43}$$

3.8.1 Definition of load ratio χ for reliability analysis

The ratios of different actions' characteristic values are defined as a parameter for interpretation of the reliability results. The influence of this parameter on the reliability level and load combination are investigated in the upcoming chapters. Permanent or dead load G, leading variable Q_1 and accompanying variable load Q_2 may be considered as three types of loadings for the reliability analysis. The ratios of these loads are defined in (3.44) and (3.45). The results are represented based on these defined ratios.

$$\chi = \frac{Q_k}{Q_k + G_k} = \frac{Q_{1\,k} + Q_{2\,k}}{Q_{1\,k} + Q_{2\,k} + G_k} \tag{3.44}$$

$$k = \frac{Q_{2\,k}}{Q_{1\,k}} \tag{3.45}$$

These values are to be applied in reliability analysis to distribute the total assumed loading in a different loading type and to observe their influence on the final reliability index result. The χ value represents the structural normalized weight, and G is the self-weight of structure; as such, the high values of χ correspond to the light-weight structures where variable loads are dominant in the structure loading. On the other hand, small values of χ represent the heavy-weight structures, where self-weight or permanent actions are the most subjected actions. Figure 3.9 shows the changing behavior schematic in loads by χ. Variable k is also the ratio between two variable loads and when its minimum equals 0, which means that there is only one variable load in the limit state. When It reaches its maximum of 1, two variable loads have the same characteristic value. In between the maximum and minimum k, 0.25, 0.5 or 0.75 values are considered.

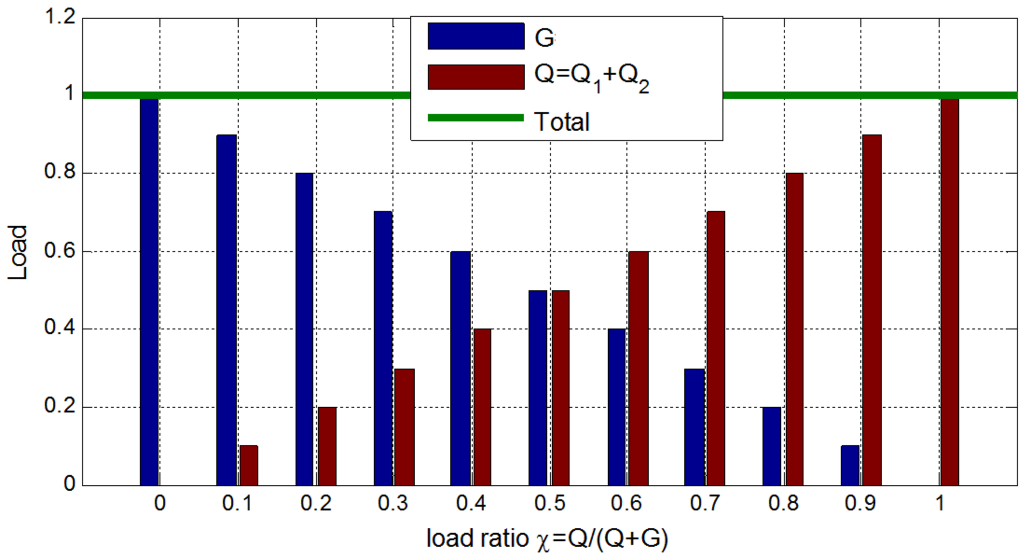


Figure 3.9: Loading diagram according to the χ

By increasing the parameter χ, the distributions of loads also vary. This variety is investigated in an arbitrary case with an assumed total load of 1, with one permanent and one imposed load. The distribution behavior for these two loads by increasing χ is presented in Figure 3.10 and Figure 3.11, respectively.

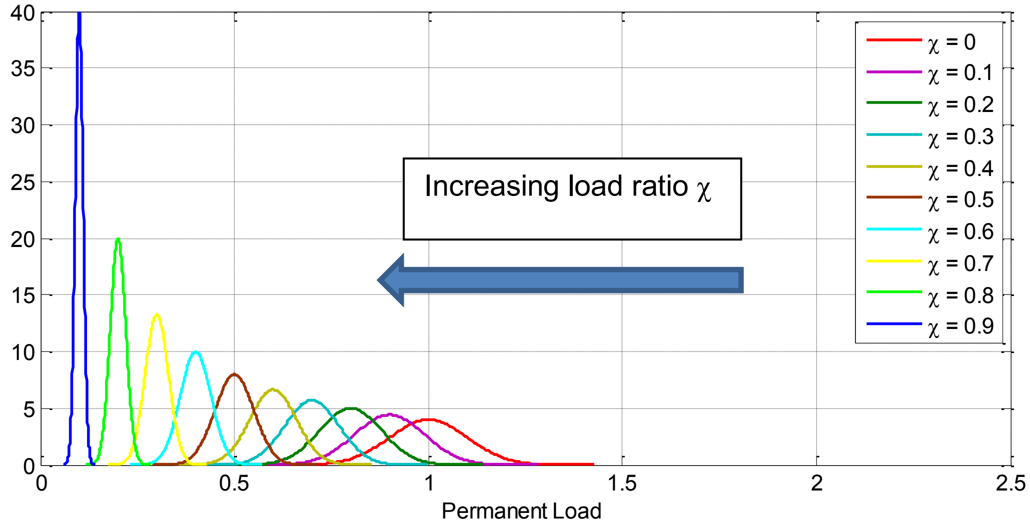


Figure 3.10: Permanent load *G* distribution with increasing of χ

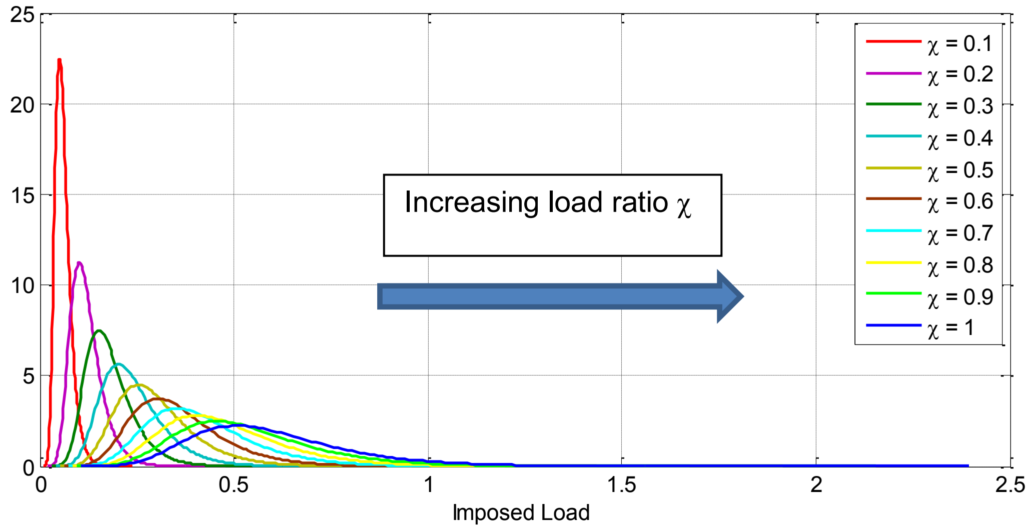


Figure 3.11: Variable load Q distribution with increasing of χ

The representation of χ factor for structural weight may also be classified based on its values. In order to categorize structures based on influence of χ, the distributions in Figure 3.10 and Figure 3.11 for each ratio are compared individually.

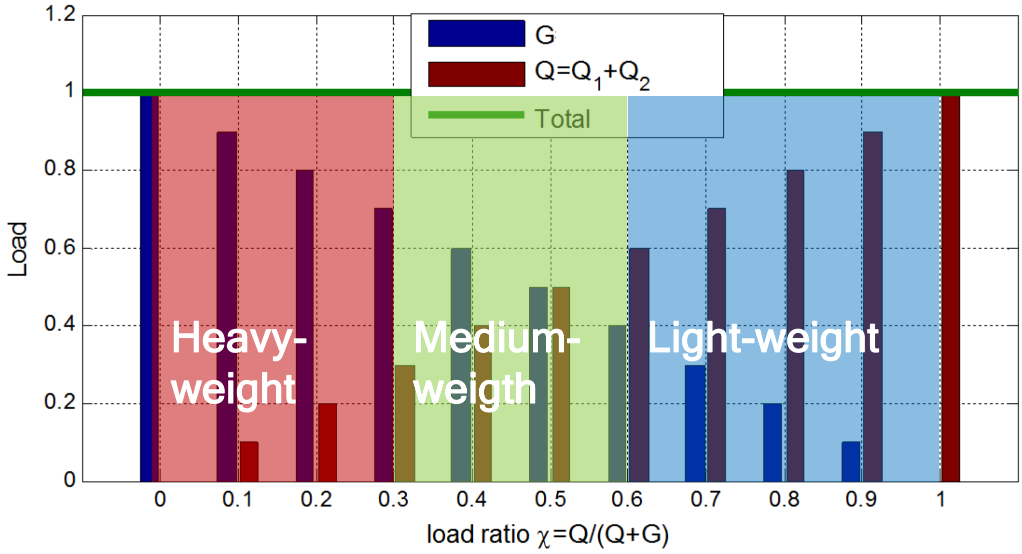


Figure 3.12: Classification of structural weight based on χ

Based on this comparison the categorization of structural weight according to χ is presented in Figure 3.12. The first part $\chi{:}\,(0-0.3)$, as it seems in Figure 3.13, is the interval which represents permanent actions as decisive one. The distributions of permanent actions are also in this interval laying in higher value with higher deviation. The values of permanent loads are lying in higher ranges in this interval therefore; it means that the structure in this interval represents the heavy-weight ones. Here structures with materials like concrete and masonry may be represented as hevy-weight structures.

On the other hand at the end of the interval for $\chi{:}\,(0.6-1)$, variable actions are higher than permanent loads. Therefore, the amounts of self-weight or permanent load are not decisive and this interval represents the light-weight structures. Based on the Figure 3.15 it can be also observed that here the distribution of variable load are decisive. Steel structures and timber structures may be represented by this category as light-weight structures.

The middle interval is representing the medium weight structures. As it seems in Figure 3.14 the distributions are close to each other. Either of permanent or variable loads may be the decisive actions in this category. Composite structures could be classified as a medium weight structures.

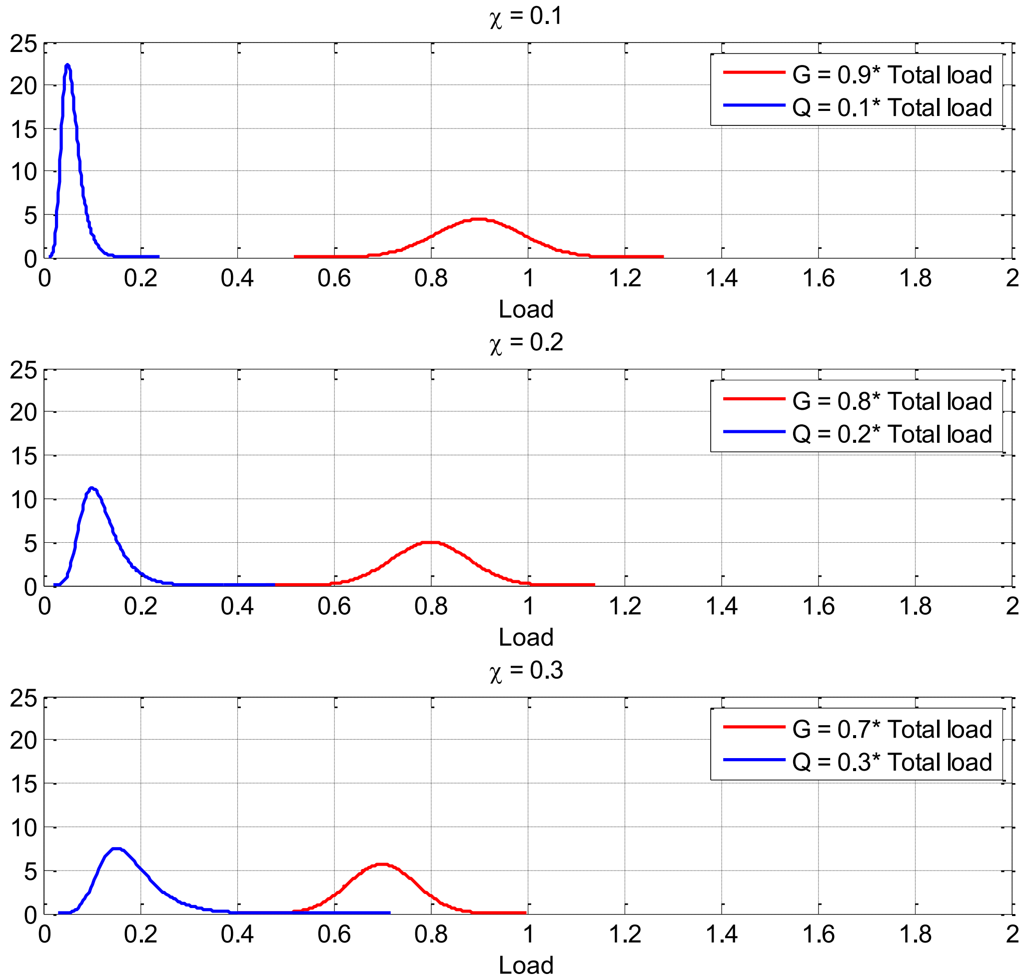


Figure 3.13: Heavy-weight distributions

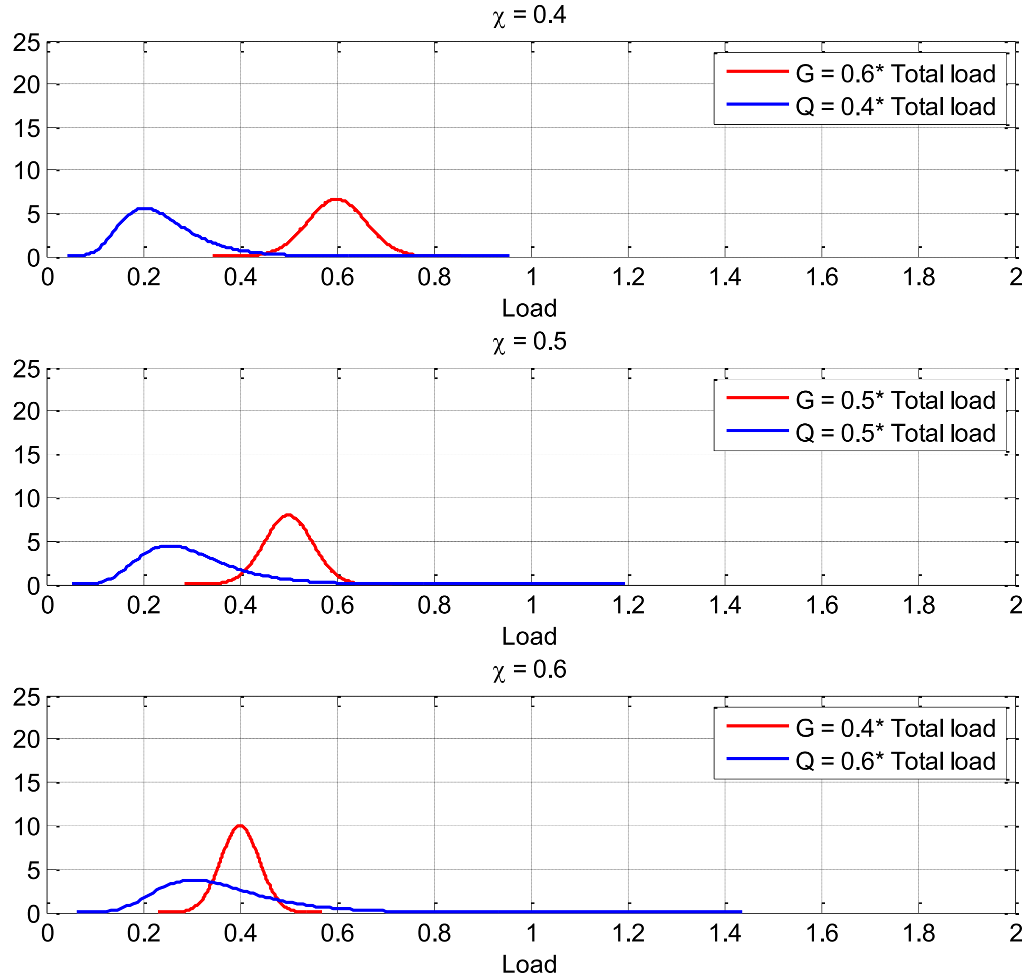


Figure 3.14: Medium-weight distributions

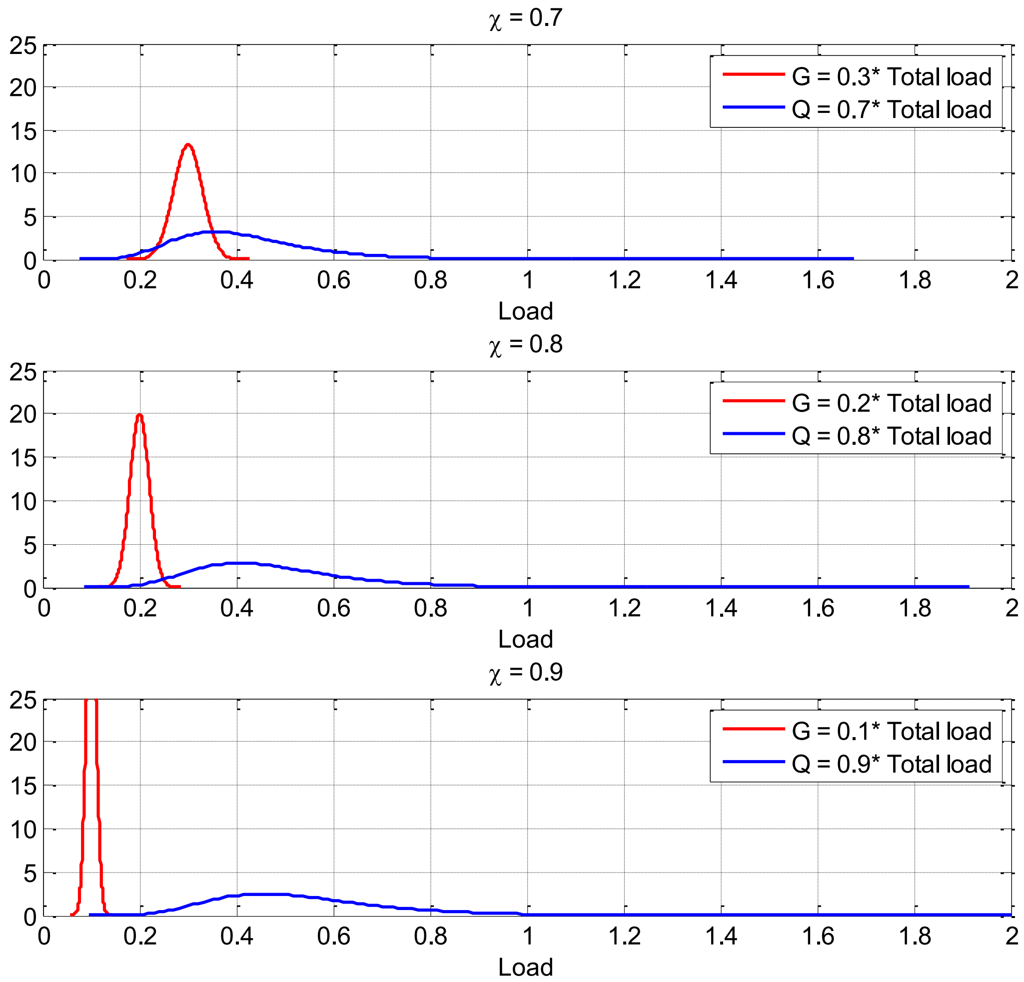


Figure 3.15: Light-weight distributions

3.8.2 Comparison result

In this part, the limit state includes only one leading variable action. The stochastic parameters for the basic variables in reliability analysis are introduced in Table 3.2. This section aims to compare the Monte Carlo method, importance sampling and the FORM method. Figure 3.16 shows the variation reliability of a complete load interval for the Monte Carlo method (with different numbers of trails) as well as FORM and the importance sampling method.

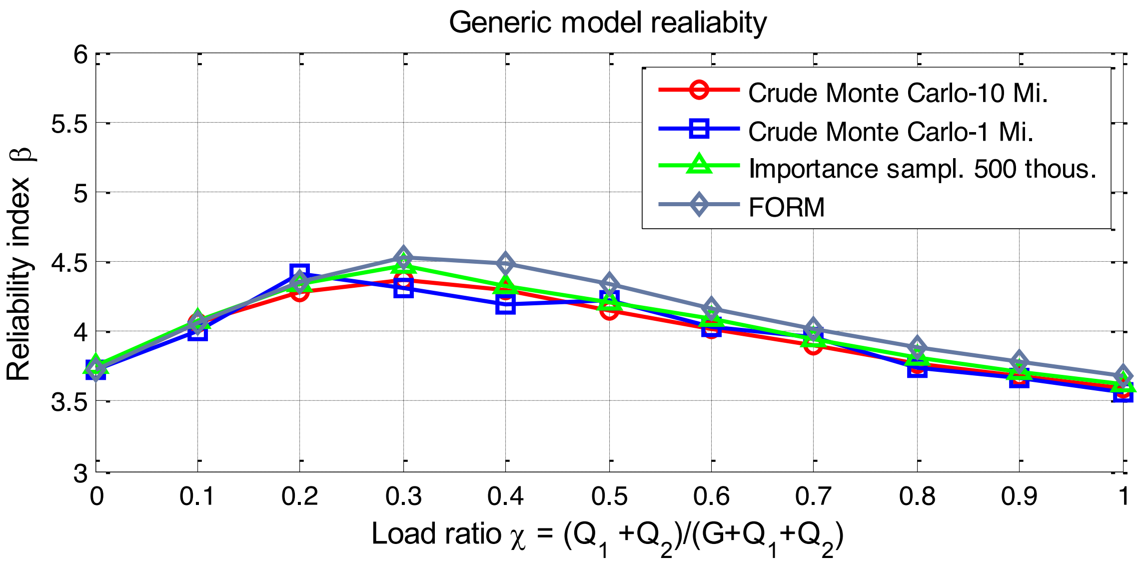


Figure 3.16: Reliability of generic model with crude Monte Carlo and importance sampling

The same behavior and result are observed with both crude Monte Carlo and importance sampling. The FORM result shows a slightly higher value, especially for higher values of χ. These higher values appear because of the approximate transformation of non-normal distributions to normal distribution. On the other hand, it is obvious in Figure 3.16 that the same level of variance is observed with a crude Monte Carlo of 10^7 sample points and importance sampling of $5 * 10^5$ sample points. In the Crude Monte Carlo method with 10^6 realizations, however, the variance of the estimated probability is unacceptable. The behavior depicted in the diagram corresponding to the crude Monte Carlo method with 10^6 points does not have a smooth shape. At some points, for instance $\chi = 0.3$ and $\chi = 0.4$, the properties of the diagram change unacceptably, and this change occurs because of the large variance in calculated reliability. This error comes from the lack of sample points in the Crude Monte Carlo. The acceptable result for the crude Monte Carlo is done with 10^7 samples. The same acceptable result has been reached through importance sampling, with a 5% sample point number in the crude Monte Carlo. Hence, by applying importance sampling, the calculation cost is so far reduced by 95%.

3.9 Stochastic parameters for calibration and code analysis with probabilistic methods

The basis of probabilistic analysis and reliability evaluation are the parameters that define the stochastic behavior of basic variables. Defining the probabilistic model of basic variables is the fundamental component of each reliability analysis. The essential part of probabilistic modeling is related to considering distribution functions. Each basic variable should have its individual PDF for implementation in reliability analysis.

Table 3.2: Stochastic parameters for reliability analysis [15], [59]

No.	Category of variables	Name of basic variables	Symb. X	Dist.	Mean	Cov. x
1	Actions	Dead load	G	Normal	G_k	0.05–0.10
2		Snow (50 years)	S	Gumbel	$1.1S_k$	0.30
3		Snow (1 years)	S	Gumbel	$0.35S_k$	0.7
4		Wind (50 year)	W	Gumbel	$0.6W_k$	0.35
5		Wind (1 year)	W	Gumbel	$0.3W_k$	0.5
6		Imposed (50 years)	Q	Gumbel	$0.6q_k$	0.35
7		Imposed (5 years)	Q	Gumbel	$0.2q_k$	1.1
8	Resistance	Structural steel	f_y	Lognormal	$f_{yk}+2\sigma$	0.08
9		Concrete	f_c	Lognormal	$f_{ck}+2\sigma$	0.17
10		Reinforcing steel	f_y	Lognormal	$f_{yk}+2\sigma$	0.05
11		Timber	f_t	Lognormal	$f_{tk}+2\sigma$	0.15
12		Masonry	f_k	Lognormal	$1.32\ f_k$	0.16
13	Uncertainty	Steel bending	θ_R	Lognormal	1,10	0.07
14		Concrete	θ_R	Lognormal	1.00	0.10
15		Timber	θ_R	Lognormal	1.00	0.10
16		Masonry	θ_R	Lognormal	1.00	0.18
17		Actions	θ_E	Lognormal	1.00	0.05

In the context of this study, reliability analysis is the tool for evaluation and assessment of the investigations. A conventional representation of stochastic parameters for different basic variables is represented in Table 3.2.

These parameters are represented based on the mean value standard deviation and the type of distribution functions. For reliability analysis with more than one variable load, Turkstra's rule is applied for considering the combination of two time-dependent variable loads in this review [60]. In upcoming sections, reliability analysis will be conducted based on the methods and parameters explained in this chapter.

4 Load combinations and partial safety factors

4.1 EN-1990 load combination

The design process in EN-1990 is conducted based on the limit state design concept. This concept requires the modeling of loads and structural components in various design cases. The limit state is divided into two categories: ultimate limit state and serviceability limit state. Limit states are the criteria defined based on the loads and structural parameters to verify the design. The verification has to be done by checking the exceedance of the limit state. All relevant load cases and structural parameters have to be considered with their design values in the procedure of verification [14].

The design value of action effects (E_d) should be determined according to the different combinations of relevant load cases. The design value of action effects is calculated by implementing the partials safety factors and the correspondent characteristic value of each individual action effect. Partial safety factors are categorized according to the source of actions, such as permanent actions, self-weight, variable actions, environmental actions, seismic loads, geotechnical loads and accidental loads.

For each critical load case, the design values of the action effects (E_d) are calculated through a combination of the action values that occur simultaneously. In structural design, several types of loads are considered, which can be defined as natural or human-made phenomena. A structure may be subjected to actions due to self-weight, loads on floors, wind, snow, thermal actions, and so on. However, only critical load cases have to be considered for the verifications. These critical load cases are compatible with the design values determined from characteristic values [14],[16].

In the case of ultimate limit states, the different forms of limit states have been defined in section 6 of EN-1990. For persistent and transit design situations, the fundamental load combination is defined by equation 6.10 in EN-1990 and (4.1):

$$\sum_{j\geq 1} \gamma_{G,j} G_{k,j} \text{"} + \text{"}\, \gamma_p P \text{"} + \text{"}\, \gamma_{Q,1} Q_{k,1} \text{"} + \text{"} \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{4.1}$$

where " $+$ " denotes “to be combined with,” Σ denotes “the combined effect of” and *P* represents action due to pre-stressing. Equation (4.1) can also be represented according to favorable (inf) and unfavorable (sup), as in (4.2).

$$\sum \gamma_{G,j,sup} G_{k,j,sup} \text{"} + \text{"} \sum \gamma_{G,j,inf} G_{k,j,inf} \text{"} + \text{"} \gamma_{Q,1} Q_{k,1} \text{"} + \text{"} \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{4.2}$$

In EN-1990, two other combinations (6.10a and 6.10b) were proposed for limit states in STR and GEO situations. The less favorable of these two combinations will be applied as a design value for the action effects. However, in the German national Annex of EN-1990, use of these two combinations is not permitted (Section 4.3 details the advantage of these combinations).

$$\sum_{j\geq 1}\gamma_{G,j}G_{k,j}"+"\gamma_p P"+"\gamma_{Q,1}\psi_{0,i}\,Q_{k,1}"+"\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad (6.10a) \qquad (4.3)$$

$$\sum_{j\geq 1}\xi_j\,\gamma_{G,j}G_{k,j}"+"\gamma_p P"+"\gamma_{Q,1}\,Q_{k,1}"+"\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad (6.10b) \qquad (4.4)$$

$G_{k,j}$ is permanent action, $Q_{k,1}$ is leading variable action and $Q_{k,i}$ is accompanying variable load, while $\gamma_{G,j}$ is permanent load partial factor, $\gamma_{Q,1}$ is partial factor for leading variable load, ξ is reduction factor for permanent load and $\psi_{0,i}$ is the combination factor for variable loads. The corresponding values for these factors are shown in EN-1990 in A1.1., Table A1.2(A), A1.2(B) and A1.2(C).

4.2 Reliability analysis of EN-1990 load combinations

According to EN-1990, various kinds of combinations and factors can be used to introduce a load combination for a structure. Different values for multiple types of partial factors and combination factors based on limit state type, load properties and structural type are proposed in EN-1990. Different selections from these factors lead to different results for structural design. In order to show the varying outcomes of each combination, reliability analysis is conducted for all possible load combinations. The algorithm of partial factors application for reliability analysis is based on the design (4.5).

$$E_d = R_d \qquad (4.5)$$

Consequently, for each load combination in EN-1990 with two variable loads and one dead load, (4.5) may be reformulated for 6.10, 6.10a and 6.10b as (4.6), (4.7) and (4.8), respectively.

$$\gamma_G G_k + \gamma_{Q1}\,Q_{k1} + \gamma_{Q2}\psi_0 Q_{k2} = \frac{R_k}{\gamma_M} \qquad (4.6)$$

$$\gamma_G G_k + \gamma_{Q1}\psi_0\,Q_{k1} + \gamma_{Q2}\psi_0 Q_{k2} = \frac{R_k}{\gamma_M} \qquad (4.7)$$

$$\xi \gamma_G G_k + \gamma_{Q1} Q_{k1} + \gamma_{Q2} \psi_0 Q_{k2} = \frac{R_k}{\gamma_M} \tag{4.8}$$

The characteristic values of all basic variables with different load ratios χ, k [eq.(3.44) and eq.(3.45)] are calculated according to (4.5). Afterwards, mean values for distribution functions are determined based on the assumption of stochastic parameters and fractile values in Table 3.2. The dead load is considered the permanent action with the highest coefficient for dead load in Table 3.2. These calculations must be made for all load combinations and partial factor sets. The overall rationale for this algorithm is presented in Figure 4.1. Material partial factors, γ_M, are selected according to the recommendation of their correspondent Eurocode as presented in Table 4.1.

Table 4.1: Material partial factor γ_M based on recommendation in Eurocodes

Concrete	Steel	Steel Rebar	Timber	Masonry
1.5	1.0	1.1	1.3	1.5

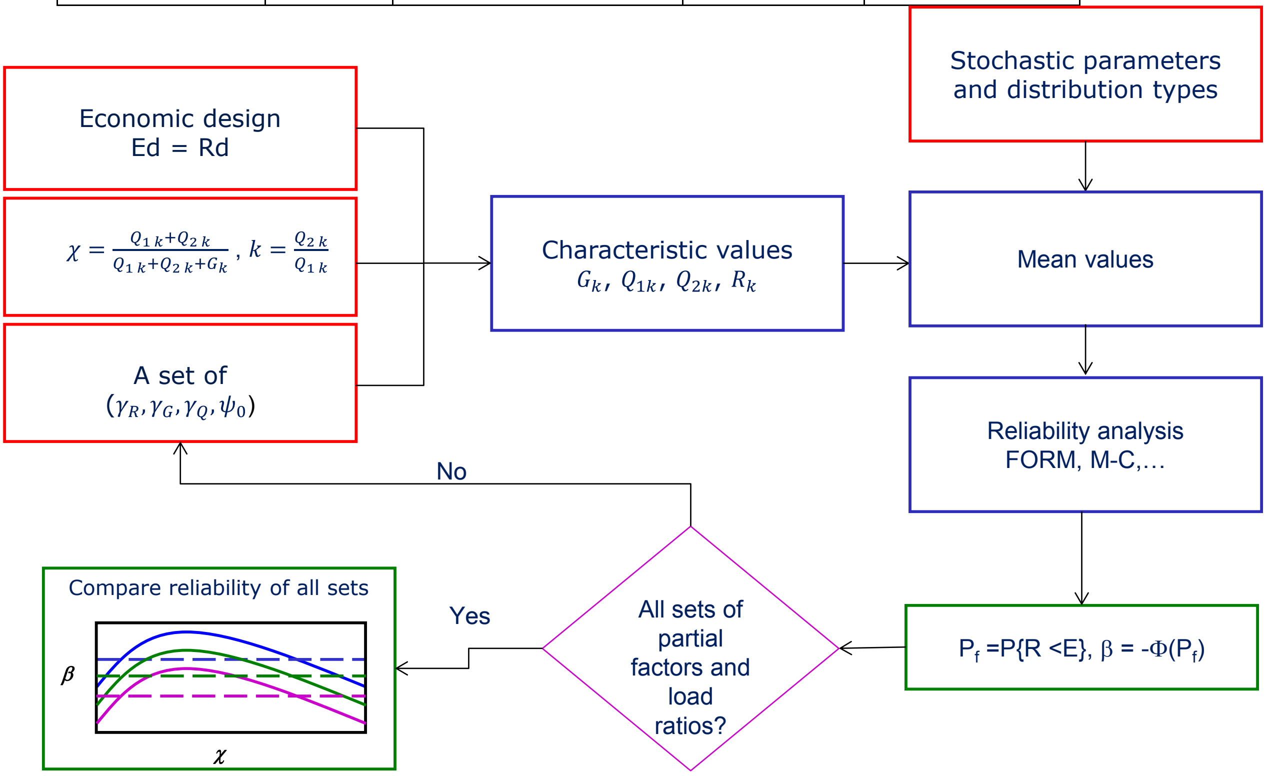


Figure 4.1: Reliability analysis of load combinations algorithm

The limit state function, which includes three types of actions and resistance parameters, is considered as (4.9) for reliability analysis of EN-1990 load combinations. All the corresponding stochastic parameters are applied according to Table 3.2,

$$g = \theta_R R - \theta_E (G + Q_1 + Q_2) \tag{4.9}$$

where

g is limit state function,

θ_R is resistance uncertainty,

R is resistance,

θ_E is actions uncertainty,

G is permanent action,

Q_1 is limit leading variable action, and

Q_2 is limit accompanying variable action.

The limit state in (4.9) may be a representative of different types of structural materials and failure modes. All structural failure modes in which the action effects and actions have a linear relation, such as a beam in bending, can be modeled by the same limit state as in (4.9). In the current study, concrete (EN-1992-1-1[61]), steel (EN-1993-1-1[62]), steel rebar (EN-1992-1-1[61]), timber (EN-1995-1-1[63]) and masonry (EN-1996-1-1[64]) are considered for investigation. For each type of material, R is represented by the correspondent stochastic parameter for the material in Table 3.2. Moreover, three types of variable actions (wind, snow and imposed load) are considered for the investigation of different load combinations. The basic variables, the load ratios χ and k as explained in (3.44) and (3.45), are also considered. In total, 4,455 reliability analyses are completed for all types of materials, loads and their combinations. The average reliabilities for all these cases for three fundamental combinations in EN-1990 are presented in Figure 4.2.

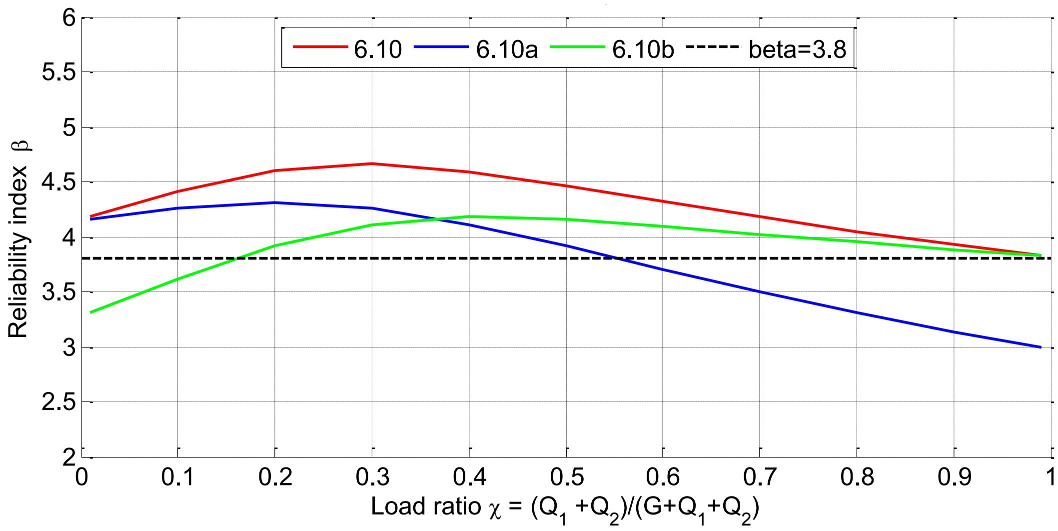


Figure 4.2: Average reliability for all cases and EN-1990 combinations

Figure 4.2 shows three different combinations, although in practice for the design process, only two combinations can be used. According to EN-1990, the less favorable combination between 6.10a and 6.10b has to be selected for the design process. Therefore, the combination with greater reliability is decisive. Consequently, the correspondent combination has to be selected between 6.10a and 6.10b. The reliability behavior depicted in Figure 4.4 can be reformatted as that depicted in Figure 4.3 with the less favorable combination from 6.10a and 6.10b.

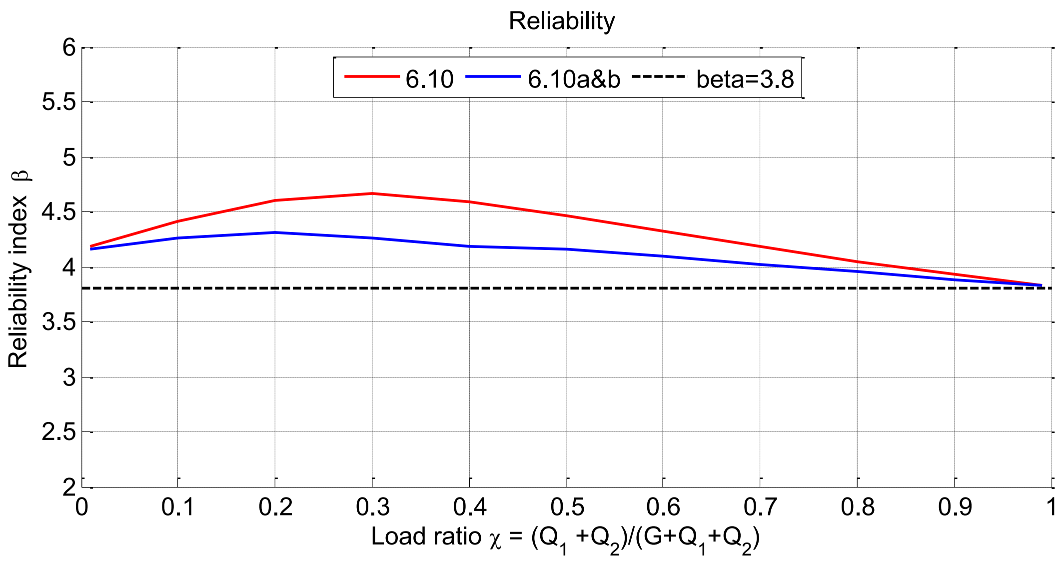


Figure 4.3: Average reliability for all cases and EN-1990 combinations

These 4,455 values can also be treated as random variables. This database of reliabilities representes a mean and coefficient of variation. The histogram for reliability indexes according to all possible combinations and basic variables is shown in Figure 4.4.

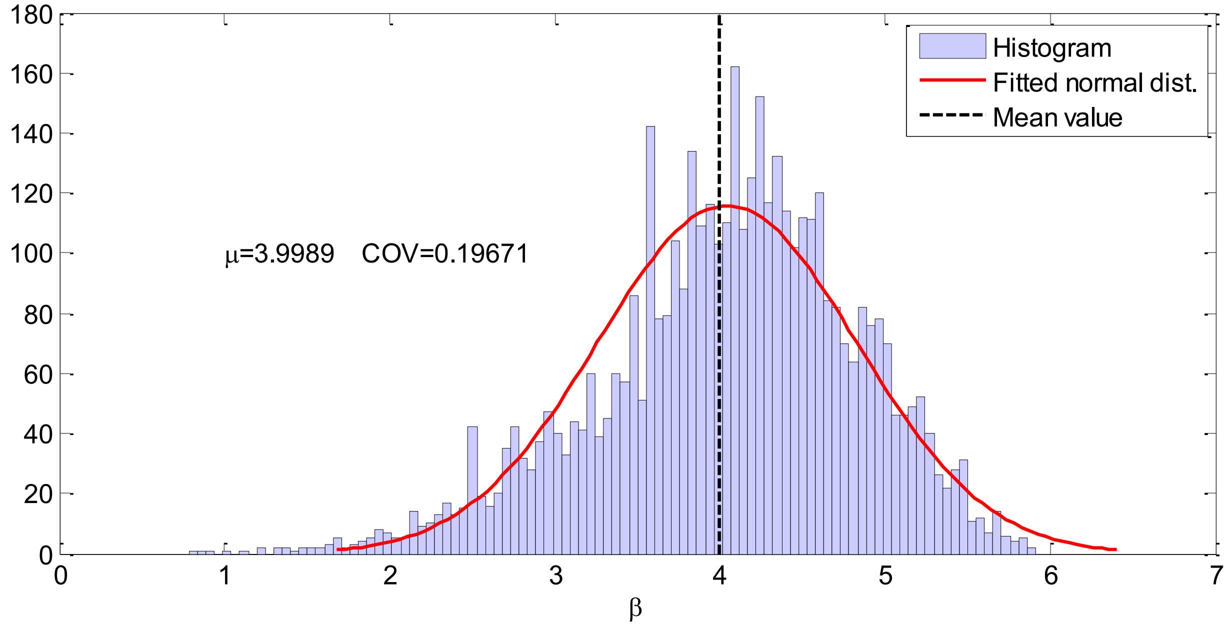


Figure 4.4: Histogram of all reliability indexes for EN-1990 combinations

The whole database may be subdivided into two possible fundamental combinations in EN-1990, 6.10 and 6.10a&b. The histogram for 6.10 and 6.10a&b are presented in Figure 4.5 and Figure 4.6, respectively.

