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ORIGINAL PAPER

Estimation of autoregressive fading channels based on two cross-coupled H_∞ filters

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Abstract This paper deals with the on-line estimation of time-varying frequency-flat Rayleigh fading channels based on training sequences and using H_∞ filtering. When the fading channel is approximated by an autoregressive (AR) process, the AR model parameters must be estimated. As their direct estimations from the available noisy observations at the receiver may yield biased values, the joint estimation of both the channel and its AR parameters must be addressed. Among the existing solutions to this joint estimation issue, Expectation Maximization (EM) algorithm or cross-coupled filter based approaches can be considered. They usually require Kalman filtering which is optimal in the H_2 sense provided that the initial state, the driving process and measurement noise are independent, white and Gaussian. However, in real cases, these assumptions may not be satisfied. In addition, the state-space matrices and the noise variances are not necessarily accurately estimated. To take into account the above problem, we propose to use two cross-coupled H_∞ filters. This method makes it possible to provide robust estimation of the fading channel and its AR parameters.

Keywords Rayleigh fading channels · Autoregressive processes · Kalman filtering · H_∞ filtering

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1 Introduction

Current wireless communication systems such as Worldwide Interoperability for Microwave Access (WiMAX) are designed to provide high data rate transmission and to support terminal mobility. This kind of transmission usually leads to severe frequency-selective fading, which can be converted to frequency-flat fading by using multicarrier modulation [1]. In addition, due to terminal mobility, the received signal is subject to Doppler shifts which result in time-varying fading.

Thus, after demodulation, matched filtering and sampling at bit rate $1/T$, the resulting received signal can usually be expressed as

$$y(n) = h(n)d(n) + v(n) \tag{1}$$

where $d(n) \in \{-1, 1\}$ is the n^{th} transmitted data bit when Binary Phase Shift Keying (BPSK) is considered, $h(n)$ is the fading process and $v(n)$ is assumed to be a complex additive white Gaussian noise process with zero-mean and variance σ_v^2.

Given the received signal $y(n)$, estimating the time-varying frequency-flat fading process $h(n)$ is a major challenge for coherent symbol detection. For this purpose, parametric approaches can be considered.

Rayleigh fading channels are usually modeled as zero-mean wide-sense stationary circular complex Gaussian processes, whose stochastic properties depend on the maximum Doppler frequency f_d. According to Jakes [2], the

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theoretical Power Spectrum Density (PSD) associated with either the in-phase or quadrature portion of the fading process $h(n)$ is band-limited and U-shaped. Moreover, it has two infinite peaks at $\pm f_d$. Its corresponding discrete-time autocorrelation function is a zero-order Bessel function of the first kind:

$$R_{hh}(n) = J_0(2\pi f_d T|n|) \quad (2)$$

where T is the symbol period and $f_d T$ denotes the Doppler rate.

When dealing with channel simulation, three main families of approaches have been proposed: Sum-Of-Sinusoids (SOS) models [2–5], frequency domain filtering combined with an Inverse Discrete Fourier Transform (IDFT) [6,7] and time-domain filtering of white noise sequence leading to Autoregressive Moving Average (ARMA) model [8–10] or Moving Average (MA) model [11]. In various papers [12–14], the fading channel is modeled as a p^{th} order Autoregressive (AR) process, denoted by AR(p) and defined as

$$h(n) = -\sum_{i=1}^{p} a_i h(n-i) + w(n) \quad (3)$$

where $\{a_i\}_{i=1,\ldots,p}$ are the AR parameters and $w(n)$ is the zero-mean complex white Gaussian driving process with variance σ_w^2.

From a theoretical point of view, as the PSD is not log-integrable, the variance of the driving process of the AR model should be equal to zero according to the Kolmogoroff-Szego formula[1] [15]. Although this way of modeling the channel may be debatable, it is of interest in practical cases because the model is simple and few parameters have to be estimated. In addition, it can be combined with an optimal filter for channel prediction and equalization.

In this latter case, a Kalman filter can be used, which is optimal in the H_2 sense if the initial state, the driving process and measurement noise are independent, white and gaussian. However, these assumptions do not always hold in practical cases. In addition, model uncertainty should be taken into account. Indeed, the AR model does not fit exactly the fading process, especially when low-order AR models are used. See Fig. 1 which shows the PSD of the theoretical Jakes model and that of the simulated AR channel when the order is set to 1, 2, 5 and 20. Moreover, the noise variances and the AR model parameters are unknown and hence need to be estimated. This results in model parameter errors. Thus, for practical systems, the performance of the Kalman estimator may suffer degradation.

