



**Deanship of Graduate Studies**

**Al-Quds University**

**Application–Based Statistical Approach for  
Identifying Appropriate Queuing Model**

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**M. Sc. Thesis**

**Jerusalem-Palestine**

**1436/2015**

# **Application–Based Statistical Approach for Identifying Appropriate Queuing Model**

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A thesis submitted in partial fulfillment of requirements for  
the Master's degree in Computer Science /Department of  
computer Science / Faculty of Graduate Studies Al-Quds  
University

**1436/2015**



**Al-Quds University**

**Deanship of Graduate Studies**

**Computer Science Department**

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Master thesis submitted and accepted on :14/5/2015

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Jerusalem-Palestine

1436/2015

## **Dedication**

This work is dedicated..

To my kids for their love  
Especially my daughter Sara

To my great professors  
For their support and encouragement

“Thanks for you all”

Naimeh Hirbawi

## **Declaration**

I certify that this thesis submitted for the Master's Degree, is the result of my own research, except where otherwise acknowledged, and that this study (or any part of it) has not been submitted for a higher degree to any other university or institution.

Signed.....

Naimeh Mohammad Salem Saleh Hirbawi

Date:14/5/2015

## Acknowledgments

My first and greatest appreciate to my thesis advisor Dr. Badie Sartawi for his encouragement and support from the beginning to the end of the research. I am extremely grateful to Dr. Badie for giving me a sense and guidance of direction, and also for helping me by his invaluable advices and insight.

*I hope that my words may have given my great Professor Dr. Badie his rights of gratitude and thanks.*

## Abstract

Queuing theory is a mathematical study of queues or waiting lines. It is used to model many systems in different fields in our life, whether simple or complex systems. The key idea in queuing theory of a mathematical model is to improve performance and productivity of the applications. Queuing models are constructed in order to compute the performance measures for the applications and to predict the waiting times and queue lengths.

This thesis is depended on previous papers of queuing theory for varies application which analyze the behavior of these applications and shows how to calculate the entire queuing statistic determined by measures of variability (mean, variance and coefficient of variance) for variety of queuing systems in order to define the appropriate queuing model.

Computer simulation is an easy powerful tool to estimate approximately the proper queuing model and evaluate the performance measures for the applications. This thesis presents a new simulation model for defining the appropriate models for the applications and identifying the variables parameters that affect their performance measures. It depends on values of mean, variance and coefficient of the real applications, comparing them to the values for characteristics of the queuing model, then according to the comparison the appropriate queuing model is approximately identified.

The simulation model will measure the effectiveness performance of queuing models  $A/B/1$  where A is inter arrival distribution, B is the service time distributions of the type Exponential, Erlang, Deterministic and Hyper-exponential. The effectiveness performance of queuing model are:

- $L$  : The expected number of arrivals in the system.

- $L_q$ : The expected number of arrivals in the queue.
- $W$ : The expected time required a customer to spend in the system.
- $W_q$ : The expected time required a customer to spend in Queue.
- $U$ : the server utilization.



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## Abbreviation and Definitions

Symbol	Name	Definition
<b>M</b>	Markov or memory-less	Poisson or exponential arrival or service process
<b>D</b>	Degenerate distribution	A fixed or deterministic Arrival or service process
<b>G</b>	General distribution	G refers to independent arrivals
<b>E<sub>k</sub></b>	Erlang distribution	Erlang distribution with
<b>H<sub>k</sub></b>	Hyper Exponential with parameter <i>k</i>	Continuous probability distribution
<b>FIFO/FCFS</b>	First in first out/First come first out	The way in which the customers are served
<b>C/S</b>	Server	Number of server
<b>λ</b>	Lambda	Arrival rate
<b>μ</b>	Mu	Service rate
<b>P<sub>n</sub></b>	Probability	Probability of exact n of customers or jobs in the

		system
<b>L</b>	Length of the system	The expected numbers of customers/jobs in the system
<b>L<sub>q</sub></b>	Length of the queue	The expected numbers of customers/jobs in the queue
<b>W</b>	Wait time in system	The expected time of a customer/job spends in the system
<b>W<sub>q</sub></b>	Wait time in queue	The expected time of customer/job spends in the system
<b>U</b>	Server utilization	A measure of how “busy” the system is
<b>σ</b>	Standard deviation	Measure the amount of variance
<b>V</b>	Variance	
<b>n</b>	Population size	number of customers /jobs
<b>CV</b>	Coefficient of variance	Ratio of σ to the mean

## Chapter One

---

### 1.1 Introduction

Queuing theory is used to deal with systems that include queues (waiting times). It enables mathematical analysis of the behavior of systems in order to evaluate of the performance measures of the systems including waiting time in system/queue, utilization of server and so on. Queuing theory is adopted in many fields to predict the expected time a customer spends in the queue likewise in the system and it is useful in defining the optimal number of servers in systems. Queuing analysis is very important in capability problems which are very popular in many applications especially in industry and one of the fundamental operations of redesign process. It requires to balance between the cost of increased capacity and the gains of increased service and productivity. It is also important to computer networking because it can predict the length of time for the data it request.

There are many queuing models in the world which are used to model the approximate real application by analyzing mathematically the behavior of the application. The first step in solving any practical problem is to define the appropriate queuing model. In queuing theory they always begin with the simple model, but if the results are not proper to this case they move to another one but more complicated [1]. In simple models the simple formulas are used because it can easily predict the effect of a given parameter on the performance

measures of the system, but if other distributions are used then the mathematical models will be more complicated. The most common models are stochastic ones.

“Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems” (Nelson, Nicole, Banks and Carson, 2009).

In queuing models, we define number of important performance measures such as the expected number of customers /jobs in the queue and system, the expected time a customer/job spent in queue and system, the probability distribution for those numbers and times, the utilization of the server whether it is full or empty, and the probability of the queue and system in steady state [1][2][3]. These performance measures are significant to improve the system and predict the effect of suggested changes in the system.

The aim of the thesis is how to identify the appropriate queuing model for the application and calculate the performance measures for it. It will help to enhance more productive and efficient systems that decrease the waiting time in system and increment more number served customers; It will help analysts not to move from model to another if the model not fit their application in order to improve the performance for it.

The thesis is based on historical data (previous papers) which analyze the behavior of the different applications and define the queuing model.

## **1.2 Research Problem Statement**

Queuing theory is used to study the waiting lines (queues) in various applications. Queuing models are used to typify the different queuing systems that occur in reality. Each model uses different formulas to denote how to accomplish the corresponding system, including the average number of customers and the average time of customers that spend in system



,under various circumstances. These queuing models are useful in developing systems more productive and efficient. They can supply more service capacity with lower limit of cost. They have ability to achieve equilibrium between the cost of system capacity and the amount of waiting time.

**Figure 1.1 queuing System**



The biggest challenge in queuing theory is defining the queuing models for different systems, so we state our research problem as follows: How can we identify the queuing models and determine the appropriate probability distribution for the characteristics of the queuing models in correspondence with the parameters real systems and improve the performance measures for them?

### 1.3 Research Motivation

Our motivation for this research are the following:

- 1- Most of the previous papers has specified their queuing models as Markov models. “ Network arrivals are often modeled as Poisson processes for analytic simplicity, even though a number of traffic studies have shown that the packet interarrivals are exponentially distributed. We evaluate ... to determine the error introduced by modeling them using Poisson processes”, (Vern Paxson and Sally Floyd, 1995).

- 2- Most of the papers used the same models that identified by the previous ones for the same application, without any analysis to be sure that they fit their application or not even if there are different parameters. For example call centers applications usually used Erlang C distribution, “ ... the Erlang C model, a queuing model commonly used to analyze call center performance... our findings indicate that the Erlang C model is subject to significant error in predicting system performance” (Robins, Medeiros and Harrison, 2010).
- 3- In queuing systems most mathematical theories suggest to use the specific probability distribution such as exponential distribution in order to model the uncertainties in arrival time of customers and service time [6][12]. The previous papers in constructing the system depend on real observations indicated that the assumption of exponentially distribution of arrival times can be invalid (Law and Kelton, 2000).

#### **1.4 Research objectives**

The objectives of this research are the following:

- 1- Using quantitative computations for defining the appropriate queuing models for the application by estimating the characteristics of the queuing model:
  - The arrival rate of customers/jobs.
  - The service rate of the system.
- 2- Increasing the understanding of the system and predicting the behavior of the future system.
- 3- Helpful in estimating the performance measures of the applications and support in identifying the parameters that affect the performance measures of the application.

- 4- Using technical tools and computations for predicting the queuing model from which enable to use the optimization model.
- 5- Valuable and very useful, especially for people who do not have enough background in probability distributions, statistical analysis and mathematical simulation. "... There are probably 40 queuing models based on different management goals and conditions... and it is easy to apply the wrong model, if one does not have a strong background in operations research" (Weber, 2006)
- 6- Most of the queuing models are Markov models, in this research we highlight the other queuing models that are more close to the real systems .

### **1.5 Area of Applications**

Queuing theory is spread wide used in different applications such as computer systems, networks and service centers.

In this research ,we dealt with various real systems that have been used in the previous papers. We have classified them as follows:

- 1- Commercial systems such as banks.
- 2- Networks systems such as compute support.
- 3- Business systems .
- 4- Service systems such as ATM
- 5- Transportation systems.
- 6- Social systems such as ER in a hospital.

## **1.6 Research Contribution**

In queuing theory, the purpose of designing a system is to make sure that the system meets the same requirements. We have modeled the system by analyzing the behavior and the properties, which enable us to improve and develop the performance of the system. Different statistical and analysis techniques are used in analysis. Also many computer simulation are used to evaluate the performance measures of the system but none of them can define the queuing model. The purpose of this thesis is to present an easy computer simulation model that defines different queuing model and estimates the performance measures for various applications, also shows how to calculate the entire queuing statistic determined by measures of variability (mean, variance and coefficient of variance) for variety of queuing systems. In order to yield more precise system measures for any static distribution by identify the approximate appropriate queuing model for different application without need for deep probabilistic background to understand the different distribution functions.

## **1.7 Synopsis of the research**

In chapter two, we present the background, related work, queuing models are described and comparison between different techniques for defining the queuing models. In chapter three, our methodology is explained ,different types of arrival rate distribution and service time distribution are identified and evaluated. In chapter four, we discussed the analysis of our study for determining the queuing models.

In chapter five, we introduced computer simulation and validations for different kinds of queuing theory. Finally, in chapter six the conclusion of the thesis and future work are presented.