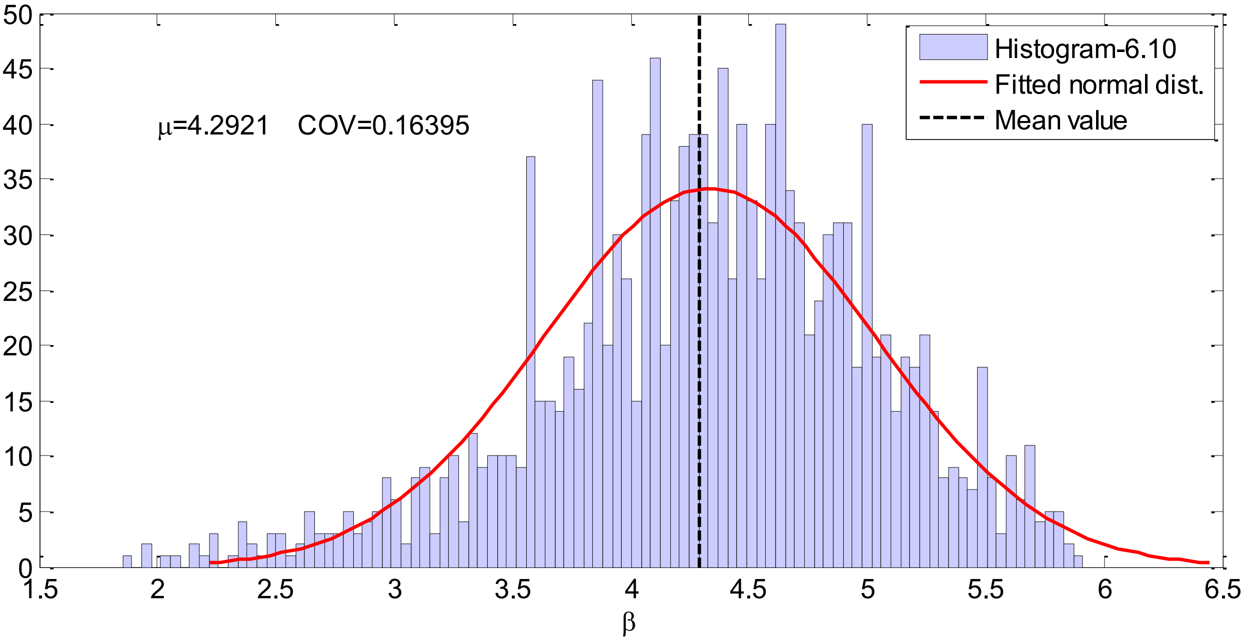


Figure 4.5: Histogram of all reliability indexes for EN-1990 combination 6.10

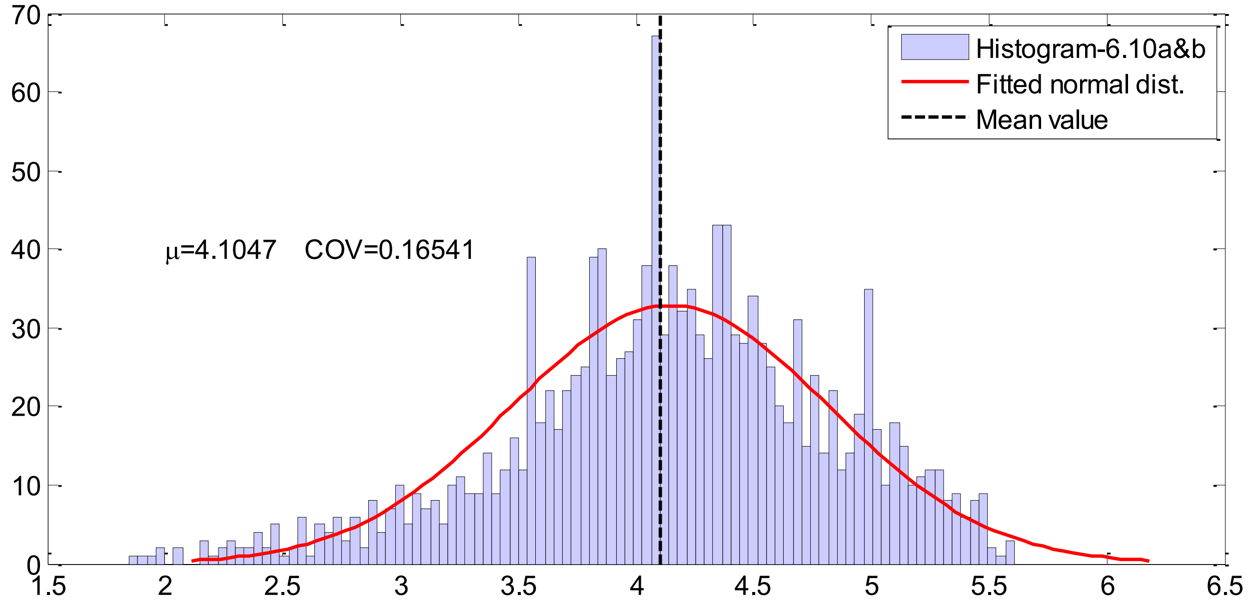


Figure 4.6: Histogram of all reliability indexes for EN-1990 combinations 6.10a&b

The mean values of both possible combinations in EN-1990 show that, on average, they provide a safe result, according to the recommended target reliability, for 50 years. According to the mean values, the combination 6.10 provides greater reliability than the 6.10a&b. The coefficient of variation in both cases are nearly the same, but with smaller mean value in case of 6.10a&b. The standard deviation of this combination is also lower than that of 6.10; thus, the results are more consistent for 6.10a&b, as can also be observed in Figure 4.3.

The reliability analysis for all resistance types provides an overview of the behavior of each material type. Figure 4.7 shows the histogram of all reliability analyses with a different type of loading. As can be observed in both Figure 4.7 and Figure 4.8, the lowest value belongs to the steel materials. Concrete shows greatest reliability: the values are considerably higher than the target value of 3.8 for reliability. On the other hand, for masonry and timber structures, the values are compatible with the target value of reliability, as is observable in the histograms. In Figure 4.7, in a few cases, such as for steel or reinforcement steel, the reliability values are smaller than 2. Figure 4.7 and Figure 4.8 correspond to the load combination 6.10. The results for combination 6.10a&b are presented in Appendix C: Additional diagrams for load combinations of EN-1990.

The analysis of these results shows that the concrete material provides the most safety because of its partial safety factor and coefficient of variation for its model uncertainty. Despite the equal value of the material partial safety factor for masonry and concrete, the reliability of concrete is higher because the COV of its uncertainty is lower than that of masonry. The same applies for steel and reinforcement steel. The bias in the model uncertainty for a steel structure makes its reliability greater than that of reinforcement steel, although the material partial factor for reinforcement steel is greater than that of steel.

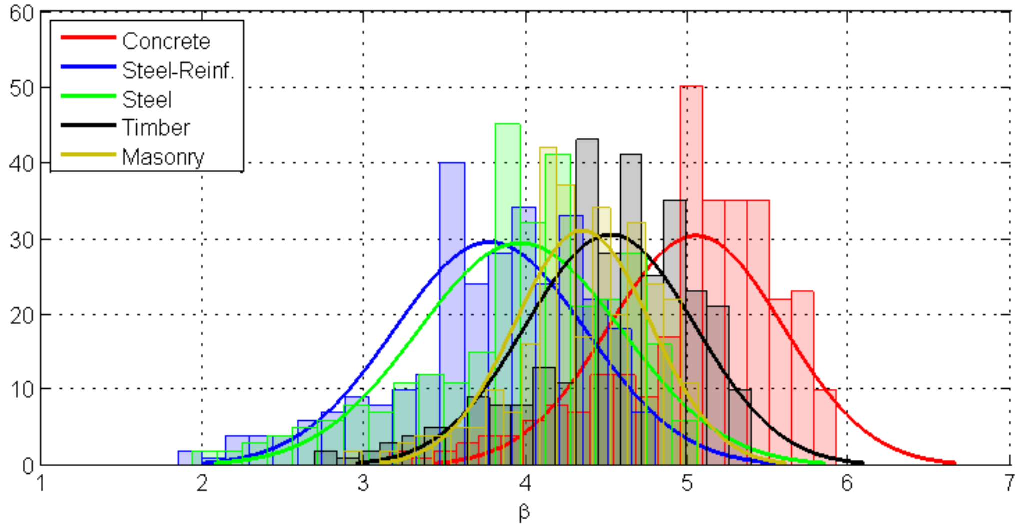


Figure 4.7: Histogram of different resistance types based on EN-1990 combination 6.10

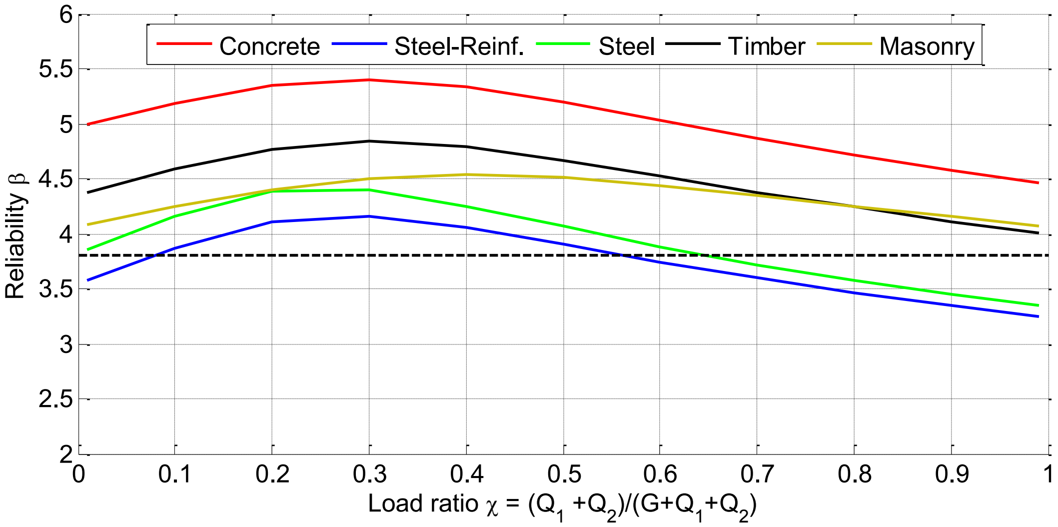


Figure 4.8: Average reliability for each resistance of EN-1990 combination 6.10 for all load cases and load ratios

The analysis based on the variable load types must also be presented. Three types of variable loads are considered in the first step without accompanying load in load combinations. Hence, the value of parameter k [eq.(3.45)] is equal to zero. The results for each type of variable load and resistance are presented for imposed, wind and snow load in Figure 4.9, Figure 4.10 and Figure 4.11, respectively.

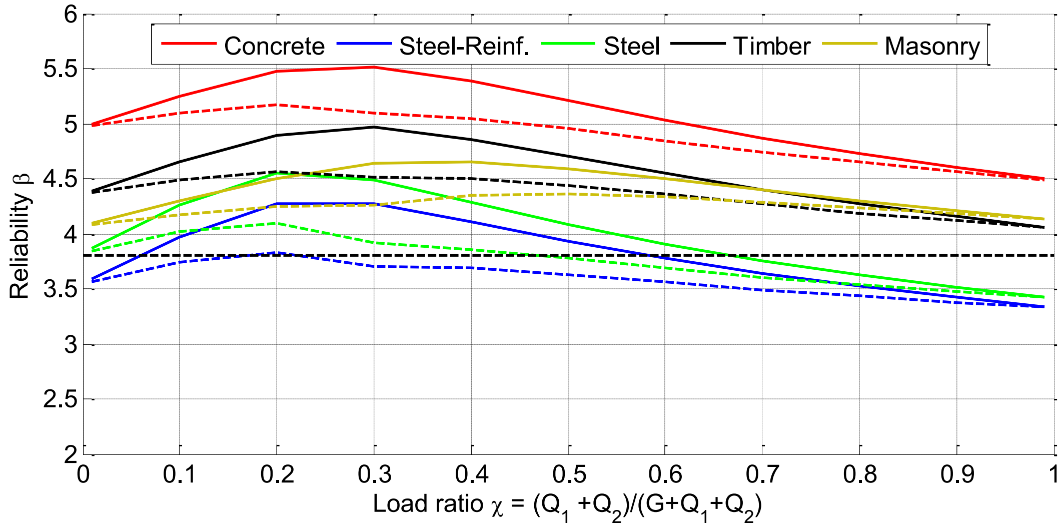


Figure 4.9: Reliability for imposed load with *k* = 0, 6.10 with line, 6.10a&b with dash

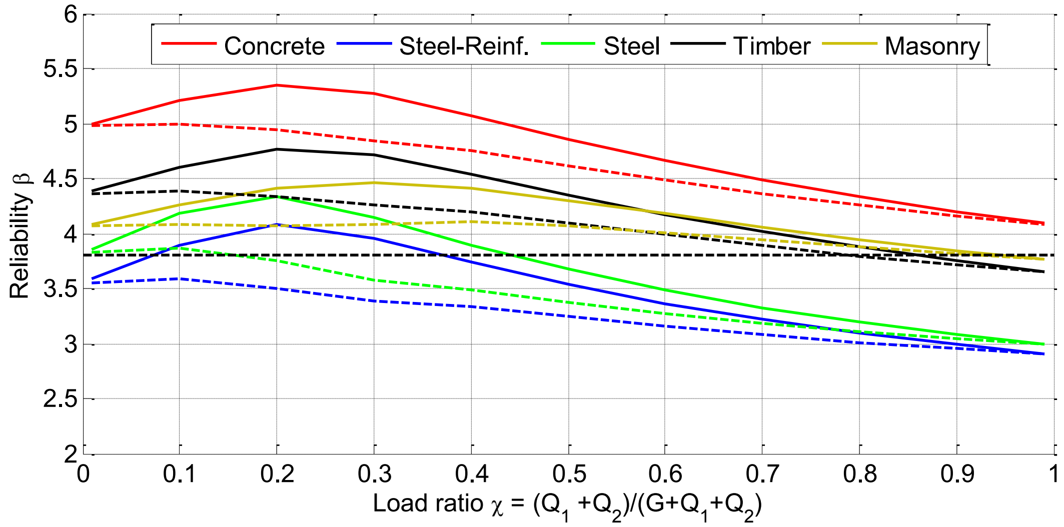


Figure 4.10: Reliability for wind load with *k* = 0, 6.10 with line, 6.10a&b with dash

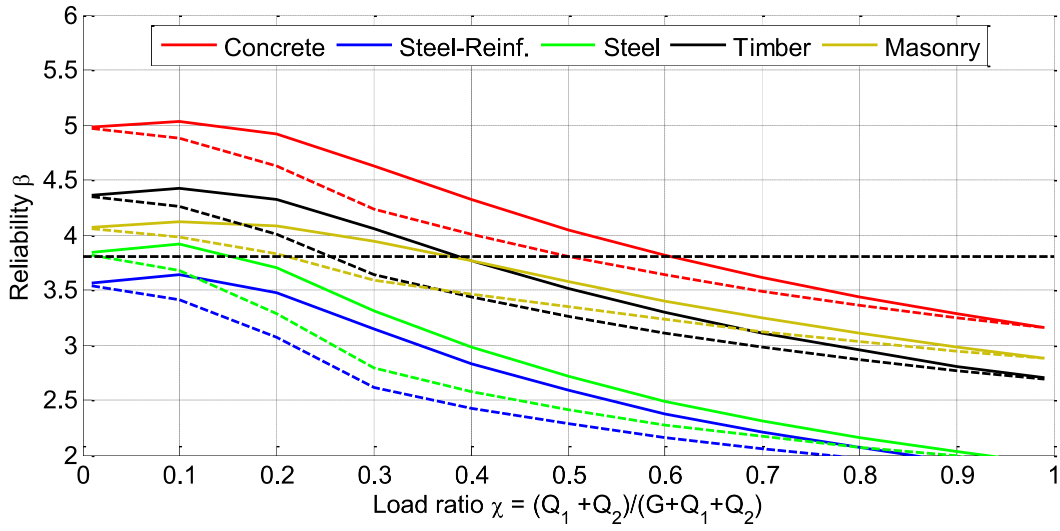


Figure 4.11: Reliability for snow load with *k* = 0, 6.10 with line, 6.10a&b with dash

As shown for all cases, the reliability index decreases in the higher value of load ratio χ, and in some cases, it is lower than the target reliability. This can be interpreted with regards to structural weight. As explained before, the load ratio χ is a representation of structural weight. Higher values of χ correspond to the light-weight structures. In the high range of χ, the variable load is more than the permanent load. Therefore, the higher deviation and COV of variable load reduces the reliability.

On the other hand, in lower ranges of χ, almost all cases are on the safe side and even higher than target reliability. Therefore, they may be considered as conservative in design. These higher values also show that the partial factors of permanent action or dead loads are conservative. The concept of selecting such high values for partial factors of permanent action was, on one hand, to compensate the safety measures with the variable action partial factor and, on the other hand, to balance permanent and variable actions according to the safety requirements. However, the balance does not occur because with higher values of χ, the permanent actions are considerably lower than variable loads. Therefore, safety measures of permanet actions do not influence the calculations. For this reason, the combination of 6.10a&b is more economical than 6.10 (see Section 4.3 for further explanation of the comparison of these two combinations with a case study of a concrete beam).

Parameter k also affects reliability. For each variable load, the other two variable loads are considered accompanying actions for analysis with non-zero *k* values. As such, there are six combination possibilities between imposed, snow and wind loads in the case of two variable loads. In each possibility, five different values of *k* between zero and 1 are considered to investigate the influence of load ratio *k*. The results of all possible combinations with different values of *k* are presented in Figure 4.12 through Figure 4.15. The other reliability results for materials and their combination may be found in Appendix C: Additional diagrams for load combinations of EN-1990.

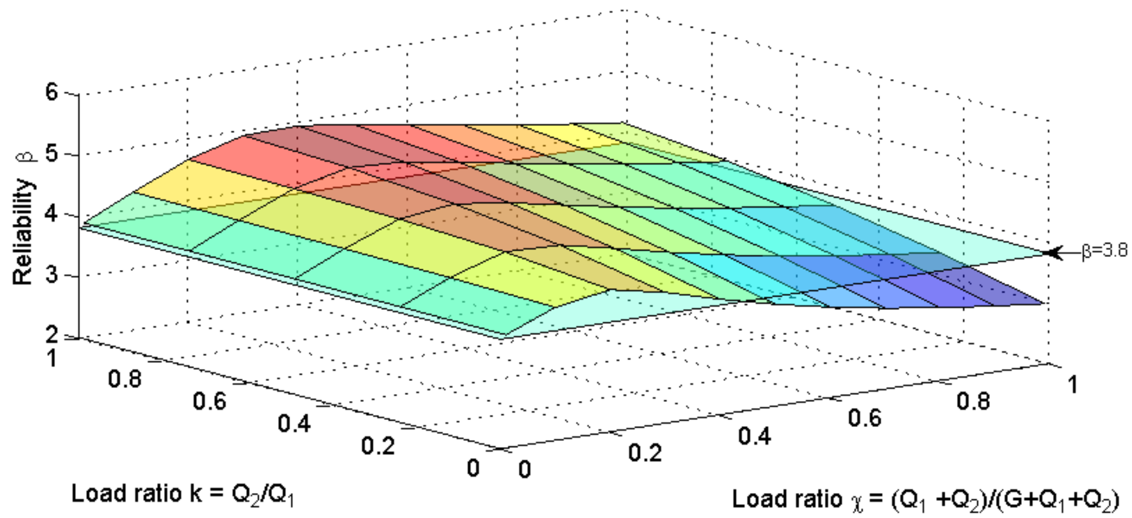


Figure 4.12: Steel reliability for wind as leading action and imposed accompanying, combination 6.10

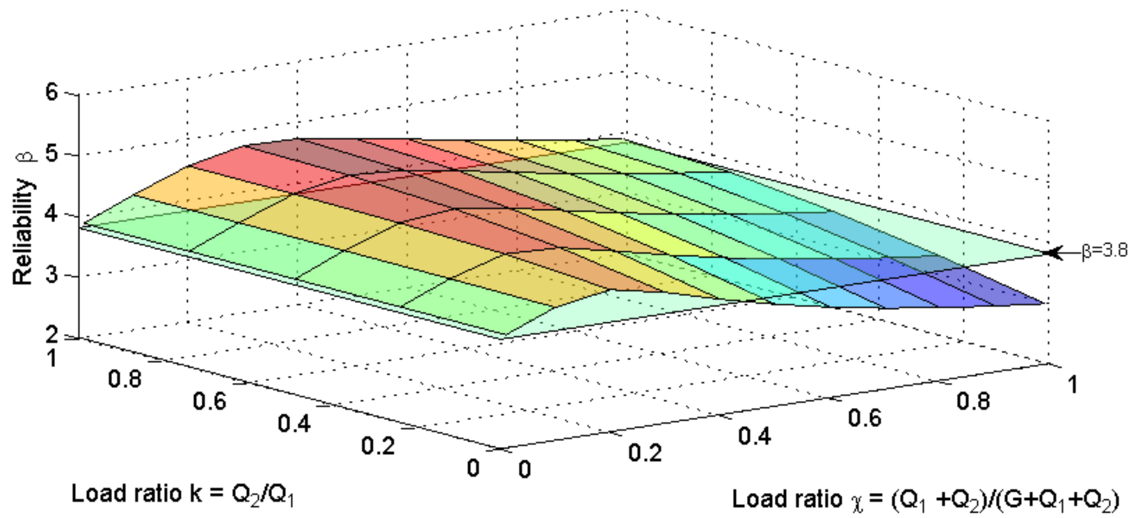


Figure 4.13: Steel reliability for wind as leading action and snow accompanying, combination 6.10

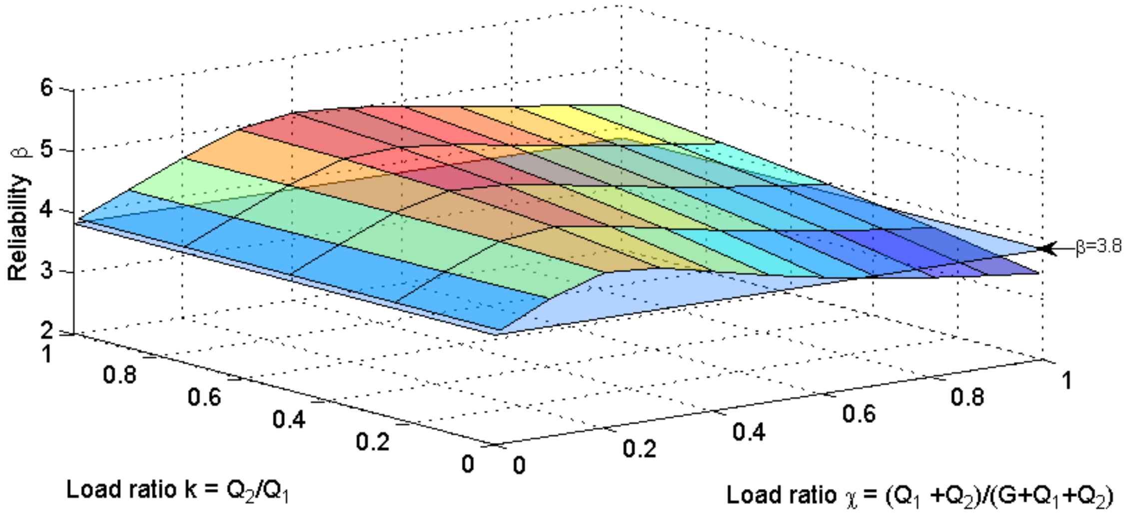


Figure 4.14: Steel reliability for imposed load as leading action and wind accompanying, combination 6.10

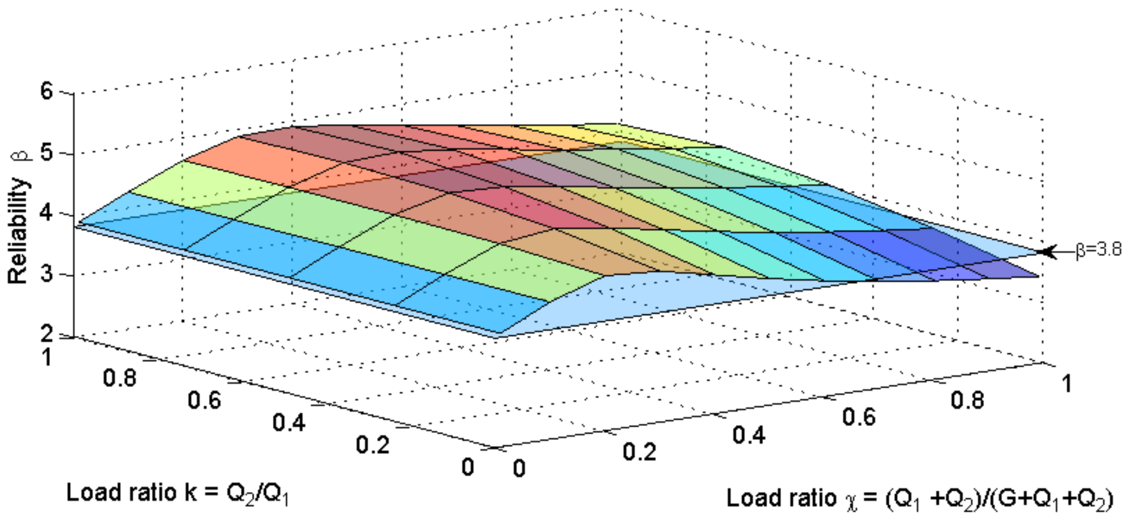


Figure 4.15: Steel reliability for imposed load as leading action and snow accompanying, combination 6.10

Parameter k affects reliability by increasing the reliability index. In Figure 4.12 through Figure 4.17, it can be seen that by increasing k, the reliability of a steel structure is also increased. By applying an extra variable load in the limit state, more safety measures are also be introduced in the calculation. Therefore, the higher values of k lead to greater reliability. The diagrams for other cases with different materials can be found in Annex A.

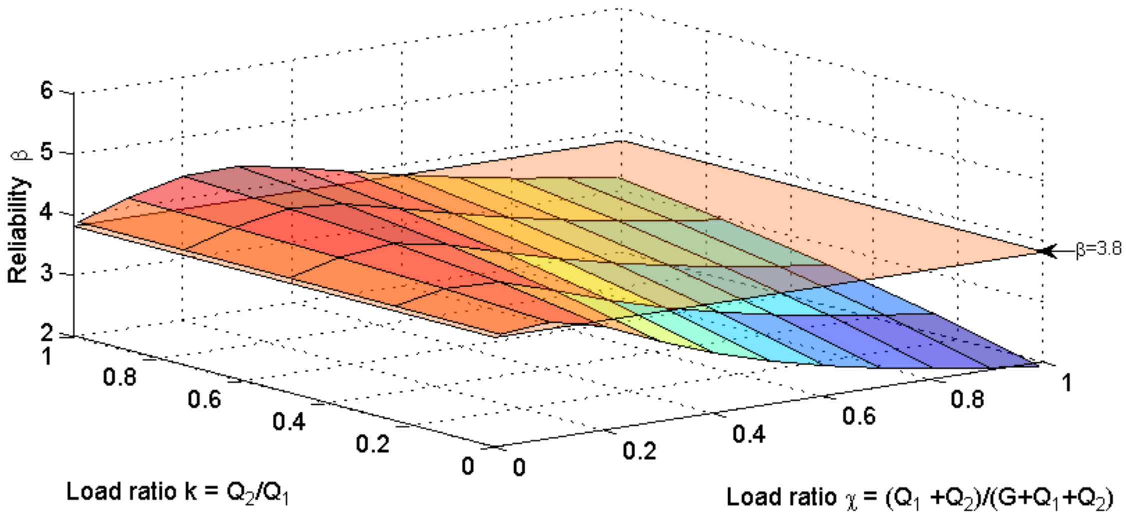


Figure 4.16: Steel reliability for snow as leading action and imposed accompanying, combination 6.10

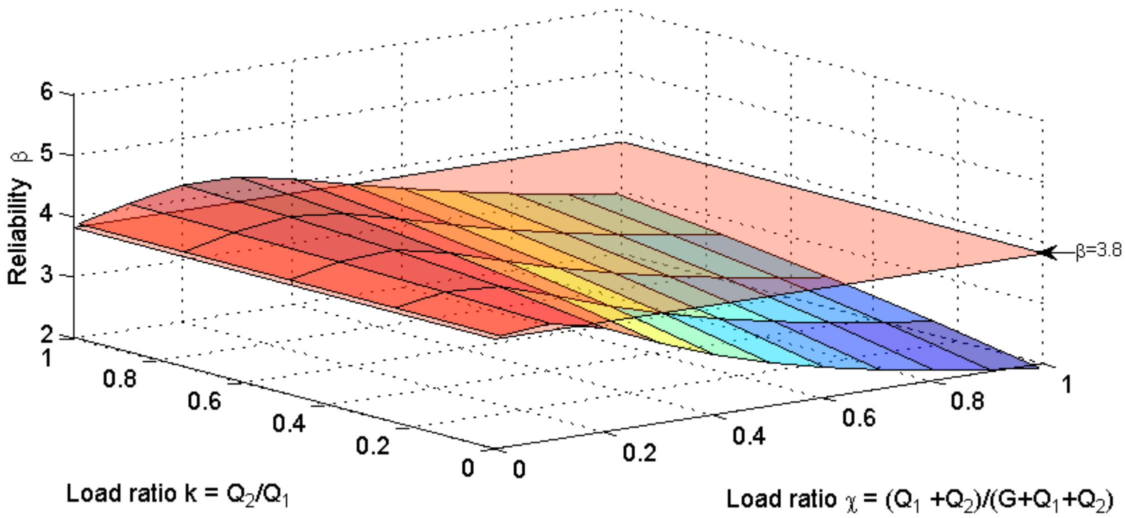


Figure 4.17: Steel reliability for snow as leading action and wind accompanying, combination 6.10

As can be observed for the case of snow load from Figure 4.16 and Figure 4.17, the reliability level is very low for the high value of χ. Therefore, it seems that an increase is needed in safety measures regarding snow loads (see Section 4.5 for a comprehensive discussion of and proposal for snow load partial factors).

4.3 Comparison of combination 6.10 and 6.10a&b in design

The previous section shows that by comparing different reliability analysis approaches, the load combination 6.10a&b gives more consistent results concerning target reliabilities. It can be concluded that this load combination leads to more economical design than the load combination 6.10. In order to compare these two combinations regarding the design, a concrete beam is considered as a case study. Concrete beams are used worldwide. Consequently, this case study aids in the goal of comparison in this study.

The geometrical properties of a concrete beam are illustrated in Figure 4.18. A concrete beam is considered to be subject to permanent, imposed and leading variable loads, and wind as accompanying variable load.

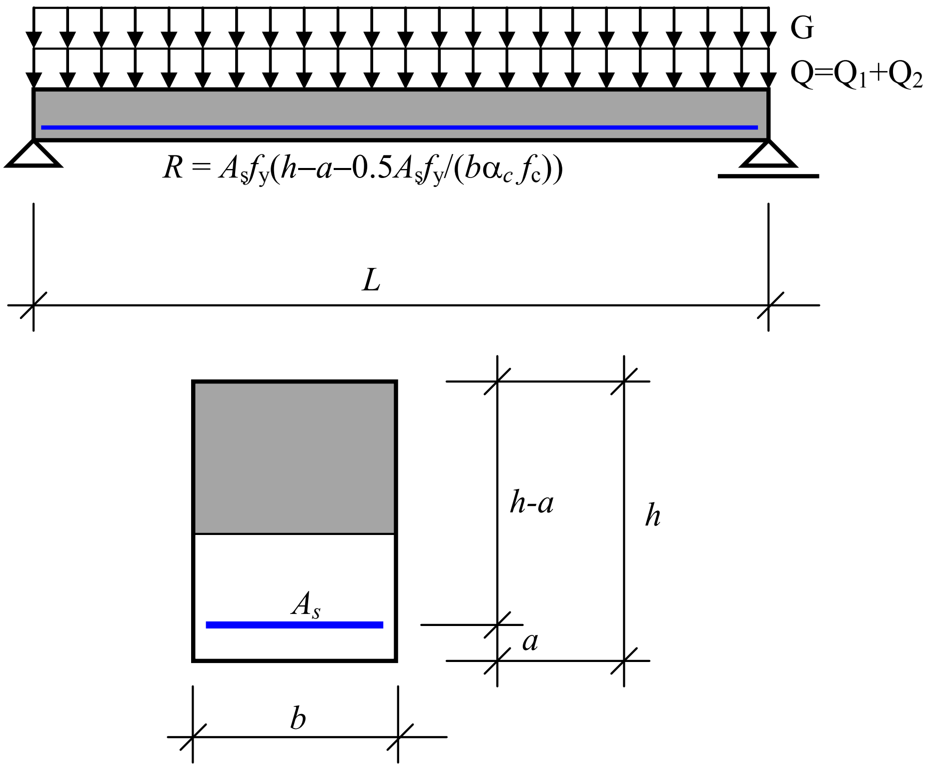


Figure 4.18: Geometrical properties of a reinforced concrete beam

Based on EN-1990 and EN-1992 [61], the design resistance of a concrete beam is calculated according to (4.10):

$$R_d = A_s\,(f_{yk}/\gamma_s)\,(h - a - 0{,}5\,A_s\,f_{yk}\,\gamma_c\,/(b\alpha_{cc}f_{ck}\,\gamma_s)), \tag{4.10}$$

where, A_s is the area of the steel reinforcement, f_{yk} is the characteristic value of the reinforcement strength, h is the height of the cross-section, a is the distance of reinforcing bars from the bottom side, f_{ck} is the characteristic value of concrete strength, and α_{cc} is the reduction factor of the concrete strength with $\alpha_{cc} = 0.85$ as a recommended value. Partial factor γ_s is the partial factor for reinforcing steel, which is γ_s = 1.15, and γ_c is the partial factor for concrete strength, which is equal to γ_c = 1.5. Hence, the limit sate function for a reliability analysis of the concrete beam is defined by (4.11) by considering uncertainty, resistance and action effects:

$$g(x) = R(x) - E(x) = \theta_R(A_s f_y \left(h - a - \frac{0{,}5\, A_s f_y}{(b\alpha_{cc} f_c)}\right)) - \theta_E(G + Q_1 + Q_2) \quad (4.11)$$

where, θ is uncertainty of resistance (R) or actions (E). Variables G, Q_1 and Q_2 represent the actions' effects as the moment in the middle of the beam. For reliability analysis, a concrete beam with a compressive strength of $f_c = 20$ [MN/m²] and yielding strength of steel $f_{yk} = 500$ [MN/m²] is considered. The beam is assumed to be subject to a total load of 30 [kN.m], which is going to be dedicated to each type of load according to the load ratio of χ and *k* from (3.44) and (3.45). Figure 4.19 shows the reliability analysis for the case with imposed load as leading and wind as accompanying action.

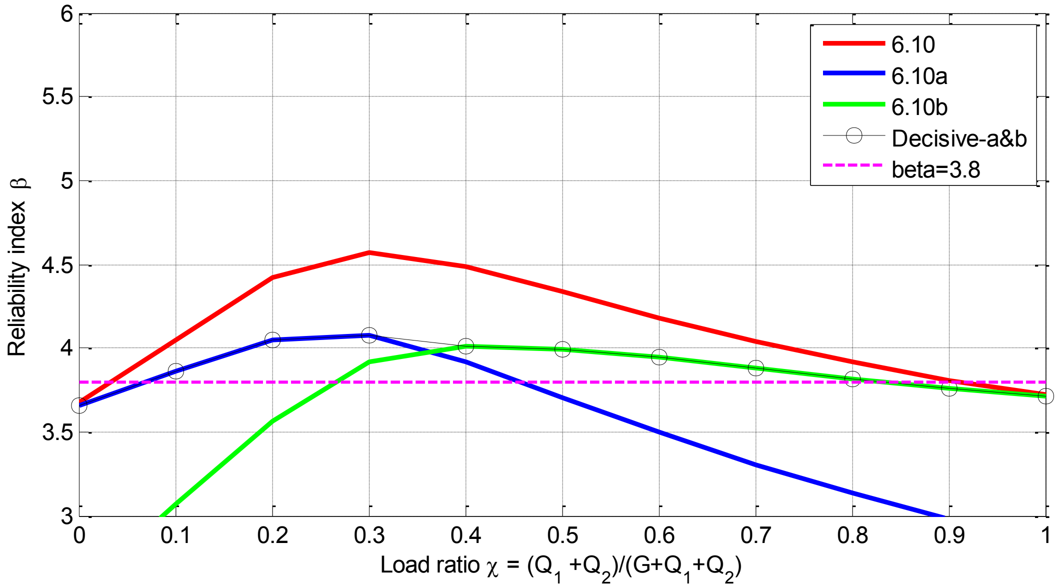


Figure 4.19: Reliability of concrete beam with imposed load and wind load with *k* = 0.25

Figure 4.19 shows that the highest result is achieved with the combination 6.10, but this amount of safety does not mean that this is an optimal choice for a load combination. According to the target reliability recommended in this code, using this load combination will provide safety greater than is required in the most ranges of load ratios. Therefore, overestimation occurs as a result of the design of the structure. If the procedure of design for these two combinations is considered correctly, the required amounts of steel cross-section are illustrated in Figure 4.20.

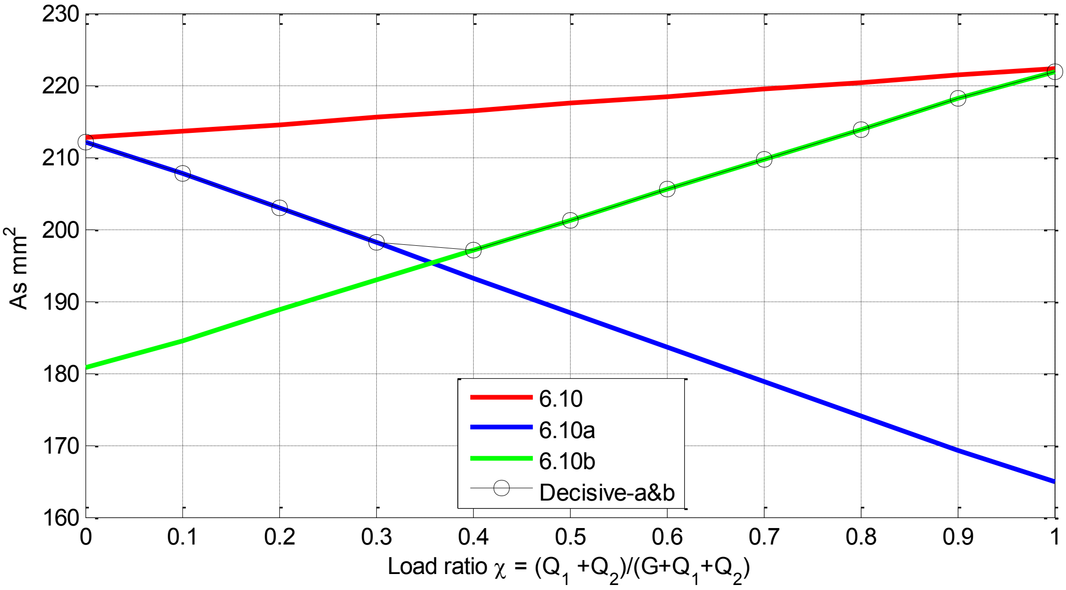


Figure 4.20: Required steel for a concrete beam design with imposed load and wind load with k = 0.25

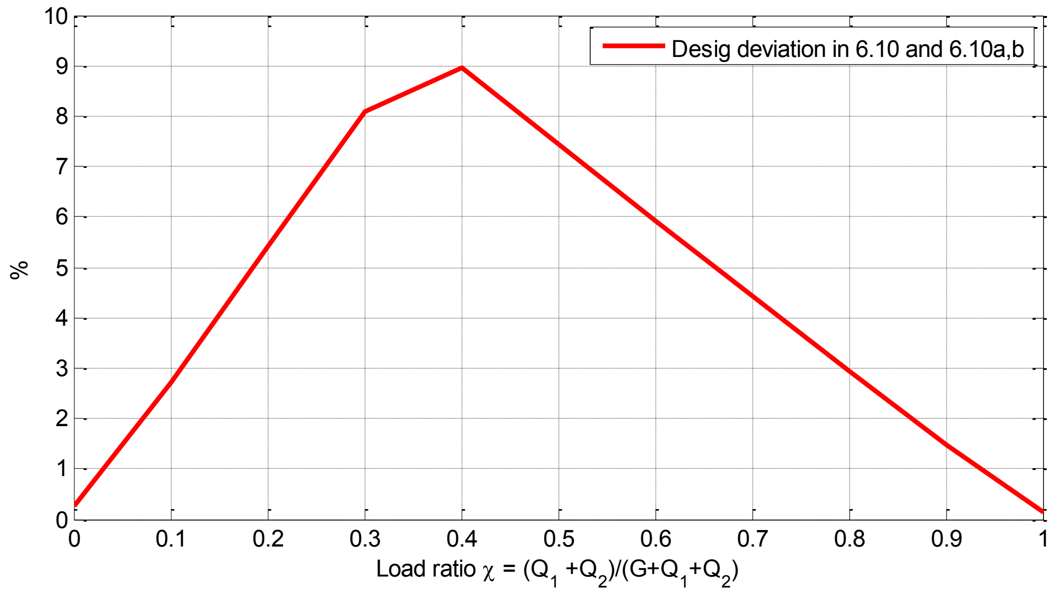


Figure 4.21: Design deviation for 6.10 and 6.10a&b for concrete beam with imposed load and wind load with k = 0.25

In Figure 4.21, the deviations for both combinations in the design are shown. The maximum point of deviation is approximately 9%. This level of deviation means that when the design procedure is applied based on a 6.10 combination, the final structure has consumed 9% more material. In other words, the final result of the design process would generate 9% waste of the material used in the construction.

The maximum deviation occurs also in the range of load ratios for structures subjected to mostly permanent actions. As such, for heavy-weight structures such as concrete structures, this deviation will significantly influence the final design and make it rather conservative. With regard to material consumption, application of 6.10 will generate 9% wasted material in the construction procedure.