Therefore, H_∞ estimation techniques, initially developed in the framework of control [16], can be considered. The

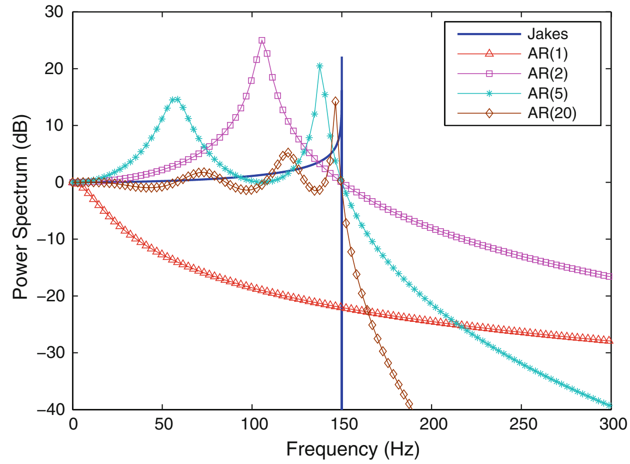

Fig. 1 Power spectrum density (PSD) of the Jakes model and that of the fitted AR process whose order is 1, 2, 5 and 20. $f_d T = 0.05$

estimation criterion is to minimize the worst possible effects of the noise disturbances (i.e., the initial state, the driving process and the measurement noise) on the channel estimation error. Furthermore, this criterion requires no a priori constraints about the noises, except that they have bounded energies. According to [16], H_∞ filtering is more robust against the noise disturbances and modeling approximations than Kalman filtering.

In [17], Cai et al. have proposed a channel estimation scheme for Orthogonal Frequency Division Multiplexing (OFDM) wireless systems based on two serially connected H_∞ filters (see Fig. 2). The first one aims at estimating the AR parameters, which then are used to estimate the fading process by means of a second H_∞ filter. Nevertheless, the AR parameter estimates are biased since they are estimated directly from the noisy data. This might result in poor estimation of the autoregressive process, as pointed out by Labarre et al. in [18].

In this paper, we propose to investigate the relevance of a structure based on two cross-coupled H_∞ filters for the joint estimation of time-varying frequency-flat Rayleigh fading channel and its AR parameters (see Fig. 3). This structure has provided significant results in the field of speech enhancement [19] and is here derived for channel estimation when training sequences are used. Thus, during the so-called training mode, the first H_∞ filter in Fig. 3 uses the training sequence $d(n)$ known at the receiver, the observation $y(n)$ and the latest estimated AR parameters $\{\hat{a}_i\}_{i=1,\ldots,p}$ to estimate the fading process $h(n)$, while the second H_∞ filter uses the estimated fading process $\hat{h}(n)$ to update the AR parameters. At the end of the training period, the receiver stores the estimated AR parameters. Then, in the so-called decision directed mode, a standard H_∞ filter provides a prediction of the fading process $\hat{h}(n+1)$ by using the observation $y(n)$, the stored AR parameters $\{\hat{a}_i\}_{i=1,\ldots,p}$ and the decision

[1] $\sigma_w^2 = \exp(\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\Psi_{hh}(\omega)d\omega)$, where $\Psi_{hh}(\omega)$ denotes the PSD of the AR process that fits the theoretical Jakes spectrum.

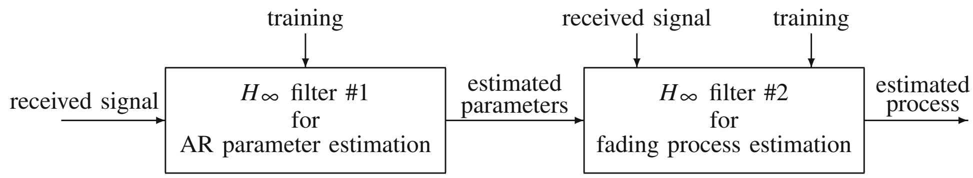

Fig. 2 Two serially connected H_∞ filter based channel estimator [17]

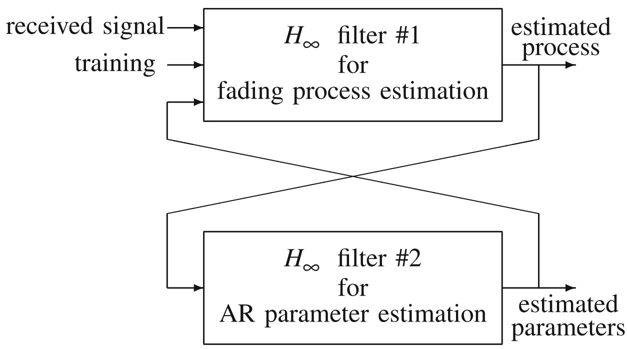

Fig. 3 Proposed two cross-coupled H_∞ filter based channel estimator

$$\hat{d}(n) = \text{sgn}(\text{Re}(\hat{h}^*(n)y(n))) \tag{4}$$

where sgn(.) denotes the signum function, Re(.) denotes the real part and $\hat{h}^*(n)$ is the complex conjugate of $\hat{h}(n)$.