## Chapter Two

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### 2.1 Background

Queuing theory was invented in the 1909s, by A.K Erlang to improve the development of telephony applications. In the 1950s, Leonard Kleinrock applied queuing theory to computer network, James Jackson studied queuing theory in network of multiple nodes . In 1953, G Kendall introduced the notation A/B/X/Y/Z type of queuing theory.

In 1957, James Jackson studied queuing theory in network of multiple nodes, and presented an open queuing network with exponential servers and Poisson distribution. Also he introduced the steady state has a product form. Parallel queues were presented by F. Haight in 1958. In 1961 J. Little a formula of mean number of customers in queue and system was proved from mean waiting time which known as Little's law. In 1963, Jackson introduced another close queuing networks that have exponential servers and queuing networks which arrival process depends on the state of the system. In 70's, most of researches in queuing theory were to evaluate the performance of the computer. In 2000, William Stallings provided a practical guide of queuing analysis [2]. Joseph M. Lancaster, Mark A, Franklin,

Jeremy Buhler and Roger D. Chamberlain used queuing theory to model streaming applications.

In recent years, they use queuing theory to model software and now the researchers are interested in modeling the insurance systems and studying the retrieval queues.

## 2.2 Related Work

Good references for fitting and estimation the queuing models are the early edition of Gross and Harris (1974), Hall (1991) and Law (2007). Hall (1991) describes goodness of fit tests in different models, (Green and Nguyen, 2001) satisfying the service time assumption of the M/M/s model in healthcare where the coefficient of variance of length of stay is very close to one. (Green, 2006) assumed that service time is follow an exponential distribution or Erlang distribution with the coefficient of variance close to one. (Noln and Thomopoulos, 2006) show how to calculate standardize system statistics for the Erlang distributed interarrival and service time which determined by the coefficient of variance and found that the queuing model (E1/E1/1 to E10/E10/1) the coefficient is range from [1-0.32] and when the model is (E10+/E10+/1) the coefficient is close to zero.

## 2.3 The Main Characteristics of Queuing Process

- **Arrival Pattern of customers:** This refers to the probability distribution of inter arrival times which means the arrival time of successive arrivals (customers/ jobs).
- **Service Time Pattern:** This refers to the probability distribution of service times which depends on many factors; such as number of customers waiting in queue for service and number of servers.
- **Numbers of Servers:** This refers number of servers which serve the system .

- **System Capacity:** The queuing can be finite or not finite, the system capacity refers to the maximum amount of the system that can contain such as waiting room, if there is not enough space in room for waiting the following customer must leave the system.
- **Queue Discipline:** This refers to the way in which the customers are served in queue, the most popular queue discipline :
  - FCFS which means first come first serve.
  - LCFS which means last come first serve.
  - RSS which means random selection service.

## 2.4 Model Notation

Queuing models can be introduced by Kendall's notation:

**A/B/X/Y/Z**

A: The distribution of inter arrival time.

B: The distribution of service time .

X: The number of servers.

Y: The capacity of the system.

Z: The queue discipline.

Common distributions used in queuing models:

M: Markovian (Memory-less).

$E_k$ : Erlang with parameter  $k$ .

$H_k$ : Hyper Exponential with parameter  $k$  (mixture of  $k$  exponentials).

D: Deterministic (constant).

G: General.

## 2.5 Queuing Models

In this section, most common of queuing models will be described with their types and applications.

### 2.5.1 Markovian models (systems)

In these models the arrival process is Poisson with exponential service times [1][2].  $\lambda$  is used to indicate the mean arrival rate, and  $\mu$  is used to indicate the service time rate. We can derive the performance measures for the system from the equilibrium probabilities such as the average number of customers in queue or system and the waiting time the customers spent in queue or system. We will review the Markov models that are popular in queuing theory.

The M/M/1 or M/M/1/ $\infty$

This model is a stochastic process, and the most commonly used in systems in which the arrival process of customers has a Poisson process, and the service times of customers are determined by an exponential distribution with a single server in system. An M/M/1 queue is a good estimation for enormous numbers of queuing systems such as customer service environment, banks, phone queuing systems and so on.

The M/M/c or M/M/c/ $\infty$

The arrival process of this model is a Poisson process and the service time is an exponential distribution with multiple servers. These models arise in many systems such as computer resources applications and phone line systems.

The M/M/c/c loss systems

The arrival process and the service time in this queue are like the M/M/c model but the queue capacity is finite. In this model, if one server is free, at least all arriving customers must be



served, but if servers are busy the newly customer must leave the systems without getting any service. In this case, the customer is considered as lost.

The M/M/ $\infty$

In this model, there is no queue because it has infinite servers, so each arriving customer receives service, as well as a customer has never to wait for service if there is a server available for each arrivals of customer so we may think of such a system as self-service system. It is a suitable theoretical for applications that have delays such as parking and warehouses.

### 2.5.2 Non Markovian models (systems)

In non Markovian models either the inter arrival time or the service time has to be non-exponentially distributed. In these models the analysis and the computation for the performance measures are more complex. The most common models in non Markovian models are G/M/1 and M/G/1 which is a classical model.

The M/G/1 model

In M/G/1 model the arrival process is Poisson process and the service time is General distribution with single server[5][9]. This model is widely used in networking applications and a large number of real-life computer.

The G/M/1 model

In G/M/1 the customers arrive according to an arbitrary distribution (General), and the service time has an exponential distribution with one service.