4.3.1 Conclusion

Based on the criteria of reliability, and as has been shown for a concrete beam, the behavior of 6.10a&b is the most compatible with the target reliability recommended by EN-1990. Applications of these two combinations, 6.10a and 6.10b, must be considered simultaneously. Hence, the less favorable of these two will be selected for the design process. Overall, the reliability of these two demonstrates constant behavior in the ranges of load ratios.

The other application of these two combinations appears in material consumption and structural costs. As has been shown for the design of a concrete beam using the load combination 6.10a&b, the final material consumption is at maximum approximately 10% lower than would be the case with combination 6.10. The result for 6.10a&b is more sustainable than that of 6.10.

Despite the advantages, these combinations are ignored in the national annex of EN-1990 in Germany. Based on this investigation, it will be recommended that in the new version of the national annex of EN-1990 these two combinations are also considered as applicable combinations in the code. With a simple calculation, it can be concluded that much consumption of unnecessary material occurs in the German construction industry; such wastage can be diminished if these two combinations are applied in the design procedure.

4.4 Reduction of permanent load partial safety factor

Permanent actions are highly under discussion during the ongoing reviewing process of next generation of Eurocodes. As one of the most important goals for next version of EN-1990, it is decided to produce a guideline of structural design which can give an economical design. Sustainability of structural design has to be considered in the introduction of safety measures.

Subsequently researchers are ought to investigate the conservative consideration of Eurocodes. A related aspect is safety factors for permanent actions. The calibration results in this investigation and other references show that the current value of 1.35 is higher than required partial safety factor for self-weight or permanent actions. Therefore a reduction is this factor has to be investigated.

4.4.1 Reliability analysis of reduced permanent partial factor

Partial safety factor of permanent actions is going to be considered as 1.25 instead of 1.35. This new value will replace as partial safety factor of permanent actions in load combinations of EN-1990. The comparison of fundamental combinations for both cases is done according to the reliability analysis. The reliability calculation is conducted the same as in section 4.2. The overall average of all reliability analysis with $\gamma_G = 1.25$ is represented in Figure 4.22.

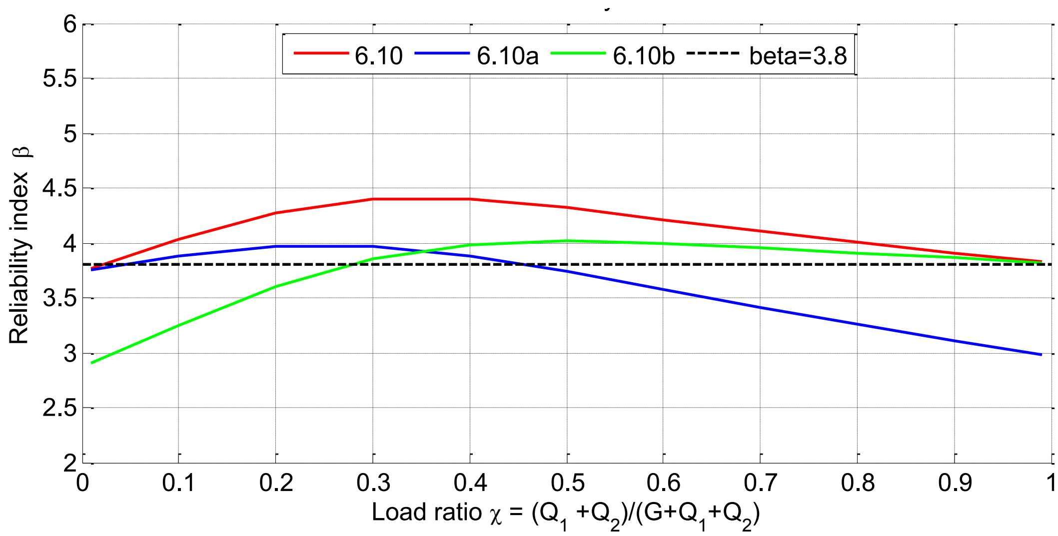


Figure 4.22: Average reliability for all cases and EN-1990 combination with $\gamma_G = 1.25$

As it seems in Figure 4.22, the results of fundamental load combinations in EN-1990 by considering $\gamma_G = 1.25$ are producing more consistent result regarding target reliability. Both combinations 6.10 and 6.10a&b are giving lower results than the case with 1.35 as partial factor in previous sections.

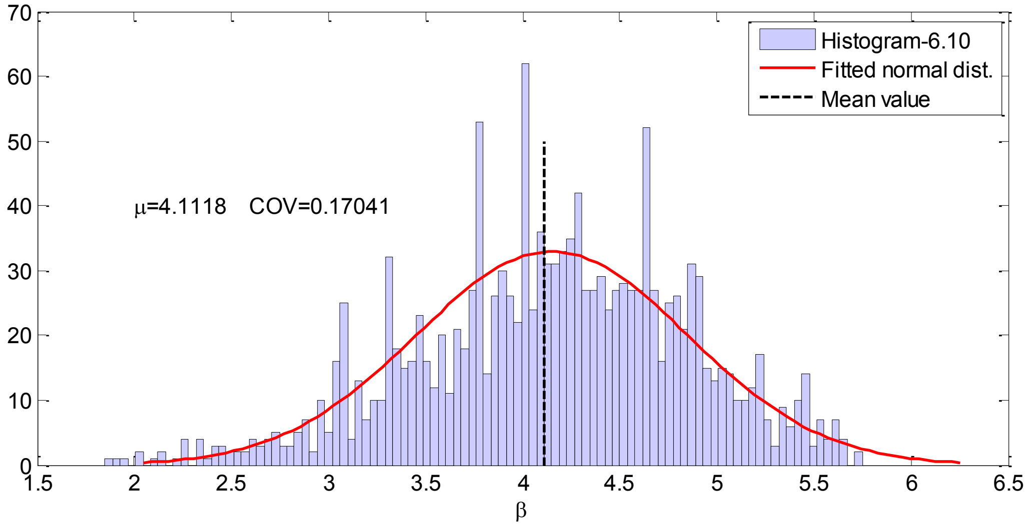


Figure 4.23: Histogram of all reliabilities for combination 6.10 with $\gamma_G = 1.25$

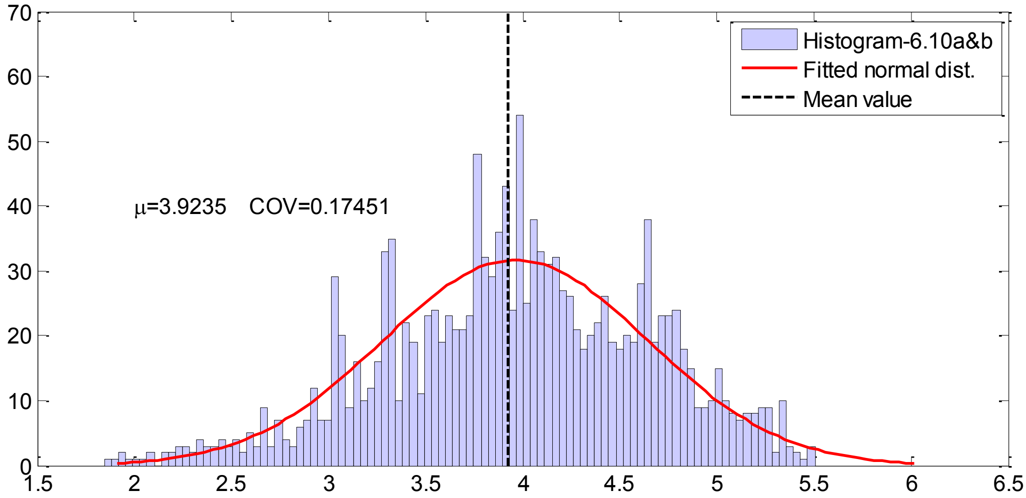


Figure 4.24: Histogram of all reliabilities for combination 6.10a&b with $\gamma_G = 1.25$

Eventually the reliability average values for both cases are compared for combinations 6.10 and 6.10a&b in Figure 4.23 and Figure 4.24 respectively. The values of cases with $\gamma_G = 1.25$ are higher than target reliability but closer to the target in comparison with $\gamma_G = 1.35$. Therefore it can be concluded that by application of 1.25 instead of 1.35, will produce economical design along with safe design. This will lead to more sustainable design for structural design.

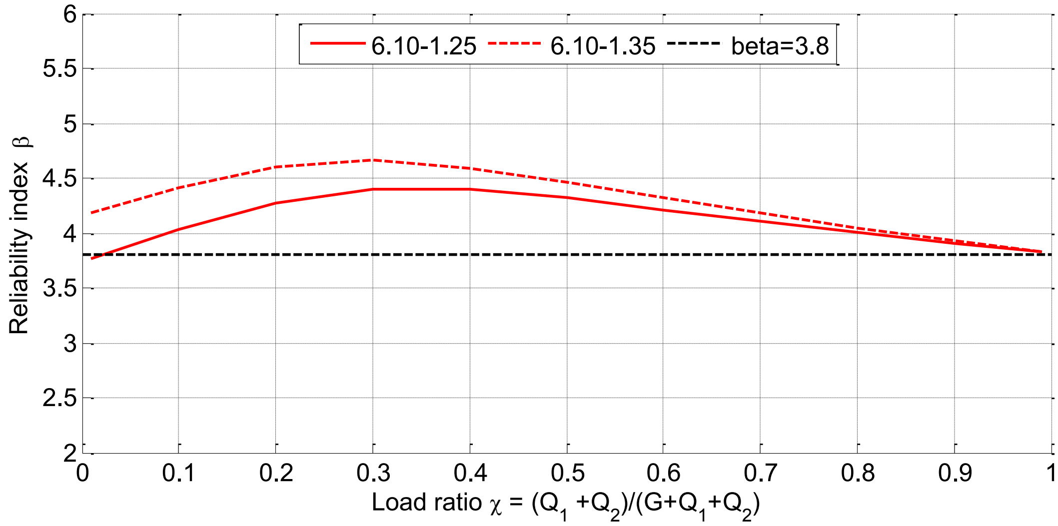


Figure 4.25: Comparison of average reliability for combination 610. with $\gamma_G = 1.25$ and $\gamma_G = 1.35$

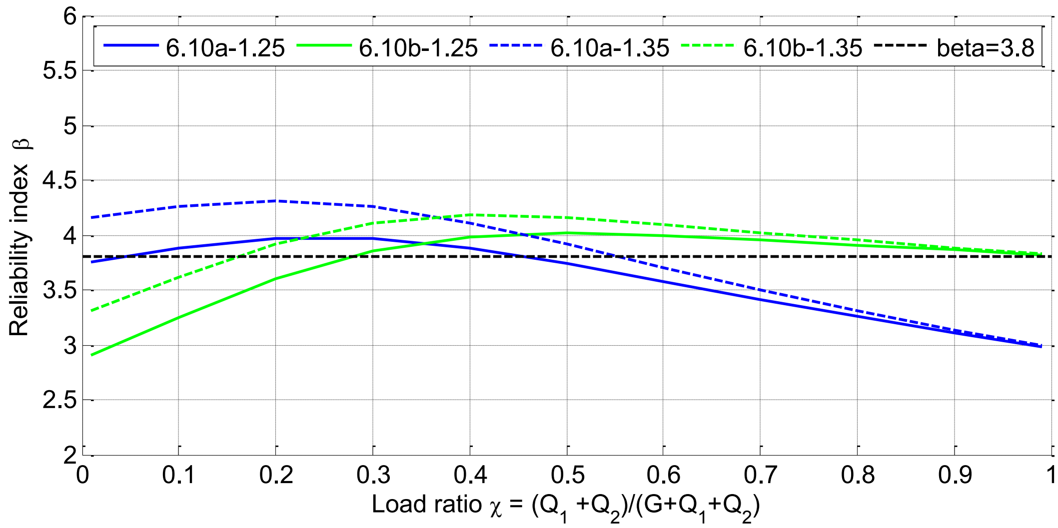


Figure 4.26: Comparison of average reliability for combination 610a&b with $\gamma_G = 1.25$ and $\gamma_G = 1.35$

4.5 Application of increase factor for snow load

4.5.1 Describing the increase factor

Considerable uncertainty must be applied for modeling snow loads because of their environmental origin. Over the last 15–20 years, the snow precipitation has varied in different ways because of the phenomena of extreme climate change [65]. During 2005 and 2006 in Europe, several failures structural failures occurred due to heavy snow load [66]. Since then, different investigations have shown inconsistent levels of safety between the designed structures and the recommended safety levels in the codes [67]. One reason for this violation of safety requirements may be the insufficient application of safety requirements in structural design codes. Therefore, more safety measures must be introduced to fulfil the minimum safety requirements.

An increase factor is proposed in this investigation to sustain the required safety measures in cases of structural design with snow load. Reliability analysis based on the combinations and partial factors of EN-1990-1-1 show that the partial factor of snow load is not enough to reach the target reliability [16] (see e.g., Figure 4.17). This study proposes and investigates a new method for calculation of structures subjected to snow load. This method will be applied and improved to get consistent results with the target reliabilities of the Eurocode. The characteristic value of snow load for a structural component is determined based on (4.12):

$$S_k = s_0 \cdot c_i \tag{4.12}$$

where

s_0 is the ground value of snow load based on the location and elevation of the structure, or it represents the characteristic value for the ground value of snow, and

c_i is a shape factor based on the form of the structure.

According to the recommended value of the characteristic value of snow load for specific location and structural type, the design value is determined by applying partial the safety factor of snow.

$$S_d = S_k \cdot \gamma_Q \quad with \quad \gamma_Q = 1.5 \tag{4.13}$$

An additional safety factor has to be applied in the case of snow loads. An increase factor is implemented to account for the cases with low amount of safety. This increase factor will be applied to the partial factor of snow and increases the design safety amount. This increase factor k_s is defined based on the ratio of snow load to the weight of the structural components themselves. According to Table 4.2, a minimum value of 1 and a maximum of 1.5 are considered for the increase factor, and a linear interpolation has to be done to determine k_s in the middle interval. The design value of snow load is determined by considering increase factor in (4.14).

$$S_d = S_k \cdot \gamma_Q \cdot k_s \tag{4.14}$$

Table 4.2: Increase factor k_s for snow load

Ratio of snow over self-weight	Increase factor k_s
$\frac{s_0}{G} \leq 0.5$	1
$0.5 \leq \frac{s_0}{G} \leq 3.0$	$0.9 + 0.2\frac{s_0}{G}$
$\frac{s_0}{G} \geq 3.0$	1.5

In order to define the ratio in a normalized format, the ratio of snow load can be represented based on the total amount of load. Instead of an open interval to infinity, the values can be assigned to the interval of ratio, the so-called S, which is between 0 and 1, as mentioned in (4.15) and Table 4.3.

$$S = \frac{s_0}{G + s_0} \qquad and \qquad \frac{s_0}{G} = \frac{S}{1 - S} \tag{4.15}$$

Table 4.3: Increase factor k_s for snow load

Ratio of snow over total load	Increase factor k_s
$S \leq 0.333$	1
$0.333 \leq S \leq 0.75$	$0.9 + 0.2 \cdot \frac{S}{1 - S}$
$S \geq 0.75$	1.5

Based on Table 4.2 and Table 4.3, the increase factor k_s corresponding to snow load can be represented for both formats of ratios in Figure 4.27.

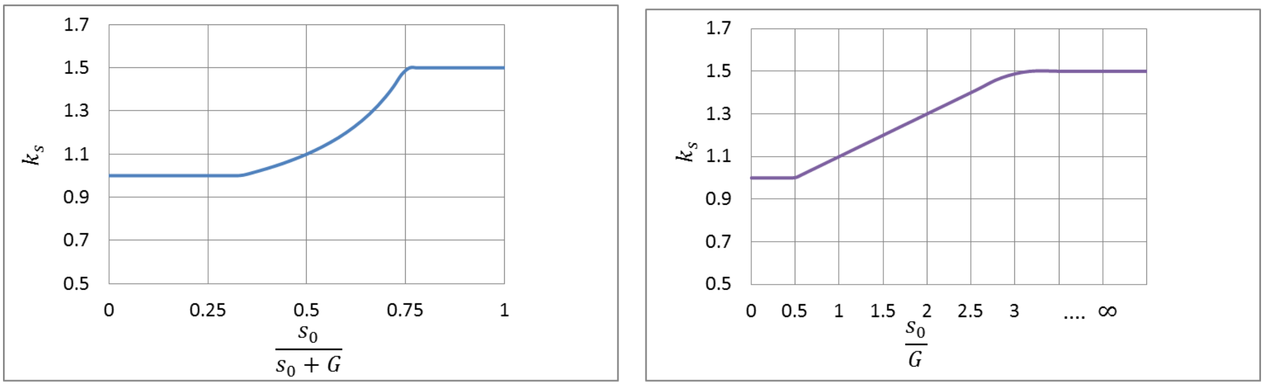


Figure 4.27: Increase factor k_s of snow load

These three intervals are separated based on the load's ratios. These ratios can be considered to represent the structural weight of the applied snow load. Small ranges of this ratio mean that the structure is heavy. For heavy structures, the amount of snow load in comparison with the dead load of the structure is small. Therefore, the increase factor is considered to be 1. In other words, there is no increase in the amount of snow load because it is not a decisive factor in the design process.

In the case of the middle interval, a linear interpolation is implemented. The factor increases with the ratio. The lighter the structure is, the higher the snow load effect will be. The last interval represents light-weight structures. In this case, the maximum value of the increase factor has been considered because the snow load has a more critical role in the design.

4.5.2 Reliability analysis of combination with increase factor

In order to compare the results of this method and to evaluate the differences from the EN-1990 combinations, reliability analysis with FORM (see Section 3.5) has also been conducted. Load combinations for structural design in EN-1990 are implemented with corresponding values for partial factors and combination factors. The dead load in this part is self-weight only, and no permanent load is involved in reliability. Therefore, the coefficient of variation for the self-weight of a steel structure as 0.05 from Table 3.2 is considered. The stochastic parameters for material and variable load are also applied according to Table 3.2.

The application of the increase factor according to Table 4.2 or Table 4.3 is done with all load combinations of EN-1990. The result of the reliability index for the case with only one variable load as snow, is represented in Figure 4.28, and Figure 4.29 represents the other cases, with snow as the leading variable and imposed load as accompanying action, with ratio $k = 0.5$.

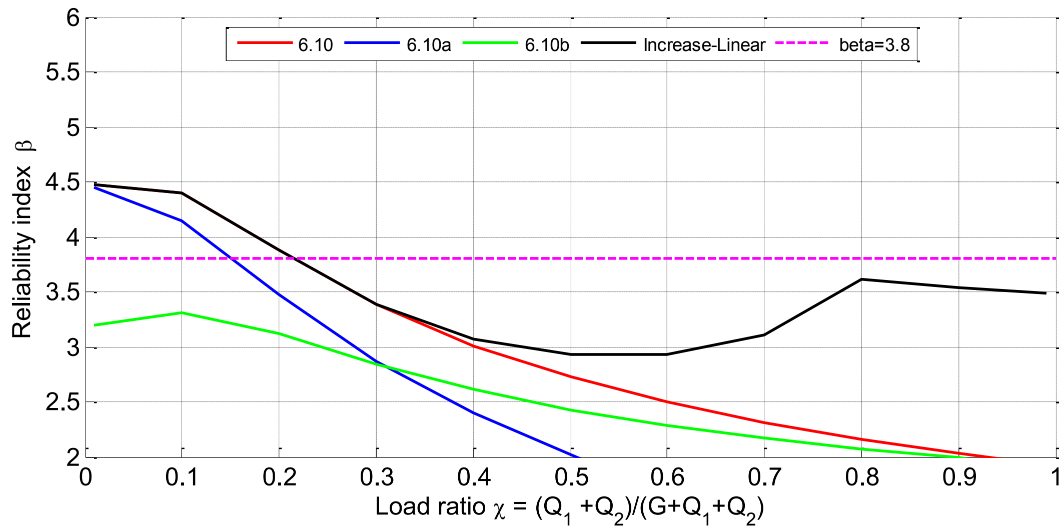


Figure 4.28: Reliability index for one-variable load, snow load

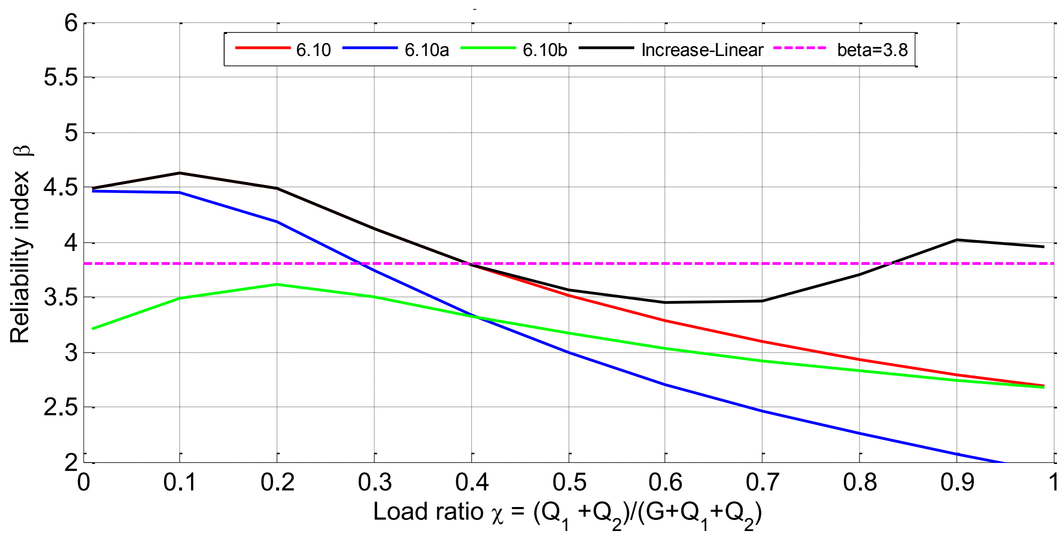


Figure 4.29: Reliability index for two variable loads, snow load leading and imposed accompanying

As observed, the application of the increase factor, based on the linear equation in Table 4.2 or Table 4.3 for variable loads, produces more consistent results than does the EN-1990 approach. The difference between the maximum and minimum values of the reliability index with an increase factor is lower than the difference of max. and min. in a fundamental combination of EN-1990. Hence, the final results demonstrate improved safety through an increased factor application method for variable load with a single combination. The reliabilities with higher ratios of χ reach values close to the target reliability.

4.5.3 Improvement of linear method

An improvement in the linear method should offer better results in the middle range ratio of χ (for example in Figure 4.28, the range between 0.3 and 0.8). In this range, the reliability index of the linear method is reduced, and it is below the target reliability level. This creates a concave shape in the reliability diagram.

In order to overcome this problem, an improvement for the calculation of increase factor in this middle range should be applied. Based on the linear recommendation in the middle range, the increase factor has to be calculated based on a linear interpolation between 1 and 1.5. To reduce the effect of this concave area and produce a result more compatible with the target reliability, the increase factor of snow has to be raised more at the beginning of the middle interval. It means that the inclination of the increase factor in the smaller values of the middle range has to be higher than at the end of the middle range. Therefore, instead of a linear function for a rising increase factor in the middle range interval, parabola functions can be applied (Figure 4.30).

$$k_s = -0.08 \cdot \left(\frac{s_0}{G}\right)^2 + 0.48 \cdot \frac{s_0}{G} + 0.78 \tag{4.16}$$

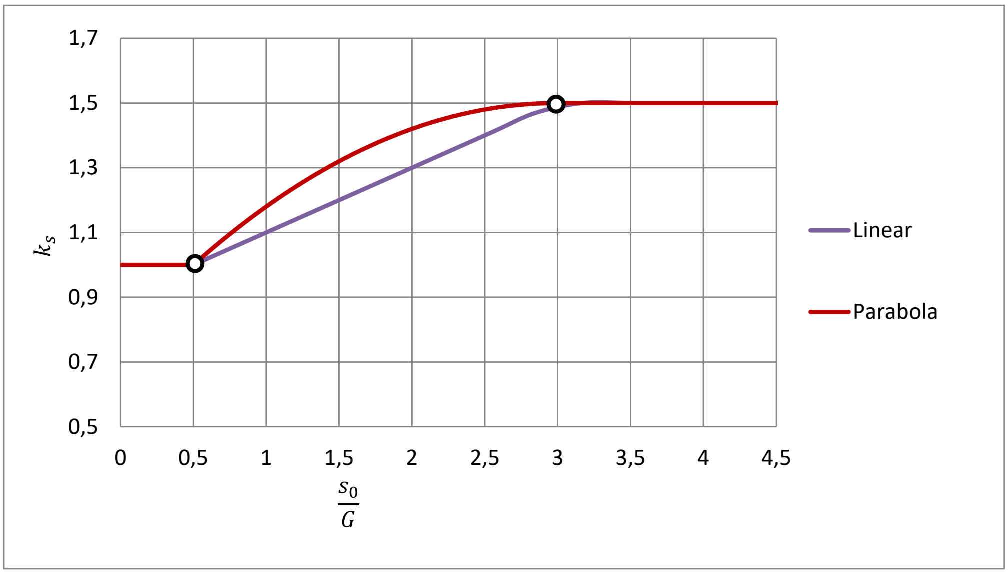


Figure 4.30: Linear and parabola models for calculation of k_s in middle range

The reliability analysis for comparison of the parabola method is shown in Figure 4.28 and Figure 4.29. The resulting reliability indexes are compared with those of 6.10, 6.10a and 6.10b of EN-1990. With higher values of reliability, the influence of a parabola application can be observed in the middle range .

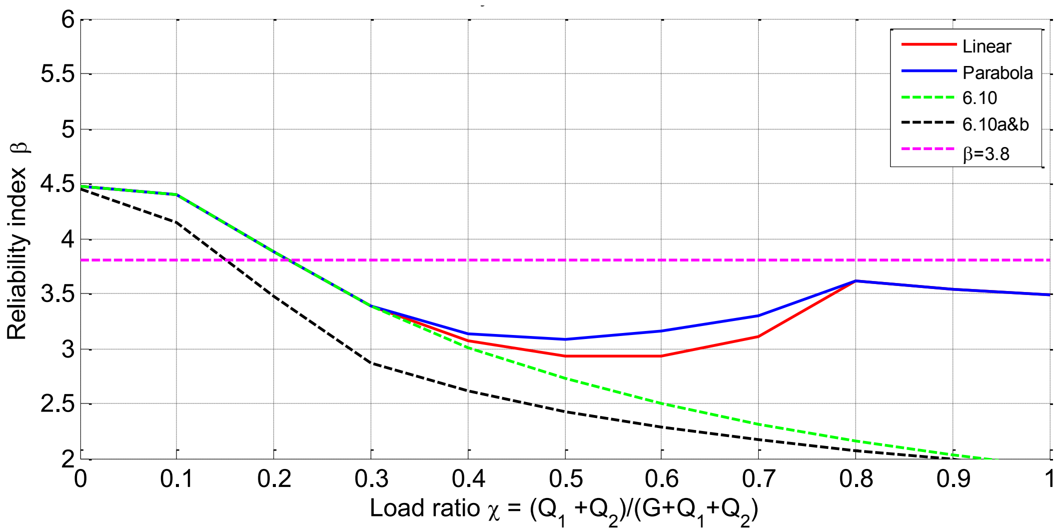


Figure 4.31: Reliability for linear and parabola methods with EN-1990 combinations for k = 0

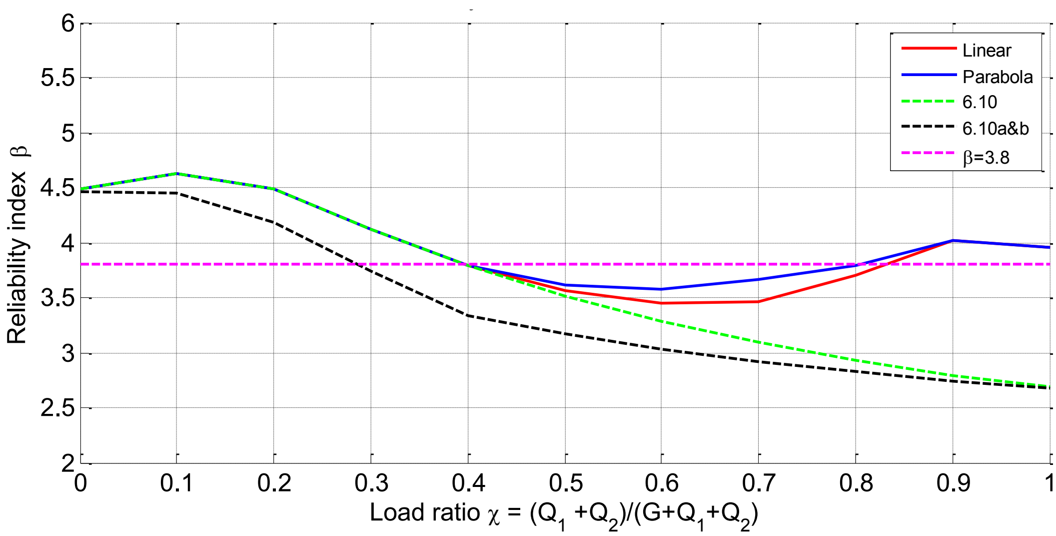


Figure 4.32: Reliability for linear and parabola methods with EN-1990 combinations for k = 0.5

In order to compare these methods with EC-1990 combinations, the deviations of the results are presented for both diagrams. The deviations are exposed in Figure 4.33 and Figure 4.34. The deviation is calculated from the combination 6.10, because in all cases this combination gives the maximum value of reliability. As seen, the deviation is considerable in cases with higher ratios of variable loads. In the case of light-weight structures, the method of an increase factor gives higher safety levels. The comparisons between the corresponding values of the parabola and the linear method show that the parabola will increase the reliability to its maximum amount of approximately 10%.

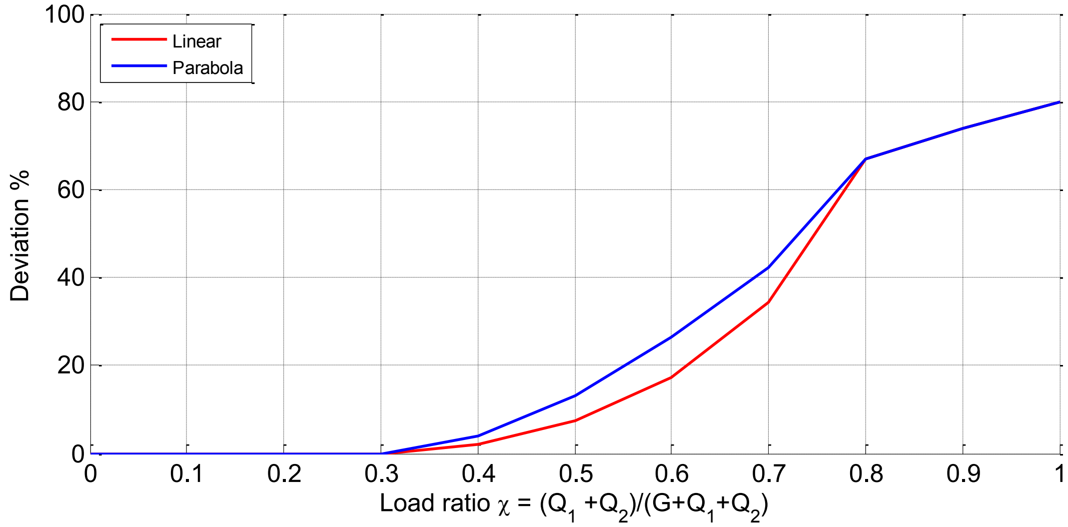


Figure 4.33: Deviation of reliability for *k* = 0

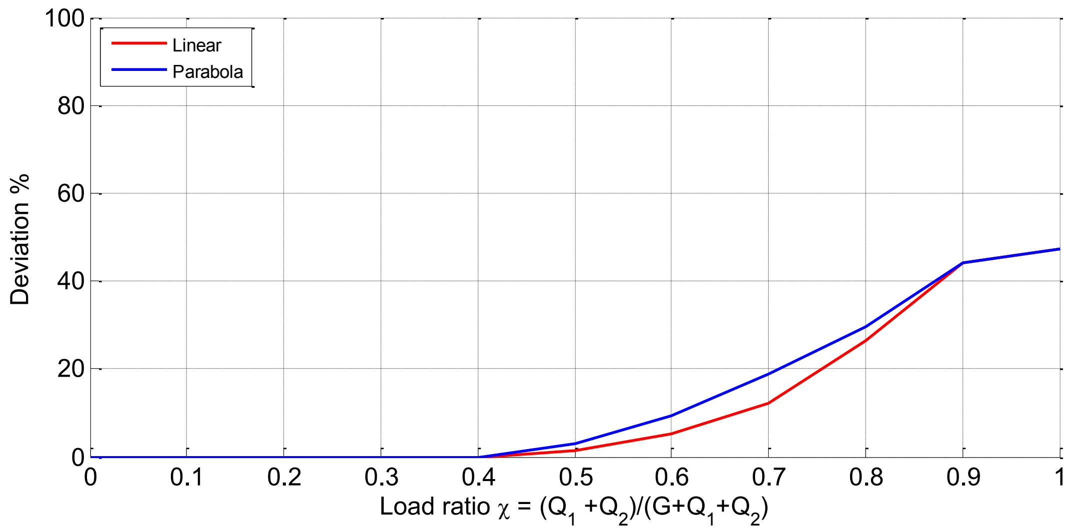


Figure 4.34: Deviation of reliability for *k* = 0.5

4.5.4 Conclusion

The goal of calibration analysis is to achieve constant reliability with respect to the target reliability value and to provide the optimum required safety in the design process. Through reliability analysis for combination in the EN-1990 for the snow load, it has been observed that the resulting values of reliability are not consistent with regard to the target reliability in the whole interval of load ratios. Moreover, the results show that the safety level provided by EN-1990 combinations is lower than the level required in the code.

Application of the recommended method as an increase factor for snow load produces considerably greater safety. The reliability levels of EC-1990 combinations show unacceptable results in case of high variable load. In these cases, the maximum deviation of increase factor method with the combination 6.10 is nearly 80%. According to this amount of deviation, the method of the increase factor seems to be needed for combinations with snow loads. The reliability behavior in the case of snow loads leads to the conclusion that the structures with low permanent actions or self-weight are more sensitive to the lack of safety. Therefore, the maximum value of the increase factor belongs to this interval of load ratio. The improvement of linear interpolation with a parabola function increases reliability. With the application of a parabola, the concave shape of the diagram for the middle range load ratio is reduced. Consequently, reliability levels are smoother and will make the design in all range of load ratios more economical.

Eventually, it can be concluded that application of this method for all kinds of variable loads will help to create more economical and safe results simultaneously. In the case of heavy-weight structures, it will prevent the design process to produce conservative results because of the high value of partial factor for variable loads. Contrarily, light-weight structures do not allow the calculation to create an unsafe result by applying higher values of partial factor for variable loads.

4.6 Time-dependent actions and partial safety factor method

Variable actions are the time-dependent parameters in the code. The classification of target reliability is also done based on different reference periods to consider the existence of different time-dependent loads simultaneously. For each critical case of loading, the design values of action effects (Ed) should be calculated based on the combination of the action values that apply concurrently. The classification of actions in Eurocode has been done based on the following characterization of actions:

- variation in time,
- origin (direct or indirect),
- variation in space (fixed or free), and
- nature or structural response (static and dynamic), or both [14].

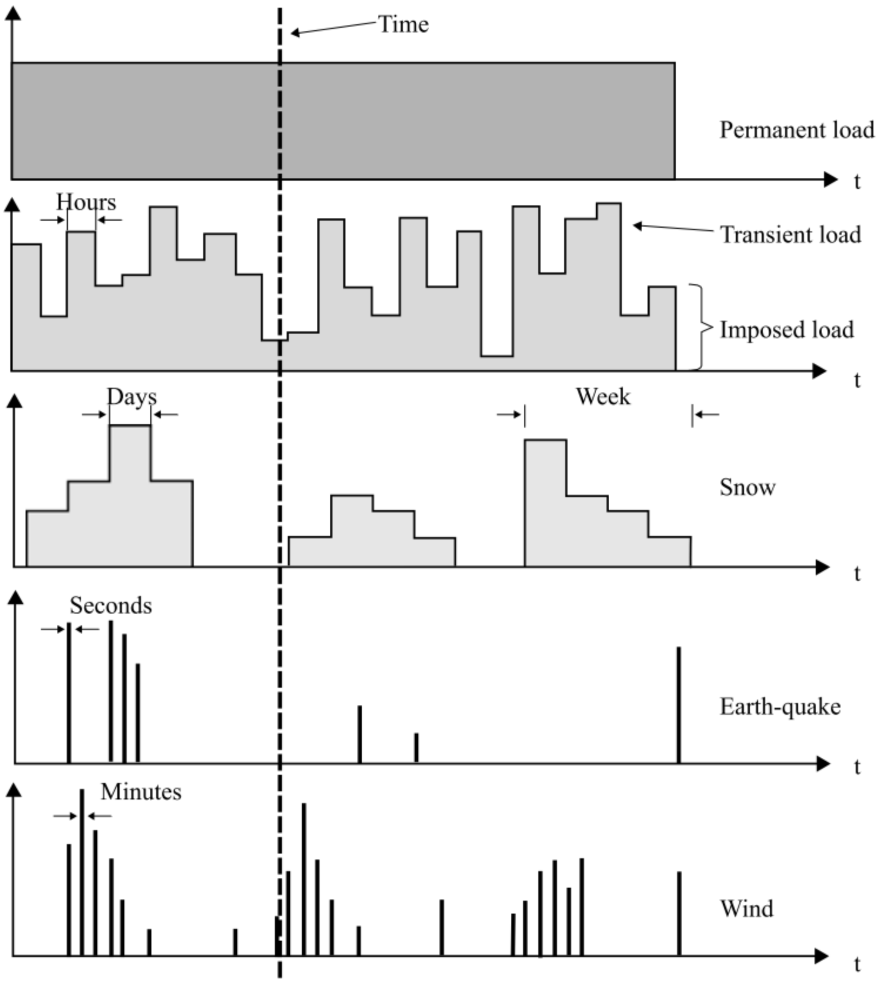


Figure 4.35 Time variation of different loads [31]

The most influential parameter for the classification of action is an action's properties during the time. The Eurocode considers the actions based on their time variability in three categories: permanent, variable and accidental actions.

Permanent actions are those whose variation during the structure's lifetime may be neglected. The self-weight of the structure or the weight of some equipment can be considered self-weight. Variable actions change significantly during their reference period and the structure's lifetime, so their time-varying properties have to be considered in the load combinations. The combination factors are defined in order to consider the time variability characteristic of these actions. Environmental actions such as snow and wind loads are considered variable. Moreover, live load or imposed loads on structures are also one of the important variable actions in structural design.

The accidental action happens in a very short time in a structure's lifetime, with a significant magnitude. Earthquake actions are classified in this category of actions [31]. A schematic representation of different loads based on the time-varying properties is illustrated in Figure 4.35.

In EN-1990, the time variability of load combinations is considered by applying combination factors ψ and defining the reference period for characteristic values of variable actions. These factors are dependent on the stochastic characteristics of variable actions. For describing the probabilistic representation of variable loads, two different distributions are involved: the point in time or instantaneous and the maximum value in the certain period of time [68]. In order to determine the characteristic value of variable actions, a certain reference period must be defined. Reference period is a time interval in which the extreme value of variable action is observed. The characteristic value is defined based on the desire probability of exceedance and distribution of maximum values in this specific time interval.

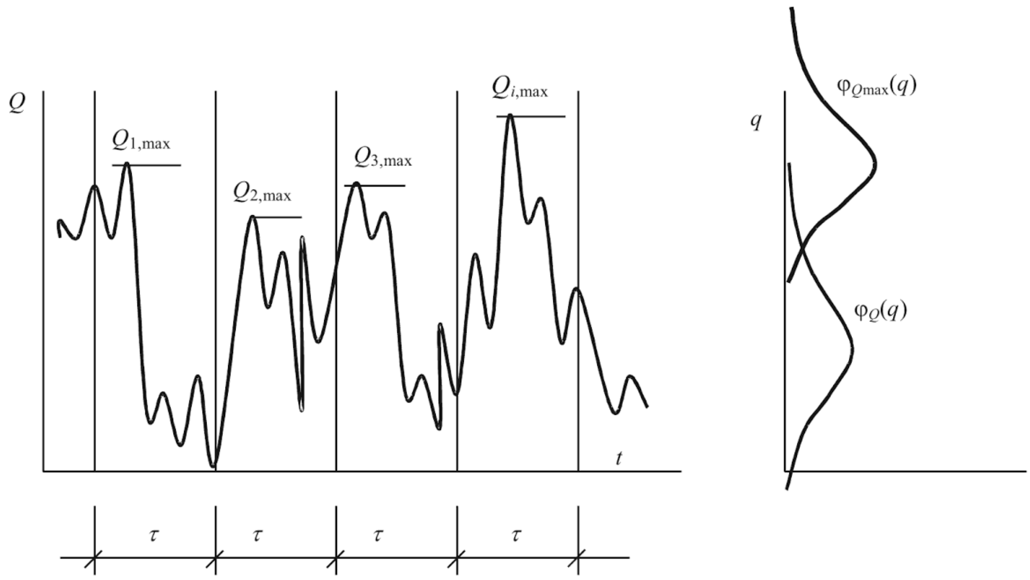


Figure 4.36: Variable action at a point in time and maximum probability density function (PDF) in reference period τ [69]

One must consider that the reference period is not necessarily equal to the design working life of the structure. In [70] the design working life of structures is categorized into four groups:

- temporary: 1–5 years,
- short life: for 25 years,
- ordinary: for 50 years, and
- long life: for 100 years.

The recommendation of EN-1990 for the reference period is the annual maximum of variable loads, and the corresponding probability of exceedance for characteristic value is 0.02 for the annual maximum. If the characteristic value is exceeded in each of the reference periods with its probability p, then after some repetitions for the reference period, the probability of exceedance for the characteristic value will be equal to 1, and this time period will be called the "return period" [23]. The relation between the reference period and probability can be expressed based on (4.17) [14].

$$T_{ret.} = \frac{\tau}{p} \quad (4.17)$$

Here, $T_{ret.}$ represents the return period, and τ is the reference period. By considering the recommended probability of 0.02 and reference period of 1 year, the return period will be determined as 1/0.02 = 50 years. Thus, the characteristic value of variable action based on this probability and reference period belongs to the mean return period of 50 years [14]. According to EN-1990, selecting a reference period depends on the characteristics of each load, but generally its recommendation is 1 year. However, [70] recommends using the 50-year maximum as reference period, because it is equal to the design lifetime of ordinary constructions, and the asymptotic distribution function of extreme value for 50 or more years has greater accuracy.