The remainder of the paper is organized as follows. The estimation of autoregressive fading channels based on H_∞ filtering is presented in Sect. 2. It includes a state of the art on AR parameter estimation from noisy observations. Simulation results are reported in Sect. 3. Conclusions are drawn in Sect. 4.

2 Joint AR parameter and channel estimation based on H_∞ filters

Before presenting the approach which aims at jointly estimating the fading process $h(n)$ and its AR parameters $\{a_i\}_{i=1,\ldots,p}$ by means of two cross-coupled H_∞ filters, let us recall various approaches that have been proposed for AR parameter estimation.

2.1 State-of-the-art on AR parameter estimation

In addition to the techniques based on support vector method [20], high-order statistics [21] or Least Absolute Deviation (LAD) techniques [22,23], two main families of approaches have been proposed for AR parameter estimation. Let us first have a look on the off-line second-order statistics solutions.

One standard method consists in solving the so-called Yule-Walker (YW) equations. For this purpose, the authors in [13,14] suggest using the theoretical autocorrelation function of the channel. Given Eq. (2), this can be done provided that the maximum Doppler frequency f_d is known. As this quantity is difficult to be estimated, the autocorrelation function of the channel can be estimated directly from the noisy observations available at the receiver. However, due to the additive noise, this approach results in biased AR parameter estimates [24]. To overcome this problem, alternative approaches have been proposed such as the Modified YW equations [25], the total least square YW approach [26], the high-order YW estimator [27], and the truncated singular value decomposition method [28]. Nevertheless, the noise variances σ_v^2 and σ_w^2, which are required to estimate the process by means of Kalman filter, are not necessarily easy to be retrieved with the above off-line methods.

To avoid this drawback, we have recently investigated the relevance of Errors-In-Variables (EIV) approaches [29]. In that case, the estimation of the AR parameter vector consists in searching the null space of the autocorrelation matrix of the vector $\left[(h(n) - w(n))\ h(n-1)\ \cdots\ h(n-p)\right]$. This matrix is unknown and hence has to be estimated from the autocorrelation matrix of noisy observations. The method aims at searching the noise variances that enable the noise-compensated autocorrelation matrices of the noisy observations to be positive semi-definite. Nevertheless, the computational cost is high.

As an alternative, on-line methods based on the Expectation-Maximization (EM) algorithm which often implies a Kalman smoothing could be used [30]. Nevertheless, since it operates repeatedly on a batch of data, it results in large storage requirements and high computational cost. In addition, its success depends on the initial conditions. Alternatively, to solve the so-called joint estimation issue [31], i.e. the estimations of both the AR process and its parameters, Extended Kalman Filter (EKF) and Sigma Point Kalman filters such as Unscented Kalman Filter (UKF) [32] and Central Difference Kalman Filter (CDKF) [33] can be used. Nevertheless, the noise variances must be a priori known. To avoid the use of approaches dedicated to non-linear state model, two recursive filters can be cross-coupled. Thus, each time a new observation is available, the first filter uses the latest estimated AR parameters to estimate the signal, while the second filter uses the estimated signal to update the AR parameters. According to Gannot [34], this dual filtering approach can be viewed as a sequential version of the EM algorithm. Recently, a variant [35] based on two interacting Kalman filters has been developed in which the variance of the innovation process in the first filter is used to define the gain of the second filter. As this solution can be seen as a recursive instrumental variable technique, it has the advantage of providing consistent estimates of the AR parameters. In the following, we suggest relaxing the Gaussian assumptions required for Kalman filtering, by using H_∞ filtering.

2.2 Estimation of the fading process

Since our purpose is to estimate the fading sequence $h(n)$ modeled by a p^{th} order AR process, the $p \times 1$ state vector is defined as

$$\mathbf{h}(n) = \left[h(n)\ h(n-1) \cdots h(n-p+1) \right]^{\mathrm{T}} \tag{5}$$

As $\mathbf{d}^{\mathrm{T}}(n)\mathbf{h}(n) = h(n)d(n)$, the resulting state-space representation of the fading channel system (3) and (1) is given by

$$\begin{cases} \mathbf{h}(n) = \Phi\mathbf{h}(n-1) + \mathbf{g}w(n) \\ y(n) = \mathbf{d}^{\mathrm{T}}(n)\mathbf{h}(n) + v(n) \end{cases} \tag{6}$$

where $\Phi = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_p \\ 1 & 0 & \cdots & 0 \\ & \ddots & & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}$, $\mathbf{g} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ and $\mathbf{d}(n) = d(n)\mathbf{g}$.