#### 2.5.4 Network models (system)

The queuing networks have become important tools in the design and analysis of the computer system, because network models fulfil a convenient balance between accuracy and efficiency for many applications.

There are two kinds of network models; open and closed network [6][7][8], in an open queuing network the customers come from outside the network and receive a service at a systems then depart the network.

In a closed queuing network, a new customer can enter the network exactly at the same time when one customer leaves, in this sort of models the number of customers must be constant.

In this section, we will mention some of the common queuing networks

##### **Jackson network**

Jackson is the simplest models of queuing networks. The external customer in this model is defined by Poisson arrival process and the service time is exponentially distributed with one or more servers. The service time rates rely on the number of customers in the system. These models have only one customer class with infinite jobs, and customers are served as first come first serve (FIFO).

##### **Gordon -Newell networks**

These models are also called closed Jackson networks, and they have the same assumption of Jackson network excepting that the customer cannot enter nor leave the network. In this queuing network the number of jobs is always fixed.

##### **Kelly's networks**

The arrival are Poisson process and the service time is exponentially distributed. Each system in Kelly's networks can serve different classes of customers with a fixed route of network. The capacity in this system is infinity.

## 2.6 Strategies used to defining the queuing models

Queuing models can provide the analysts a robust technique to redesign and resolve the queuing systems. The basic techniques are mathematical methods and computer simulation, but all these techniques have their advantages and disadvantages. When we want to choose the appropriate technique, many factors have to be taken in our consideration such as the nature of the system, the goals that fit the needs of the system and the less analytically for its properties.

### 2.6.1 Mathematical method

Mathematical method is preferred to use if the modelling process itself is an iterative process; which has a number of separate steps that usually must be repeated. Mathematical models are more convenient for mathematical analysis, which can make good predictions about the behaviour of the system, and also mathematical methods have an efficient rules to evaluate the performance of the system and provides a real understanding of the effect of parameter changes on the properties of the system.

The following sequence steps of the mathematical method:

- 1- Data collection and empirical observation.
- 2- Functional relationship, formalization of properties and mechanisms.
- 3- Abstraction or the results of a mathematical model (e.g. boundary conditions and/or differential equations with constraints).
- 4- Model analysis which uses mathematical techniques.
- 5- Explanation and comparison the results that obtained from step four with the real system.

In mathematical methods, the probability distribution is defined for the characteristics of the queuing model by analytical method, the properties of the distribution and graphs with visual data analysis are used as the following steps:

- 1- Data is plotted for a visual representation of the data type.
- 2- Data distribution and the type of equations in modelling data are determined by excluding what cannot be as the following:
  - A) If the data set has not any peaks, then it cannot be a discrete uniform distribution.
  - B) If the data set has only one peak, then it is not a Poisson or Binomial distribution.
  - C) If the data set has one curve and has a slow slope on each side without any secondary peaks, the expected distribution is Poisson or Gamma but cannot be discrete uniform distribution.
  - D) If the distribution of the data is equal without any skew towards any side, then the a Gamma or Weibull distribution will put away.
  - E) If the function is even distributed with a peak in the graphed results, then it cannot be an exponential distribution or Geometric distribution.
  - F) If the incidence of a factor changes with the variables of the system's environment, then it is probably not the Poisson distribution.

Using mathematical methods to model systems have some difficulties and limitations

- 1- Data availability and accuracy.
- 2- The mathematical model analysis.
- 3- Assumption that supposed model is a real system.
- 4- Communication in interdisciplinary efforts.
- 5- Use of representation methods that are unfit for thorough system.

### 2.6.2 Simulation Techniques

Simulation technique is used to model the systems for its valuable aspects:

- 1- Better understanding of the system by permitting to study it in real and compressed time period, and also study and experiment the system's interactions.
- 2- Provide the analyst with obvious definition and identification for the variables in a system.
- 3- Designing a simulation is more worthy than simulation itself because the designer needs an overall understanding of the system when the simulation model is constructed, this understanding leads to discover the hidden relations between the interaction of the variables.

“The system contains a mixture of discrete events, discrete and different magnitudes, and continuous processes. Such mixed processes have generally been difficult to represent in continuous simulation models, and the common resource has been a very high level of aggregation which has exposed the model to serious inaccuracy“ (Coly, 1982 )

Computer simulation as any technique has also its disadvantages, the most common disadvantages are:

- 1- Requires high degree of details in the model which leads to a high degree of correspondence between a model and the real system.
- 2- Computer simulation does not give us a better insight of the impact of parameter change on properties in the system, such as stability and optimality of the system performance.
- 3- More time consuming and more expensive than the mathematical method, despite that the two techniques obtain the same data.

The decision of choosing one of the strategies whether mathematical method or computer simulation depend on the accuracy of the model that we want to get and the availability of techniques to analysis them. Therefore when we want to study the behavior and the properties of some applications we use mixture of the both techniques; the analytical method and simulation technique.

## Chapter Three

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### 3.1 Methodology

Queuing models are mathematical description of the queuing systems which make some specific assumptions about the probabilistic nature of the arrival and service processes, the queue discipline and number of servers. Queuing models are used to model the applications by analysing the behaviour of them. It is very important to identify the appropriate queuing model that fit the real application in order to improve the system performance and achieve a good prediction about the behavior in the future.