The target reliability for different reference periods is defined on the one-year target value. This calculation is made possible by assuming the independent maximum for variable action. The value of target reliability for *n*-year reference period is calculated based on (4.18).

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n \quad (4.18)$$

For a 1-year reference period, different target reliabilities may be calculated for different reference periods. Figure 4.37 shows the relation of 1-year target reliability to different reference periods. In the code, based on different RCs and consequence classes, various target reliabilities are recommended for 1 year. Figure 3 shows the behavior of target reliability after transformation from the 1-year reference period to *n*-year reference period for different classes.

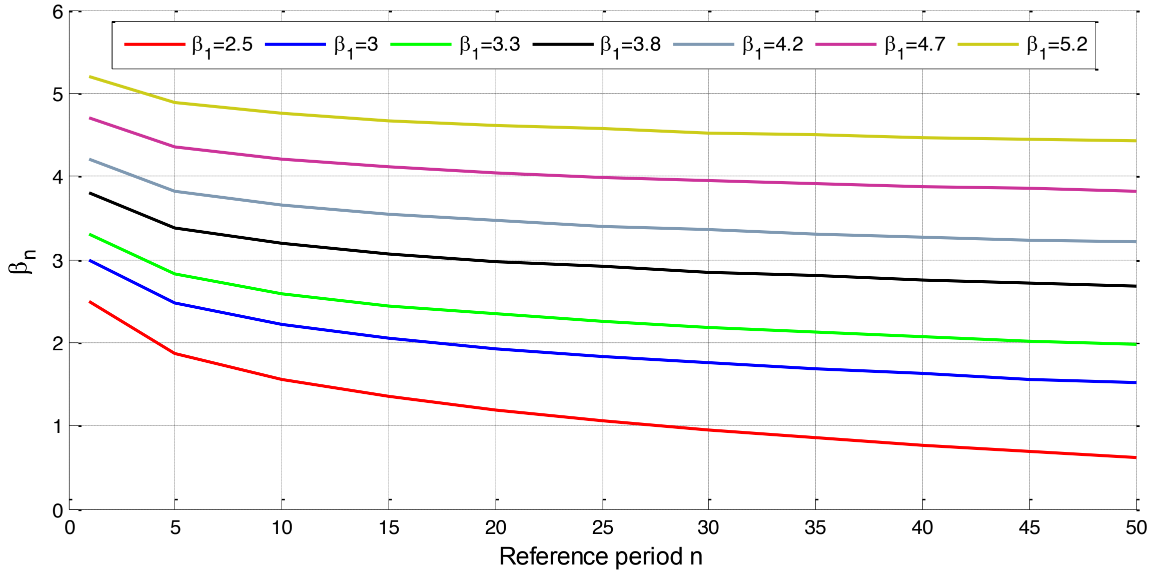


Figure 4.37: Target reliabilities and reference period

4.6.1 Combination of variable loads

The load combination process is required to be considered in order to determine the equivalent loading system in cases with two or more variable loads [52]. This issue must be considered in all reliability and risk analysis. Through the analysis, the extreme of all applied loads has to be determined in the selected reference period. The maximum load subjected to the structure during the specific reference period T can be determined according to the maximum of the sum for all individual variable actions as in (4.19) [31].

$$X_{max}(T) = max_T\{X_1(t) + X_2(t) + \cdots + X_n(t)\} \tag{4.19}$$

Generally, solving (4.19) requires complex calculations because it is the combination of various random processes with different properties. In a special case of variable loads being represented as a stationary stochastic process and mutually independent, the linear sum of variables can be represented by outcrossing rate based on Rice's formula. A detailed explanation can be found in [24] and [52]. In safety standards, such a complex calculation for finding combination values of actions is impossible. Therefore, the solution was simplified for the combination of variable actions.

4.6.1.1 Ferry Borges-Castanheta (FBC) method

An approximation of the combination problem and time-dependent variable action was recommended based on the rectangular wave renewal process in [71]. This simplified representation of random processes will be applied to solve the (4.19). The modeling of rectangular wave process in the FBC model is illustrated in Figure 4.38.

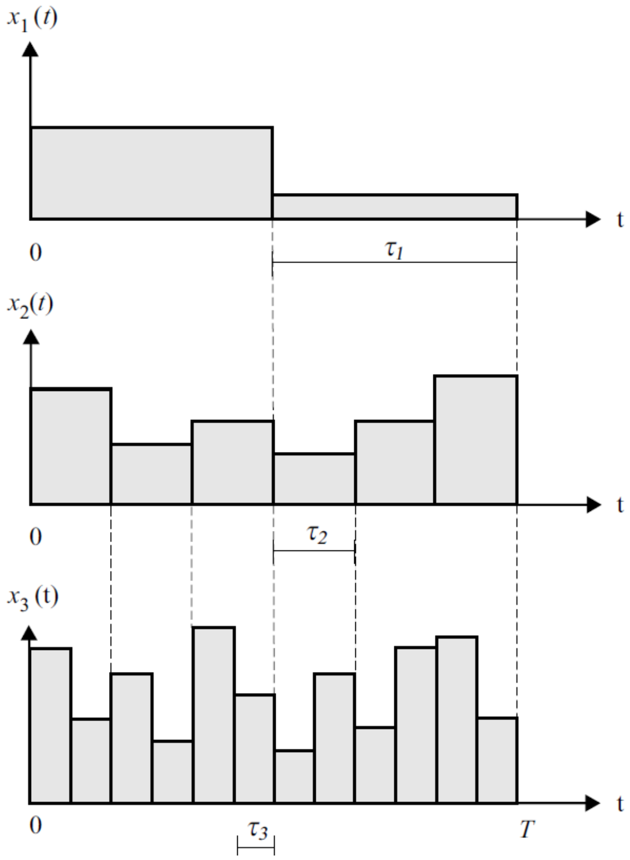


Figure 4.38: Ferry Borges-Castanheta (FBC) load process [31]

In order to implement the FBC model, some assumptions have to be made:

- $\{X_n(t)\}$ is stationary, ergodic stochastic processes;
- all intervals τ_n with constant load for load process $\{X_n(t)\}$ are equal, and τ_n here is duration of each pulse for load X_n;
- $\tau_1 \geq \tau_2 \ldots \geq \tau_n$;
- T represents the reference period;
- $r_n = T/\tau_n$ are integers;
- r_n/r_{n-1} is an integer;
- X_n are constant during each interval τ_n;
- the values of X_n for different intervals are mutually independent [24]; and
- X_1,X_2...X_n are independent.

Each load process may be represented by three distributions: point in time or instantaneous $F_X(x)$, the combination distribution $F_{X_c}(x)$, and the maximum distribution in reference period T $F_{X_{max,T}}(x)$. Figure 4.39 represents these three distribution functions for one variable action.

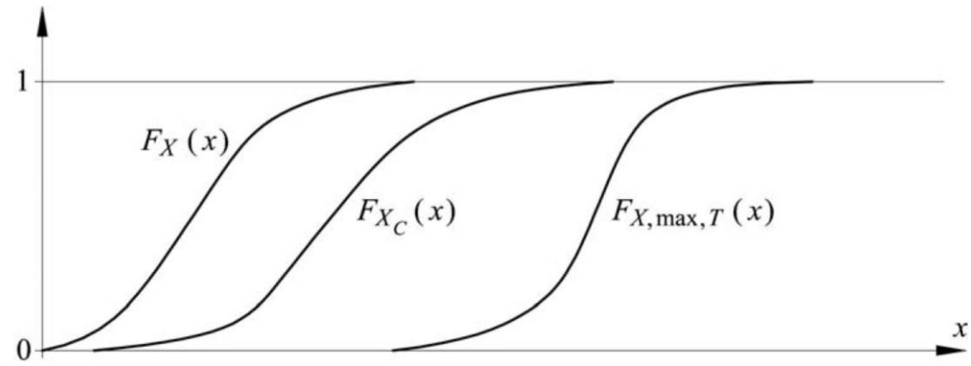


Figure 4.39: Distribution function for combination of action [24]

The maximum distribution for the reference period T is determined based on the arbitrary point in time distribution according to (4.20).

$$F_{X_{max,T}}(x) = [F_X(x)]^r \tag{4.20}$$

In the case of the combination of two variable loads X_1 and X_2, and with consideration of the assumption of the FBC model, the combination distributions will be calculated in two cases of combination as (4.21) and (4.22).

- X_2 is combination load, X_{2_c} is the maximum of X_2 during an interval of τ_1 as one pulse of X_1.

$$F_{X_{2c}}(x_2) = \left[F_{X_2}(x_2)\right]^{r_2/r_1} \tag{4.21}$$

- X_1 is combination load, X_{1_c} is equal to the arbitrary point in time distribution of X_1.

$$F_{X_{1c}}(x_1) = F_{X_1}(x_1) \tag{4.22}$$

The combination distributions may also be represented based on the maximum distribution [72]. For the same case with two variable loads, the representation of loads 1 and 2 are respectively calculated with (4.23) and (4.24).

$$\begin{cases} F_{X_{1,max,T}}(x_1) = \left[F_{X_1}(x_1)\right]^{r_1} \\ F_{X_{1c}}(x_1) = F_{X_1}(x_1) \end{cases} \rightarrow \quad F_{X_{1c}}(x_1) = \left[F_{X_{1,max,T}}(x_1)\right]^{\frac{1}{r_1}} \tag{4.23}$$

$$\begin{cases} F_{X_{2,max,T}}(x_2) = \left[F_{X_2}(x_2)\right]^{r_2} \\ F_{X_{2c}}(x_2) = \left[F_{X_2}(x_2)\right]^{\frac{r_2}{r_1}} \end{cases} \rightarrow \quad F_{X_{2,max,T}}(x_2) = \left[\left(F_{X_{2c}}(x_2)\right)^{\frac{r_1}{r_2}}\right]^{r_2}$$
$$\rightarrow F_{X_{2c}}(x_2) = \left[F_{X_{2,max,T}}(x_2)\right]^{\frac{1}{r_1}} \qquad (4.24)$$

The relation of combination distribution and the maximum distribution in certain reference periods in (4.23) and (4.24) show an important characteristic of the FBC method. It seems that the combination distribution of X_2 depends only on the repetition rate of X_1; and r_2 has no influence on the combination distribution in consideration of the maximum of reference period.

This method can be applied for any number of loads and combination formats, considering all mentioned assumptions. Table 4.4 shows the different cases for the application of the FBC model with three variable loads.

Table 4.4: Ferry Borges-Castanheta (FBC) load combination for three variable loads [72]

Load combination	**Load 1**	**Load 2**	**Load 3**	
1	r_1	r_2/r_1	r_3/r_2	Load 1 dominating next load 2
2	1	r_2	r_3/r_2	Load 2 dominating
3	1	1	r_3	Load 3 dominating
4	r_1	1	r_3/r_1	Load 1 dominating next load 3

4.6.1.2 The Turkestra's load combination rule

As can be seen in Figure 4.35, only rarely do the maxima of all variable loads occur simultaneously. Therefore, it is rather conservative to determine the maximum of the loads during the reference period with (4.25).

$$X_{max}(T) = max_T\{X_1(t)\} + max_T\{X_2(t)\} + \cdots + max_T\{X_n(t)\} \qquad (4.25)$$

If the probability of the simultaneous occurrence of two loads is negligible, then the combination problem can be solved based on the recommendation of Turkestra [60] by determining the maximum of each individual load in the reference period [31].

$$Z_1 = max_T\{X_1(t)\} + X_2(t^*) + \cdots + X_n(t^*)$$
$$Z_2 = X_1(t^*) + max_T\{X_2(t)\} + \cdots + X_n(t^*)$$
$$\cdots$$
$$Z_n = X_1(t^*) + X_2(t^*) + \cdots + max_T\{X_n(t)\}$$

The maximum value of the load combination will then eventually be determined by (4.26).

$$X_{max}(T) \approx max\{Z_i\} \tag{4.26}$$

Turkestra's model is applied in most of the codified load combinations in structural standards.

4.6.2 Analysis of stochastic parameter for maximum variable load in a reference period

The transformation of distribution from the arbitrary point in time to the maxima in the reference period is a complicated procedure that depends on the correlation properties of variable load in the time interval. In the case of a stationary Gaussian process with individual mean and standard deviation, the maxima will follow the Rayleigh distribution. In reality, it is not common to have the Gaussian and stationary process as a representation of a random process. Therefore in these cases, the maximum distribution of variable actions will be described well based on the Gumbel distribution of extreme value distribution type one, as in (4.27) [68].

$$F_{Q,max} = exp\left[-exp - \left(\left(\left(\frac{\pi}{\sqrt{6}}\right) \cdot \frac{(x-\mu_Q)}{\sigma_Q}\right) + 0.577\right)\right] \tag{4.27}$$

Here μ_Q and σ_Q correspond to the mean and standard deviation of variable load in the certain reference period.

According to the characteristic of variable loads, it is not possible to have an accurate database by observation of the maximum variable loads value during reference period of 50 years. Commonly, the database for maximum in reference periods such as 1 year is available. Consequently, the maxima distribution in the short reference period may be considered to determine the statistical properties of the load in a longer reference period of 50 years. In the case of the Gumbel distribution, by assuming the independency of maximum values in years, the transformation from t_1 to t_2 will be done based on the (4.28) and (4.29). Evidently, the standard deviation will remain the same, and the mean value will shift according to the ratio of two reference periods. Despite the constant standard deviation, the coefficient of variation will change because of differing mean values.

$$\begin{aligned} \mu_2 &= \mu_1 + \left(\frac{\sqrt{6}}{\pi}\right) \cdot \sigma_1 \cdot ln(N) = \mu_1 + \left(\frac{\sqrt{6}}{\pi}\right) \cdot (\mu_1 \cdot COV_1) \cdot ln(N) \\ &= \mu_1 \left[1 + \left(\frac{\sqrt{6}}{\pi}\right) \cdot (COV_1) \cdot ln(N)\right] \quad with\ N = \frac{t_2}{t_1} \end{aligned} \tag{4.28}$$

$$\sigma_2 = \sigma_1 \tag{4.29}$$

Using (4.28) and (4.29), the coefficient of variation for the reference period of t_2 can be represented in (4.30) based on the values correspond to t_1. The influences of transformation on the coefficient of variation and mean values are exposed, respectively, in Figure 4.40 and Figure 4.41. This diffrentiontion is the effect of the transformation between different reference periods.

$$COV_{t2} = \frac{COV_{t1}}{1 + \left(\frac{\sqrt{6}}{\pi}\right) \cdot (COV_{t1}) \cdot ln(N)} \tag{4.30}$$

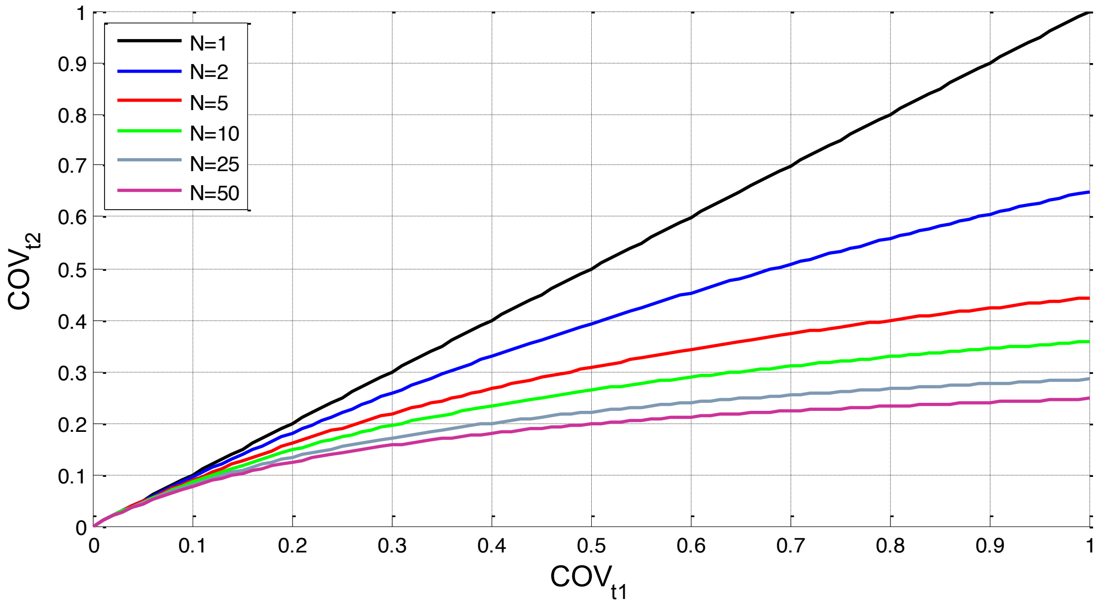


Figure 4.40: COV_{t1} versus COV_{t2}

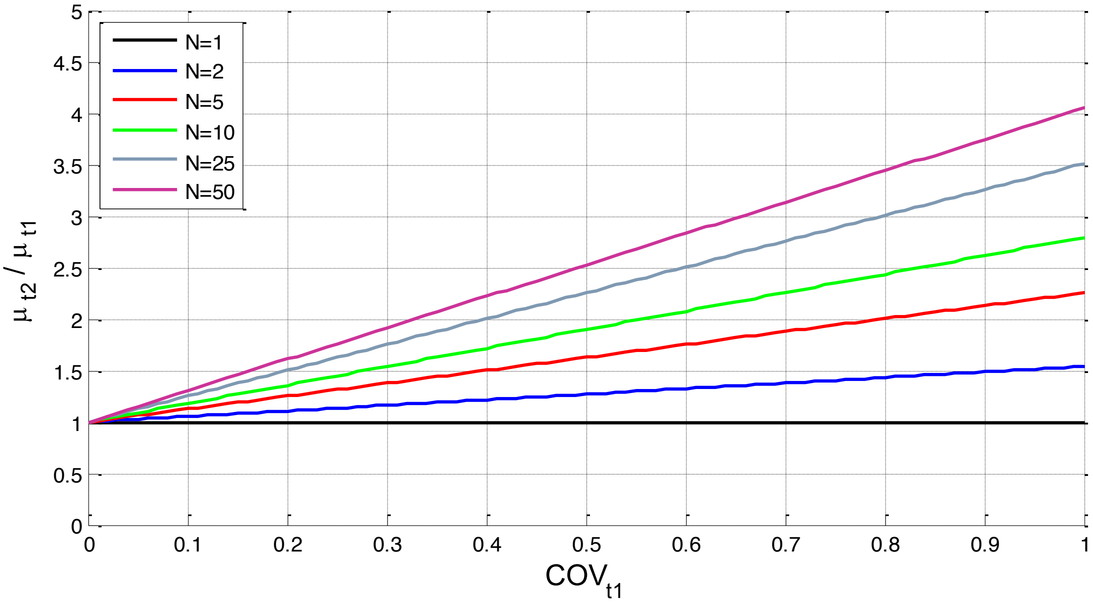


Figure 4.41: Ratio of mean values for t_1 and t_2

The reduction of COV_{t2} in comparison with COV_{t1} is significantly large in higher values of COV_{t1}. As a typical case, the transformation from data for the 1-year maxima to the 50-year maxima will reduce the COV by approximately 70%. On the other hand, in the case of mean values, greater increases in mean value will occur for the cases with higher value of COV_{t1}.

4.6.3 Probabilistic calibration of combination factor with design method

The partial factors and combination factors are the essential components of the design process based on EN-1990. There are two general methods for determination of safety factors: calculation based on the reliability analysis and the method based on the design value format [24]. Design value format is an approach recommended in [16] and [19]. Based on the design method, the exceedance probability of design load in the reference period t for target reliability β can be represented by (4.31),

$$P\{Q > Q_d\} = \Phi(\alpha_E \cdot \beta), \tag{4.31}$$

with $\alpha_E = -0.7$ is the sensitivity factor for actions and β target value of reliability.

In the case of the combination of two actions based on the FBC model, the probability of exceedance for the interval of τ_1, as the pulse with the longest duration, with a repetition rate of r_1, has to be considered. The probability in the case of combination can be found in (4.32) [16].

$$P\{Q_c > Q_{c,d}\} = \Phi(\alpha_E \cdot \beta)/r_1 \tag{4.32}$$

Subsequently, the corresponding target reliability in the case of combination will be (4.33).

$$\beta_c = -\Phi^{-1}\{\Phi(\alpha_E \cdot \beta)/r_1\} \tag{4.33}$$

Afterwards, the design ratio value determined by the combination value of action will give a combination factor value as in (4.34), which is based on the corresponding combination distribution and probability of exceedance. In order to present a combination factor according to the maxima distribution in the reference period, (4.34) is replaced by (4.35).

$$\psi_0 = \frac{F_{Q,c}^{-1}\{\Phi(0.4 \cdot \beta_c)\}}{F_{Q,c}^{-1}\{\Phi(\beta_c)\}} \tag{4.34}$$

$$\psi_0 = \frac{F_{Q,max}^{-1}\{\Phi(0.4 \cdot \beta_c)^{r_1}\}}{F_{Q,max}^{-1}\{\Phi(\beta_c)^{r_1}\}} = \frac{F_{Q,max}^{-1}\{\Phi(0.4 \cdot \beta_c)^{r_1}\}}{F_{Q,max}^{-1}\{\Phi(0.7 \cdot \beta)\}} \tag{4.35}$$

The approximation of (4.35) for the case of normal distribution and Gumbel distribution are expressed in (4.36) and (4.37), respectively. The COV represents the coefficient of variation in the reference period.

$$\psi_0 = \frac{1 + (0.28 \cdot \beta - 0.7 \cdot \ln(r_1)) \cdot COV}{1 + 0.7 \cdot \beta \cdot COV} \tag{4.36}$$

$$\psi_0 = \frac{1 - 0.78 \cdot COV \cdot [0.58 + \ln(-\ln\Phi(0.28 \times \beta)) + \ln((r_1)]}{1 - 0.78 \cdot COV \cdot [0.58 + \ln(-\ln\Phi(0.7 \times \beta))]} \tag{4.37}$$

Figure 4.42 presents the approximation diagrams. The difference between the exact and approximated method is negligible. By increasing the value of COV for the reference period, the deviation between the approximation and the exact method is increased.

The calibration method based on design value that is explained in this section is the method recommended in the EN-1990. Figure 4.43 depicts the calibration result for the combination factor with the design method. The combination factor for high repetition rate and big COV will be zero. For higher repetition rates, the combination factor will become smaller. This is the same in the case of COV. Greater variance in the variable load will prevent the possibility of the maximum value accompanying load at the same time as the leading action. Therefore, the combination factor will be small.

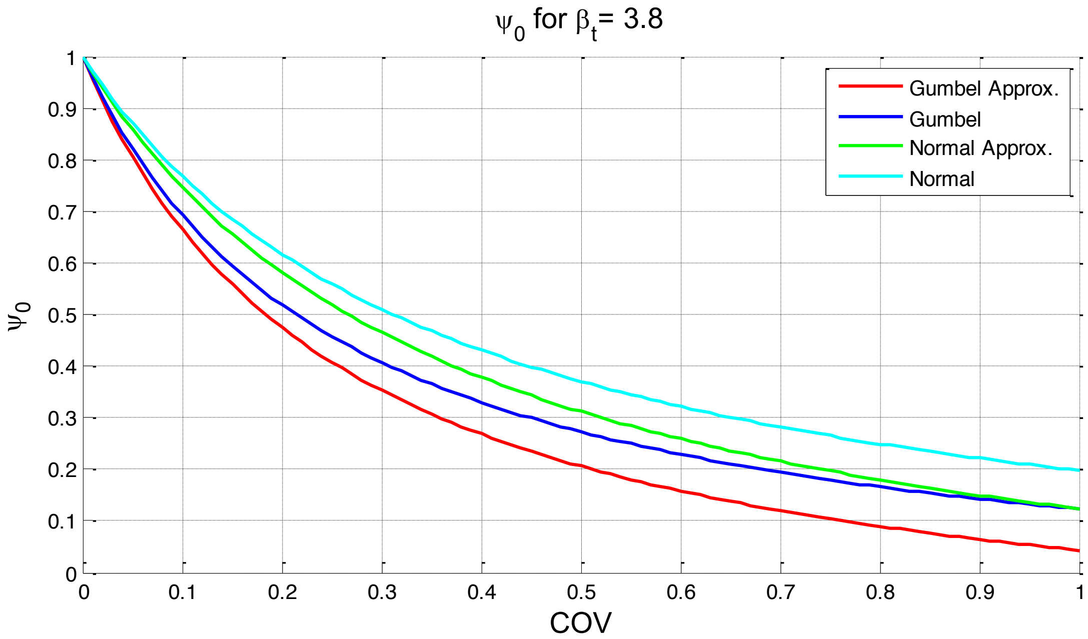


Figure 4.42: Comparison of different distribution for r_1 = 5

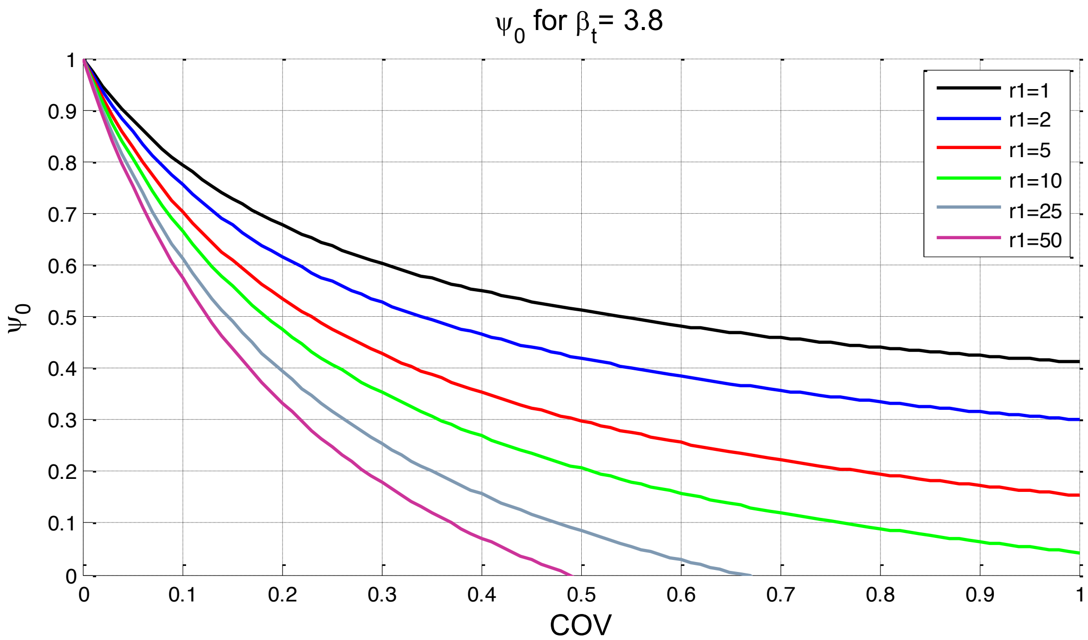


Figure 4.43: Combination factor for different COV and r_1 for Gumbel and 50-year reference

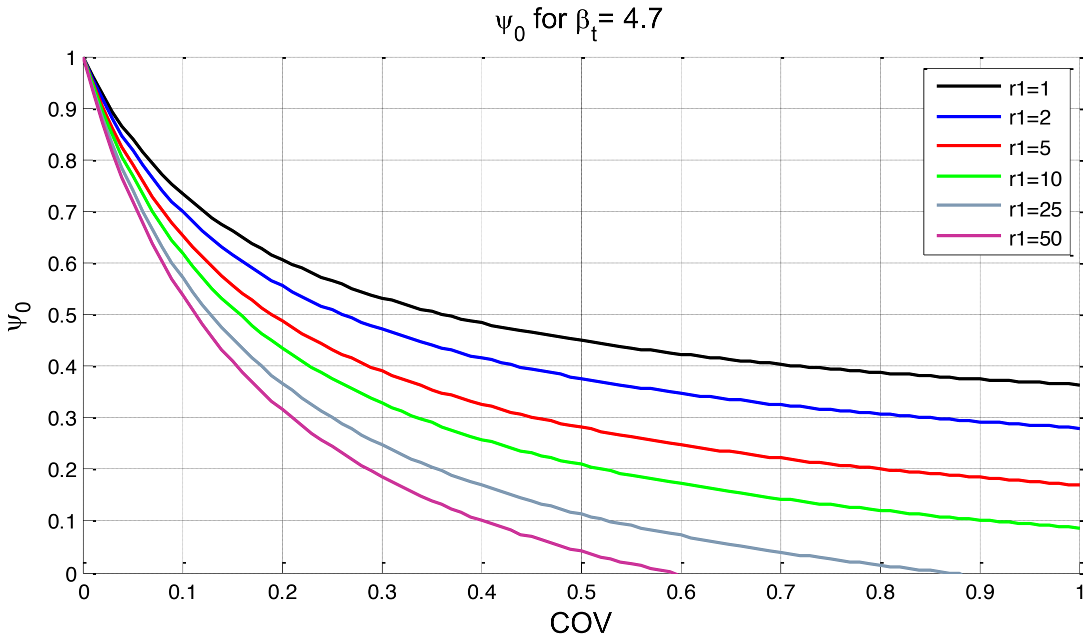


Figure 4.44: Combination factor for different COV and r_1 for Gumbel and 1-year reference

4.6.4 Conclusion

According to Figure 4.43 and Figure 4.44 and the comparison of these results with the recommended values in EN-1990, it may be concluded that the combination factors in the EN-1990 are conservative. It is also mentioned in [73] and [74] that the values in EN-1990 are in general conservative. This is because of a rough calibration according to previous design methods, such as allowable stress [75].

In the safety concept recommendations of Germany in 1980s, the same method was applied to calibrate the partial factor for German code [76]. A detailed explanation is provided for the background information in [75] and [77].

4.7 Simplified load combination

4.7.1 A proposal for simplified load combination

Through the new review of the Eurocodes, a concept that must be considered by the committees is ease of use. The new generations of Eurocodes must be more convenient for engineers to be applied in practice. The confusing parts in the current versions have to be omitted. In order to satisfy this aim, a simplified load combination format for the fundamental combinations of EN-1990 is proposed. This recommendation solves the complexity of choosing different selections of combination factors for variable loads.

The combination factors are defined in the EN-1990 to include the occurrence of maximum variable load at the same time. Based on different categories of variable loads, different values of combination factors must be selected based on Table 4.4.

Table 4.5: ψ factors recommended in Annex A of EN-1990-1-1

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings, category (see EN 1991-1-1)			
Category A: domestic, residential areas	**0,7**	**0.5**	**0.3**
Category B: office areas	**0.7**	**0.5**	**0.3**
Category C: congregation areas	**0.7**	**0.7**	**0.6**
Category D: shopping areas	**0.7**	**0.7**	**0.6**
Category E: storage areas	**1.0**	**0.9**	**0.8**
Category F: traffic area,	**0.7**	**0.7**	**0.6**
vehicle weight $\leq 30kN$			
Category G: traffic area,	**0.7**	**0.5**	**0.3**
$30kN$ $<$ **vehicle weight** $\leq$ $160kN$			
Category H: roofs	**0**	**0**	**0**
Snow loads on buildings (see EN 1991-1-3)			
Finland, Iceland, Norway, Sweden	**0.7**	**0.7**	**0.2**
Remainder of CEN Member States, for sites located at altitude $H > 1000\, m\, a.s.l.$	**0.7**	**0.7**	**0.2**
Remainder of CEN Member States, for sites located at altitude $H \leq 1000\, m\, a.s.l$	**0.5**	**0.2**	**0**
Wind loads on buildings (see EN 1991-1-4)	**0.6**	**0.2**	**0**
Temperature (non-fire) in buildings (see EN 1991-1-5)	**0.6**	**0.5**	**0**

The main concept for conducting simplification in load combinations is applied to the accompanying variable actions. Based on the current version of EN-1990, the multiplication of partial factors for variable actions $\gamma_Q = 1.5$ to the combination factors ψ_0 in almost all cases will be a value approximately equal to 1. Therefore, in fundamental combinations of EN-1990, $\gamma_Q \cdot \psi_0$ will be replaced by a factor of 1. It follows that the fundamental combinations in EN-1990 (6.10, 6.10a and 6.10b) in the case of simplified combinations will be replaced by the following combinations:

6.10
$$E_d = \sum_{j\geq 1} \gamma_{G,j} G_{k,j} \text{"}+\text{"} \gamma_p P \text{"}+\text{"} \gamma_{Q,1} Q_{k,1} \text{"}+\text{"} \sum_{i>1} \mathbf{1}\, Q_{k,i}, \tag{4.38}$$

6.10a
$$E_d = \sum_{j\geq 1} \gamma_{G,j} G_{k,j} \text{"}+\text{"} \gamma_p P \text{"}+\text{"} \mathbf{1}\, Q_{k,1} \text{"}+\text{"} \sum_{i>1} \mathbf{1}\, Q_{k,i} \tag{4.39}$$

6.10b
$$E_d = \sum_{j\geq 1} \xi_j\, \gamma_{G,j} G_{k,j} \text{"}+\text{"} \gamma_p P \text{"}+\text{"} \gamma_{Q,1}\, Q_{k,1} \text{"}+\text{"} \sum_{i>1} \mathbf{1}\, Q_{k,i}. \tag{4.40}$$

In simplified cases, the characteristic value of the accompanying action will be considered as its design value. This simplification can be applied for all fundamental combinations in persistent and transient design cases in EN-1990.

In order to investigate the application of simplified load combination, the same analyses as in Section 4.2 are done for all fundamental combinations of EN-1990 and their simplified formats. The average value of calculated reliability for simplified load combination 6.10 and 6.10a&b are the same as those for EN-1990 in Figure 4.5 and Figure 4.6. The application of this simplification is thus totally compatible with the results obtained from EN-1990 fundamental combinations. For representing the results, the combination of wind and imposed load are selected exemplified. The comparison of simplified load combination and the combination of EN-1990 are shown in Figure 4.45 and in Figure 4.46 for leading wind load and accompanying imposed load. Figure 4.47 and Figure 4.48 also representing the case of snow as leading variable load and wind as accompanying.

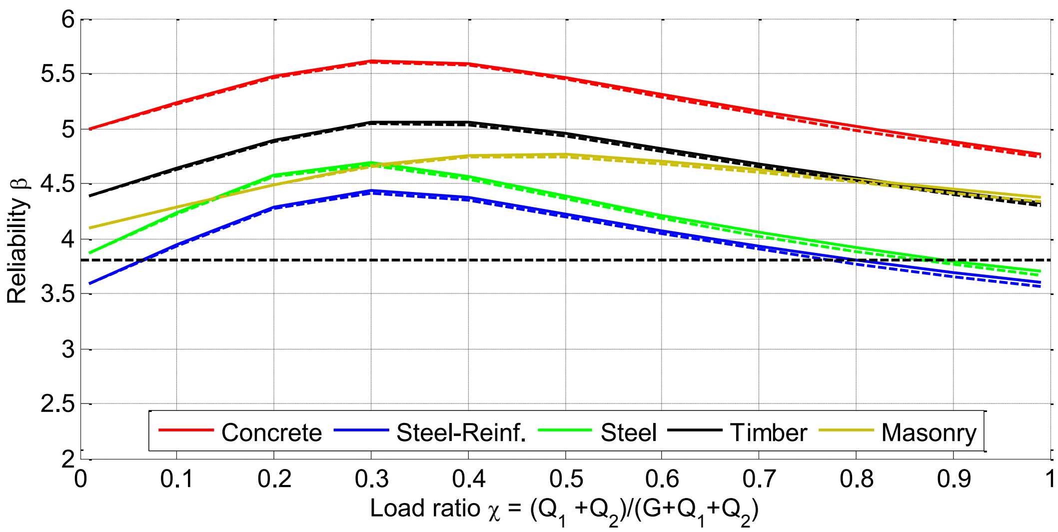


Figure 4.45: EN-1990 combination in line and simplified in dash for k = 0.5 and combination 6.10

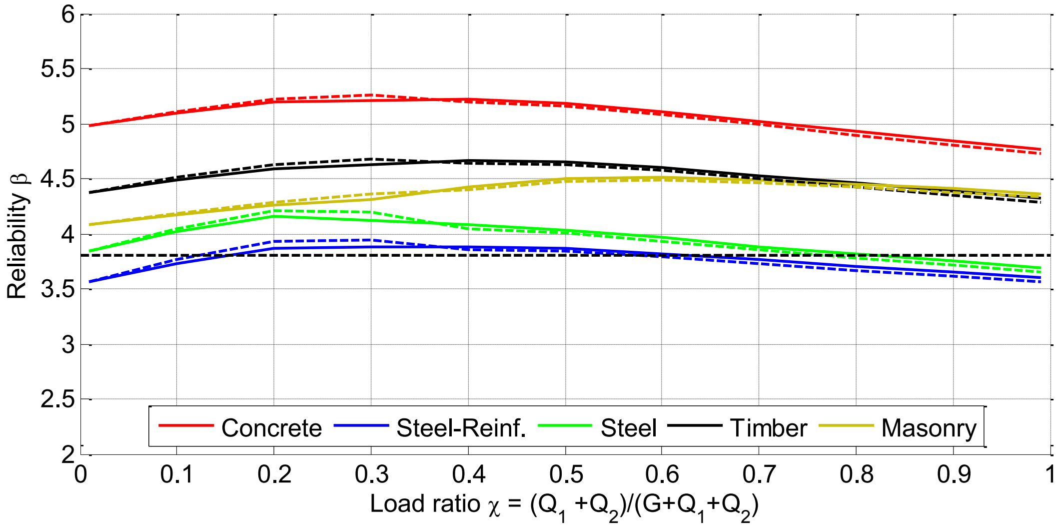


Figure 4.46: EN-1990 combination in line and simplified in dash for $k = 0.5$ and combination 6.10a&b

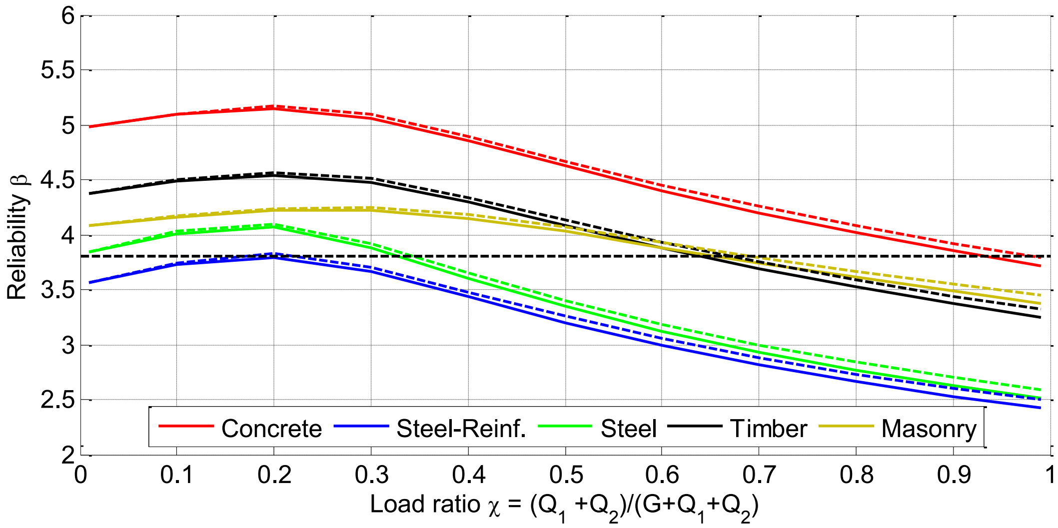


Figure 4.47: EN-1990 combination in line and simplified in dash for $k = 0.5$ and combination 6.10

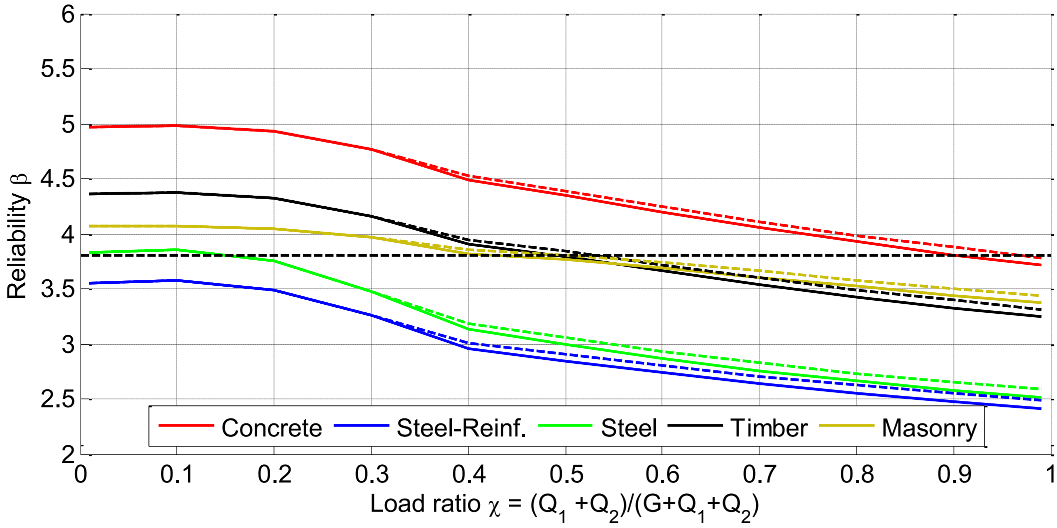


Figure 4.48: EN-1990 combination in line and simplified in dash for $k = 0.5$ and combination 6.10a&b

As is observable, the differences between the simplified and original combinations are not considerable. It can be concluded that this simplification proposes an acceptable method for ease of use because it prevents the complication of selecting different combination factors in calculations. More detailed results may be found in [78].

4.7.2 Recommendation of simplified combination in last draft of EN-1990

After serious discussions of the application of simplified load combination, it has been decided to recommend this method as a note in the table of load combination in the code. In last draft of the new generation of EN-1990 [18], the simplified load combination is recommended in Note 4 for Table A1.3 as follows [18]:

"For persistent and transient design situations, when $\gamma_Q \cdot \psi_0 \approx 1$ the design value of the accompanying variable action can be approximated by its characteristic value."