Unlike Kalman filtering, the H_∞ filtering not only deals with the estimation of the state vector $\mathbf{h}(n)$, but also makes it possible to focus on the estimation of a specific linear combination of the state vector components:

$$z(n) = \boldsymbol{l}\mathbf{h}(n) \tag{7}$$

where $\boldsymbol{l}$ is a $1 \times p$ linear transformation operator. Here, as we aim at estimating the fading process $h(n)$, this operator is defined as

$$\boldsymbol{l} = \left[1\ 0 \cdots 0 \right] \tag{8}$$

Given Eqs. (6)–(7) and Fig. 4, the H_∞ filtering provides an estimation of the fading process $\hat{h}(n) = \boldsymbol{l}\hat{\mathbf{h}}(n)$, by minimizing the H_∞ norm of the transfer operator $\mathcal{T}$. This operator maps the discrete-time noise disturbances $w(n)$, $v(n)$ and the initial state error $\mathbf{e}_0 = \mathbf{h}(0) - \hat{\mathbf{h}}(0)$ to the channel estimation error $e(n) = \boldsymbol{l}\mathbf{h}(n) - \boldsymbol{l}\hat{\mathbf{h}}(n)$, as follows:

$$J_\infty = \sup_{w(n), v(n), \mathbf{h}(0)} J \tag{9}$$

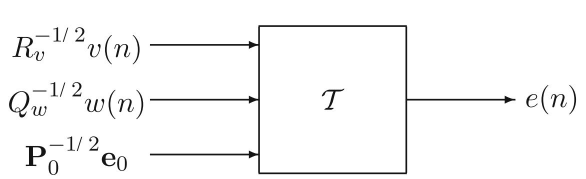

Fig. 4 Transfer operator $\mathcal{T}$

where

$$J = \frac{\sum_{n=0}^{N-1} |e(n)|^2}{\mathbf{e}_0^H \mathbf{P}_0^{-1}\mathbf{e}_0 + \sum_{n=0}^{N-1}\left(Q_w^{-1}|w(n)|^2 + R_v^{-1}|v(n)|^2 \right)} \tag{10}$$

with N the number of available data samples. In addition, $Q_w > 0$ and $R_v > 0$ are weighting parameters which often correspond to the instantaneous power of the sequences $w(n)$ and $v(n)$, respectively. Furthermore, $\mathbf{P}_0 > 0$ denotes a positive definite matrix that reflects a priori knowledge on how small is the initial state error $\mathbf{e}_0 = \mathbf{h}(0) - \hat{\mathbf{h}}(0)$. These weighting parameters are usually tuned by the designer to achieve performance requirements.[2]

However, as a closed-form solution to the above optimal H_∞ estimation problem does not always exist, the following suboptimal design strategy is usually considered:

$$J_\infty < \gamma^2 \tag{11}$$

where $\gamma > 0$ is a prescribed level of disturbance attenuation.

Following the method presented in [36], there exists an H_∞ channel estimator $\hat{h}(n)$ for a given $\gamma > 0$ if there exists a stabilizing symmetric positive definite solution $\mathbf{P}(n)$ to the following Riccati-type equation

$$\mathbf{P}(n+1) = \Phi\mathbf{P}(n)\mathbf{C}^{-1}(n)\Phi^{\mathrm{T}} + \mathbf{g}Q_w\mathbf{g}^{\mathrm{T}}, \quad \mathbf{P}(0) = \mathbf{P}_0 \tag{12}$$

where

$$\mathbf{C}(n) = \mathbf{I}_p - \gamma^{-2}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{l}\mathbf{P}(n) + \mathbf{d}(n)R_v^{-1}\mathbf{d}^{\mathrm{T}}(n)\mathbf{P}(n) \tag{13}$$

This leads to the following constraint:

$$\mathbf{P}(n)\mathbf{C}^{-1}(n) > 0 \tag{14}$$

If the condition (14) is fulfilled, the H_∞ channel estimator exists and is defined by

$$\hat{h}(n) = \boldsymbol{l}\hat{\mathbf{h}}(n) \tag{15}$$

$$\hat{\mathbf{h}}(n) = \Phi\hat{\mathbf{h}}(n-1) + \mathbf{K}(n)\alpha(n), \quad \hat{\mathbf{h}}(0) = \mathbf{0} \tag{16}$$

where the so-called innovation process $\alpha(n)$ and the H_∞ gain $\mathbf{K}(n)$ are respectively given by

$$\alpha(n) = y(n) - \mathbf{d}^{\mathrm{T}}(n)\Phi\hat{\mathbf{h}}(n-1) \tag{17}$$

$$\mathbf{K}(n) = \mathbf{P}(n)\mathbf{C}^{-1}(n)\mathbf{d}(n)R_v^{-1} \tag{18}$$

As mentioned in [37], the matrix $\mathbf{P}(n)$ can be seen as an upper bound of the error covariance matrix in the Kalman filter theory, i.e. $E[(\mathbf{h}(n) - \hat{\mathbf{h}}(n))(\mathbf{h}(n) - \hat{\mathbf{h}}(n))^H] \leq \mathbf{P}(n)$. Moreover, the H_∞ channel estimator (12)–(18) has similar observer structure as the Kalman one. However, due to (13), the H_∞ channel estimator has a computational cost slightly higher than Kalman's one. If the weighting parameters Q_w, R_v and $\mathbf{P}_0$ are respectively chosen to be σ_w^2, σ_v^2 and the initial error covariance matrix of $\mathbf{h}(0)$, then the H_∞ estimator reduces to a Kalman one when $\gamma \longrightarrow +\infty$.