We have two kinds of analysis; transient and equilibrium analysis, but we can get exact analytical results for the mean performance parameters under equilibrium conditions, also in some special cases, we can obtain results on higher moments, such as variance or probability distributions. In contrast, transient analysis is impractical in general only for some very simple cases, so simulation methods are preferred to use.

Queuing models depends on the nature of the applications and their behavior. The most common models which depend on application's behavior:

- 1- Dynamic model: The state variables are changing over time , e.g. discrete event and dynamic systems.

- 2- Static models (steady state): The state of system parameter at a specific time instant and computes the system in equilibrium, e.g. optimization models.
- 3- Stochastic model: The behavior of the system cannot be completely predictable.
- 4- Deterministic models: The behavior of the system can be completely predictable and perfect understanding for the comprehensive system, e.g. insurance industry.
- 5- Discrete models: The state variables are changing only when an event occurs or change in state over time .
- 6- Continuous models: The state variables are changing in continuous manner, e.g. electric field.

### **3.2 Assumption of the study**

Constructing the queuing models depend heavily on the assumptions, our assumption for this study:

- 1- Departure and arrival rate are in steady state, which means arrival rate is less than service time in the system, and its properties are independent of time.
- 2- Assume that the application has only single server, so if the application has more than one the service time is divided on number of servers.
- 3- Assume that the size of buffer is infinite.
- 4- The queue should be unconstrained which means no balking, reneging, reworks, abandonment, or state-dependent behavior.



The most of queuing models depend on the assumption that the concerned random variables are independent with exponential distribution. This assumption is artificial because in practice the exponential distribution is rare but the memory-less (Markov) property of exponential distribution makes the analysis simple and easy, and also has only one parameter. Although it provides a good fit for inter arrival times if the service provided is random than if it involves a fixed set of tasks. It is common in client/server systems.

Memory-less property refers to the state of the system at future time which is decided by the system state at the present time and does not depend on the state at earlier time instants. Therefore, the most widely used queuing models for which relatively simple closed analytical formulas have been developed are specified as M/M/S/K/FCFS type (Hall, 1990; Lawrence and Pasternak, 1998) which refers :

M: Markov process that assumes the arrival process follows the Poisson distribution and the time service follows the exponential distribution.

S: Number of the servers which can be one server as in the simplest case or multiple.

K: The system capacity but for this model it is infinity.

FCFS: The queue discipline.

We must take into consideration the three main key characteristics of queuing process to model the real system. We must estimate each of these characteristics of the real system in accordance with them of the queuing process; arrival process distribution, service time distribution and number of servers.

### 3.3 Parameters of queuing models

$\lambda$ : Arrival rate (average number of customers /jobs arriving to the system per period of time).

$1/\lambda$  : average interarrival time.

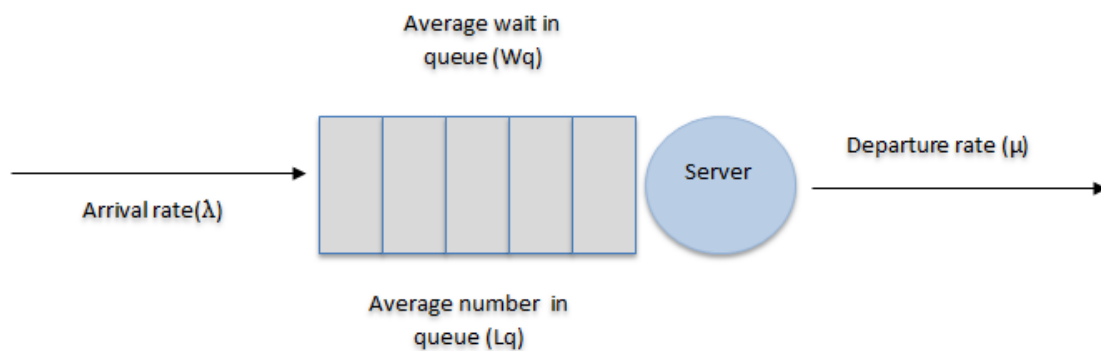
$\mu$ : Service rate (average number of customers/ jobs served in a system per period of time).

$1/\mu$ : average service time.

$c\mu$ : Service rate when c more than one (c: number of servers).

$\rho$ : Traffic intensity or utilization factor, the average utilization of the server,  $\rho$  is calculated as  $(\lambda/\mu)$ .

Figure 3.2. parameters of queuing models



### 3.4 Arrival Process and service times probability distributions

Arrivals rate means number of (customers / jobs /requests ) per period of time. Arrivals are arriving to the system in various ways. They are arriving individually or in a group, in steady rate or unsteady, independently or according to some kind of correlation and are characterized by a probability distribution.

Service times means the average amount of time required to provide the customer a service, but service rate means average number of customers which can be served per period of time. Service rate can be calculated by inverting the value of service time or vice versa. Service times are also characterized by probability distribution. From a modeling standpoint, the operational characteristics of service important more than the physical characteristics. For examples we care about whether service time are long or short, whether arrivals are served in first come first service order (FCFS) or according to some kind of priority rule, whether they are regular or highly variable.

In steady state or statistical equilibrium state, the arrival rate is less than service rate but if the opposite happened, then the system will be blocked. The service rate is sometimes constant as in many manufacturing processes. In other cases, the service times are variable depending on variations in the service requirements and customer inputs.