Table A1.3 presents load combinations for the ultimate limit state in the new draft of EN-1990. Table 4.6 presents exactly as in the draft of EN-1990 [18].

This method has been criticized by people from countries like Denmark, which use a smaller value for the combination factor in the case of wind and snow. For example, in the national annex of Denmark for EN-1990 [79], the combination factor for snow and wind load is $\psi_0 = 0.3$. Then the resultant value of $\gamma_Q \cdot \psi_0 = 0.45$ will not approach 1. In this case, if they want to apply the simplified method in their design, this method will lead to an economically unattractive result. The conservative results by application of simplified load combinations are the main concern of such countries.

Table 4.6: Table A1.3, load combinations for ultimate limit states [18]

Design situation	Persistent and transient	Accidental	Seismic	Fatigue
Clause	6.3.7.2	6.3.7.3	6.3.7.4	6.3.7.5
Equation	6.29	6.31	6.32	6.33
Permanent ($G_{d,j}$)	$\gamma_{G,j} G_{k,j}$	$G_{k,j}$	$G_{k,j}$	$G_{k,j}$
Leading variable ($Q_{d,1}$)[1]	$\gamma_{Q,1} Q_{k,1}$	$\psi_{1,1} Q_{k,1}$ or $\psi_{2,1} Q_{k,1}$	$\psi_{2,i} Q_{k,i}$	$\psi_{1,1} Q_{k,1}$
Accompanying variable ($Q_{d,i}$)	$\gamma_{Q,i} \psi_{0,i} Q_{k,i}$	$\psi_{2,i} Q_{k,i}$		$\psi_{2,i} Q_{k,i}$
Prestress (P_d)[2]	$\gamma_P P_k$	P_k	P_k	P_k
Accidental (A_d)	-	A_d	-	-
Seismic (A_{Ed})[3]	-	-	A_{Ed}	-
Fatigue (Q_{fat})	-	-	-	Q_{fat}

….

NOTE 4: For persistent and transient design situations, when $\gamma_{Q,i}\ \psi_{0,i} \approx 1$ the design value of the accompanying variable action can be approximated by its characteristic value.

4.7.3 Comments on simplified load combination method

Considering the note for recommendation of simplified load combination, the comment from Denmark will be rejected because the comment is not valid for this simplified rule. The code writers recommend this method when the condition $\gamma_Q \cdot \psi_0 \approx 1$ is not satisfied. Therefore, it cannot allowed to be applied for certain countries. Consequently, there will be no conflict with their design.

Application of simplified combinations is an optional method in the code. In other words, if the engineers want to skip the complexity of choosing the combination factor based on the different variable loads, this simplification can be applied. Therefore, there is no need for this optional clause to apply in all cases. Engineers must decide based on the condition of the simplified load combination.

Nevertheless, another alternative may be proposed to allow the application of simplified load combinations for Denmark as well. However, it is described that the current version for implementation of simplified load combination has no conflict with the national annexes. According to the values of the combination factor, the note in the context of the code may be modified to the following:

> For persistent and transient design situations, when $\gamma_Q \cdot \psi_0 \approx 1$ and $\gamma_Q \cdot \psi_0 \approx 0.5$ the design value of the accompanying variable action can be approximated respectively by its characteristic value and 50% of characteristic value.

This alternative proposal will solve the comment from Denmark. They can apply the simplified load combination in the case of snow and wind based on the mentioned recommendations. The result will be an acceptable approximation of the exact combination in their national annexes.

The reliability analysis is conducted for the case of combination for different variable loads. The most critical case, which is $\psi_0 = 0.3$ for snow and wind, will be considered to verify the recommended simplified method. Combinations 6.10 and 6.10a&b according to the EN-1990 and Denmark national annex will be compared with the fundamental combinations. The results of the comparison for this alternative and combination in EN-1990 are presented in Figure 4.49–Figure 4.52. The results are shown for both 6.10 and 6.10a&b for two cases. The first case represents the one with imposed load leading and wind load accompanying. The second is with wind load leading and snow load accompanying.

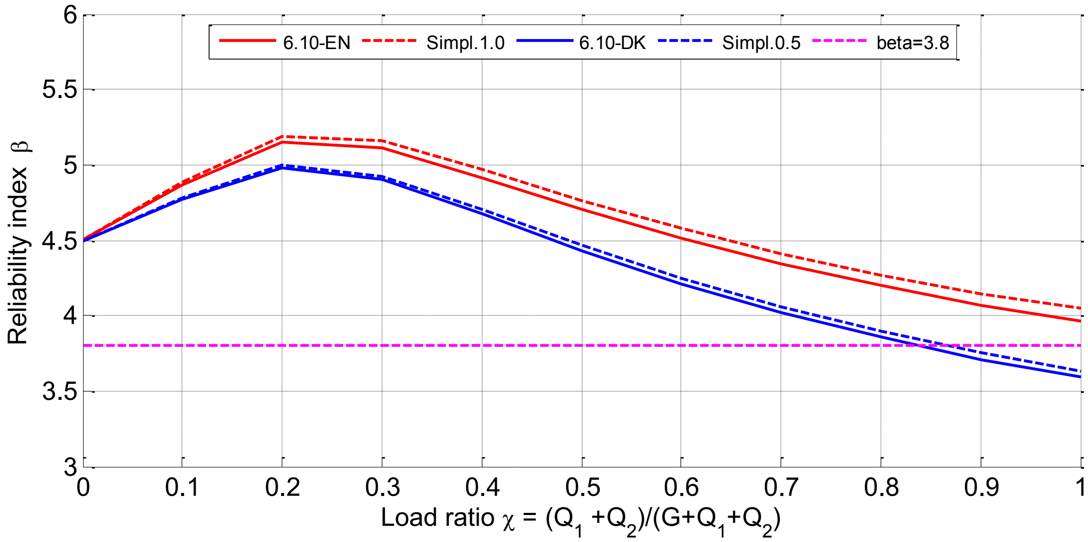


Figure 4.49: Load combination 6.10 for imposed load with wind and *k* = 0.5

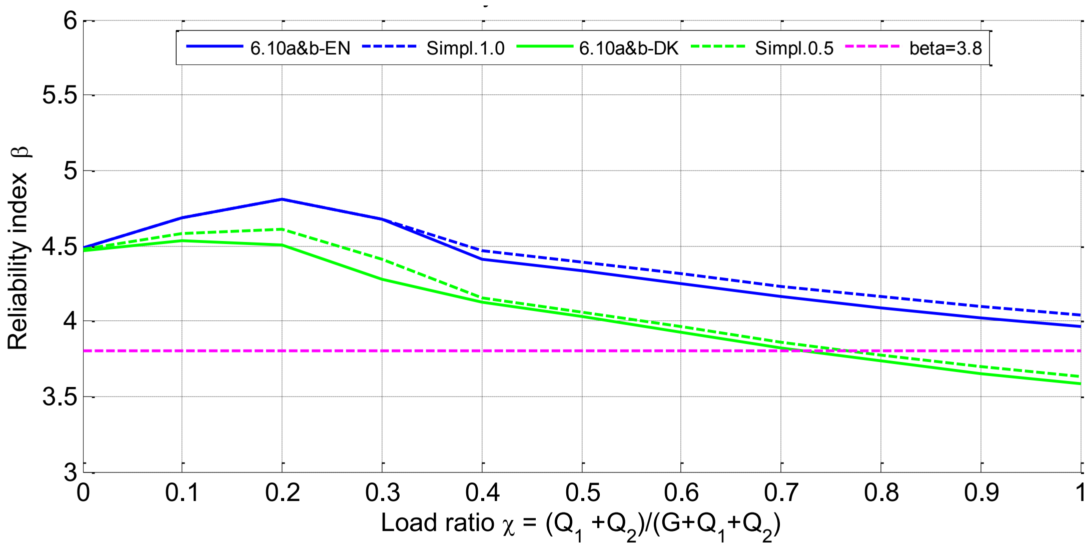


Figure 4.50: Load combination 6.10a&b for imposed load with wind and k = 0.5

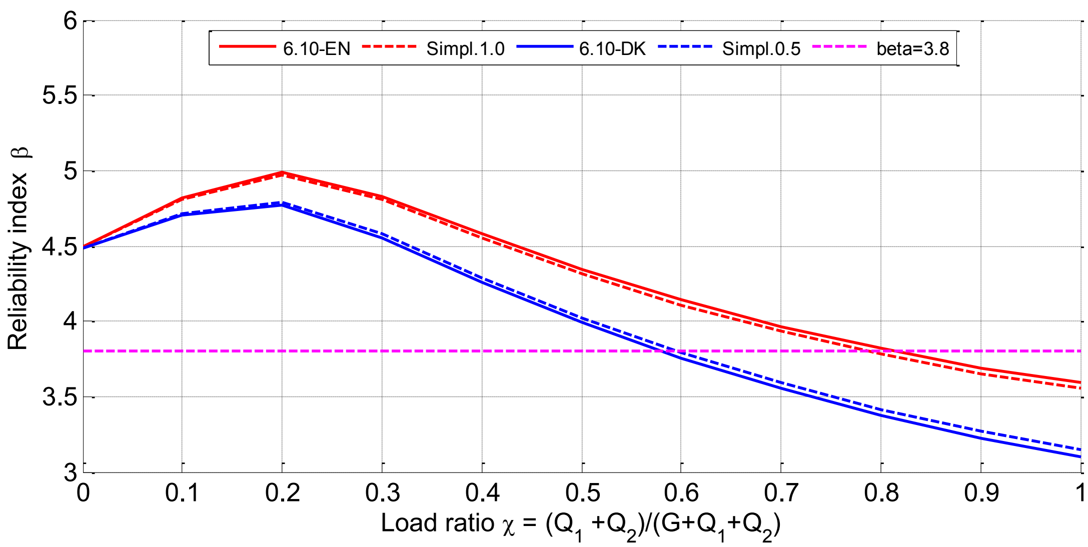


Figure 4.51: Load combination 6.10 for wind with snow and k = 0.5

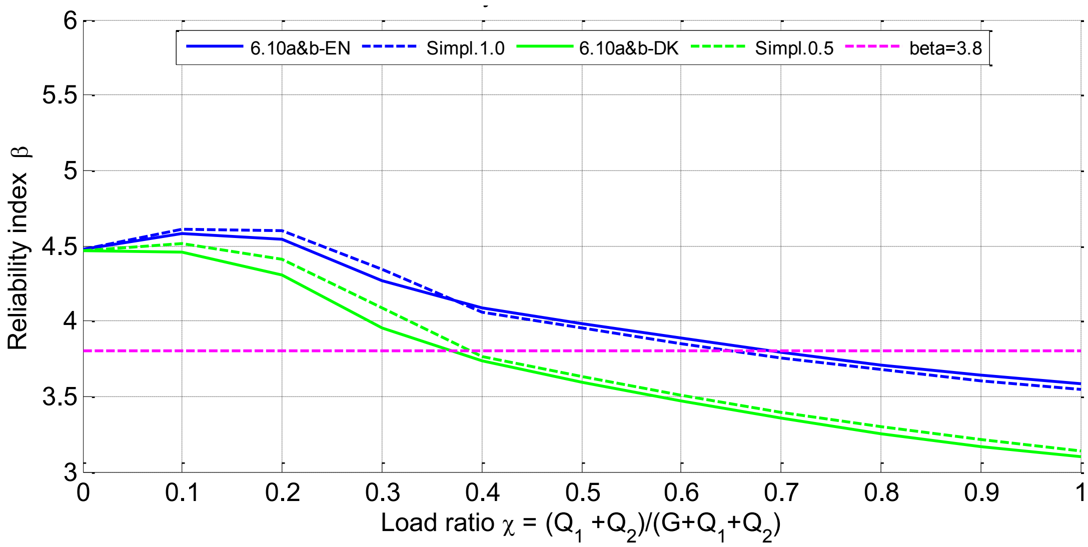


Figure 4.52: Load combination 6.10a&b for wind with snow and *k* = 0.5

4.7.4 Conclusion

The simplified load combination allows engineers to deal with the design problem in a more convenient way. The selection of different combination factors for different cases of variable load will be skipped. This method completely satisfies the ease of use concept in the new generation of the Eurocodes. This is an approximation of fundamental load combinations in the code, but it will be an optional note in the code. Therefore, engineers have both options to conduct their design, in a simplified way and the complete combinations approach.

It has been shown that the application of simplified load combination will lead to a result that is compatible with the load combinations in the code. The difference in reliability values can be neglected, and this method is an acceptable approximation for fundamental combinations.

The recommendation is also compared with other national annexes, such as that of Denmark. The simplified load combination with the current format is not valid in this country, but with an alternative proposal, it may be applicable based on the country's national annex. The reliability analyses show the compatibility of alternative solutions with the original combination in the national annex for Denmark.

4.8 A new method for partial factor calibration based on Monte Carlo method

4.8.1 Interest band method

A new concept for the calibration of partial factors is defined within this research. Since the dawn of the partial safety factor method's implementation, an engineers' knowledge of background uncertainties has been improved to a great extent. Year by year, more data have been collected in databases, and they represent more realistic behavior with respect to actions and resistance parameters. Subsequently, the statistical quantification of different sources of uncertainty seems to be necessary. Calibration of partial factors will upgrade structural codes based on the modern requirements of the community, such as sustainability and economical design [80]. The precise calibration results will on the one hand prevent the waste of material and on the other provide sufficient safety.

The primary methods for the calibration of partial factors are conducted based on the FORM in initial calibrations [81]. In this study, a new concept is introduced based on the crude Monte Carlo method; this new method consists of three main parts:

1. reliability analysis and finding the resistance distribution for each set of load characteristic values which gives a reliability index equal to target reliability,
2. determining the interest band and corresponding interest points, and
3. calculating partial factors.

In the first step, reliability analysis has to be performed for calibration. The Monte Carlo reliability analysis for calibration should provide the required amount of safety. By adjusting this requirement, the calibration results can be made compatible with the safety requirements. The safety criteria are defined based on the target reliability values in EN-1990 [16].

For the reliability analysis in this section, two types of load—permanent (G) and variable (Q)—are considered with a resistance parameter for different types of material. Stochastic parameters of basic variables for reliability analysis are applied according to Table 3.2. In reliability analysis, there is no accompanying variable action. Therefore, all variable loads are considered with their 50 years reference period distribution (Table 3.2).

The algorithm of reliability analysis is conducted with a limit state function in (4.9). For each value of χ factor, a resistance has to be determined in which the reliability index has to be the same as target value of reliability ($\beta_t = 3.8$).

Afterwards, the calibration can be performed for random realizations of last-reliability analyses with the target value. In the Monte Carlo method, the random points represent possible realizations of basic variables in reality, which means that each of the random points produced can be the material or load in the structure. Conversely, in the process of design based on concept of a partial safety factor, the limit state is the criteria where the engineers look after for failure. Limit state represents those realizations of load and resistance where these two values are equal. Subsequently, the points located in the area where loads are lower than the resistance are considered safe cases. Otherwise, they are considered failure cases. In a design with the partial safety factor method, the design values actions or action effects have to be equal to or smaller than the design value of resistance.

The calibration process must satisfy not only the safety of the design, but also the economical design. If only safety is considered as the decisive parameter in the calibration process, the partial safety factor may provide a conservative result. In order to overcome this problem, the optimum number of design values must to be selected for calibration. The ideal place for a design point or a realization of basic variables in a design situation is near the limit state.

In the calibration process, interest band is formed according to the average distance of the failure points from the limit state (i.e., parameter "*a*"). The limit state will extend from both sides to a "band" with a width of 2a (Figure 4.53). This band is the place where the interest points for calibrations are located. All the points in this interest band are considered for the calibration of the partial safety factor. Each point represents a realization of design value for its corresponding basic variable. In other words, the limit state is extended only with parameter "a," which forms a band and covers an area called "the interest band." The boundary between failure and safety is defined by an area instead of a line.

Each point in the interest band is determined by the random realization of all basic variables involved in the limit state equation. The correspondent realizations of these basic variables are treated as design values.

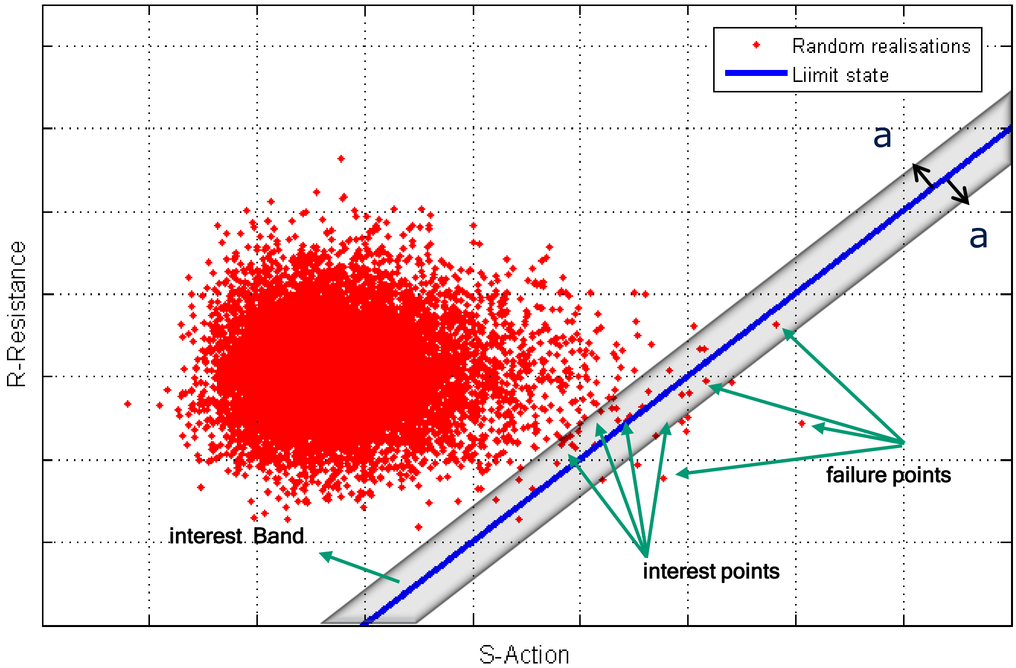


Figure 4.53: Interest band method for calibration of partial factors

Eventually, the ratio of design value over the characteristic value leads to the partial safety factor for basic variables. For resistance, the ratio of characteristic value over design value can be considered a partial factor. Moreover, the realization of model uncertainty has to be considered in the calculation of partial factors as the contribution of model uncertainty. The averages of determined partial factors from all interest points represents the calibrated partial factor. The parameters related to the action part are considered only for calibration. Equation (4.41) shows the calculation formula for a partial factor of actions.

$$\gamma_E = \theta_E \frac{E_d}{E_k} \tag{4.41}$$

The interest band method is applied for five types of material and three types of variable loads. The resistance parameters are concrete (EN-1992-1-1[61]), steel (EN-1993-1-1 [62]), steel rebar (EN-1992-1-1 [61]), timber (EN-1995-1-1 [63]) and masonry (EN-1996-1-1 [64]). Wind, imposed and snow loads are three variable loads in the investigation. The calibration process is implemented only for the load part. Therefore, the reliability indexes are determined based on the calibrated values of load partial factors and the recommended partial safety factor of resistance in Eurocode system. The corresponding values to the material partial factors based on the Eurocodes are listed in Table 4.1.

The overall results of the calibration for all resistance types and load categories are presented in Table 4.4. The average reliability level for a range of load ratios χ is also illustrated in Figure 4.54. It can be observed that the deviation of reliability from the target value is not significant.

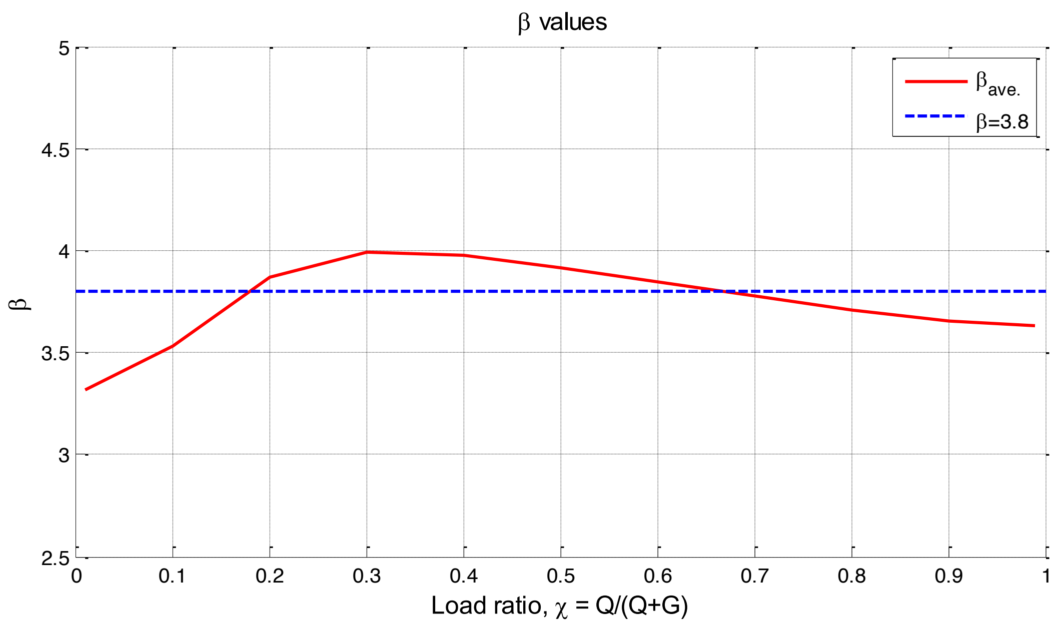


Figure 4.54: Average reliability with calibrated partial factor

Table 4.7: *Overall calibration results*

γ_G	$\gamma_{imposed}$	γ_{wind}	γ_{snow}	$\beta_{ave.}$
1.11	1.32	1.57	2.32	3.74

The highest value of the partial factor is related to the partial factor of snow loads. By comparing the results with the recommended value of γ_Q = 1.5 in the EN-1990, the calibrated partial factor of wind is nearly the same as the code value. For imposed load, it can be considered a conservative application of partial factor in comparison with the value of EN-1990. In the case of permanent action, the calibrated values show the lower amount in comparison with γ_G = 1.35 in EN-1990.

The calibrated partial factors of actions may also be represented based on different types of materials. The results in Table 4.4 represent the average for the whole database. The categorized results of action partial factors are shown in Figure 4.55.

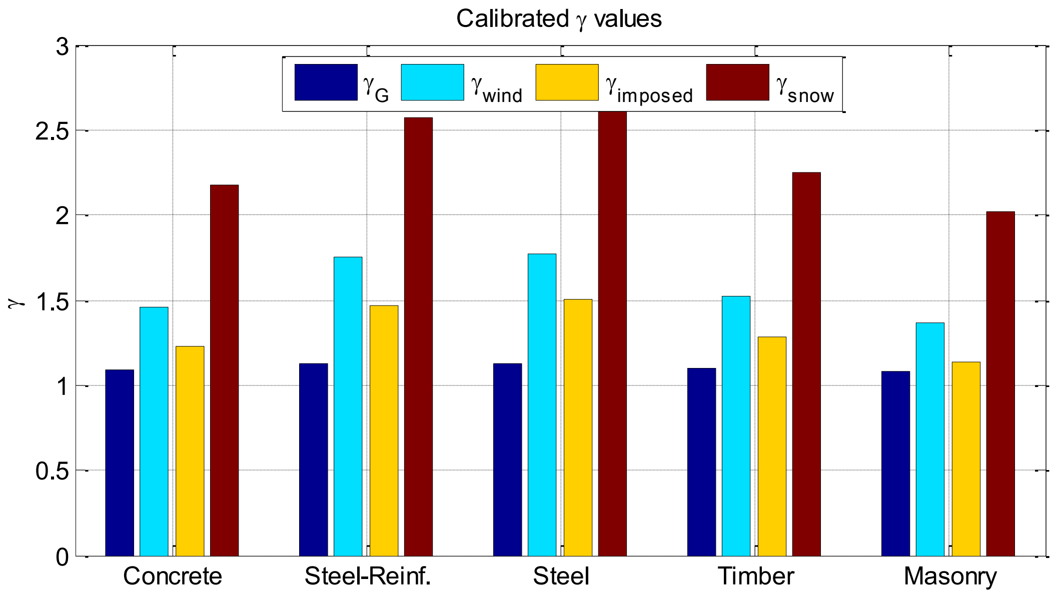


Figure 4.55: Calibrated partial factor for different resistance types

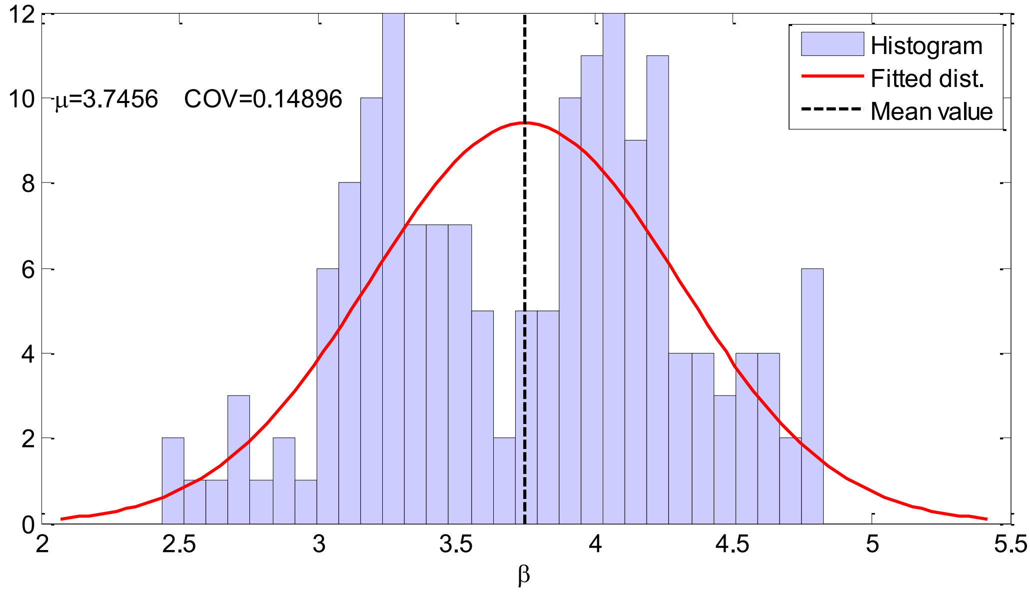


Figure 4.56: Histogram and normal fitted distribution of reliability

The reliability representing the average of the whole database for all cases of resistance and loading is compatible with the target reliability of the calibration process. The reliability values may be illustrated as a distribution (Figure 4.56).

These values are distributed based on the parameters involved in the calculations. The database of reliability indexes for calibrated partial factor has a mean value of 3.74 and standard deviation of 0.55. It leads to a coefficient of variation that equals 15%.

The reliability level of calibrated partial factors may also be represented separately for each type of material for the load cases. The classified reliability indexes based on the resistance type and variable actions are presented in Figure 4.57.

The reliability behavior shows that the greatest safety with calibrated values is given by concrete materials. The lowest values correspond to steel structures. This behavior may be explained by the partial factors of material in Table 4.1. Concrete has the maximum partial factors, and steel has the minimum. In the case of masonry, despite the same partial factor of material with concrete, the reliability is lower, which is mainly due to the higher COV value of masonry material parameter.

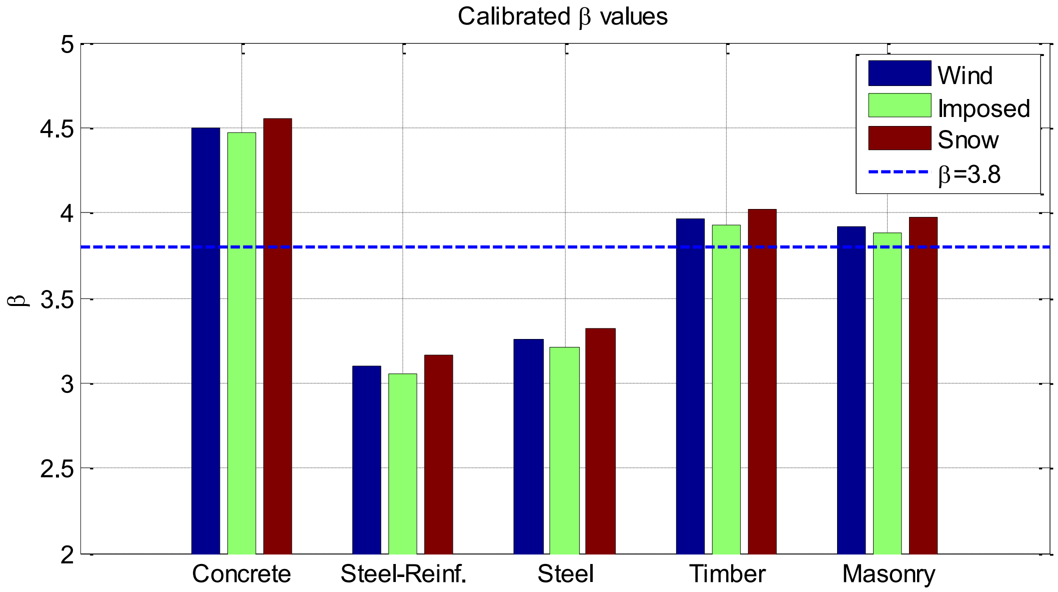


Figure 4.57: Reliability of different resistance types with calibrated partial factor

4.8.2 Conclusion

Calibration is the requirement of all available codes of structural engineering in practice. The recommended values of partial factors have to be calibrated in a periodical review of design codes. In this study, a new method for calibration of partial factors has been introduced. The so-called interest band method is based on the Monte Carlo method.

The calibration results for different types of resistance models and loads show that this method presents an acceptable approach for the calibration of partial factors. Consideration of random realization in the interest band as a design value of basic variables is reasonable. The algorithm of calibration shows compatible behavior regarding the target reliability index. The calibration process is applied for actions involved in the limit state function with different resistance models.

The calibration approach has been applied with the assumption of EN-1990 safety requirements. The calibrated partial factor for permanent actions is smaller than the value specified in EN-1990. The results of the calibration show that in case of snow load, the recommended partial factor of EN-1990 does not cover the uncertainty of snow loads. A higher value of partial factor has to be applied for snow loads to reach the target reliability. In the case of wind loads, the partial factor of EN-1990 is nearly the same as the calibrated value. For imposed loads and permanent actions, the recommendation of EN-1990 seems to be a conservative value.

These values are the representations of the overall averages for all cases. Calibrated partial factors and resultant reliability indexes vary for different types of resistance and actions.

Moreover, for different values of load ratios, the result may change. The ratio of permanent load and variable load play an essential role in the calibration process. In cases of high proportion of permanent loads, the partial factor of permanent action is the decisive parameter, because the variable loads do not have considerable influence on reliability. Contrarily in the case of a high ratio variable load due to the fact that the permanent loads do not affect reliability in these cases variable loads are decisive parameters.

The interest band approach has the advantage of full probabilistic methods. The random realization represents the more realistic behavior of basic variables in the Monte Carlo method. The interest band method has the same disadvantage as the Monte Carlo method. The calculation costs are higher with this method in comparison with a calibration process based on FORM, and this method also takes more time.

The calibrated values are highly dependent on the assumption of stochastic parameters and probabilistic models of loads. The parameters here are the ones proposed for code calibration. The calibrated values may vary with the choice of other stochastic parameters and probabilistic models for actions. Further sensitivity analysis based on the assumption of stochastic parameters can offer perspective on their effect on final calibration results.

Overall, compensation between permanent actions and variable loads has to be implemented to obtain the consistency of results with target reliabilities. The result of the calibration shows that for light-weight structures and heavy-weight structures (maximum and minimum range of load ratio χ), the reliabilities do not reach the target value ranges.

5 Resistance partial safety factor

5.1 Introduction

Loads, materials, dimensions and models are the most important parameters in structural analysis. Chapter 4 has discussed the components of limit states related to loads and actions. In this chapter, resistance and its partial factor as the second term in limit state functions is investigated.

In most cases, the verification or evaluation of each material property in structures is conducted according to experiments. Therefore, every calculation, including test results, has an uncertainty coming from experiments. The uncertainty is taken into account by introducing stochastic properties to the test results. Eventually, the parameter will no longer be a deterministic or single value, but it will be defined within a range based on the PDF and its parameters, such as mean value and standard deviation. The most important parameter in structural engineering evaluated by experiments is material properties.

Test results directly influence the current design's methodology in Eurocodes, including partial factor methods or limit state design. All recommended values for proposed partial factors are based on the probabilistic analysis and tests. Different methods based on probabilistic analysis and stochastics are introduced in EN-1990 to determine of partial factors based on the stochastic parameters of material and test results (see Annex C and D in [16]).

5.2 Test number influence on partial factor

5.2.1 General

In the testing procedure, one of the most effective approaches to obtain more accurate results is to increase the number of test attempts, making the probabilistic model an accurate representation of material properties. Thus, the influence of test number should be considered in the probabilistic analysis of partial factors, which are the outcomes of probabilistic methods. The values of partial factors depend on the coefficient of variation for the material parameters, and the coefficient of variation for parameters resulting from numerous tests. For cases in which the design is performed based on experiments, the influence of test numbers on the partial factors should also be considered. In the first step, it seems that the partial factor can be reduced based on the test number. More tests lead to the smaller partial factor, supposedly. The influence of the test number on the partial factor is investigated throughout this section.

In EN-1990 [16] Section 5.2, a brief explanation is offered about the design assistance by testing, and Annex D of EN-1990 can be consulted for further detail. This topic is described in its context in ISO 2394 [19]. These two explanations differ only with respect to how they deal with the topic's details. In ISO 2394, one finds more detailed information. The basic statistical analysis and background of these formulations are mentioned in the international standard ISO 12491 [82], [83].

5.2.2 Basic statistical analysis of test number

The mean value of a set of tests can be represented as a random variable M. If the true mean value and the true standard deviation of population for random variables X are μ and σ, respectively, then using expectation (E) and variance operator (D), the mean of mean values and standard deviation of mean value predicted by a set of tests will be formulated as follows:

$$m = E(M) = E\left(\frac{\sum(X_i)}{n}\right) = \frac{(\sum E(X_i))}{n} = \frac{n\mu}{n} = \mu \tag{5.1}$$

$$s^2 = D(M) = D\left(\frac{\sum(X_i)}{n}\right) = \frac{(\sum D(X_i))}{n^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}, \tag{5.2}$$

where m and s are mean and standard deviation for the sample average [83].

The coefficient of variation for random variable M (sample average) will be represented with the coefficient of variation for random variable X with (5.3).

$$V_M = \frac{s}{m} = \frac{\sigma}{\mu\sqrt{n}} = \frac{V_X}{\sqrt{n}} \tag{5.3}$$

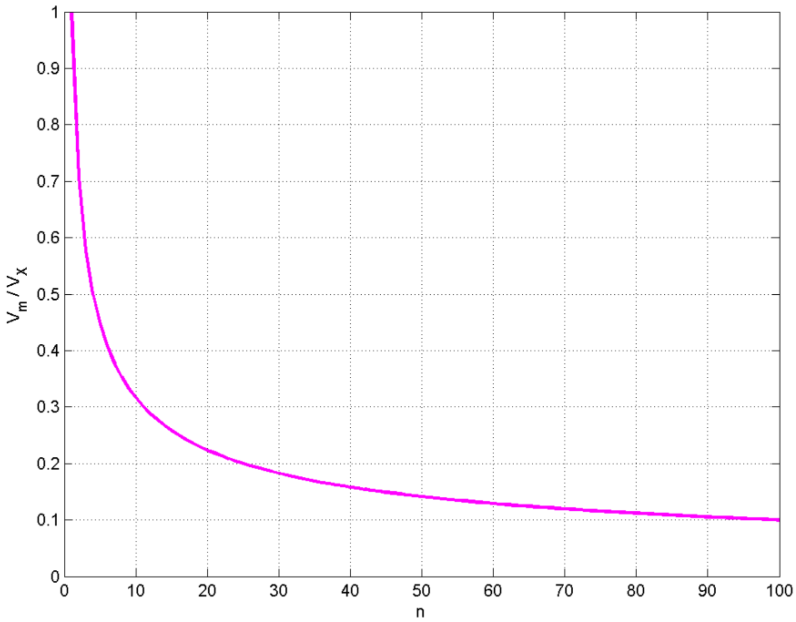


Figure 5.1: COV proportion of sample average and true mean value of random variable X

As indicated in (5.3), by increasing the test numbers to infinity, the coefficient of variation for mean value of sample average inclines to zero. Contrarily, the mean value of the sample average will be the true mean value of random variable *X*, and it will be a deterministic value because of zero standard deviation. It is a reasonable phenomenon; more test numbers lead to more accurate results.

5.2.3 Coverage method for fractile estimation

In order to determine the partial factors of material properties, both the characteristic value and the design value of corresponding random variables should be calculated. The partial factor is then calculated based on (5.4).

$$\gamma_R = \frac{characteristic\ value}{design\ value} \tag{5.4}$$

Characteristic value and design value represent a specific fractile value of the random variable based on its statistical properties. Therefore, the coverage method is used in order to consider the influence of test numbers on the partial factor.

Estimations are made based on this method for a population with a limited number of samples (n). The aim is to find probability fractile p for n samples with confidence level γ, which represents the probability that this estimation covers the fractile [83]. For lower fractiles such as resistance parameters, all predicted values $x_{p,cover}$ are ensured to be smaller than the fractile value x_p with confidence level γ.This expression is represented as (5.5) [83].

$$P(x_{p,cover} < x_p) = \gamma \tag{5.5}$$

Without information on the coefficients of variation of random variables, the predicted fractile will be represented based on the sample mean, standard deviation and a fractile factor.

$$x_{p,cover} = m - k_p s \tag{5.6}$$

In (5.6), m and s are respectively mean and standard sample deviations , and k_p is the fractile coefficient for probability p. Coefficient k_p will be determined based on non-central t distribution with degrees of freedom equal to $n-1$ and non-centrality parameter $\sqrt{n} \cdot u_p$; and u_p is the p fractile of standard normal distribution [84].

$$k_p = \frac{1}{\sqrt{n}} t_\gamma\left(\sqrt{n} \cdot u_p, n-1\right) \tag{5.7}$$

The factor k_p will be determined by considering three parameters: confidence level γ, fractile probability, and sample numbers.

Equation (5.6) is valid for those random variables with normal distribution. Equation (5.6) represents the left-hand fractile. According to the symmetry properties of normal distribution, the right-hand fractile with a probability of $1 - P$ will be determined by adding $k_p s$ to the mean value instead of subtracting.

If the random variable is log-normally distributed, then the equation should be reformulated using a transformation between normal and lognormal variable. If the random variable X is lognormally distributed, then the logarithm for this random variable Y is normally distributed.

$$Y = \ln(X) \tag{5.8}$$

Equation (5.6) is valid for the random variable Y with normal distribution. The estimated fractile of random variable Y will be transformed to the lognormal fractile of random variable X with lognormal distribution by transformation, as below:

$$x_p = e^{m_Y - k_p \cdot s_Y}, \tag{5.9}$$

where m_Y and s_Y are mean and standard sample deviations, respectively, for random variable Y and where x_p is the fractile corresponding to the probability of p.

5.2.3.1 EN-1990 and coverage method

In EN-1990, the recommended confidence level is 75% for fractile estimation. In order to find the characteristic and design value, probabilities corresponding to these values are recommended in EN-1990. For characteristic value, 5% (or 95%) is recommended as a fractile value. In the case of design value, the fractile value of about 0.1% (or 99.9%) will be considered as the design value. Figure 5.2 shows different values of k_p based on various probabilities and test numbers. This parameter is applied in later sections for the evaluation of test data.

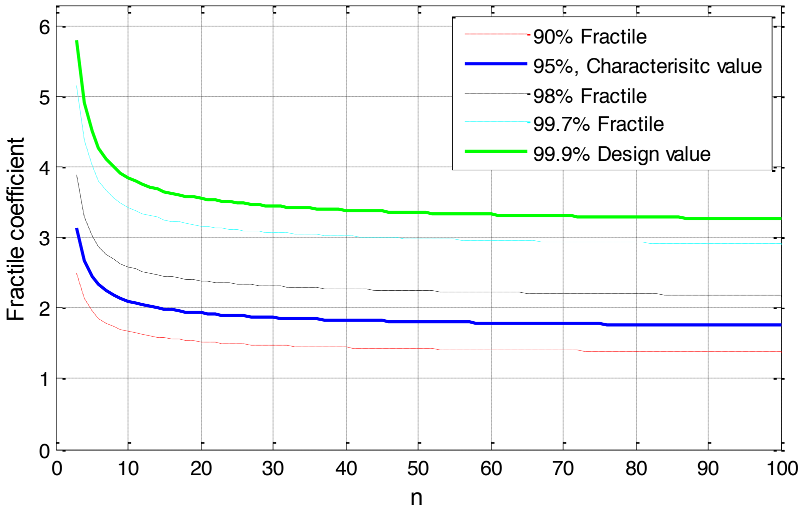


Figure 5.2: Coefficient k_p for different probabilities and test numbers with confidence level 75%

5.2.3.2 Partial factor of material

For the log-normally distributed random variable, the partial factor will be calculated based on (5.4), and characteristic and design values correspond to (5.9). By considering different coefficients of variation for test samples, the theoretical partial factor is determined as in Figure 5.3.

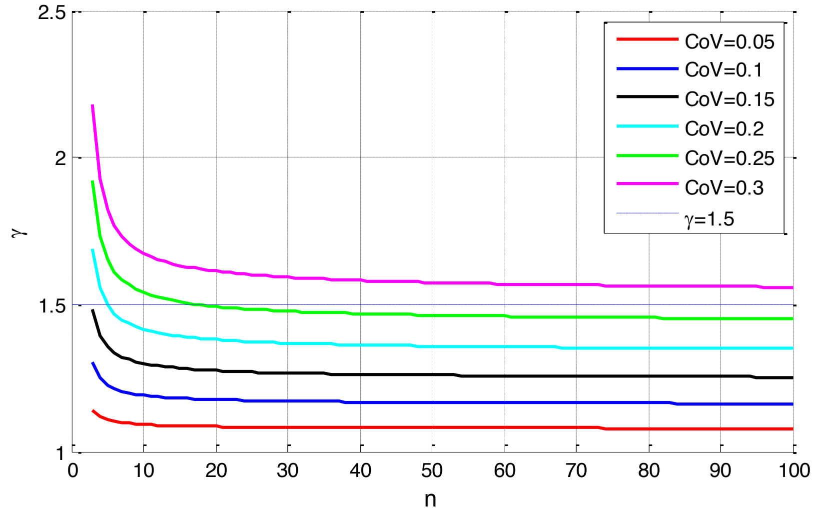


Figure 5.3: Partial factor for lognormal distributed random variable with confidence level 75%

As depicted in Figure 5.3, after test number 10, all curves show relatively constant behavior, meaning that the result of 10 tests is an acceptable representation of the material partial factor. The partial factor deviations from the correspondent value for 10 samples will show the amount of difference in various test numbers. If the determined value of a partial factor in Figure 5.3 is considered as the selected value, its comparison with other values for different test numbers is shown in Figure 5.4.