[2] We will explain how to tune them in Sect. 2.4.

It should be noted that the level attenuation factor γ must be carefully selected to satisfy the condition in (14) as [38]

$$\gamma^2 > \max\left(\text{eig}\left[\boldsymbol{l}^{\mathrm{T}}\boldsymbol{l}\left[\mathbf{P}^{-1}(n) + \mathbf{d}(n)R_v^{-1}\mathbf{d}^{\mathrm{T}}(n)\right]^{-1}\right]\right) \quad (19)$$

where $\max(\text{eig}[\mathbf{A}])$ is the maximum eigenvalue of the matrix $\mathbf{A}$. The level attenuation factor γ mainly depends on the weighting parameter R_v and the matrix $\mathbf{P}(n)$. Given Eq. (19), γ should be updated at each iteration. In practical cases, to guarantee that the matrix $\mathbf{P}(n+1)$ is not singular, γ is selected as

$$\gamma^2 = \zeta \max\left(\text{eig}\left[\boldsymbol{l}^{\mathrm{T}}\boldsymbol{l}\left[\mathbf{P}^{-1}(n) + \mathbf{d}(n)R_v^{-1}\mathbf{d}^{\mathrm{T}}(n)\right]^{-1}\right]\right) \quad (20)$$

with $\zeta > 1$.

2.3 Estimation of the AR parameters

In this subsection, we propose to estimate the AR parameters $\{a_i\}_{i=1,\ldots,p}$ from the estimated fading process $\hat{h}(n)$. For this purpose, Eqs. (15) and (16) are firstly combined to express $\hat{h}(n)$ as a function of the AR parameters:

$$\begin{aligned}\hat{h}(n) &= \boldsymbol{l}\Phi\hat{\mathbf{h}}(n-1) + \boldsymbol{l}\mathbf{K}(n)\alpha(n) \\ &= \hat{\mathbf{h}}^{\mathrm{T}}(n-1)\boldsymbol{\theta}(n) + u(n) \end{aligned} \quad (21)$$

where $\boldsymbol{\theta}(n) = [-a_1\ -a_2\ \cdots\ -a_p]^{\mathrm{T}}$ is a vector of the AR parameters and $u(n) = \boldsymbol{l}\mathbf{K}(n)\alpha(n)$.

When the channel is assumed stationary, the AR parameters are time-invariant and satisfy the following relationship:

$$\boldsymbol{\theta}(n) = \boldsymbol{\theta}(n-1) \quad (22)$$

Equations (21) and (22) hence define a state-space representation for the estimation of the AR parameters. By defining the AR parameter estimation error as $e_{\boldsymbol{\theta}} = \hat{\mathbf{h}}^{\mathrm{T}}(n-1)\boldsymbol{\theta}(n) - \hat{\mathbf{h}}^{\mathrm{T}}(n-1)\hat{\boldsymbol{\theta}}(n)$, a second H_∞ filter can be used to recursively estimate $\boldsymbol{\theta}(n)$ as follows:

$$\hat{\boldsymbol{\theta}}(n) = \hat{\boldsymbol{\theta}}(n-1) + \mathbf{K}_{\boldsymbol{\theta}}(n)\alpha_{\boldsymbol{\theta}}(n), \quad \hat{\boldsymbol{\theta}}(0) = \mathbf{0} \quad (23)$$

$$\alpha_{\boldsymbol{\theta}}(n) = \hat{h}(n) - \hat{\mathbf{h}}^{\mathrm{T}}(n-1)\hat{\boldsymbol{\theta}}(n-1) \quad (24)$$

$$\mathbf{K}_{\boldsymbol{\theta}}(n) = \mathbf{P}_{\boldsymbol{\theta}}(n)\mathbf{C}_{\boldsymbol{\theta}}^{-1}(n)\hat{\mathbf{h}}(n-1)R_u^{-1} \quad (25)$$

$$\begin{aligned}\mathbf{C}_{\boldsymbol{\theta}}(n) &= \mathbf{I}_p - \gamma_{\boldsymbol{\theta}}^{-2}\hat{\mathbf{h}}(n-1)\hat{\mathbf{h}}^H(n-1)\mathbf{P}_{\boldsymbol{\theta}}(n) \\ &\quad + \hat{\mathbf{h}}(n-1)R_u^{-1}\hat{\mathbf{h}}^H(n-1)\mathbf{P}_{\boldsymbol{\theta}}(n)\end{aligned} \quad (26)$$

$$\mathbf{P}_{\boldsymbol{\theta}}(n+1) = \mathbf{P}_{\boldsymbol{\theta}}(n)\mathbf{C}_{\boldsymbol{\theta}}^{-1}(n), \quad \mathbf{P}_{\boldsymbol{\theta}}(0) = \mathbf{P}_{\boldsymbol{\theta}0} \quad (27)$$

where $R_u > 0$ and $\mathbf{P}_{\boldsymbol{\theta}0} > 0$ are the weighting parameters. In addition, $\gamma_{\boldsymbol{\theta}} > 0$ is the disturbance attenuation level. To guarantee the existence of this H_∞ filter, the attenuation level $\gamma_{\boldsymbol{\theta}}$ should be selected in the same manner as γ in (19).