In this section, we will review a brief summary of the common probability distributions for arrival process and service time:

### **3.4.1 Poisson Distribution**

Poisson distribution is the most common distribution in queuing models for arrivals process. It is a discrete distribution which means the number random of events occur in fixed intervals of time and these intervals of times between successive events are independent random variables. In queuing models, **M** denotes that customers or requests for disk access behave according to a Poisson process, which refers to stochastic or Markov process thus the use of "M".

Exponential distribution means the intervals of time between successive events in a Poisson process. **M** denotes that the time between completing service for a customer is independent and follows an Exponential distribution.

If a queuing model has M for both of the inter -arrival times and the service times then the arrival rate follows Poisson distribution with Parameter  $\lambda$  and service times follow the exponential distribution with parameter  $\mu$ .

### **3.4.2 General Distribution**

In queuing models, general distribution is described G when the behavior of queue has unknown rate of customer arrivals with unknown service time distribution. We assume that the arrivals or service time distribution or both of them have general distribution when we are not able to determine which distribution would take the place of them.

### **3.4.3 Deterministic (constant) Distribution**

In deterministic model, a given input always gets the same output and does not include random variables. This distribution treats all of the input parameters as constant. Every time, the deterministic model will get the same results when we run it with the same initial conditions.

Deterministic models are typified most simple mathematical models of everyday situations.

### **3.4.4 Erlang Distribution**

The Erlang distribution is a continuous probability distribution developed by A.K Erlang for telephone calls application. It is particular case of Gamma distribution.

In queuing theory Erlang distribution is most common used to describe waiting times of telephone engineering systems.

### **3.4.5 Hypo exponential Distribution**

Hypo exponential distribution is a continuous probability distribution, also called generalized Erlang distribution. This distribution can be used in many domains of

applications. Moreover, it can be used in the same fields as Erlang distribution, such as telephone traffic engineering systems and more generally in stochastic processes.

### 3.5 Steps of fitting a theoretical distribution:

There are three steps of fitting a theoretical distribution to a sample data [15][20]:

- 1- In first step, a histogram is constructed for a sample of data. The overall shape of the histogram is compared with probability function from theoretical distributions. The aim of this step is to identify several candidate distributions for further processing. It is important to include all likely theoretical distribution candidates at this step; and those that do not fit well to the sample data will be eliminated in the third step.
- 2- The second step is to determine the distribution parameters in order to obtain the best fit of the theoretical distribution to the sample data. This should be done for all the candidates of the theoretical distributions from the first step. In this step there are two approaches that are often used: the maximum likelihood estimation method and the method of moments. The first approach identifies the distribution parameters that make the resulting distribution the most likely to have produced the sample data. The second approach equates the first  $q$  population moments with the first  $q$  sample moments ( $q$  is equal to the number of distribution parameters). The result of this step is a set of parameters for each candidate distribution that make the distribution fit the sample data very closely.
- 3- The third step is to identify the best fit theoretical distribution, from the candidate distributions to represent the sample data which depends on the use of statistical goodness of fit tests. In this step two goodness of fit tests are commonly used: the first one the chi square test which is a measure of the squared distance between the

sample data histogram and the fitted theoretical probability density function. The second one is the Kolmogorov-Smirnov (K-S) test which is a measure of the largest vertical distance between an empirical distribution and the fitted theoretical cumulative distribution function. The test results of both tests are reported as p-value (p-value is a measure of the probability that compare another data sample with the same as the present data sample given that the distribution is appropriate).

### 3.6 Approaches of the study

This study is depended on previous papers of queuing theory for varies application, the previous papers analyze the behavior of the applications and define the queuing model for them using the methods that mention above, only 5% of these papers used the measure of variability the coefficient of variance in their analysis and exploit it in defining the probability distribution for both interarrival and service time. The papers especially those which *not hinted or used* the coefficient of variance in the analysis, we reanalyze them by including histogram and tables of mean and variance for interarrival time, also tables of coefficient of variance for service time. In this paper we estimating the probability distribution for both arrival and service processes based on the measures of variability to estimate the queuing model.

Computer simulation model is presented which defines the most appropriate queuing model for a system based on results of the measures of variability and calculates the performance measures for it.

*This study is based on the role of approximate analysis of queuing systems.*

### 3.7 Evaluating Arrival Distribution

In this section we present how to calculate the entire queuing models based on mean and variance to estimate the arrival process. Many papers based the coefficient of variance to estimate the interarrival time and service time distributions [27] especially the Erlang distribution.

Interarrival times most commonly fall into one of these probability distributions (Dharmawirya and Adi, 2011):

- 1- Poisson distribution
- 2- General distribution
- 3- Deterministic distribution

In specifying the queuing model for any application, we must make assumption about the probabilistic nature of the arrival rate and service time. Since the most common assumption about arrivals processes is Poisson distribution, So if  $N(t)$  is the number of arrivals per period of time  $t$  and  $N(t)$  is Poisson distribution then,

$$\text{Probability } \{N(t) = n\} = e^{-\lambda t} (\lambda t)^n / n!$$

Where  $\lambda$  is the expected number of arrivals per period of time,  $t$  is period of time, Another way to characterize the Poisson distribution is the inter-arrival time (time between the consequence arrivals) which has exponential distribution. If  $I$  is the interarrival time of a Poisson Process with rate  $\lambda$  and  $1/\lambda$  is the average time between arrivals then,

$$\text{Probability } \{I \leq t\} = 1 - e^{-\lambda t}$$

The exponential distribution has an important property that is “memoryless”, which means the occurrence of next arrival (customer) is independent of the last arrival. Poisson process is considered the most “random “ arrival process for this reason.