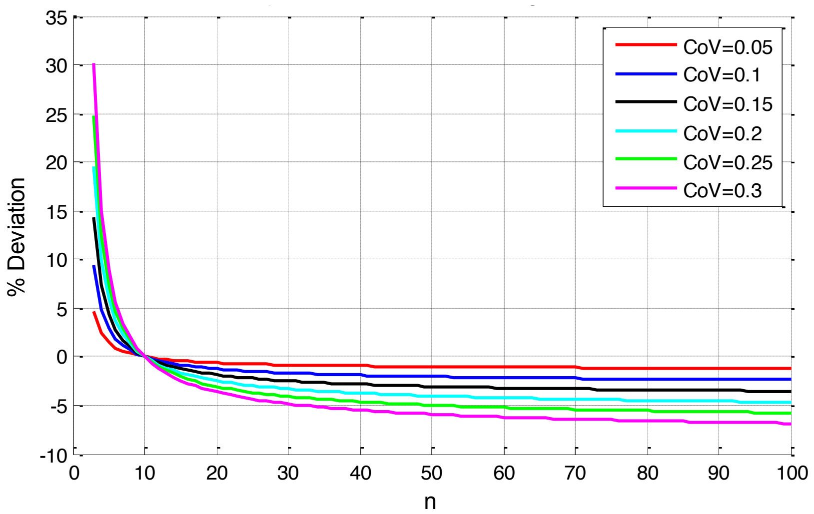


Figure 5.4: Partial factor deviation for the value corresponding to n = 10 test numbers

The maximum deviation value after test number 10 (as observed in Figure 5.4) is around 6%, which corresponds to the maximum coefficient of variation 30%. On the other hand for the other coefficients of variation, the deviations between 10 and 100 numbers of tests are approximately lower than 5%. Therefore, after 10 test numbers, the calculated partial factor will not change significantly. This amount of deviation can be also ignored and considered a constant value after

10 tests. As such, 10 is the optimum number of tests to give nearly perfect perspective to material uncertainty. In other words, exceeding 10 test attempts wastes material and costs.

5.2.4 Analysis of concrete compression tests series

A series of compression tests were completed in different governmental constructions projects in Hong-Kong [85]. Tests are done for different grades of concrete. The results of the cube compression concrete test are implemented to investigate the influence of test numbers during construction work on the design results.

These tests were done during the construction of each project stage. In each test, two specimens were considered from each batch of concrete. Three sets of tests for concrete with grade 20, 30, and 40 MPa are considered here. The histogram and fitted log-normal distribution for all classes are illustrated in Figure 5.5–Figure 5.7.

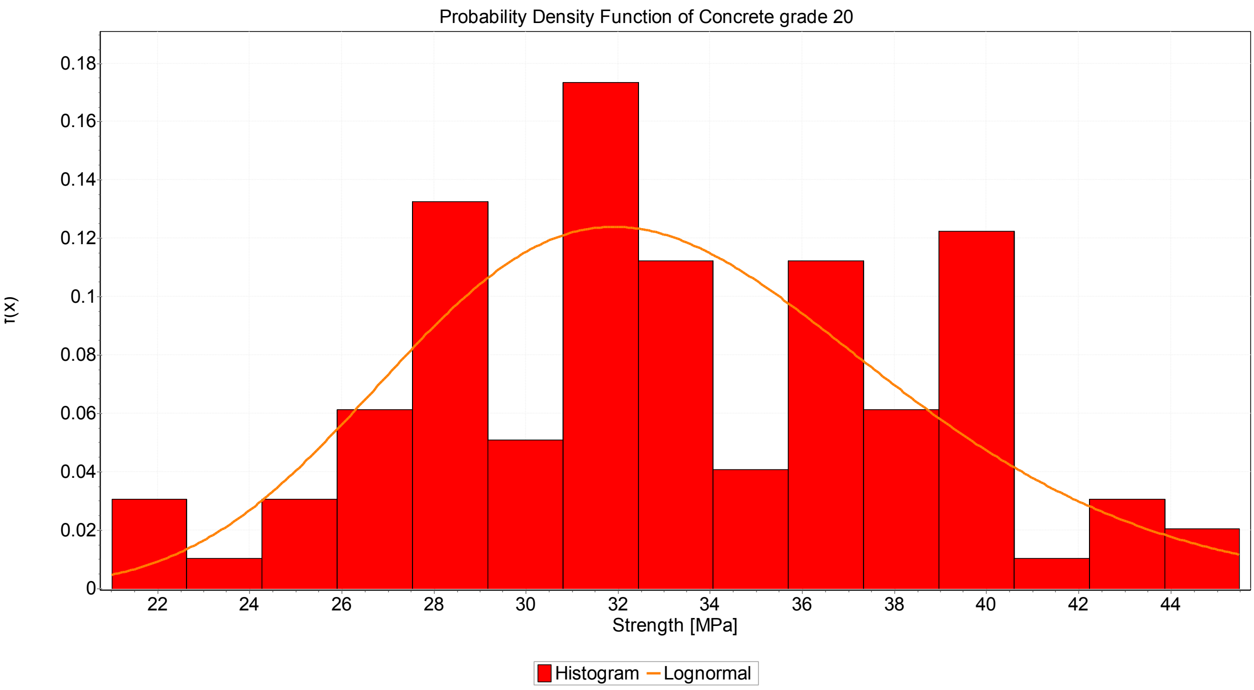


Figure 5.5: Fitted lognormal distribution with parameters σ = 0.1626 and μ = 3.4896

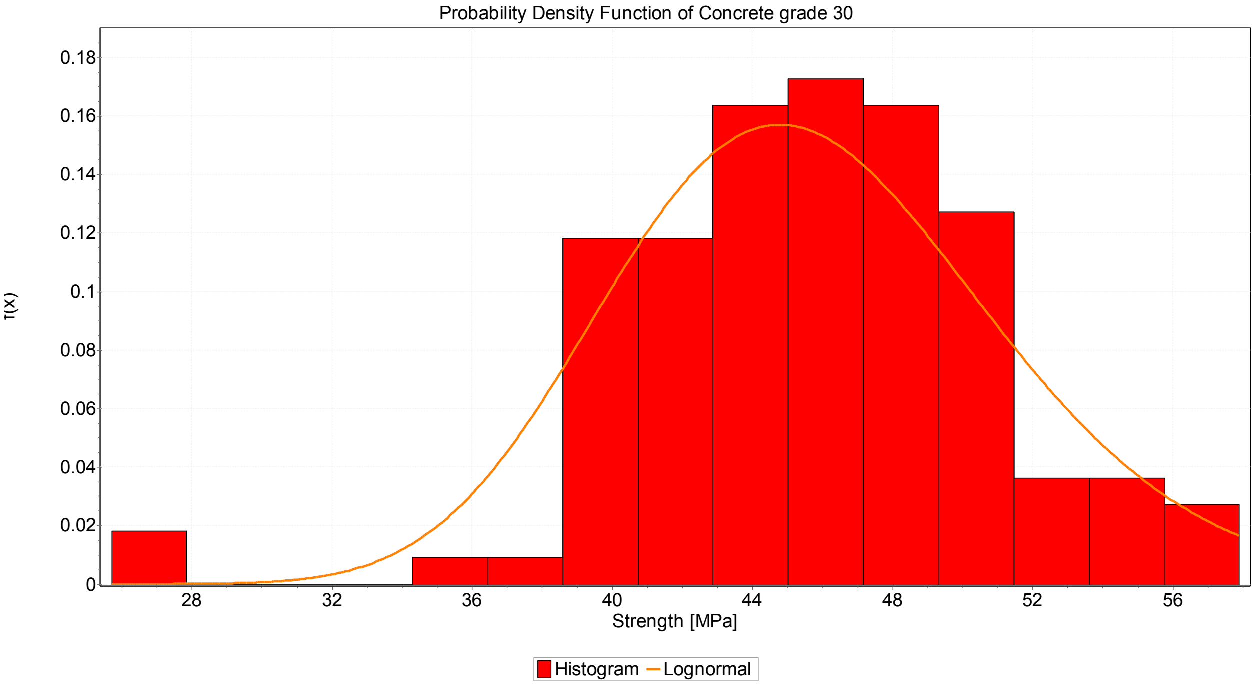


Figure 5.6: Fitted lognormal distribution with parameters σ = 0.1209 and μ = 3.8165

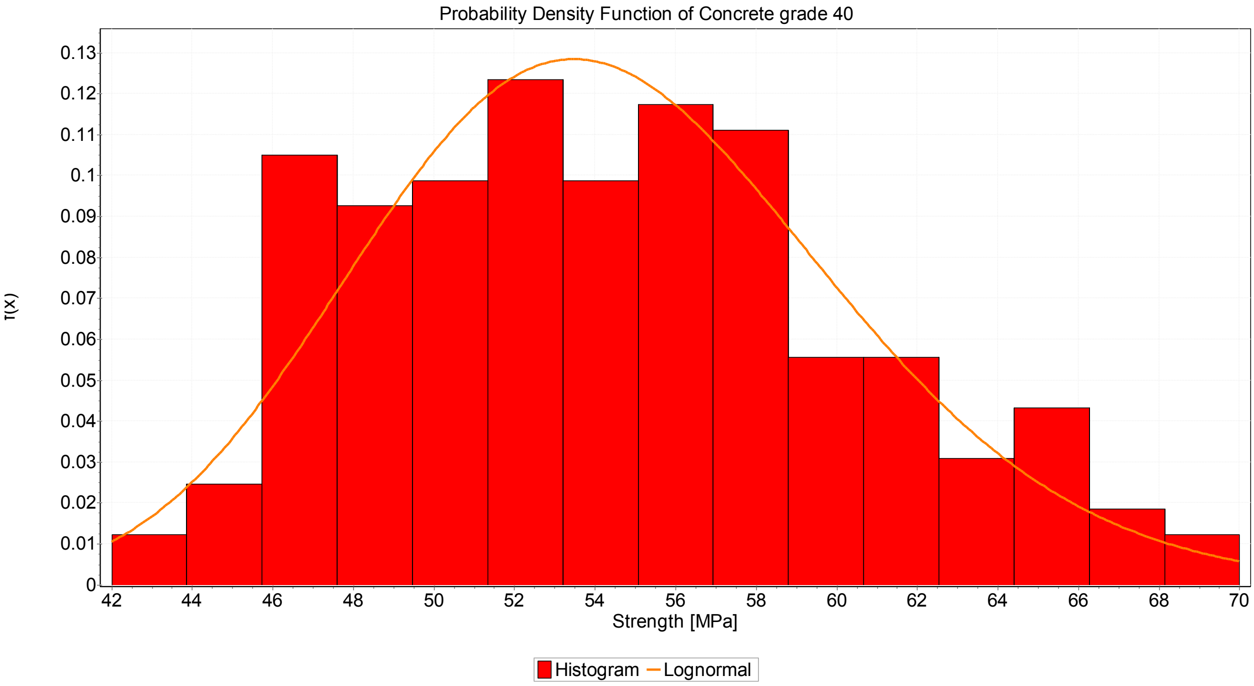


Figure 5.7: Fitted lognormal distribution with parameters σ = 0.1078 and μ = 3.9909

In Figure 5.8 and Figure 5.9, the coefficient of variation and partial factors for each test step and for each concrete grade are illustrated. Based on the random behavior, the concrete grade 20 has the greatest coefficient of variation. It has also the highest deviation in the range of tests numbering lower than 10. It seems that the grade 20, which has the most significant coefficient of variation also has also the greatest deviation in the first set of tests. Consequently, the partial factor value for this type is the highest. As in the previous section, the curves show the same behavior after step 10 tests here as well. They reach approximately a constant value, which is the adequate representation of material properties.

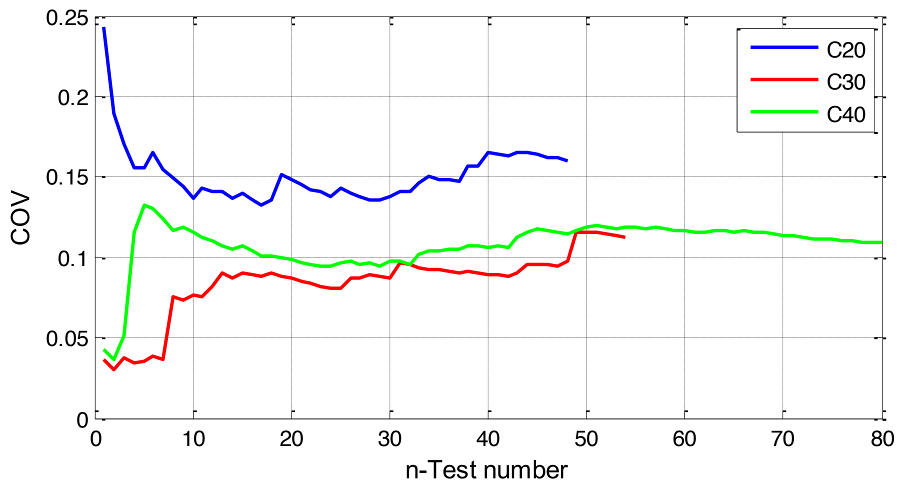


Figure 5.8: Coefficient of variation for different concrete grades

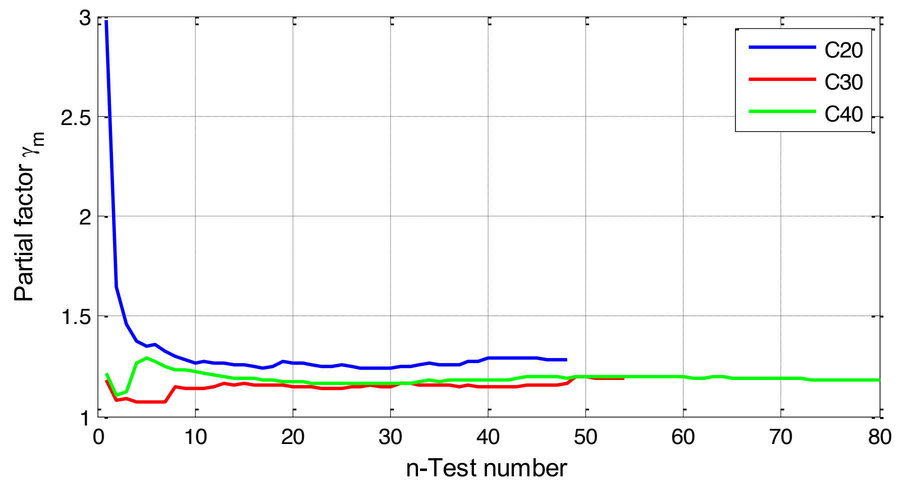


Figure 5.9: Partial factor corresponds to each step for different concrete grades

5.2.5 Design of concrete beams and columns

The design of concrete components will be done based on the partial factors, determined based on the methods explained in the previous section and the characteristic value of the concrete grade. The geometry and loading are considered as expressed in Table 5.1. The partial reinforcement factor is considered based on EN-1992 as γ_s = 1.15 ([61], [86]).

Table 5.1: Beam design

Parameter	Unit	Value
Design bending moment	kN.m	170
Width	mm	450
Effective depth of the tension reinforcement	mm	500
f_{yk}	MPa	500

The design of the required reinforcement area for the concrete beam is done in each step based on the partial factors for each test number in Figure 5.9. The same comparison as in Figure 5.4 has been made for the required steel area with the design according to 10 test numbers, as a benchmark for comparison. The deviations of all results from step 10 are shown in Figure 5.10.

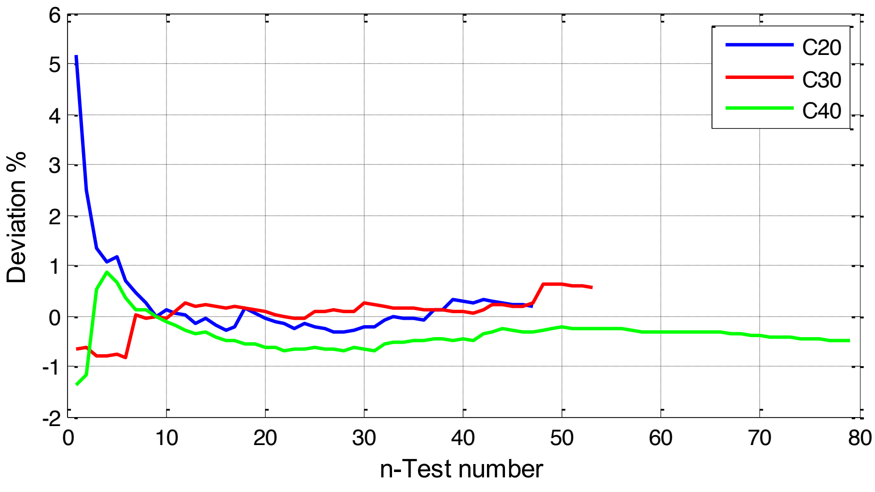


Figure 5.10: Deviation of designed A_s form the value of step n = 10

As observed after approximately 10 tests, the deviation is approximately zero, but in ranges lower than that, high deviations are observed. The differences in these curves are the consequence of random behavior. Testing is a random phenomenon, so it is reasonable that the behaviors of three different sets of tests also vary. The constant behavior of these curves after 10 test numbers supports the assumption that 10 is the optimum number of tests for finding the corresponding material partial factor.

The other structural component is a concrete column. In the determination of the concrete area in the column, the concrete partial factor has significant influence. It is assumed that the proportion of reinforcement area to the concrete area is 2%. The behaviors of curves in Figure 5.11 are the same as in Figure 5.10 (because both of them come from same test results), and consequently they have the same partial factor. In the case of a concrete column, as expected, the effect of the concrete strength partial factor is greater than that of beam design. The deviation in the highest value is nearly three times bigger than in beam design. The behavior here is also similar after 10 tests. They follow a rather constant form after 10.

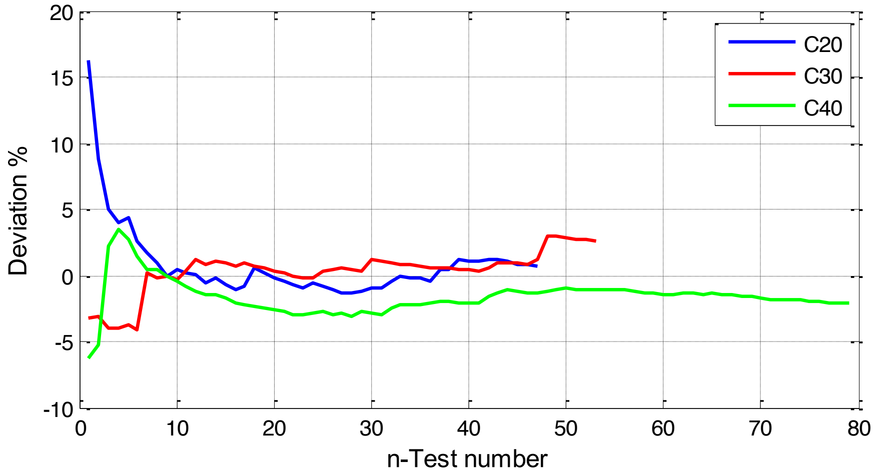


Figure 5.11: Deviation of designed A_c form the value of n = 10 step with 75% confidence level

5.2.6 Conclusion

In the case of a partial factor, as has been shown from a certain number of tests, increasing test numbers will not affect the value of the partial factor much. Ten has been determined to be the optimal number of attempts for tests. After 10, increasing the number of tests numbers no longer affects the coefficient of variation, partial factor and the design significantly. It can be concluded that running more than 10 test samples is not effective, and it would result in wasting time and resources.

The other source of uncertainty that must be considered is model uncertainty. The next section discusses the evaluation of model uncertainty and its partial factor based on the test.

5.3 Determination of model and resistance partial factor with Annex D of EN-1990

5.3.1 General

Experimental studies and test evaluations are fundamental in structural analysis and structural component designs. Precise interpretation of a test result will lead to an acceptable level of prediction of structural behavior. Currently, most design codes such as the Eurocodes are based on probabilistic methods and reliability analysis. An experimental database is one of the essential components of probabilistic methods. With probabilistic methods, engineers deal with a set of representative values instead of a single value for each property of a structural component. These sets of values come from the probability distribution functions for each basic variable, predicted by evaluating the test data. In addition to material parameters and geometry, the modeling of structural behavior significantly influences design safety. A recommended method for evaluating the test results with regard to the structural behavior model is provided in Annex D of EN-1990. This method is used to determine the partial factor for model uncertainty (γ_{Rd}) and the partial resistance factor (γ_M).

5.3.2 Recommendation in Annex D of EN-1990

The statistical method is recommended in Annex D of EN-1990 for determination of the resistance model based on the test results. The main idea of this method is based on comparison of the experimental data with the prediction of the resistance model. The calculation is undertaken in several steps. The first step is to consider a theoretical model for the structure represented by basic variables ($\underline{X}$).

$$r_t = g_{rt}(\underline{X}) \tag{5.10}$$

The prediction of the resistance model is then determined based on the selected theoretical model. The comparison of the theoretical values and experimental values is shown in Figure 5.12.

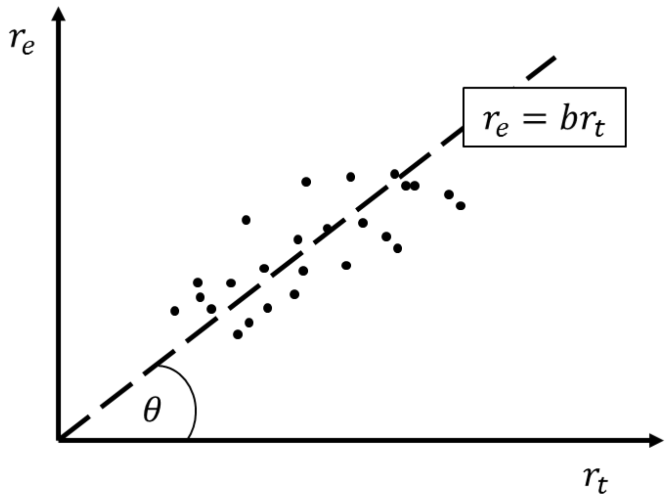


Figure 5.12: Experimental (r_e) and theoretical (r_t) diagram [16]

The ideal model will predict the resistance of the structure so that all the points lie on a line with θ = 45°. The scatter of the points from this line shows the error or deviation from the theoretical value.

For a statistical determination, a probabilistic model has to be defined according to the resistance model. The probabilistic model to be applied to the test data according to EN-1990 is given in (5.11):

$$r = br_t\delta, \tag{5.11}$$

where b is the "least square" and best-fit to the slope. It represents the model bias corresponding to the parameter θ, and is calculated with (5.12).

$$b = \frac{\sum r_e r_t}{\sum {r_t}^2} \tag{5.12}$$

The error term δ is defined for each experimental observation, and its ratio to the theoretical prediction is given by (5.13).

$$\delta = \frac{r_{ei}}{br_{ti}} \tag{5.13}$$

Based on the recommendations in Annex D of EN-1990, it is obvious that the code considers a lognormal distribution for the error term δ. The following parameter is defined by the logarithm of δ values. The coefficient of variation corresponding to the error parameter is calculated in accordance with this transformation, which is undertaken with (5.14).

$$\Delta_i = \ln(\delta_i) \tag{5.14}$$

Consequently the mean and standard deviation for parameter Δ are obtained from (5.15) and (5.16).

$$\overline{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\Delta_i \tag{5.15}$$

$$s_\Delta^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\Delta_i - \overline{\Delta}\right)^2 \tag{5.16}$$

The coefficient of variation for error in the model is then obtained from (5.17).

$$V_\delta = \sqrt{exp(s_\Delta^2) - 1} \tag{5.17}$$

The final aim of Annex D to EN-1990 is to determine the characteristic value or the design value of the resistance parameter. By taking advantage of calculated design and characteristic values, the partial factors can be determined. This is the main concept for this study, based primarily on the recommendations of Annex D to EN-1990. The process for calculating the characteristic value or design value in this annex is performed by considering uncertainty contributions both from the basic variables and from model uncertainty [87]. The contributions of these uncertainties are implemented in the calculation process by means of the coefficient of variation for the basic variables V_{X_i} and the coefficient of variation for model error V_δ.

The calculation of the coefficient of variation for the resistance model V_r, which is shown in the product function form (5.18), will be obtained from (5.19).

$$r = br_t\delta = b\{X_1 \times X_2 \dots X_j\}\delta \tag{5.18}$$

$$V_r = \left({V_\delta}^2 + 1\right)\left[\prod_{i=1}^{j}\left({V_{Xi}}^2 + 1\right)\right] - 1 \tag{5.19}$$

There is an alternative expression [eq. (5.20)] in Annex D of EN-1990 for calculation of (V_r) in case of small values for V_δ^2 and V_{Xi}^2.

$${V_r}^2 = {V_\delta}^2 + {V_{rt}}^2 \tag{5.20}$$

The parameter V_{rt}^2 is calculated with (5.21) with simple production form for the resistance models, and the in case of more complex models, (5.22) is used.

$$V_{rt}^2 = \sum_{i=1}^{j} V_{Xi}^2 \tag{5.21}$$

$$V_{rt}^2 = \frac{VAR\left[g_{rt}\left(\underline{X}\right)\right]}{g_{rt}^2\left(\underline{X}_m\right)} \cong \frac{1}{g_{rt}^2\left(\underline{X}_m\right)} \times \sum_{i=1}^{j} \left(\frac{\partial g_{rt}}{\partial Xi} \sigma_i\right)^2 \tag{5.22}$$

The characteristic value or design value has to be determined in accordance with the determined coefficient of variation. The calculation differentiates two cases, the former with limited numbers of tests (*n* < 100) and the latter with large numbers of tests (*n* ≥ 100).

In the former case, the statistical uncertainty in parameter Δ is considered by assuming the *t*-distribution for this parameter with n as the number of tests. In the latter case, the characteristic value is obtained with (5.23):

$$r_k = b g_{rt}\left(\underline{X}_m\right) exp(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\delta Q_\delta - 0.5\, Q^2), \tag{5.23}$$

with

$$Q_{rt} = \sigma_{ln(rt)} = \sqrt{ln\left(V_{rt}^2 + 1\right)}, \tag{5.24}$$

$$Q_\delta = \sigma_{ln(\delta)} = \sqrt{ln\left(V_\delta^2 + 1\right)}, \tag{5.25}$$

$$Q = \sigma_{ln(r)} = \sqrt{ln\left(V_r^2 + 1\right)}, \tag{5.26}$$

$$\alpha_{rt} = \frac{Q_{rt}}{Q}, \text{ and} \tag{5.27}$$

$$\alpha_\delta = \frac{Q_\delta}{Q}, \tag{5.28}$$

where

k_n is the characteristic fractile from Table 5.2,

k_∞	is the value for k_n when $n \to \infty$ [$k_\infty = 1.64$], and
α_δ, α_{rt}	are weighting factors for Q_δ and Q_{rt} respectively.

In the case of large numbers of tests, the calculation is performed with (5.29).

$$r_k = b g_{rt}(\underline{X}_m) exp(-k_\infty Q - 0.5\, Q^2) \tag{5.29}$$

Table 5.2: k_n for 5% fractile value

n	**1**	**2**	**3**	**4**	**5**	**6**	**8**	**10**	**20**	**30**	∞
V_X Known	2.31	2.01	1.89	1.83	1.80	1.77	1.74	1.72	1.68	1.67	1.64
V_X Unknown	-	-	3.37	2.63	2.33	2.18	2.00	1.76	1.76	1.73	1.64

The determination of design values is similar to that for characteristic values, but the values of k_n and k_∞ in (5.28) and (5.29) are replaced by $k_{d,n,}$ and $k_{d,\infty}$. These values are shown in Table 5.3.

Table 5.3: $k_{d,n}$ for ultimate limit state design value

n	**1**	**2**	**3**	**4**	**5**	**6**	**8**	**10**	**20**	**30**	∞
V_X Known	4.36	3.77	3.56	3.44	3.37	3.33	3.27	3.23	3.16	3.13	3.04
V_X Unknown	-	-	-	11.4	7.85	6.36	5.07	4.51	3.64	3.44	3.04

5.3.3 Unreinforced shear wall database

5.3.3.1 Test database

The main structural elements in masonry construction are walls. Currently, unreinforced walls are more commonly used than are reinforced walls. The main requirement for this type of structure is to resist normal forces, but there are also some cases in which the verification of the wall under lateral loading is necessary. There are different references to the analysis of shear wall behavior based on probabilistic approaches (see [26], [88], [33], [89]), but the evaluation of test data based on the recommendations of Annex D for EN-1990 has not yet been considered. An adequate assessment has been carried out in [90] for the determination of the probability distribution function of uncertainty in masonry shear wall models. The determination of a compatible model for wall behavior requires experimental data. The databases can be calibrated by comparing the model prediction with the test result. The test data is then used in order to determine the partial factor for the model or the uncertainty from the calculation with the model.

In order to determine the model factor, experimental data from the European research program Enhanced Safety and Efficient Construction of Masonry Structures in Europe (ESECMaSE) [91] for masonry structures is used. The ESECMaSE was a vast experimental and fundamental program carried out in 2004 – 2008, as a collaboration of various European partners. The project was mainly concerned with the shear resistance and deformation of masonry walls built of different types of units and mortar [89]. The collected database is well presented in detail in [92] and [89]. The database consists of 129 tests including three different masonry units, 44 tests on clay brick (CB), 51 tests on autoclaved aerated concrete (AAC), and 34 tests on calcium silicate (CS). The test results are evaluated based on a comparison with values predicted by the German National Annex DIN EN 1996-1-1/NA [93].

5.3.3.2 Shear load capacity of URM wall based on DIN EN-1996-1-1/ NA

The theoretical model used in this study for comparison with the test data is the recommended method in DIN EN 1996-1-1/NA. As mentioned in [89] and [94], there are various types of failure modes for masonry shear walls:

- friction failure of the bed joint,
- tensile failure (cracking) of the units,
- overturning of single unit,
- flexural (bending) failure of masonry,
- shear compression failure of masonry, and
- compression failure of masonry (crashing).

The national recommendations in Germany for the verification of masonry walls are given in DIN EN 1996-1-1/NA. Aside from the method in the main context of DIN EN 1996-1-1/NA, a specific method is also proposed in DIN EN 1996-1-1/NA Annex K for the evaluation of wall slenderness. The shear resistance for each test sample in the database is determined based on the various failure modes according to DIN EN 1996-1-1/NA and its Annex K. The comparison between theoretical values predicted by the code and observed values from real test data leads to the evaluation of uncertainty from the model in the design process.

5.3.3.2.1 Shear wall verification in DIN EN 1996-1-1/NA

The DIN EN-1996-1-1/ NA recommendation for shear resistance determination in the case of friction and tensile failure of the units can be seen in (5.30):

$$V_{Rdlt} = l_{cal} \cdot f_{vd} \cdot \frac{t}{c}, \tag{5.30}$$

where

t is the thickness of the wall;

c is the shear stress distribution factor and determined as

$1.0 \quad for \quad \frac{h}{l} \leq 1,$

$0.5(1 + \frac{h}{l}) \quad for \quad 1 < \frac{h}{l} < 2,$

$1.5 \quad for \quad \frac{h}{l} \geq 2;$

l_{cal} is the calculated length of the wall, as follows:

$$l_{cal} = min\left(1.125 \cdot l \, , 1.333 \cdot \; l_{c,lin}\right) \tag{5.31}$$

$$l_{c,lin} = \frac{3}{2}\left(1 - 2 \cdot \frac{M_{Ed}}{N_{Ed} \cdot l}\right) \cdot l \leq l; \tag{5.32}$$

M_{Ed} is the design moment;

N_{Ed} is the design normal force;

l is the wall length;

f_{vd} is the design value of shear strength with $f_{vd} = \frac{f_{vk}}{\gamma_M}$;

γ_M is the partial factor of masonry.

The characteristic values of shear strength f_{vk} shall be determined for friction and tensile failure in order to be applied in (5.30). The friction characteristic strength for in-plane shear resistance, in case head joints are filled with mortar, may be considered as expressed in (5.33), and for tensile failure, (5.34) will be implemented:

$$f_{vk1} = f_{vk1} + 0.4 \cdot \sigma_{Dd} \tag{5.33}$$

$$f_{vk2} = 0.4 \cdot f_{bt,cal} \cdot \sqrt{1 + \frac{\sigma_{Dd}}{f_{bt,cal}}}, \tag{5.34}$$

where

f_{vk0} is the characteristic initial shear strength of masonry,

σ_{Dd} is normal stress, and

$f_{bt,cal}$ is the computational tensile strength of unit. It may be assumed as a ratio of unit compressive strength.

The other failure mode is shear compression failure, which occurs when the compressive strength in the diagonal strut is exceeded [89]. In the case of element masonry with thin-layer mortar for bed joints and a ratio of overlapping length over unit height of less than 0.4 ($^{l_{ol}}/_{h_u} \leq 0.4$), equation (5.35) has to be considered in checking shear compression failure:

$$V_{Rdlt} = \frac{1}{\gamma_M \cdot c} \cdot (f_{vd} \cdot l_c \cdot t - \gamma_M \cdot N_{Ed}) \cdot \frac{l_{ol}}{h_u} \tag{5.35}$$

where

l_c $\quad l_c = \left(1 - 2 \cdot \frac{M_{Ed}}{N_{Ed} \cdot l}\right) \cdot l,$

l_{ol} is overlapping length,

h_u is the unit height, and

f_k is the characteristic value of masonry compressive strength.

In masonry structures with element masonry, non-grouted head joints and a ratio of $h_u > l_u$, failure on single unit due to the opening of the bed joint constitutes another possible failure scenario. The calculation of shear resistance for the overturning of single units will be done according to (5.36) [89].

$$V_{Rdlt} = \frac{2}{3} \cdot \frac{1}{\gamma_M} \cdot \left(\frac{l_u}{h_u} - \frac{l_u}{h_u}\right) \cdot \frac{l_{ol}}{h_u} \tag{5.36}$$

The criteria of the flexural failure of the walls subjected to the vertical and horizontal loads may be determined simultaneously based on the ultimate limit state of the wall in axial forces (5.37).

$$N_{Ed} \leq N_{Rd} = \varphi \cdot \frac{f_k}{\gamma_M} \cdot l_w \cdot t \tag{5.37}$$

$$\varphi = 1 - 2 \cdot \frac{M_{Ed}}{N_{Ed} \cdot l} \tag{5.38}$$

Here, the Φ is the reduction factor for considering the slenderness and eccentricity of loadings on the wall, and it will be determined based on the assumption of rectangular stress blocks with (5.38). According to both (5.37) and (5.38), the shear resistance based on flexural failure mode may be calculated with (5.39).

$$V_{FLd} = \frac{l \cdot N_{Ed}}{2 \cdot h} - \frac{\gamma_M \cdot {N_{Ed}}^2}{2 \cdot h \cdot f_k \cdot t} \tag{5.39}$$

5.3.3.2.2 Annex K- DIN EN-1996-1-1/NA

The effective height of the wall is calculated based on a factor ψ, which is introduced for different types of boundary conditions. The background to this factor may be found in [95]. The factor considers, particularly, the restraint ratios at the top and bottom of the wall.

The parameter ψ is applied to the height of the wall in the process of slenderness calculation, as shown in (5.3). A general classification of the ψ factor may be done according to the restraint condition of the wall, the case with the fully restrained boundary condition at top and bottom of the wall with $\psi = 0.5$, and the case of a cantilever wall or no restraint at top with $\psi = 1$. A representation of the eccentricity and the wall is illustrated in Figure 5.13.

$$\lambda_v = \frac{\psi \cdot h_w}{l_w} \tag{5.40}$$

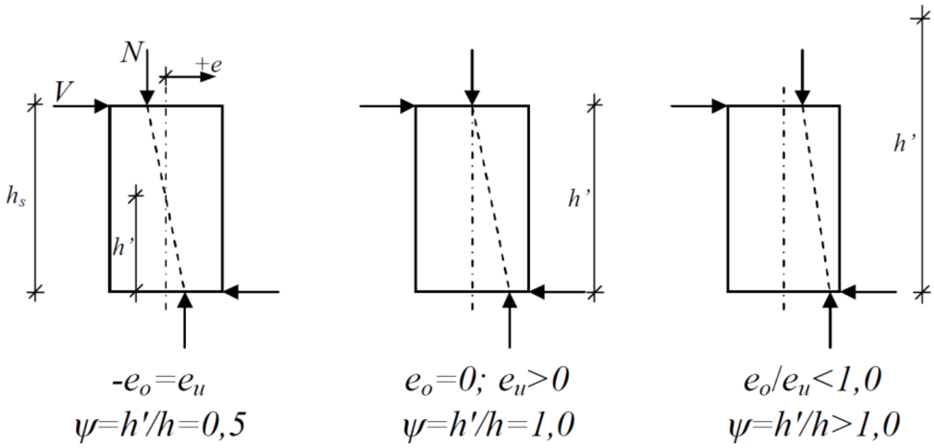


Figure 5.13: Wall eccentricity at top and bottom and ψ factor [95], [93]

In verification with Annex K instead of $\frac{M_{Ed}}{N_{Ed} \cdot l}$ in all formulas, $\frac{V_{Ed}}{N_{Ed}} \cdot \lambda_v$ will be replaced. The other difference lies in the calculation of friction and diagonal tension. For this verification, based on (5.30), the compressive length of the wall will be calculated according to (5.32).

5.3.4 Model partial factor γ_{Rd}

5.3.4.1 The whole population of database

In the first step of the test evaluation, the entire database is considered a general representation of masonry shear wall behavior under horizontal and vertical loading. The diagram in Figure 5.14 compares predicted values and experimental values for all test data.

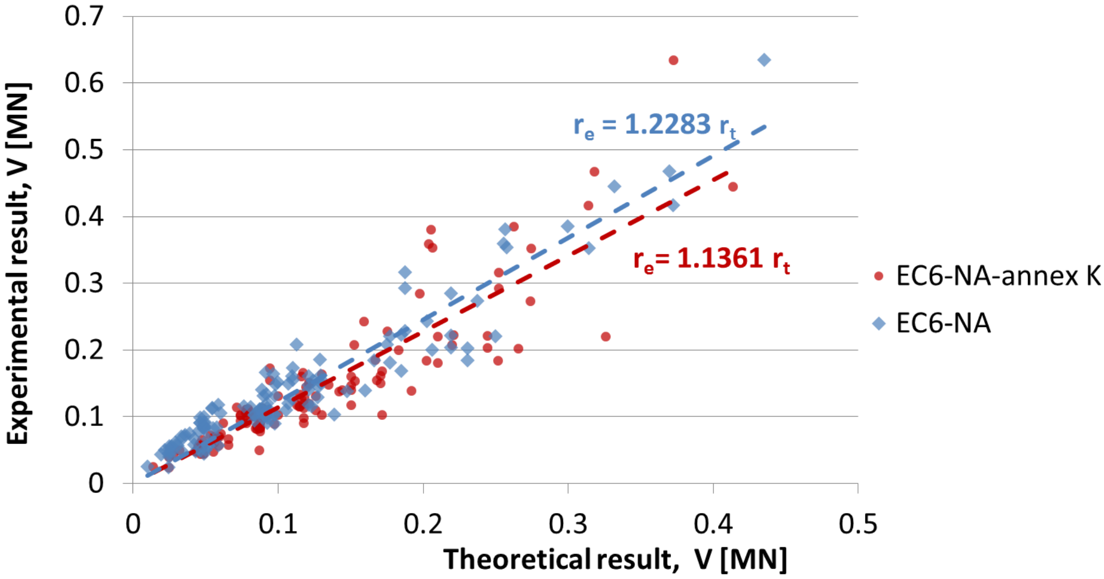


Figure 5.14: Experimental and theoretical values for masonry wall

It seems in Figure 5.14 that the lines fitted to the database express the overall comparison between experiment and theory. The factors 1.1361 and 1.2283 were calculated using (5.12) and represent the inclination of the line. These values indicate a conservative prediction strategy in the model for the calculation of shear wall resistance. This is also a bias in the model; in other words, most of the experimental data have more capacity than that predicted by the theoretical model. This feature may be interpreted as our model underestimating the resistance of the wall and the real resistance of the wall being always more than the expected value. The other parameter that must be considered as contributing to model uncertainty is the spread of the predictions from the fitted line. The representative value for this parameter is the coefficient of variation of model error V_δ, obtained from (5.13)–(5.17). The determined results for model error statistical parameters for the database are presented in Table 5.4.

Table 5.4: Statistical parameter of model error for database

	$\overline{\Delta}$	s_Δ^2	V_δ	b
EC6-NA Annex K	−0.0162	0.2257	0.2286	1.1361
EC6-NA	0.0932	0.2664	0.2712	1.2283

As explained in previous sections, the aim of first step is to determine the partial factor corresponding to the model error. Therefore, in accounting for uncertainties, only the contribution of the model error is considered. Consequently the term V_{rt} in (5.20) is ignored because it relates to the material and other basic variables' uncertainty, and the term V_δ, coefficient of variation for model error, is the only parameter in the calculation.

Finally, considering the calculated statistical parameters in Table 5.4 and (5.29), the design value and characteristic value of resistance are determined. The population of experiments in the database are more than 100, and this can be considered as a large amount of test data, so (5.29) is used for calculation. Then, by considering the bias of the model in the calculation process, the partial factor for the resistance model is obtained from (5.41).

$$\gamma_{Rd} = \frac{R_k}{R_d} \cdot \frac{1}{b} \tag{5.41}$$

In the first case, for the whole population of the experimental database, the partial factors of model will be as in Table 5.5.