It should be noted that our approach is different from the one in [17] where two serially connected H_∞ filters are used to estimate the fading process and its AR parameters. On the one hand, the approach in [17] yields biased values of the AR parameters which may result in poor estimation of the fading process. On the other hand, our approach can provide better estimation quality as it makes it possible to estimate the AR parameters from the estimated fading process.

2.4 Tuning the parameters Q_w, R_v, R_u, $\mathbf{P}_0$ and $\mathbf{P}_{\boldsymbol{\theta}0}$

As stated in Sect. 2.2, the weighting parameters Q_w and R_v in the first H_∞ filtering algorithm (12)–(18) often correspond respectively to the instantaneous power of the sequences $w(n)$ and $v(n)$.

In [17], the authors mentioned that in practical wireless communication systems the weighting parameters Q_w and R_v can be chosen respectively as the variance of the driving process and the additive sequence (i.e., $Q_w = \sigma_w^2$ and $R_v = \sigma_v^2$). Thus, by analogy with the Kalman filter theory, the weighting parameter Q_w can be recursively tuned as [35]:

$$\hat{Q}_w(n) = \lambda\hat{Q}_w(n-1) + (1-\lambda)\mathbf{f}\mathbf{M}(n)\mathbf{f}^{\mathrm{T}} \quad (28)$$

where $\mathbf{M}(n) = \mathbf{P}(n) - \Phi\mathbf{P}(n-1)\Phi^{\mathrm{T}} + \mathbf{K}(n)|\alpha(n)|^2\mathbf{K}^H(n)$, $\mathbf{f} = [\,1\ 0\ \cdots\ 0\,]$ and λ is the forgetting factor.

Here, the parameter R_v is assigned to the true value of σ_v^2. In addition, we suggest setting R_u to the power of the process $u(n)$:

$$R_u = \boldsymbol{l}\mathbf{K}(n)|\alpha(n)|^2\mathbf{K}^H(n)\boldsymbol{l}^{\mathrm{T}} \quad (29)$$

Furthermore, as we have no a priori knowledge about the initial state error, the weighting matrices $\mathbf{P}_0$ and $\mathbf{P}_{\boldsymbol{\theta}0}$ are assigned to the identity matrix (i.e., $\mathbf{P}_0 = \mathbf{P}_{\boldsymbol{\theta}0} = \mathbf{I}_p$).

It should be noted that, although the selected values of the weighting parameters might not correspond to the true values (i.e. instantaneous powers of the corresponding sequences), the H_∞ filtering by its nature is robust to this deviation [16].

3 Simulation results

In this section, we carry out a comparative simulation study on the estimation of the fading channel AR parameters between several methods:

1. The two cross-coupled H_∞ filters.
2. The two cross-coupled Kalman filters [35].
3. The two serially connected H_∞ filters [17].
4. The two serially connected Kalman filters [17].
5. The YW estimator: routine *aryule* of MATLAB 7.1.

In the first experiment, the fading process $h(n)$ is generated according to the autoregressive channel simulator [14] and a given Doppler rate $f_d T$. It is normalized to have a unit variance, i.e. $\sigma_h^2 = 1$. A zero-mean complex white Gaussian