Determining whether the arrival process is Poisson distribution is according to its properties. The properties (or conditions) of Poisson distribution are:

- 1- The arrivals must be identical and independent of other arrival.
- 2- The arrivals are processed in sequence and not concurrent .
- 3- The number of arrivals cannot be predictable, if the time of arrivals is known.
- 4- Arrivals can have known peaks.

If the arrival process satisfied these conditions, then the arrival process follows Poisson distribution.

The mathematical method is based on statistical equations to determine the arrival distribution as the following steps:

- 1- Pick carefully intervals of overall time.
- 2- Compute the arrival rate  $\lambda$ .
- 3- Compute the frequency of observed arrival rate which must occur over large time intervals.
- 4- Calculate the mean and variance using the following formulas:

$$\text{Mean} = \sum_1^n (\lambda * f) / N$$

$$\text{Variance} = (\sum (\lambda^2 * f) - (N * \text{Mean}^2)) / (N-1)$$

- $N$  : The total observations
  - $\lambda$  : The arrival rate.
  - $f$  : The frequency observed for an arrival rate
- 5- Compare the values of mean and variance from step four.

As a result of comparison between those values of mean and variance, the appropriate arrival rate distribution is determined approximately.



### 3.8 Evaluating Service Time Distribution

Service times can follow one of the following probability distribution:

- 1- Exponential distribution.
- 2- General distribution.
- 3- Erlang distribution.
- 4- Deterministic distribution.
- 5- Hypo exponential distribution.
- 6- Hyper exponential distribution.

This section shows how to determine the service time distribution using the coefficient of variance and for more accuracy the 95% confidence interval for service time is used. We analyzed the data from papers (*all the papers below did not used coefficient of variance or even hinted it in their analysis*) following the steps which described below.

Since the most widely used distribution for service time is an exponential distribution, first we must verify that the service time distribution is exponential or not according to its properties:

- 1- The service time based on the contents of the arrivals (customers /requests).
- 2- Most arrivals have different service times.

if the service time distribution is not exponential, it should be ruled out and search for another distribution using the mathematical method as a following step:

- 1- Compute the observations of service times.
- 2- Calculate the mean (average service time), standard deviation and variance by using the following formulas:

$$\text{Average service time } (m) = \sum_{i=1}^n \frac{S_i}{n}$$

$$\text{Standard deviation } (\sigma) = \sqrt{\sum_{i=1}^n (S_i - m)^2 / n}$$

$$\text{Variance } (V) = \sum_{i=1}^n (S_i - m)^2 / n$$

Whereas:

$n$  : number of service times

$S_i$ : service time of the observation  $i$

- 3- Compute the coefficient of variance for the service times by this formula:

$$\text{Coefficient of Variance } (CV) = \sigma / m$$

- 4- For more accuracy results the confidence of intervals for service time is used:

95% confidence intervals for service times:

$$\text{Mean (service time)} - (1.96 \text{ (SE (service time))})$$

$$\text{Mean (service time)} + (1.96 \text{ (SE(service time))})$$

$$SE = \sigma \sqrt{n}$$

Appropriate service time distribution is approximately determined in accordance with the value of the coefficient of service time.

### 3.9 Analysis problems in queuing systems

The fundamental objective of analysis the queuing systems is to get a deep understanding for the behavior of their underlying processes in order to predict the future behavior of these systems and therefore an intelligent decision can be taken to improve the performance and productivity of the systems. But many problems encountered in analysis the queuing systems which can be classified in three types:

1- Behavior problems: in queuing theory, most results based on the behavior problems, which can be avoided by understanding how system behaves under variety of conditions. Mathematical models are used in analysis to display the probability relations among the various elements of the underlying process. In queuing system, a collections of random variables ( number of customers or requests) are indexed by a parameter, such as time is known a stochastic process. We should study well the properties and dependence characteristics between the customers and time. Stochastic process under certain conditions is located to what is called steady state and it is easy to deal with system in which its distribution properties are independent of time than system with dependent time. So, if we want to get a perfect analysis a deep understanding is needed for the behavior of distribution characteristics of the stochastic process , and the random variables. The information which are obtained from this idealism analysis will be helpful in the decision making process.

2- Analytical problems: These problems mean the analysis of experimental data so as to define the proper mathematical model, and using validation methods to decide whether the proposed model is appropriate for desired system. Statistical study which can derive the properties of the mathematical model is used to choose the correct model.

Analysis and modeling of any system based on assumptions of the basic characteristics (elements ) of the queuing model, there should be analytical mechanisms to verify all these assumptions, assess all the

elements of the model and examine hypotheses which are related to the system behavior.

In analytical study, it is important to determine the dependencies among elements in the model and the dependence of the system on time. Most of the previous papers had Monrovia models, which assume Poisson distribution for arrival process and Exponential distribution for service time. Historically, these models are used in the early stage of queuing theory because they had only one parameter, and consequently easy to analysis and provide a useful results for decision makers.

- 3- Decision Problems: These problems are related to people who choose the same queuing models for their system from previous studies, without trying to re-analyze the system and investigate if the both systems have the same parameters and behave under the certain conditions.