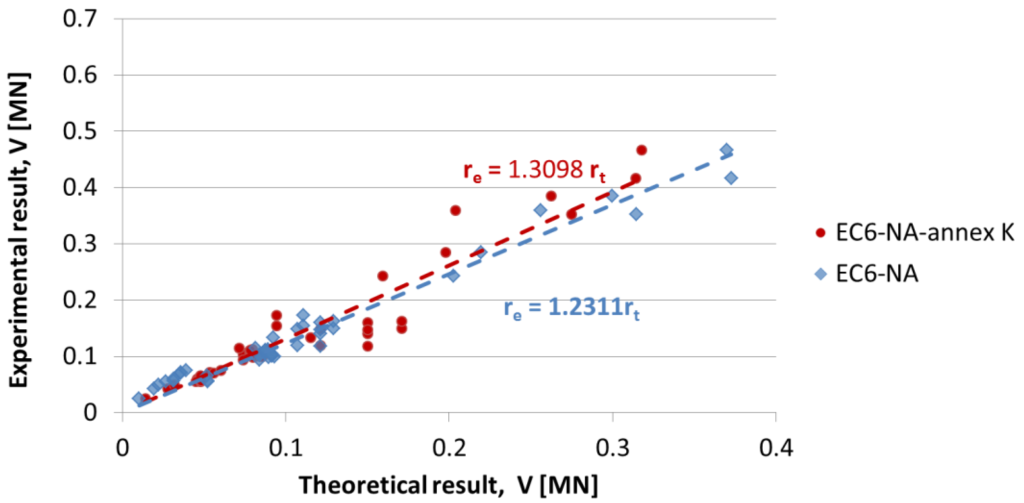
Table 5.5: Model partial factor for data base

	γ_{Rd}
EC6-NA Annex K	1.207
EC6-NA	1.182

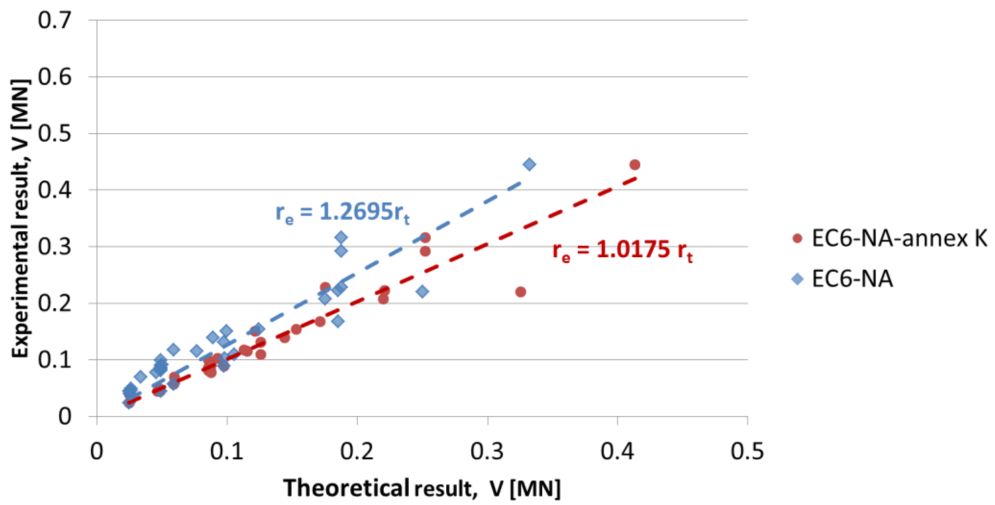
5.3.4.2 Individual masonry unit type

In order to determine more compatible values of partial factors based on the masonry unit types, the database is classified into subsets based on the type of units. As mentioned, tests were conducted on three types of units: CB, CS and AAC. The same procedure of statistical evaluation is implemented for each subset of masonry unit.

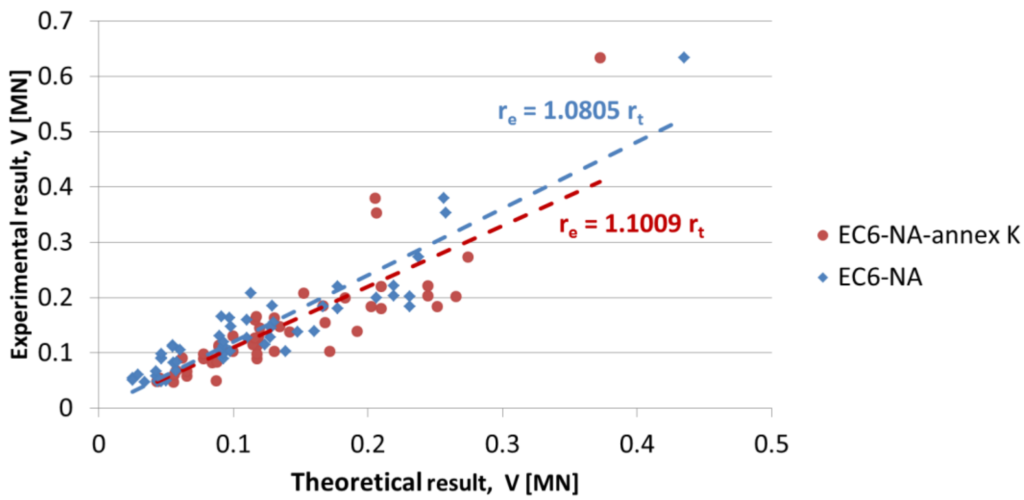
Dividing the original database into several subsets will reduce the amount of data or the population in the statistical evaluation. A recommendation appears in Annex D in EN-1990 regarding this problem. It is suggesting that for the determination of factor k_n from Table 5.2 or Table 5.3, the number of tests has to be considered the original database. Therefore, the same value of k (maximum) is taken for each subset of masonry unit, because the original test database comprises a large number of test datasets.



a) Clay brick (CB)



b) Calcium silicate (CS)



c) Autoclaved aerated concrete (AAC)

Figure 5.15: Experimental and theoretical values for each unit type

The analysis of each unit type in the database is shown in Figure 5.15. The analysis produced the parameters for the calculation of the model's partial factor in Table 5.6 and Table 5.7. The final values of the partial factors for each of these unit types differ. A difference also appears between the partial factor for the whole database and the partial factor for each unit type. The difference is explained by the various coefficients of variation for model error and model bias.

Table 5.6: Statistical parameters and model partial factor for each unit type in EC-NA Annex K

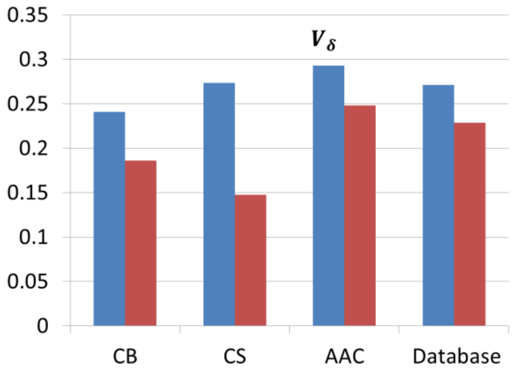
Unit type	$\overline{\Delta}$	s_{Δ}^2	V_{δ}	b	γ_{Rd}
CB	−0.016431	0.1844	0.186	1.3098	0.99
CS	0.004090	0.1468	0.1476	1.0175	1.21
AAC	−0.04769	0.2444	0.2481	1.1009	1.28

Table 5.7: Statistical parameters and model partial factor for each unit type in EC-NA

Unit type	$\overline{\Delta}$	s_{Δ}^2	V_{δ}	b	γ_{Rd}
CB	0.0955	0.2375	0.2409	1.2311	1.1327
CS	0.1110	0.2685	0.2734	1.2695	1.1471
AAC	0.0735	0.2870	0.2930	1.2062	1.2391

Figure 3.7 shows the calculated values for coefficient of variation for model error and model bias. Both of these values for AAC are near to those for the whole database. The maximum values for COV are for AAC.

The bias model for CS in EC6-NA Annex K is nearly 1, which means that on average, the resistance model of the shear wall is neither overestimated nor underestimated for CS units. For the other unit types and also for the whole database, the bias is higher and greater than 1, which means that the model underestimates the material. The scatter of the data for EC6-NA is in all cases wider than that of EC6-NA Annex K.



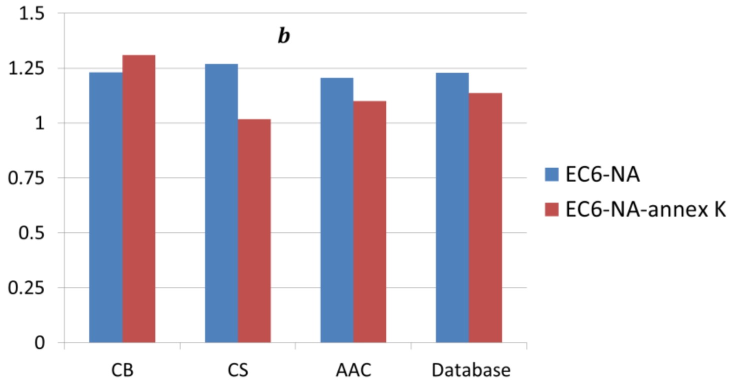


Figure 5.16: Coefficient of variation for model error and model bias for units and database

By using the calculated stochastic parameters with (5.41), the model's partial factors are calculated for all types of units and for the whole database. The results are shown in Figure 5.16 for all unit types for both cases of EC6-NA and EC6-NA Annex K.

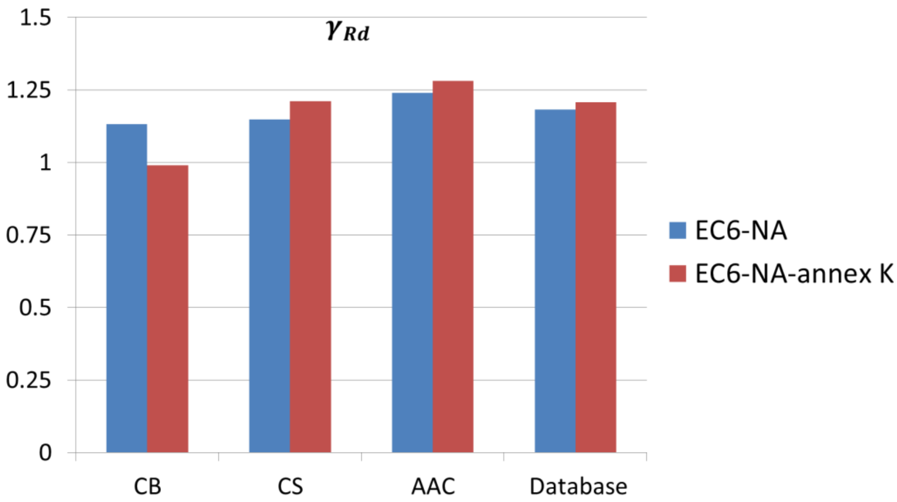


Figure 5.17: Model partial factor

5.3.5 Resistance partial factor γ_M

To determine the partial resistance factor, other basic variables must be considered for the calculation of the coefficient of variation for resistance in (5.20) and (5.21). The other coefficient of variations for basic variables for the masonry shear wall, aside from model uncertainty, can be described as follows:

- geometry,
- material properties, and
- loads.

Each of these variables has a spread, and therefore they contribute in the final COV of resistance based on (5.20) and (5.21).

Geometrical data are commonly categorized as basic variables with low variability. According to the recommendations of the JCSS probabilistic model code [20], the COV of geometrical variables is suggested to be equal to 4%.

Normal actions are involved in the recommendations for shear wall calculations in DIN EN-1996-1-1/NA. Thus, the parameter variations should contribute to the resistance COV. The sources of normal actions are considered self-weight and variable actions. For self-weight and variable actions, 5% and 20% COV are applied respectively.

Material properties play an essential role in the whole process of design. According to the test database, different types of failure modes are observed during the experiments.

Table 5.8: Observed failure modes in experiments

Failure mode	Number, $N_{f,i}$
Friction failure	3
Tensile failure	96
Tensile and friction	11
Flexural (bending) failure	3
Shear compression failure	3
Overturning of single unit	2
Unknown	11

In order to consider the contribution of the material's strength in the resistance COV, a weighting average of COV for all material strength based on different failure modes is applied. Using this kind of average, the influence of different types of failure is considered for the determination of partial factors. In calculating the average, unknown failures are not considered. The overturning of single units is not also considered in the average, because only geometrical parameters are involved in its limit state. The weighting factors include the number of failures, as shown in Table 5.8. The weighting average of material COV is calculated with (5.42).

$$COV_{m.avg} = \frac{\sum N_{f,i} \cdot COV_{m,i}}{\sum N_{f,i}} \tag{5.42}$$

Different material properties have to be considered for different failure modes. The correspondent COV values for each parameter are implemented in (5.42) for calculation of weighting average. The parameters and COV values are shown in Table 5.9.

Table 5.9: COV values of material parameters [90], [26]

Failure mode	Material parameter	COV_i
Friction failure	f_{v0}	0.35
Tensile failure	f_{bt}	0.2
Flexural (bending) failure	f_k	0.1
Shear compression failure	f_k	0.1

With the application of the COV values for material properties, the weighting average is determined to be 19.8%. This value with other basic variables—geometrical parameters and normal action—are applied in (5.20) and (5.21). Finally, the calculated resistance COV is approximately 28.9%. The same procedure as a modeled partial factor is performed to determine the resistance partial factor γ_M. The results of the model's partial factor and resistance partial factor for EC6-NA and EC6-NA Annex K are shown in Figure 5.18 and Table 5.10.

Table 5.10: Resistance partial factor for data base

	γ_M
EC6-NA Annex K	1.45
EC6-NA	1.39

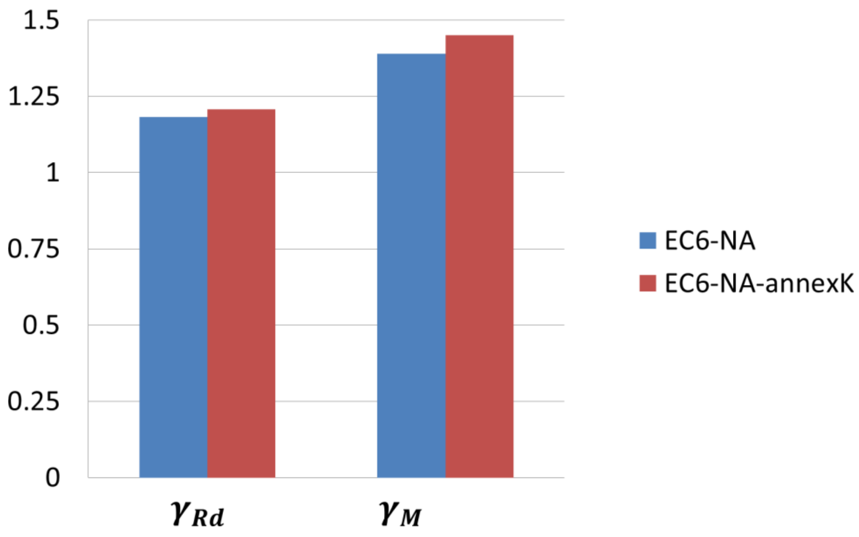


Figure 5.18: Model partial factor and resistance partial factor

5.3.6 Conclusion

For the definition of partial factors for the resistance side in structural design, the code divides the source of uncertainty into two categories: the uncertainty from material properties and uncertainty from structural behavior modeling. The determination of material uncertainty can also be undertaken by testing every single material. The recommendation in Annex D of EN-1990 has been applied in this section as a practical process for the determination of model uncertainty. These values have been determined according to the experimental database for the masonry shear wall and based on failure modes in German national Annex DIN EN-1996-1-1/ NA. Moreover, the resistance partial factor is also calculated by considering other basic variables such as material, geometry and action variations.

The results indicate that this method in Annex D of EN-1990 is a reliable one for the calculation of model partial factor and resistance partial factors. The advantage of this method is separate determination of model partial factor and resistance partial factors. The recommended value of the partial factor for the model may be applied in the shear wall calculation based on the material properties tests.

Eventually, based on the parameters involved in the calculation process, it can be concluded that two parameters influence the partial factors of the model, the scatter of the model error and the bias of the model. In the case of the last of these, the coefficient of variation is the effective parameter in the calculation of partial factors. The higher values of COV will lead to higher values for partial factors. For the bias model, values of more than 1 will be considered models that underestimate the resistance and decrease the value of a partial factor. A bias factor of less than 1 means that the model needs more safety, so the partial factor will be increased by considering the bias.

5.4 Application of partial safety factor for resistance (cases study flexural failure of masonry shear wall)

5.4.1 Design value of resistance

Limit state design consists of the determination of actions design values (E_d) and resistance (R_d) based on the load combinations and limit state functions (eq. (2.1)]). Detailed analyses appear in Chapter 4 regarding the design value of actions. In Chapter 5, however, the application of partial safety factor for material is investigated. The recommendations for implementation of partial safety factors for material and resistance are described in Section 6 of EN-1990. Equation (5.43) shows the application of the material partial safety factor based on recommendations in EN-1990:

$$R_d = R\left\{\eta_i \frac{X_{k,i}}{\gamma_{M,i}} \,; a_d\right\} \; i \geq 1, \tag{5.43}$$

where

$X_{k,i}$ is the characteristic value of the material or product property;

η_i is the mean value of the conversion factor, taking into account
- volume and scale effects,
- effects of moisture and temperature, and
- any other relevant parameters;

$\gamma_{M,i}$ is partial factor considering the uncertainty of the model (γ_{Rd}) and the partial factor of material properties(γ_m)—$\gamma_{M,i} = \gamma_{Rd} \cdot \gamma_m$

a_d design values of geometrical data.

Application of the resistance partial factor in limit state functions is an important aspect of recommendations in the EN-1990. Investigation of this application is going to be done according to a case study in this section. In this part, a flexural failure of masonry shear wall is considered as a case study to investigate the application of the partial resistance safety factor. The flexural failure limit state for this kind of wall is not directly mentioned in the context of EN-1996-1-1, and it has to be determined based on the limit state function for normal forces. Therefore, this will make the application of partial safety factors a critical case for this limit state. Moreover, this limit state is a nonlinear limit state function regarding the normal forces, and it brings extra complexity in its application. A short explanation was given in (5.37), but here the formula will be represented based on a normalized value of normal forces [96].

A masonry wall is selected for defining a nonlinear limit state. The flexural resistance of this masonry wall (Figure 5.19), subject to vertical and horizontal loading, will be calculated based on geometrical parameters and the eccentricity of loading.

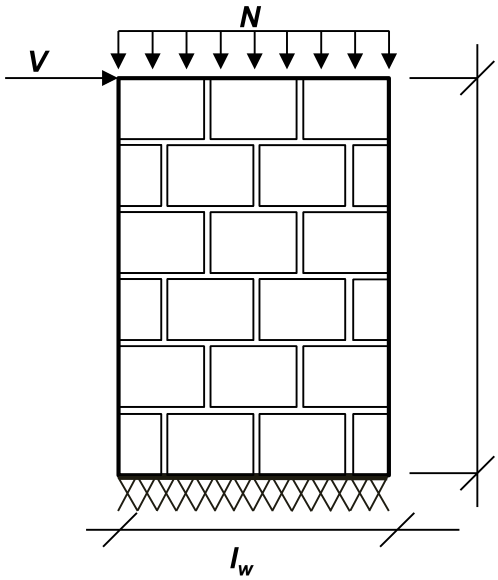


Figure 5.19: Geometry of the wall

According to the EN-1996-1-1 [64] in the ultimate limit state, the limit state function can be represented as (5.44). This equation describes the model for the wall behavior without considering the design situation.

$$N = \varphi \cdot f \cdot l_w \cdot t \tag{5.44}$$

Here, φ is determined based on the formula (5.38), which may be reformulated based on the eccentricity as (5.45).

$$\varphi = 1 - 2\frac{e}{l_w} \quad with \quad e = \frac{V \cdot h}{N} \tag{5.45}$$

The normal forces are represented with a normalized value according to (5.46)

$$n = \frac{N}{t \cdot l_w \cdot f} \tag{5.46}$$

Substituting (5.45) in (5.44) and using the normalized value of normal forces, the limit state function of flexural failure of unreinforced masonry against horizontal loading is illustrated as in (5.47).

$$V = \frac{t \cdot {l_w}^2 \cdot f}{2 \cdot h} \cdot (n - n^2) \tag{5.47}$$

The input data for an exemplary masonry wall is selected as mentioned in Table 5.11. These values are related to a wall with CS bricks. The model uncertainty for this kind of material is represented in [89] in a lognormal distribution with COV = 0.33 and mean = 1.171.

Table 5.11: Properties and parameter of the wall

Parameter	Unit	Value
wall length, l_w	mm	1250
wall height, h_w	mm	2500
strength, f_k	kN/mm^2	0.015
thickness, t	mm	175

The wall is subjected to normal force, which consists only of permanent actions. The lateral loading is considered as wind load. The stochastic parameters for conducting reliabilities are selected from Table 3.2.

5.4.2 Utilization of partial safety factor of material

Three different methods of partial factor utilization are defined in this section:

- nonlinear,
- linear-nonlinear, and
- linear.

These terms do not represent the nonlinearity in the limit state function or the material or other structural nonlinearities; rather, the terms are based on the influence of partial factors on design results.

5.4.2.1 Nonlinear

The term “nonlinear,” in the case of partial factor utilization, means that the influence of the partial factor does not change all values of resistance by an individual factor. In other words, it does not map all values by a simple factor. In this case, (5.44) in the design situation, which is the current format for calculation based on EN-1996, can be represented as follows:

$$N_d = \varphi \cdot \frac{f_k}{\gamma_M} \cdot l_w \cdot t. \tag{5.48}$$

Therefore, by using (5.44), the design value of shear resistance in flexural failure criteria is

$$V_{FLd} = \frac{t \cdot l_w{}^2 \cdot f_k}{2 \cdot h} \cdot (n - \gamma_M \cdot n^2). \tag{5.49}$$

Thus the deterministic function of flexural failure in order to implement the probabilistic analysis is represented in (5.50), which is equal to (5.47).

$$V = \frac{t \cdot l_w{}^2 \cdot f}{2 \cdot h} \cdot (n - n^2) \tag{5.50}$$

According to the above equations, the random points and design values of actions and resistance can be represented as in Figure 5.20, which illustrates that the real behavior of the wall, presented by green points, is entirely different from the design behavior. It seems that the diagram in case of design resistance shifts in both directions by application of partial factors. This behavior creates a substantial gap between the real behavior of the wall and the estimated behavior according to the design conditions.

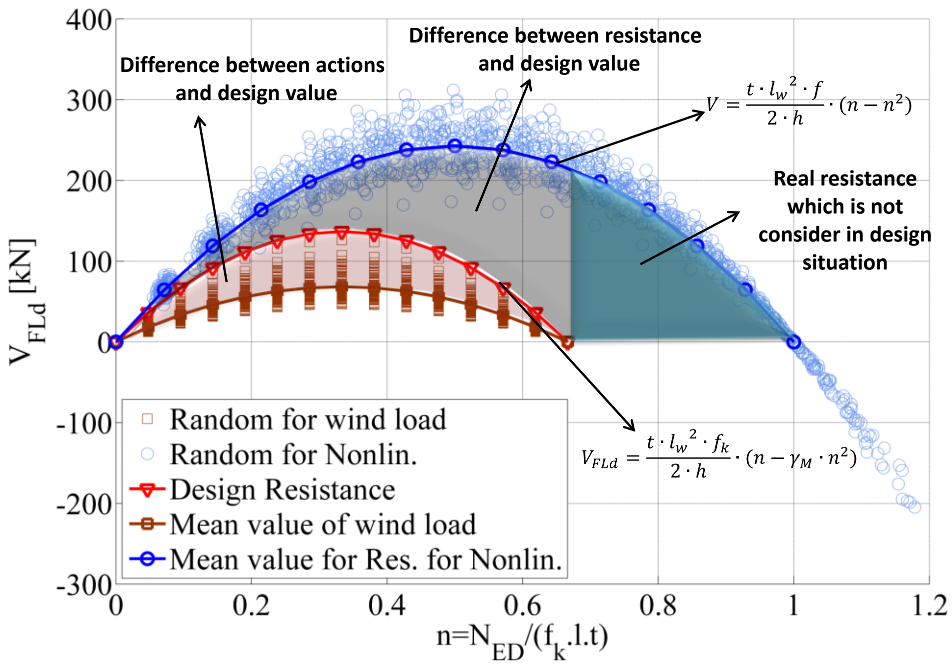


Figure 5.20: Representation of design value and the random points of action and resistance for nonlinear method

The transition of design value into resistance and action with probabilistic form for flexural failure in the so-called nonlinear method is shown in Figure 5.20. As illustrated, the two areas between design value, resistance and actions are not proportional to each other. In the next two methods for utilization of partial factors, this problem will be solved to get proportional behavior between design value, action and resistance. A part of the real resistance of the wall according to the probabilistic model of flexural behavior is also not considered in the design model, and it corresponds to the values of n greater than $n = 0.666$. The calculation and analysis are therefore done for only part of the whole interval, since in values of n greater than 0.666, the shear resistance in the case of flexural failure is negative, which is unacceptable.

5.4.2.2 Linear-nonlinear

In this case, the partial factor implementation in design situation is similar to the nonlinear method based on the current version of EN-1996. Thus, the design value of shear resistance in the case of flexural failure for this case would be the same as (5.49). The difference will arise in the deterministic function of resistance in the probabilistic form. To ensure the same behavior between resistance and design value in this method, the design value relation will be used for the deterministic solution, and it will be mapped by a factor equal to the material partial factor to get the final value for the probabilistic model of resistance, (5.51).

$$V = \gamma_M \cdot \left[\frac{t \cdot {l_w}^2 \cdot f}{2 \cdot h} \cdot (n - \gamma_M \cdot n^2) \right] \tag{5.51}$$

According to Figure 5.21, the design value behavior in this case is similar to the behavior of action and resistance. It seems that the transition of design value to the probabilistic form is done by mapping the design value with a factor. The difference between the real behavior (probabilistic model) of resistance and the design model that has been observed in the nonlinear case is avoided in the linear-nonlinear method by considering a linearization assumption, as explained in (5.51). Therefore, it can be mentioned that in the first step, a partial factor will create a nonlinear effect for the design value, but in the second part of the probabilistic model, given a linearization assumption [eq. (5.51)], the resistance behavior, action and design model will be compatible.

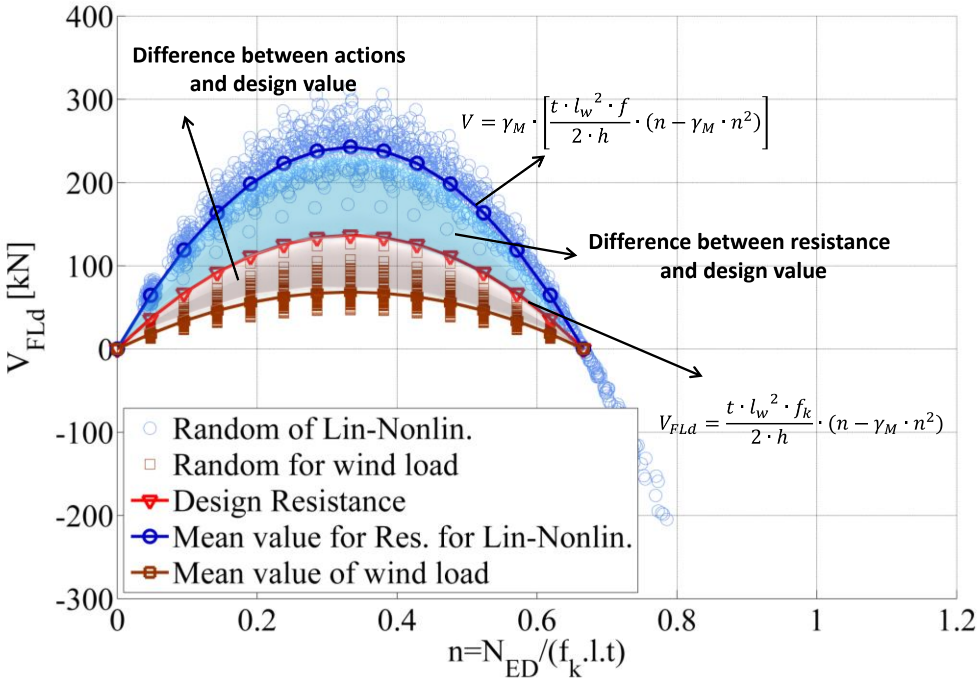


Figure 5.21: Representation of design value and the random points of action and resistance for linear-nonlinear method

5.4.2.3 Linear

In this case, the design value will be calculated by a linear mapping of the characteristic resistance by the partial factor of material [see (5.52)]. Therefore, using a partial factor of material does not affect the value of normal force in the design situation. As such, the model of resistance for flexural failure has been evaluated based on (5.52). After finding the resistance model, the safety parameters will be implemented by applying the partial factor for the resistance part. In other words, the transition from (5.44) to (5.47) has been done without considering safety parameters. The safety parameters are applied to the characteristic value of the resistance model.

$$R_d = \frac{R_k}{\gamma_M} \qquad R_k = \frac{t \cdot l_w{}^2 \cdot f_k}{2 \cdot h} \cdot (n - n^2) \tag{5.52}$$

Therefore, the design value of the shear resistance in the case of flexural failure with a linear effect of partial factor utilization would be as shown in (5.53).

$$V_{FLd} = \frac{t \cdot l_w{}^2 \cdot f_k}{\gamma_M \cdot 2 \cdot h} \cdot (n - n^2) \tag{5.53}$$

For a probabilistic model, the resistance will be modeled like the nonlinear method, based on (5.50).

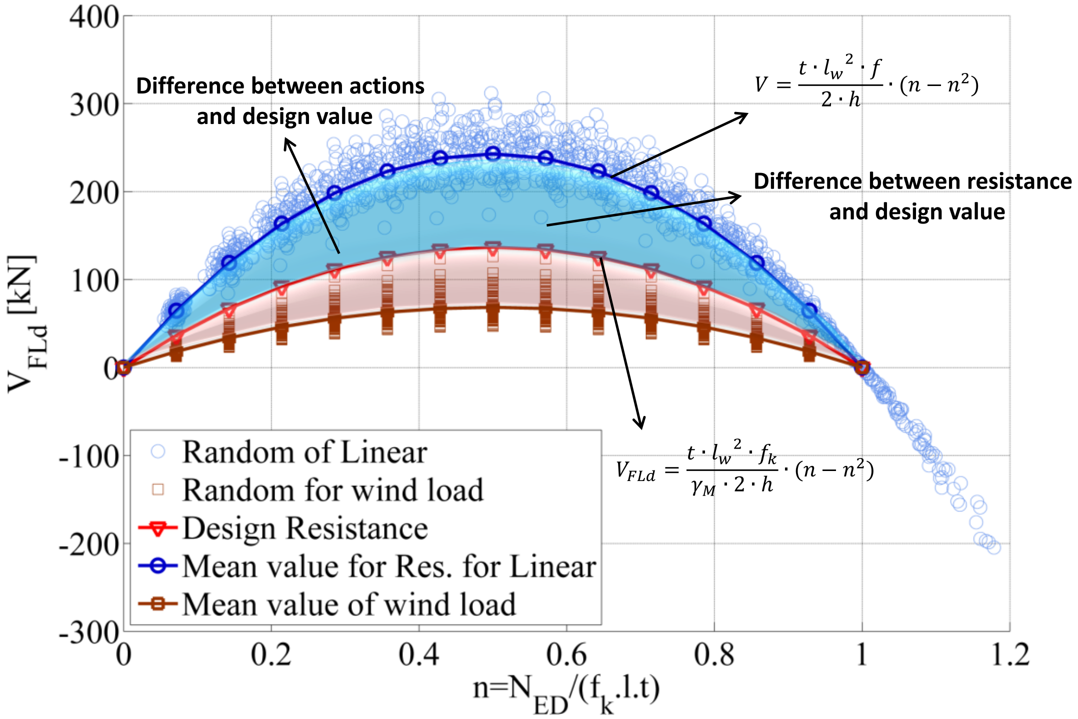


Figure 5.22: Representation of design value and the random points of action and resistance for linear method

In linear method, as it seems in Figure 5.22, the behavior of action, resistance, and design value of shear resistance in the flexural case are the same. In this case, all ranges of values for *n* will be covered.

5.4.3 Comparing the methods

According to the explanation of each method in previous sections, each of them has the individual function of limit state design and a deterministic function for creating a probabilistic model of material resistance. The action probabilistic model is also created for reliability analysis with a basic assumption of economic design ($V_{FLd} = V_{Ed}$). Therefore, the model behavior of wind action or horizontal action will be similar to the design value in each method. Table 5.12 summarizes for each method the corresponding deterministic function for the probabilistic model of resistance and the limit state design function. In this table, the utilization of a partial factor for each of these functions is illustrated.

Table 5.12: Design resistance and deterministic function for probabilistic resistance model in different methods of partial factor utilization

Method of partial factor utilization	Design value of shear resistance for flexural failure	Deterministic function for creating probabilistic resistance model in flexural failure
Nonlinear	$V_{FLd} = \frac{t \cdot {l_w}^2 \cdot f_k}{2 \cdot h} \cdot (n - \boldsymbol{\gamma_M} \cdot n^2)$	$V = \frac{t \cdot {l_w}^2 \cdot f}{2 \cdot h} \cdot (n - n^2)$
Linear-nonlinear	$V_{FLd} = \frac{t \cdot {l_w}^2 \cdot f_k}{2 \cdot h} \cdot (n - \boldsymbol{\gamma_M} \cdot n^2)$	$V = \boldsymbol{\gamma_M} \cdot \left[\frac{t \cdot {l_w}^2 \cdot f}{2 \cdot h} \cdot (n - \boldsymbol{\gamma_M} \cdot n^2)\right]$
Linear	$V_{FLd} = \frac{t \cdot {l_w}^2 \cdot f_k}{\boldsymbol{\gamma_M} \cdot 2 \cdot h} \cdot (n - n^2)$	$V = \frac{t \cdot {l_w}^2 \cdot f}{2 \cdot h} \cdot (n - n^2)$

5.4.4 Reliability analysis

The next step is the comparison of different methods regarding their reliability indexes. For each method, reliability analyses are completed to observe the influence of partial factor utilization in the limit state of shear resistance for flexural failure.

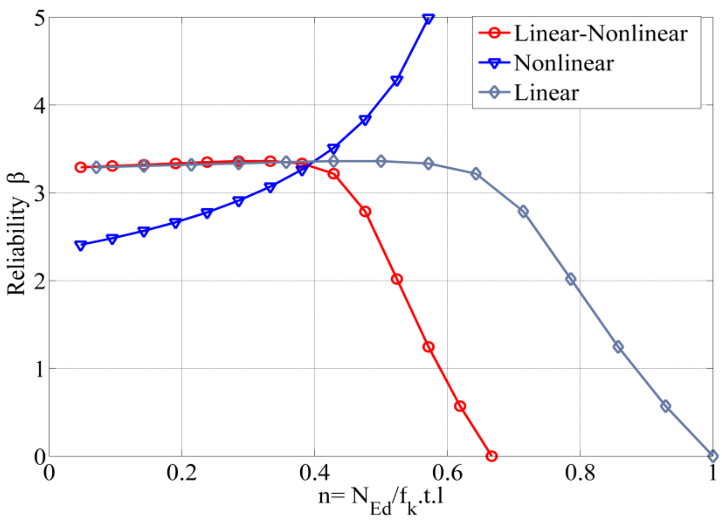


Figure 5.23: Reliability of different method

The reliability result is given in Figure 5.23. The reliability analysis is done for each method in the range of *n* for which the result is acceptable. Thus, in the nonlinear method and the linear-nonlinear method, reliability analysis is done for a range of between 0 and 0.666, and for the linear method, reliability is available for *n* between 0 and 1.

In the first step of comparison, the nonlinear method and linear-nonlinear method are considered. In both methods, the design value of resistance is equal. Therefore, according to the economic design situation ($V_{FLd} = V_{Ed}$), the behaviors of the wind load or action part of the limit state are the same. On the other hand, the deterministic function in the nonlinear [eq. (5.50)] and linear-nonlinear [eq. (5.51)] methods that create the probabilistic model of material are different. Therefore, Figure 5.24 shows that the probabilistic model of resistance for these two methods differ.

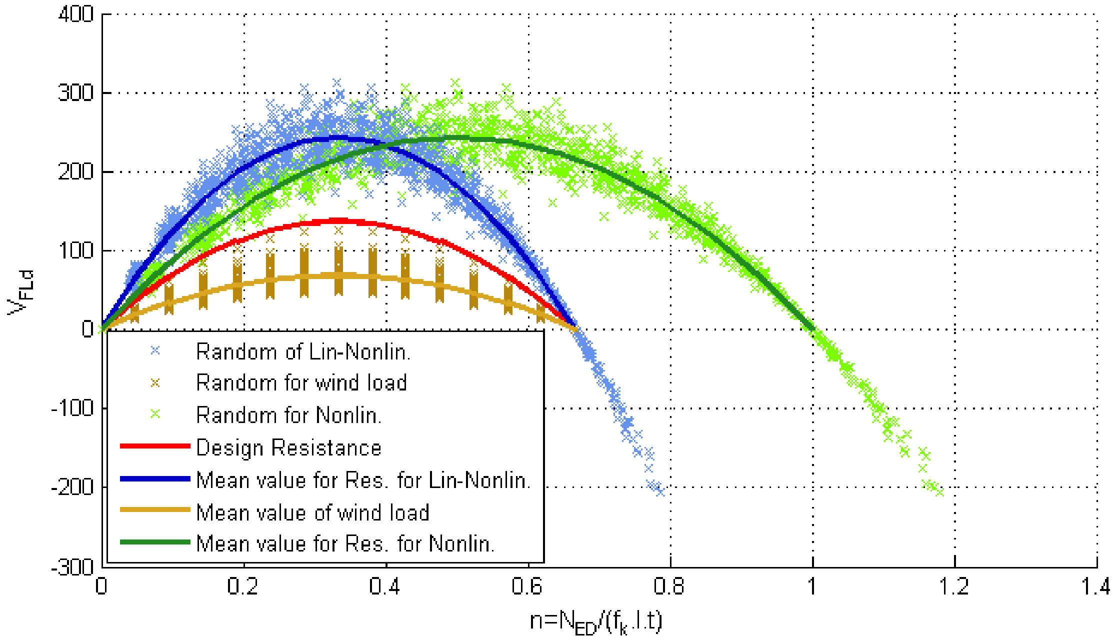


Figure 5.24: Comparison of linear-nonlinear and nonlinear method

The design value for both nonlinear and linear-nonlinear methods cover the range of 0 and 0.666 for *n*. Therefore, in reliability analysis, only this interval has been calculated for these two methods. In values of *n* lower than 0.4, the resistance model in the nonlinear model lies on values lower than the linear-nonlinear method. Thus, with the same loading model for these two methods for these values of *n*, the reliability index of the nonlinear method will be smaller than for the linear-nonlinear method (see Figure 5.23).

According to Figure 5.24, it can be observed that in values bigger than 0.4 for *n*, the probabilistic resistance model in the nonlinear method differs significantly in comparison with the action model. This difference causes ta considerable increase in the reliability index for the nonlinear method. In the linear-nonlinear method, a small difference between probabilistic model of resistance and actions leads to a significant decrease in the reliability index of the shear wall in the flexural failure mode (see Figure 5.23).

Figure 5.25 shows the behavior of actions and resistance in linear and linear-nonlinear methods. It can be observed that the diagrams of the linear-nonlinear method, in this case, are stretched to the maximum value, from n = 0.666 to n = 1. For this reason, the reliability indexes for these two methods are similar, but the value for the reliability index varies for these two cases.

In the case of the linear and nonlinear method in Figure 5.26, a similar model for probabilistic resistance is observed, but the design values of resistance in flexural failure differ. Consequently, the action probabilistic models for these two methods will have different behavior.