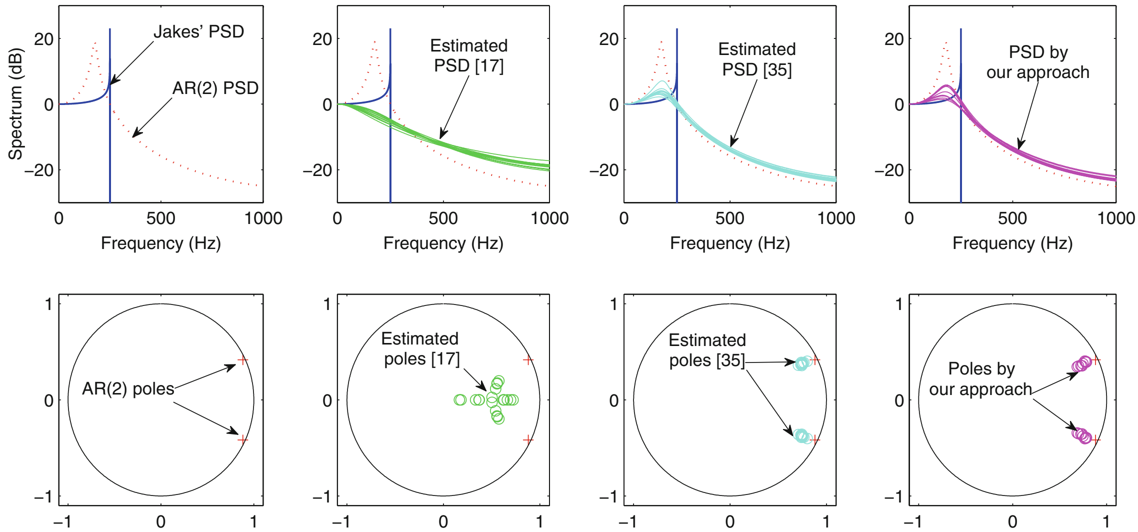

Fig. 5 Estimations of AR(2) PSD and corresponding poles[3] in the z-plane obtained by using the different methods. For each method, 10 realizations are plotted. $+$ corresponds to a pole to be retrieved whereas $\circ$ is related to the estimated pole for one realization. $f_d T = 0.1$, SNR = 10 dB and $N = 500$

noise $v(n)$ with variance σ_v^2 is then added to $h(n)d(n)$. Thus, the SNR is defined as follows:

$$\text{SNR} = 10\log_{10}\left(\frac{\sigma_h^2}{\sigma_v^2}\right) = 10\log_{10}\left(\frac{1}{\sigma_v^2}\right) \tag{30}$$

Here, the additive noise variance σ_v^2 is assumed to be available. The level attenuation factor γ in the H_∞ filters is updated at each iteration to satisfy Eq. (19). In our simulations, to avoid any singularity of the matrix $\mathbf{P}(n+1)$, the attenuation factor γ is selected to be larger than the minimum allowable value multiplied by at least a factor of $\zeta > 2$. This factor depends on the AR model order. The smaller the order, the lower the factor.

When $f_d T = 0.1$, $N = 500$ and $p = 2$, Baddour's simulator [14] provides a channel process whose AR parameters are $a_1 = -1.7625$ and $a_2 = 0.9503$. According to Fig. 5 and Table 1, the two cross-coupled H_∞ filter based approach provides approximately the same results as the two cross-coupled Kalman filter based one proposed in [35]. In addition, the two cross-coupled Kalman and H_∞ filter based approaches yield much better estimates than the other approaches, especially at low SNR. Indeed, the YW estimator and the two serially connected Kalman or H_∞ filter based estimators result in biased AR parameter estimates and smoothed spectra. Similar results are obtained with high-order AR models. Therefore, although the two cross-coupled H_∞ filter based approach does not provide better results than the two cross-coupled Kalman filter based one, it has the advantage of relaxing the Gaussian assumptions required by Kalman filtering.

Table 1 Average AR(2) parameters and driving process variance estimates based on 1,000 realizations. The true values are $a_1 = -1.7625$, $a_2 = 0.9503$ and $\sigma_w^2 = 0.0178$. $N = 500$ and $f_d T = 0.1$

SNR		10 dB	15 dB	20 dB	40 dB
Two cross-coupled	$\hat{a}_1$	−1.4878	−1.6746	−1.7400	−1.7576
H_∞ filters	$\hat{a}_2$	0.6975	0.8724	0.9336	0.9454
	$\hat{Q}_w$	0.0882	0.0192	0.0066	0.0031
Two cross-coupled	$\hat{a}_1$	−1.4908	−1.6348	−1.7001	−1.7557
Kalman filters [35]	$\hat{a}_2$	0.6991	0.8326	0.8926	0.9434
	$\hat{\sigma}_w^2$	0.0682	0.0344	0.0280	0.0167
Two serially connected	$\hat{a}_1$	−1.0274	−1.4018	−1.6184	−1.7567
H_∞ filters [17]	$\hat{a}_2$	0.2601	0.6043	0.8107	0.9443
Two serially connected	$\hat{a}_1$	−1.0293	−1.4009	−1.6177	−1.7560
Kalman filters [17]	$\hat{a}_2$	0.2616	0.6033	0.8100	0.9436
Yule–Walker	$\hat{a}_1$	−1.0318	−1.3958	−1.6031	−1.7359
Routine *aryule* of	$\hat{a}_2$	0.2647	0.5995	0.7970	0.9249
MATLAB 7.1	$\hat{\sigma}_w^2$	0.3284	0.1506	0.0704	0.0252

In the second simulation experiment, the fading process $h(n)$ is generated according to the modified Jakes simulator [3] with $f_d T = 0.05$. It is normalized to have a unit variance (i.e., $\sigma_h^2 = 1$). A complex Gaussian white noise $v(n)$ is then added to $h(n)d(n)$. According to Fig. 6, the proposed approach yields lower Bit Error Rate (BER) than the Kalman based one [35] when considering AR(1) and AR(2) models. Hence, the proposed method is more robust to modeling approximations than the one proposed in [35]. For high-order AR models (e.g., $p = 5$, 10 or 20), both approaches provide approximately similar results.

[3] $H(z) = \frac{1}{1+\sum_i a_i z^{-i}} = \frac{1}{\prod_i (1-p_i z^{-1})}$, where p_i is the ith pole. Hence, the positions of the poles in the z-plane are related to the AR parameters and the shape of the process PSD.

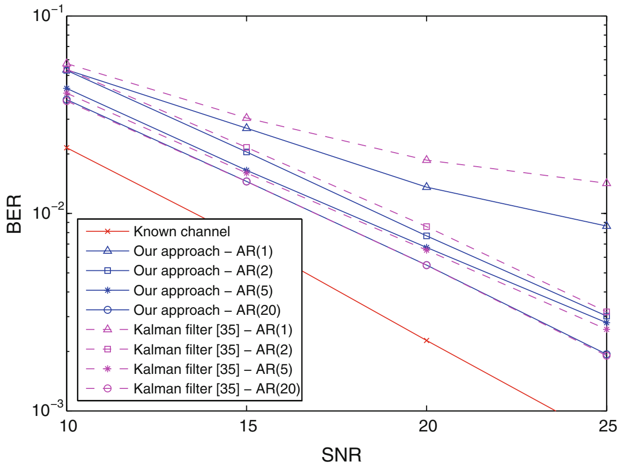

Fig. 6 Performance of the proposed H_∞ filter based channel estimator and the one based on two cross-coupled Kalman filter [35], with various order AR models. $f_d T = 0.05$ and $f_d = 250$ Hz

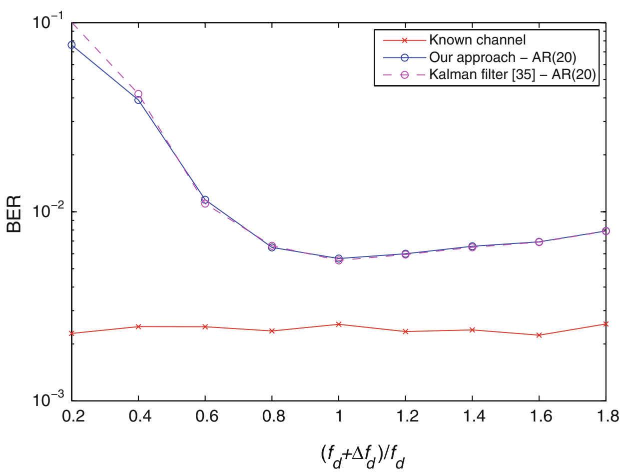

Fig. 7 Performance of the proposed two cross-coupled H_∞ filter based channel estimator and the one based on two cross-coupled Kalman filters [35], when using AR(20) model. $f_d = 250$ Hz, $-0.8 f_d \le \Delta f_d \le 0.8 f_d$ and SNR $= 20\,dB$

In the above simulation experiments, we consider fast fading processes with fixed Doppler frequency. However, due to mobile movement, the Doppler frequency might change and, hence, the AR parameters should be re-estimated. According to the various tests we have carried out, it takes about 200 training symbols for the proposed H_∞ estimator to converge to the true AR parameter values. To evaluate the robustness of the Kalman and H_∞ filter based approaches against the changes in Doppler frequency, we consider a scenario in which we used the AR parameter estimates obtained during the last training cycle to estimate in the decision directed mode the fading process when the Doppler frequency changes. In that case, the AR parameter estimates correspond to Doppler frequency $f_d + \Delta f_d$ with $-0.8 f_d \le \Delta f_d \le 0.8 f_d$, while the fading process is estimated when the Doppler frequency is f_d. Figure 7 shows the BER performance of the Kalman and H_∞ filter based estimators versus $(f_d + \Delta f_d)/f_d$ when $f_d = 250$ Hz. When the Doppler frequency changes (i.e., $\Delta f_d \neq 0$) both approaches suffer from a degradation in BER performance especially when $\Delta f_d < 0$. Nevertheless, one can notice that when $(f_d + \Delta f_d)/f_d$ is lower than 0.4, our approach outperforms the Kalman based solution.

4 Conclusion

In this paper, we present a method for the joint estimation of the time-varying fading processes and their corresponding AR parameters based on two cross-coupled H_∞ filters. The estimation criterion of the proposed approach is based on the minimization of the worst possible effects of the noise disturbances (i.e., the initial state, the driving process and the measurement noise) on the estimation error. As this criterion requires no a priori constraints about the noises except that they have bounded energies, the presented method is more robust against the noise disturbances and modeling approximations than existing methods based on Kalman filtering. According to the comparative simulation study we carried out on fading channel estimation, the proposed approach outperforms the existing two serially connected H_∞ filter based method.

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