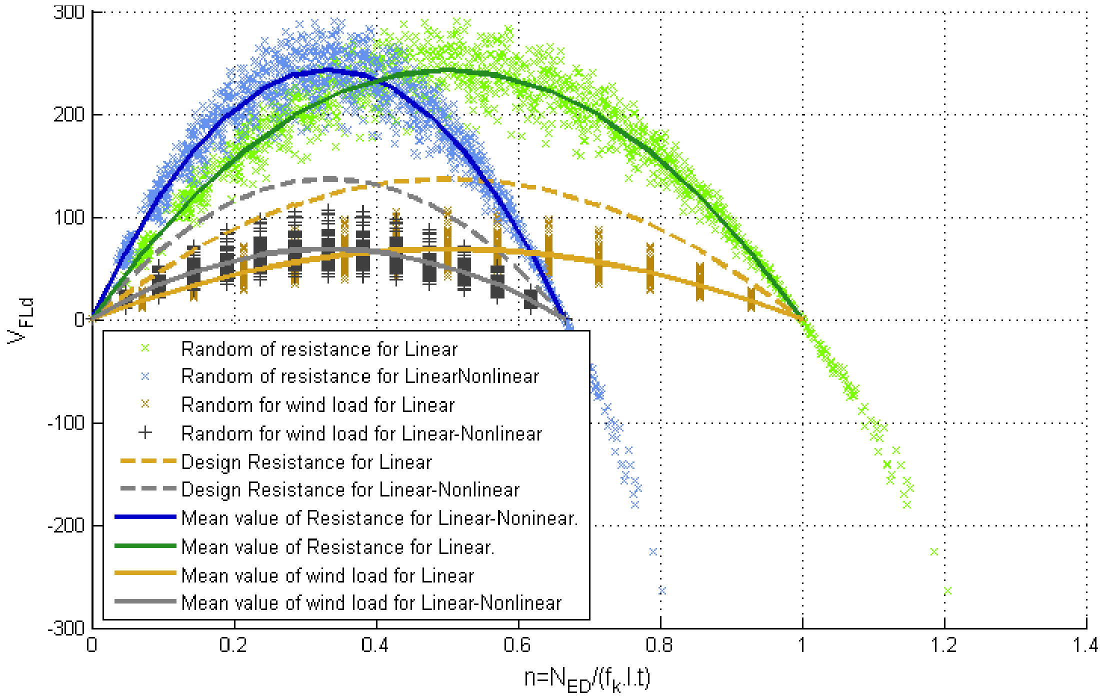


Figure 5.25: Comparison of linear-nonlinear and linear method

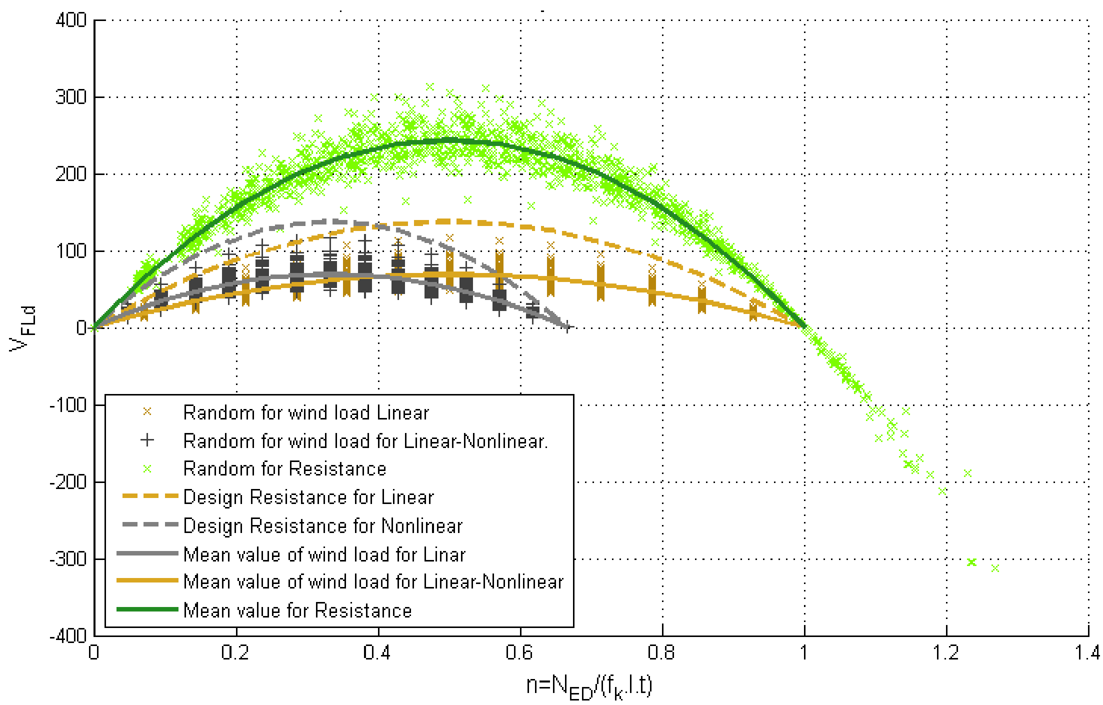


Figure 5.26: Comparison of linear and nonlinear method

5.4.5 Conclusion

The most reasonable method should be selected based on the different cases of comparison for resistance model, action model, reliability index and design situation. In this case, the nonlinear method could not be a reasonable method because it does not consider a part of wall resistance. The transition of the design model to the probabilistic model of resistance and action is not happening properly, and the resistance and actions model are not compatible with each other.

The linear-nonlinear method provides compatible behavior in the resistance model and action model, but in this case, the transition from design situation to probabilistic model is forced to be linear to secure compatible results for both probabilistic models, for action and resistance. Thus, resistance as it acts in reality is only partially represented by this form of transition.

By comparing all of the results and behaviors in different methods, one can conclude that the most reasonable method, in this case, is the linear method. Because in this case the transition of the design value to the probabilistic model for action and resistance has been done linearly, the action model, resistance model, and design value that result from this transition are completely compatible with each other.

Different behaviors and different results ensue because of different methods of material partial factor utilization. By comparing the most reasonable method (i.e., linear) with the other methods, it can be seen that this method performs similarly to the global resistance factor method.

In nonlinear limit states, as here for the flexural failure of the wall, utilization of a partial factor will lead to some critical points. An explicit explanation of this problem must be given to determine where and when the partial factors should be used to avoid these problems. In this case, Section 6.3.5(3) from EN-1990 [16] can be used to circumvent these problems: “the design resistance may be obtained directly from the characteristic value of a material or product resistance, without explicit determination of design values for individual basic variables”, using (5.52). The method would then be the same as the linear method used in this study.

6 Summary and outlook

Various sources of uncertainty arise in structural design, and among them the partial safety factor method is a reasonable measure to evaluate the safety of a construction. Partial factors cover individual uncertainties from corresponding basic variables in the limit state of structures. The background of the partial safety factor approach comprises the prerequisite discussion for this study.

In Chapter 2, the partial safety factor method and its aspects in connection with the Eurocode system are explained. EN-1990 is the basis code for other relevant Eurocodes and the recommended safety concept in this code is partial safety factors. Thus, this study's investigations have been done following the EN-1990 framework. The objective for the application of recommended partial factors in standards is to reach the target reliabilities proposed in each code. Therefore, reliability analyses are made to investigate the different approaches of the partial safety factor method.

The applied methods in this study are presented briefly in Chapter 3: Monte Carlo method, FORM and importance sampling. Different applications of these methods are also compared according to the provided reliability level and the calculation process. Furthermore, stochastic parameters are presented as the main input data for reliability analysis and for the application of reliability analysis. Two parameters for load ratios are also defined in this chapter for the reliability analysis. Load ratio χ is the first, defining the proportion of variable load to the total load, including self-weight or permanent load. Higher values of this parameter represent the light-weight structures, and lower values are for heavy-weight structures. This definition helps in further analysis to interpret the results based on the weight of structure. Variable k is another parameter related to load ratio, and it represents the ratio of leading and accompanying variable loads.

Limit states are the main components that define the failure of a structure or its member. In a general format, limit states mainly consist of two basic components: load and resistance. Therefore, the investigations of different aspects of the partial safety factor method according to the reliability analysis are also subdivided into two main chapters corresponding to loading and resistance.

In Chapter 4, the aspects of partial the safety factor method related to the loading and their partial factors are reviewed. The application of different load combinations and recommended partial factors according to EN-1990 is briefly investigated. Analyses are made of five different types of material parameters: concrete, steel, reinforcement steel, masonry and timber. In addition, three types of variable actions are also considered in the analysis of load combinations. Load combination 6.10 and 6.10a&b, based on EN-1990, are compared according to the provided reliability indexes. The results considering the average point of view expose values slightly higher than the target reliability value. Combination 6.10a&b shows more consistent results regarding target reliability. The economic effects of the implementation of 6.10a&b rather than 6.10 are verified through a case study of the design of a concrete beam. The results prove that the application of 6.10a&b produces more economical results. In other words, designing with 6.10a&b is more sustainable than with 6.10.

Although the combined average of reliabilities for EN-1990 produces a result compatible with the target value, certain cases yield low reliability. In case of snow load, the recommended partial factors of EN-1990 are not enough to reach the required safety level. As a result, a so-called increase factor method is proposed to apply higher safety measures in cases of snow load. The reliability analysis based on this method proves that its application increases the reliability level to close to the target values.

Different types of variable loads are involved in the structural designs. The combination of these time-depended loads has to be considered in the calculations. The combination factors in EN-1990 are proposed to cover the possibility of the simultaneous occurrence of two time variable actions. The representation of these loads has to be based on maxima for the chosen reference period. The transformation of stochastic parameters for a different reference period is investigated in Chapter 4 as well. A deterministic formula is likewise derived for the calculation of COV values for various reference periods.

Furthermore, the calibration results based on the design value method prove that the recommended values of combination factors in the code are conservative. Selecting the combination factors for different types of variable loads for different load combinations is one of the aspects in the current version of EN-1990 that is in contrast with the concept of ease of use. Therefore, a simplified load combination by means of choosing the appropriate combination factors is proposed within this study. This simplified load combination is compared with original combinations in EN-1990 through numerous reliability analyses. The results prove that the application of a simplified method is completely compatible with the original combinations in the code. According to this analysis, this method is mentioned in the most recent draft of the update to EN-1990 as a note in the table of load combinations.

In the final section of Chapter 4, a new method for the calibration of partial safety factors is proposed. This new method, which is called “interest band,” is based on the full probabilistic methods and Monte Carlo reliability analysis. The random realizations near the limit state function are considered design values for basic variables. The partial factors are calibrated based on the target reliability. The calibrated values show the compatible reliability level in comparison with the selected target reliabilities and reduce the variation of reliability indexes at the same time.

In Chapter 5, the resistance partial factor is the objective of the investigation. Due to the fact that resistance parameters and models are highly dependent on experiments, a stochastic analysis is first done for the relation of test numbers and the partial resistance factor. According to the probabilistic background of test numbers, 10 is recommended as the optimum number for material tests. It is observed that beyond 10 tests, the variations in the resultant partial safety factor of material or its coefficient of variation are approximately constant.

The partial safety factor of resistance has two main contributions form material uncertainty and model uncertainty. The evaluation of material uncertainty may be done according to different types of tests for various material properties. Contrarily, the assessment of partial factor for model uncertainty was always a challenging point in probabilistic modeling of resistance. In this study, the probabilistic evaluation of test results is implemented according to Annex D in EN-1990 for calculation of both partial factors for model uncertainty and for resistance as a whole. The method is applied on an experimental databased for unreinforced masonry shear walls. The theoretical model for the resistance of a masonry shear wall from DIN EN1996-1-1/NA and its Annex K is considered. The most important advantage of the probabilistic evaluation in Annex D of EN-1990 is found to be the separation of model bias and of model error. In this research, the bias model is considered in the process of partial factor calculation in addition to the coefficient of variation. The results of the analysis for the masonry wall model in DIN EN1996-1-1/NA shows the design with 1.5 as partial factor is relatively safe and slightly conservative.

In the last part of section 5, the utilization of partial safety factors is investigated in a case study of a masonry shear wall in a flexural failure. This limit state has to be derived indirectly from the recommended limit state function in EN1996-1-1. The partial factor utilization has considerable influence on the reliability level of this limit state. Three different methods for utilization of partial factor are considered. It is concluded that the application of the resistance factor method is the best approach to neglect the improper influence of partial factor utilization.

Generally, it can be concluded that partial factor method covers different types of uncertainty in the design. This methodology deals with sources of individual uncertainty by indicating relevant partial safety factors for each type of uncertainty. Although the implementation of this safety method leads to economical design, there remain some aspects that can be improved to reach more sustainable results. The probabilistic analysis is based on the stochastic parameters selected from various references. According to new phenomenon worldwide, such as climate change and new technologies, a detailed analysis based on new databases for environmental load probabilistic models seems essential in this matter. Moreover, a detailed reliability analysis can be performed according to the latest probabilistic modeling of basic variables. The reliability analysis has to be conducted for a structural system or a structural component without an explicit limit state function. In other words, a reliability analysis for the practical projects of structural design according to the Eurocodes recommendations requires further comprehensive investigation. According to the ongoing improvements in computational technologies, the application of full probabilistic design methods will be convenient to perform. Therefore, a comprehensive study of the fundamental approaches for this kind of calculation is necessary for the future of structural design.

References

[1]. Randall Jr, F.A.: Historical Notes on Structural Safety*. in ACI Journal Proceedings. 1973. ACI.
[2]. Randall, F.A.: The safety factor of structures in history. Professional Safety, (1976), pp. 12-28.
[3]. Ellingwood, B.: Probability-Based Load Criteria for Structural Design, Vol. Chapter., CRC Press. 2018, pp. 283-288.
[4]. Blockley, D.I.: The nature of structural design and safety. 1980.
[5]. Beeby, A.W.;Narayanan, R.S.: Introduction to Design for Civil Engineers. CRC Press. London, 2000.
[6]. Freudenthal, A.M.; Garrelts, J.M.;Shinozuka, M.: The analysis of structural safety, DTIC Document1964.
[7]. Cornell, C.A.: A Probability-Based Structural Code*. Journal Proceedings, 66 (1969) 12, pp. 974-985.
[8]. Ravindra, M.K.;Galambos, T.V.: Load and resistance factor design for steel. Journal of the Structural Division, 104 (1978) 9, pp. 1337-1353.
[9]. C.E.B: Recommendations for an international code of practice for reinforced concrete. English edition: American Concrete Institute and Cement an Concrete Association ed. Comité Européen du Béton (CEB) Paris, 1964.
[10]. Basis of design of structures, proposals for modification of partial safety factors in eurocodes, N.C.a.W. Reports, SAKO Joint Committee of NKB and INSTA-B: Oslo,1999.
[11]. Ellingwood, B.: Development of a probability based load criterion for American National Standard A58: Building code requirements for minimum design loads in buildings and other structures. Vol. 577. US Department of Commerce, National Bureau of Standards 1980.
[12]. Narayanan, R.;Beeby, A.: Designers' Guide to EN 1992-1-1 and EN 1992-1-2. Eurocode 2: Design of Concrete Structures: General Rules and Rules for Buildings and Structural Fire Design. Thomas Telford 2005.
[13]. Castro, P.M.; Delgado, R.M.;de Sá, J.M.C.: A partial factor methodology for structural safety assessment in non linear analysis. COMPUTERS AND CONCRETE, 2 (2005) 1, pp. 31-53.
[14]. Gulvanessian, H.; Calgaro, J.A.;Holický, M.: Designer's Guide to EN 1990: Eurocode: Basis of Structural Design. Thomas Telford 2002.
[15]. Gulvanessian, H.: An Independent Technical Expert Review of the SAKO Report – FINAL CEMBUREAU,BIBM,ERMCO2003.
[16]. EN 1990: Eurocode: Basis of structural design (EN 1990:2002 + A1:2005 + A1:2005/AC:2010). European Committee for Standardisation (CEN),Brussels, 2002/2010
[17]. Gulvanessian, H.;Holicky, M.: Eurocodes: using reliability analysis to combine action effects. Structures & Buildings, 158 (2005) SB4, pp. 243–252.
[18]. prEN 1990: Eurocode basis of structural and geotechnical design-Working document. European Committee for Standardisation (CEN),Final draft, 2017
[19]. ISO 2394 General principles on reliability for structures. International Organization of Standardization Switzerland, 2014
[20]. JCSS Probabilistic model code, Joint Committee on Structural Safety: www.jcss.ethz.ch,2001.

[21]. Rackwitz, R.: Optimization — the basis of code-making and reliability verification. Structural Safety, 22 (2000) 1, pp. 27-60.
[22]. Klüppelberg, C.; Straub, D.;Welpe, I.M.: Risk-A Multidisciplinary Introduction. Springer 2014.
[23]. Holický, M.: Reliability analysis for structural design. AFRICAN SUN MeDIA 2009.
[24]. Sørensen, J.D.: Notes in Structural Reliability Theory And Risk Analysis Aalborg, 2004
[25]. Ang, A.: Practical assessments of risk and its uncertainty. Reliability and Optimization of Structural Systems: Assessment, Design, and Life-Cycle Performance, (2007), pp. 45.
[26]. Brehm, E.: Reliability of Unreinforced Masonry Bracing Walls-Probabilistic Approach and Optimized Target Values. TU Darmstadt 2011.
[27]. Skjong, R.;Ronold, K.: Societal indicators and risk acceptance In: Proceedings of 17th International Conference on Offshore Mechanics and Arctic Engineering OMAE, 1998.
[28]. Rackwitz, R.: Zuverlässigkeit und Lasten im konstruktiven Ingenieurbau. Lecture notes, (2004), pp.
[29]. Ang, A.H.-S.;Cornell, C.A.: Reliability bases of structural safety and design. Journal of the Structural Division, 100 (1974) Proc. Paper 10777, pp.
[30]. Salehi, H.; Jäger, W.;Montazerolghaem, M.: Die Anwendung der Monte-Carlo-Methode zur Bestimmung der Zuverlässigkeit von Mauerwerksbauteilen. Mauerwerk Kalender, (2016), pp. 317-331.
[31]. Faber, M.H.: Risk and safety in civil engineering. Swiss federal institute of technology. Zurich, 2007.
[32]. Huntington, E.: Frequency distribution of product and quotient. The Annals of Mathematical Statistics, 10 (1939) 2, pp. 195-198.
[33]. Schueremans, L.: Probabilistic evaluation of structural unreinforced masonry,PhD-Thesis, Katholieke Universiteit Leuven. 2001
[34]. Choi, S.K.; Grandhi, R.V.;Canfield, R.A.: Reliability-based Structural Design. Springer. London, 2010.
[35]. Ravindra, M.K.; Heaney, A.C.;Lind, N.C.: Probabilistic evaluation of safety factors. IABSE reports of the working commissions, 4 (1969), pp. 35-46.
[36]. Thoft-Christensen, P.;Baker, M.J.: Structural reliability theory and its applications. (1982), pp.
[37]. Ditlevsen, O.;Madsen, H.O.: Structural reliability methods. Vol. 178. Wiley New York 1996.
[38]. Hasofer, A.M.;Lind, N.C.: Exact and invariant second-moment code format. Journal of the Engineering Mechanics division, 100 (1974) 1, pp. 111-121.
[39]. Rackwitz, R.;Fiessler, B.: Structural reliability under combined random load sequences. Computers & Structures, 9 (1978) 5, pp. 489-494.
[40]. Hammersley, J.: Monte carlo methods. Fletcher & Son Ltd. England, 1964.
[41]. Sobol, I.M.: A primer for the Monte Carlo method. CRC press 1994.
[42]. Metropolis, N.;Ulam, S.: The monte carlo method. Journal of the American statistical association, 44 (1949) 247, pp. 335-341.
[43]. Sobol', I.i.a.M.: The Monte Carlo Method. The University of Chicago 1974. 72.
[44]. Dhillon, B.S.: Bibliography of literature on safety factors. Microelectronics Reliability, 29 (1989) 2, pp. 267-280.
[45]. Jahani, E.; Shayanfar, M.A.;Barkhordari, M.A.: A new adaptive importance sampling Monte Carlo method for structural reliability. KSCE Journal of Civil Engineering, 17 (2013) 1, pp. 210-215.
[46]. Kececioglu, D.;Cormier, D.: Designing a specified reliability directly into a component, SAE Technical Paper1964.
[47]. Cohen, A.C.;Whitten, B.J.: Parameter estimation in reliability and life span models. M. Dekker 1988.
[48]. Kelton, W.D.;Law, A.M.: Simulation modeling and analysis. McGraw Hill Boston 2000.

[49]. Raychaudhuri, S.: Introduction to monte carlo simulation. in Simulation Conference, 2008. WSC 2008. Winter. 2008. IEEE.

[50]. Kroese, D.P.; Taimre, T.;Botev, Z.I.: Handbook of Monte Carlo Methods. Vol. 706. John Wiley & Sons 2013.

[51]. Zio, E.: The Monte Carlo simulation method for system reliability and risk analysis. Springer 2013.

[52]. Melchers, R.E.: Structural reliability analysis and prediction 2ed. Wiley. London, 1999.

[53]. Engelund, S.;Rackwitz, R.: A benchmark study on importance sampling techniques in structural reliability. Structural Safety, 12 (1993) 4, pp. 255-276.

[54]. Melchers, R.E.: Importance sampling in structural systems. Structural Safety, 6 (1989) 1, pp. 3-10.

[55]. Harbitz, A.: An efficient sampling method for probability of failure calculation. Structural Safety, 3 (1986) 2, pp. 109-115.

[56]. Bucher, C.G.: Adaptive sampling — an iterative fast Monte Carlo procedure. Structural Safety, 5 (1988) 2, pp. 119-126.

[57]. Papadrakakis, M.;Lagaros, N.D.: Reliability-based structural optimization using neural networks and Monte Carlo simulation. Computer Methods in Applied Mechanics and Engineering, 191 (2002) 32, pp. 3491-3507.

[58]. Salehi, H.; Montazerolghaem, M.;Jäger, W.: Application of crude Monte Carlo and Adaptive importance sampling in reliability assessment of URM shear walls. In: Proceedings of 16th International Brick And Block Masonry Conference (IBMAC), Padova,Italy, 2016.

[59]. Holicky, M.;Sykora, M.: Conventional probabilistic models for calibration of codes. in MH Faber, J. Köhler & K. Nishijima (eds.), Proceedings of 11th International Conference on Applications of Statistics and Probability in Civil Engineering ICASP11. 2011.

[60]. Turkstra, C.J.;Madsen, H.: Load combinations in codified structural design. The structural division, (1980), pp. 2527-2543.

[61]. EN 1992-1-1, Design of concrete structures European commitiee for standardization Brussels, 2004

[62]. EN 1993-1-1 Design of steel structures European committee for standardization-CEN,2005

[63]. EN 1995-1-1 Design of timber structures. European committee for standardization-CEN,2004

[64]. Eurocode 6 - Design of masonry structures - Part 1-1: General rules for reinforced and unreinforced masonry structures. European committee for standardization-CEN,Brussels, 2005

[65]. Severyn, V.; Pashchenko, A.;Mytrofanov, P.: Probabilistic Analysis of Structures Under Snow Load. International Journal of Engineering & Technology; Vol 7, No 3.2 (2018): Special Issue 2DO - 10.14419/ijet.v7i3.2.14431, (2018), pp.

[66]. Holicky, M.;Sykora, M.: Failures of roofs under snow load: Causes and reliability analysis, Vol. Chapter. 2010, pp. 444-453.

[67]. Kozak, D.L.;Liel, A.B.: Reliability of steel roof structures under snow loads. Structural Safety, 54 (2015), pp. 46-56.

[68]. Background documentation, Eurocode 1 (ENV 1991), Part 1: Basis of design, JCSS, 1996.

[69]. Sedlacek, G.; Cajot, L.; Haller, M., et al.: Probabilistic quantification of safety of a steel structure highlighting the potential of steel versus other materials, EUR 21695, European commission: Luxembourg,2005.

[70]. Murzewski, J.: Upper bound for combination of action effects. IABSE report, 74 (1996), pp. 279-290.

[71]. Ferry-Borges, J.;Castanheta, M.: Structural safety. 2nd ed. Lisbon, 1971.

[72]. Sørensen, J.D.: Load combination factor, JCSS probabilistic workshop: Ghent,2015.

[73]. Vrouwenvelder, T.: Reliability based code calibration the use of the jcss probabilistic model code. In: Proceedings of JCSS Workshop on Code Calibration, Zurich, 2002.
[74]. Vrouwenvelder, T.;Gulvanessian, H.: Basis of Design and Actions on Structures: Background and Application of Eurocode 1, IABSE: Delft 1996.
[75]. Borges, J.F.; Thielen, G.; Rackwitz, R., et al.: Conceptional Preparation of Future Codes - Progress Report, CEB Bulletins 174, CEB: Paris,1982.
[76]. Grundlagen zur Festlegung von Sicherheitsanforderungen für bauliche An-lagen. Deutsches Institut für Normung,DIN-Beuth Verlag, Berlin—Köln, 1981
[77]. Schobbe, W.: Konzept zur Definition und Kombination von Lasten im Rahmen der deutschen Sicherheitsrichtlinie. Wilhelm Ernst & Sohn 1982.
[78]. Breinlinger, F.;Jäger, W.: Verbesserung der Praxistauglichkeit der Baunormen durch pränormative Arbeit - Teilantrag 1: Sicherheitskonzept und Einwirkungen, F 2957, Frauenhofer IRB Verlag, Bundesinstitut für Bau-, Stadt- und Raumforschung: Stuttgart 2015.
[79]. DS/EN 1990 DK NA: National Annex to Eurocode: Basis of structural design. Danish Standards Foundation,2013
[80]. A P Mann, L.J.M.: Rationalisation of safety and serviceability factors in structural codes, CIRIA1977.
[81]. Fischer, L.: Das neue Sicherheitkonzept im Bauwesen. Ernst & Sohn. Berlin, 2001. 186.
[82]. ISO 12491 - Statistical methods for quality control of building materials and components, Beuth1997.
[83]. Holichky, M.; Materna, A.; Sedlacek, G., et al.: Implementation of the Eurocodes: Handbook 2 - Reliability backgrounds, European Commission: Prague,2005.
[84]. ISO 16269-6: 2014 Statistical Interpretation of Data, Part 6-Determination of Statistical Tolerance Intervals. International Organization of Standardization,2014
[85]. Leung, W.C.;Ho, K.S.: Report on strength comparison of 100 mm and 150 mm cubes Hong-Kong,1996.
[86]. Arya, C.: Design of structural elements: Concrete, steelwork, masonry and timber designs to British standards and Eurocodes. CRC Press 2009.
[87]. Salehi, H.: Determination of partial factor for model uncertainty for unreinforced masonry shear walls. Mauerwerk, 22 (2018) 2, pp. 103-112.
[88]. Glowienka, S.: Zuverlässigkeit von Mauerwerkswänden aus großformatigen Steinen. Probabilistische Analyse von großformatigem Mauerwerk aus Kalksandstein und Porenbeton mit Dünnbettvermörtelung. TU Darmstadt, 2007.
[89]. Montazerolghaem, M.: Analysis of Unreinforced Masonry Structures with Uncertain Data,PhD-Thesis, TU Dresden Faculty of Architecture: Chair of Structural Design, Dresden. 2015
[90]. Montazerolghaem, M.;Jäger, W.: Characterization of uncertainty (probabilistic models) in verification of unreinforced masonry shear wall / Charakterisierung der Unschärfe (probabilistische Modelle) beim Nachweis von Wandscheiben aus unbewehrtem Mauerwerk. Mauerwerk, 19 (2015) 4, pp. 287-297.
[91]. ESECMaSE: Enhanced Safety and Efficient Constructionof Masonry Structures in Europe: http://www.esecmase.org 2006.
[92]. Jäger, W.; Ortlepp, S.; Schöps, P., et al.: Vergleich der normativen Ansätze zum Nachweis von Aussteifungsscheiben im Gebäude nach DIN 1053-1/-100, E-DIN 1053-13, EN 1996-1-1/NA und dem Forschungsvorhaben ESECMaSE hinsichtlich des Sicherheitsniveaus: TU Dresden,Unpublised
[93]. DIN EN 1996-1-1/NA-Nationaler Anhang-Eurocode 6: Bemessung und Konstruktion von Mauerwerksbauten – Teil 1-1: Allgemeine Regeln für bewehrtes und unbewehrtes Mauerwerk. DIN Deutsches Institut für Normung,Berlin, 2012
[94]. Mann, W.;Muller, H.: Failure of Shear-Stressed Masonry. An Enlarged Theory, Tests and Application to Shear Walls. in Proc. Br. Ceram. Soc. 1982.

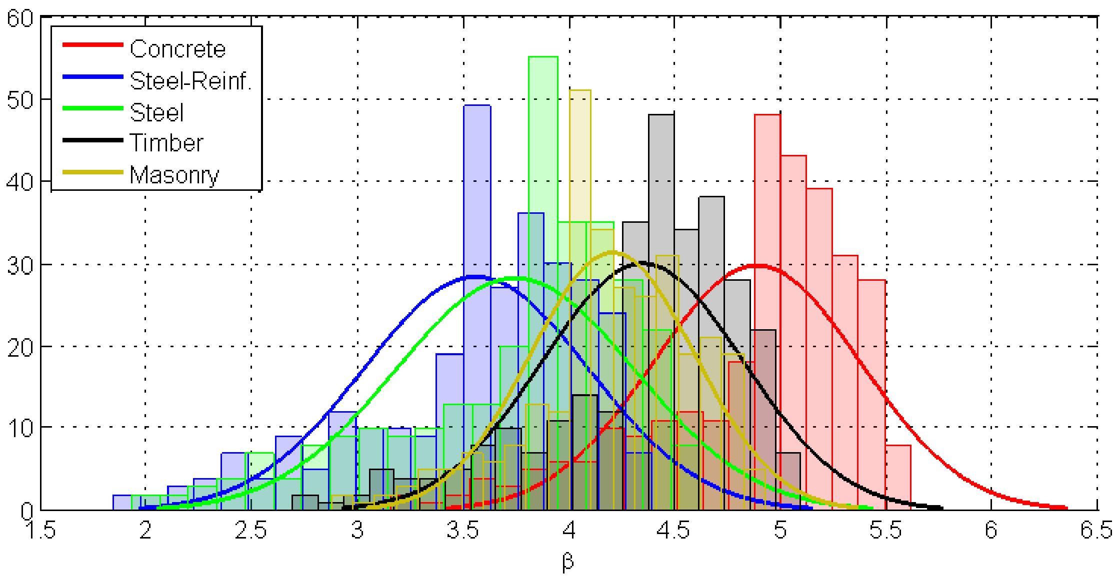
[95]. Kranzler, T.: Tragfähigkeit überwiegend horizontal beanspruchter Aussteifungsscheiben aus unbewehrtem Mauerwerk,PhD-Thesis, Inst. für Massivbau TU Darmstadt. 2008

[96]. Salehi, H.; Montazerolghaem, M.;Jäger, W.: Reliability analysis of methods for utilization of partial factors in flexural failure mode of masonry shear walls. In: Proceedings of 16th International Brick And Block Masonry Conference (IBMAC), Padova,Italy, 2016.

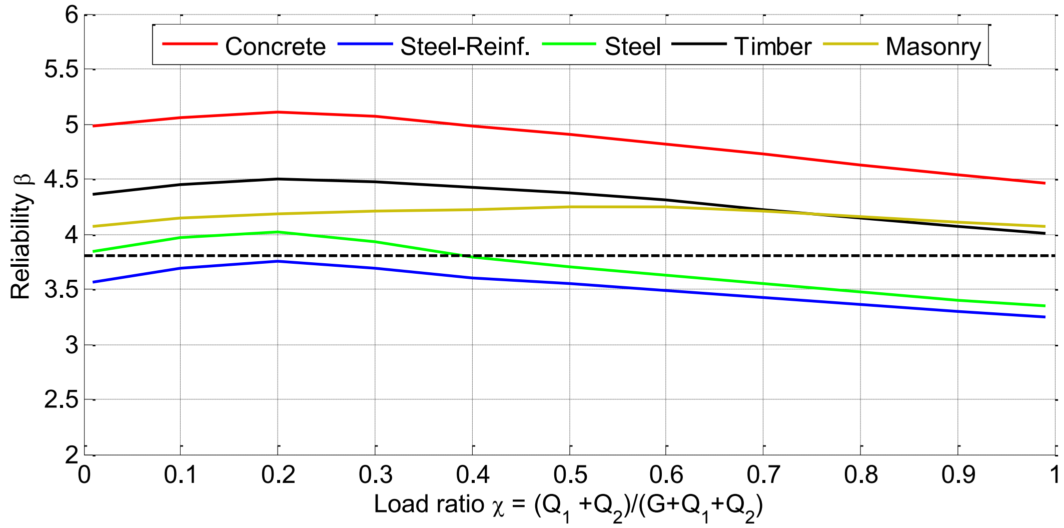
Appendix A: List of figures

Appendix B: List of tables

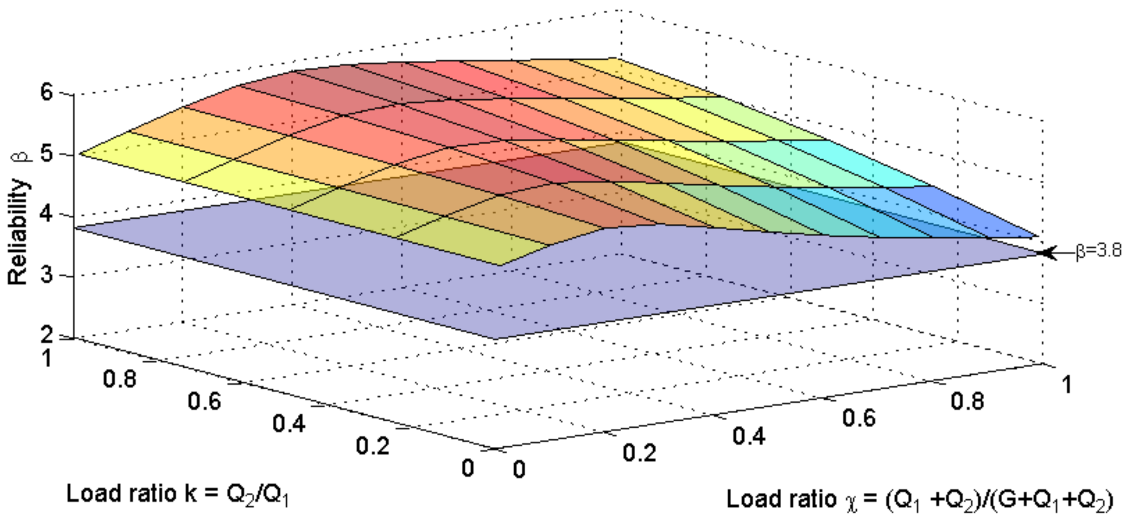
Appendix C: Additional diagrams for load combinations of EN-1990 reliabilities



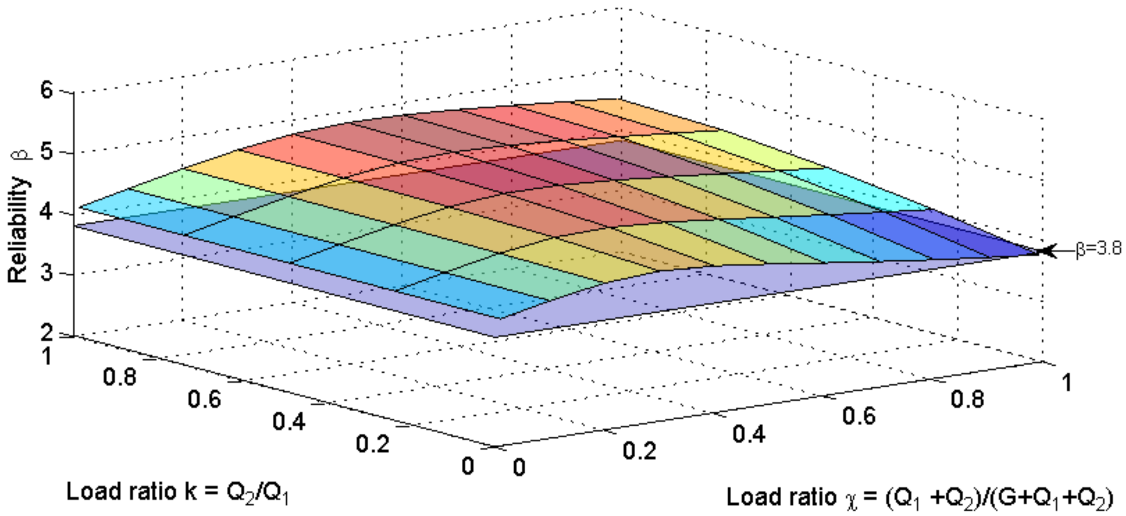
Histogram of different resistance types based on EN-1990 combination 6.10a&b for all load cases and load ratios



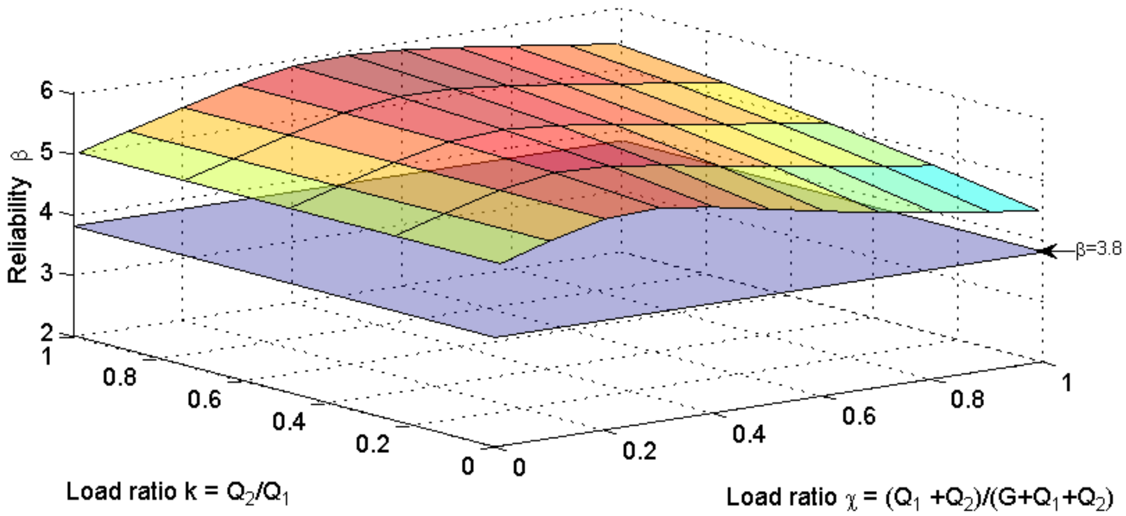
Average reliability for each resistance of EN-1990 combination 6. 10a&b for all load cases and load ratios



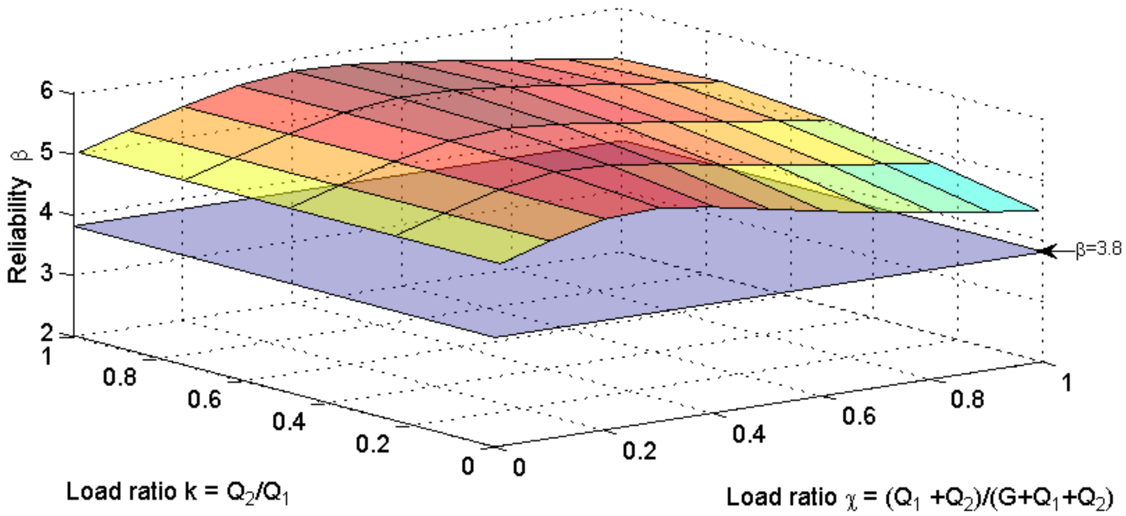
Concrete reliability for wind leading and imposed accompanying, combination 6.10



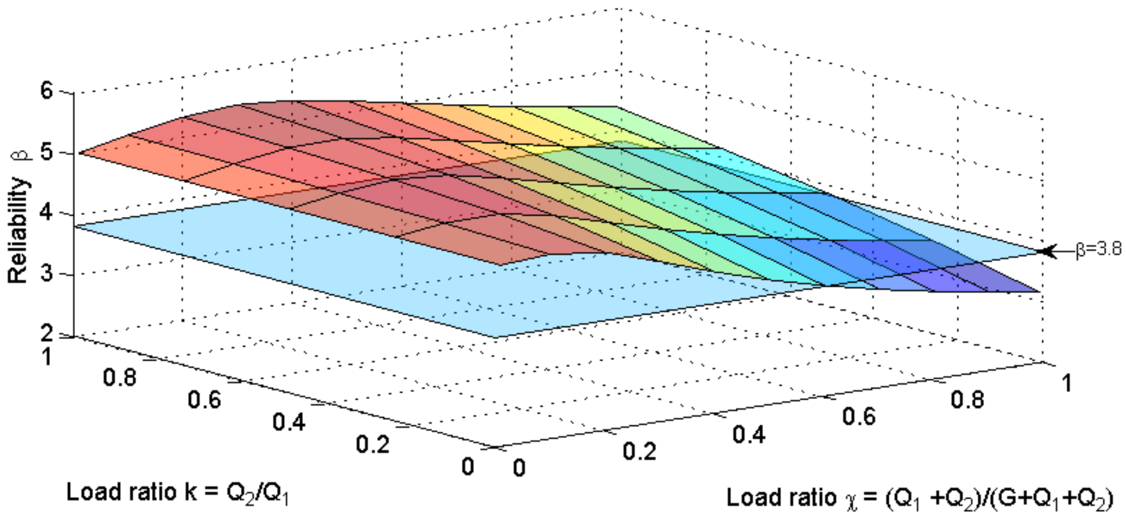
Concrete reliability for wind leading and snow accompanying, combination 6.10



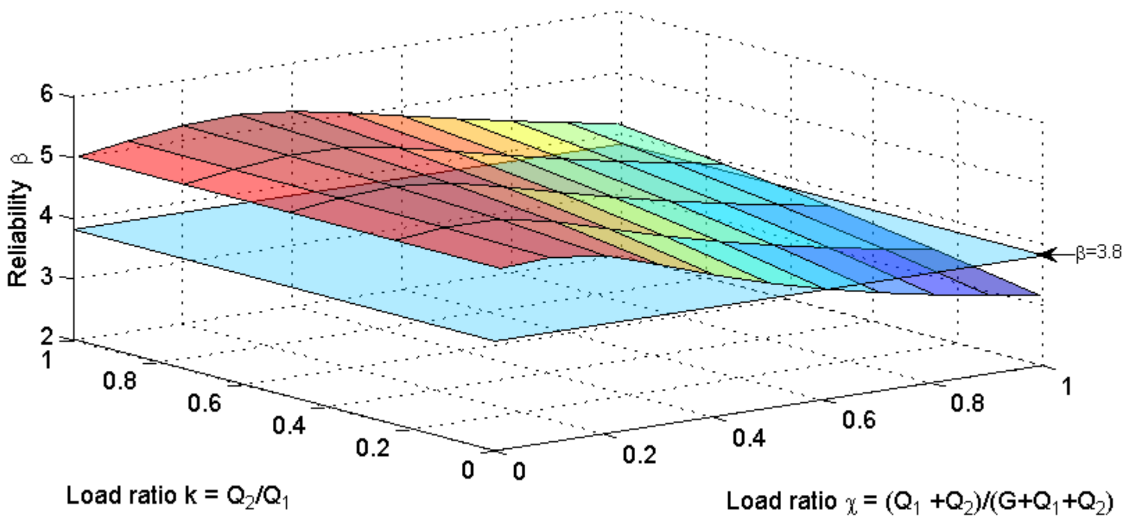
Concrete reliability for imposed leading and wind accompanying, combination 6.10



Concrete reliability for imposed leading and snow accompanying, combination 6.10



Concrete reliability for snow leading and imposed accompanying, combination 6.10



Concrete reliability for snow leading and wind accompanying, combination 6.10

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