Other problems are related to the design of the system and the measurement of the system performance.

## Chapter Four

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### 4.1 Analysis of the study

This research is based on a large number of previous studies in queuing theory, especially those papers which are *not* used the measures of variability (mean, variance and coefficient of variance). The purpose of this chapter is to obtain tables of numeric results of measures of variability for different queuing models of large number of applications, in order to get ability of designing a proposed computer simulation model for identifying the appropriate a queuing model.

### 4.2 Analysis Arrival Rate Distribution

Data is collected from previous studies and analyzed by mathematical method, we defined the parameters of the queuing model from empirical data. We calculated the arrival rates for the application and compared it with the same previous study, then the mean and variance for arrival rates are computed. According to the values of mean and variance of the arrival rate, we estimate the appropriate arrival rate distribution approximately.

Now, we will discuss a research paper [31] “The sea port application” as an example, the objective of the research was to define the queuing model for seaport and improve its the measures performance. They collected data of arrival rate (ships per day) for one year, they used actual the number of days and predicted number of days for arrival rate as illustrated in table 4.1. They determined that the arrival rate follows the Poisson distribution.

**Table 4.1 Arrival rate for seaport application**

**Table 1.** Comparison of actual versus predicted ship arrival distribution.

Arrival rate (Ships/day)	Actual number of days (A)	Predicted number of days (B)	Minimum (A) or (B)
0	1	1	1
1	6	7	6
2	18	20	18
3	27	37	27
4	49	54	49
5	73	62	62
6	60	59	59
7	61	47	47
8	37	34	34
9	15	21	15
10	9	12	9
11	5	6	5
12	2	3	2
13	2	2	2
Total	365	365	336

We analyzed this application again using the measures of variability:

- 1- The mean and variance for arrival rates (ships per day) are calculated twice using the same equations formulas that reported in chapter three.
- 2- The first one we used the actual number days to compute the mean and variance for arrival rate ,the second one we used the predicted number of days ,as clarified in table 4.2 .
- 3- The values of mean and variance for arrival rate are compared to identify the appropriate arrival rate distribution.

**Table 4.2 Table of our calculations for mean and variance of arrival rate**

paper name :Application of queuing theory to the container terminal at Alexandria seapor									
Arrival rate (A)	Actual number of days(B)	predicted number of day ( C )	mininum of	A*B	A8C	A**2	A**2*B (L)	A**2*C(M)	
0	1	1	1	0	0	0	0	0	
1	6	7	6	6	7	1	6	7	
2	18	20	18	36	40	4	72	80	
3	27	37	27	81	111	9	243	333	
4	49	54	49	196	216	16	784	864	
5	73	62	62	365	310	25	1825	1550	
6	60	59	59	360	354	36	2160	2124	
7	61	47	47	427	329	49	2989	2303	
8	37	34	34	296	272	64	2368	2176	
9	15	21	15	135	189	81	1215	1701	
10	9	12	9	90	120	100	900	1200	
11	5	6	5	55	66	121	605	726	
12	2	3	2	24	36	144	288	432	
13	2	2	2	26	26	169	338	338	
<b>91</b>	<b>365</b>	<b>365</b>	<b>336</b>	<b>2097</b>	<b>2076</b>	<b>819</b>	<b>13793</b>	<b>13834</b>	
			Mean	5.745205479	5.687671233	Variance	5.567017914		
	<b>N=365</b>		mean**2	33.007386	32.34960405				
			<b>N*mean*2 (P)</b>	<b>12047.69589</b>	<b>11807.60548</b>				

In this application, the value of mean approximately equals the variance for the arrival rate, which means that the arrival rate distribution is Poisson.

After we analyzed a large number of previous studies in queuing models and adopted the measures of variability in order to identify approximate the appropriate arrival rate distribution, we have tables of numeric results for mean and variance for varies distributions of different applications which means:

- If the mean and variance for the arrival rates are equal ,the arrival rate follows approximate Poisson distribution .
- If the arrival rates do not change over time, then the arrival rate follows approximate deterministic distribution.
- If there is no pattern of distribution ,the arrival rate follows approximate General distribution .

The table below demonstrates the values of mean and variance of the arrival for different applications from previous studies and their proposed distribution and , also the expected distribution.

**Table 4.3 Table of values of mean and variance for arrival rate for different applications**

Application Name	$\lambda$	$\mu$	Mean (M)	Variance (V)	The proposed Distribution by previous studies	The expected Distribution by our study	conclusion
seaport	5.67	0.18	5.68	5.57	Poisson	Poisson	$m = V$
Health care	28	14	6.277 2	7.07171 9	Poisson	Poisson	$m = V$
Electronic data system	1	1.428571	2	0	Deterministic	Deterministic	$\lambda$ stays constant

### 4.3 Analysis of Service Time

Each probability distribution has its characteristics that influences determining the service times distribution :

- 1- If the service time based on the content of the arrivals and most of them have different service times ,then the service time distribution is exponential .



- 2- If the service time is not based on the content of arrivals , then the service time distribution is constant.
- 3- If the service time based on the content of the arrivals and the same arrivals have the same service time , then the service time distribution is Hyper exponential .
- 4- If the entire service times are equal to the entire of all parameters of the service time ,then the service time distribution is Hypo exponential distribution .
- 5- If the arrival rate is unknown with unknown service time , then the service time distribution is General distribution , which is the most complex model .

the service time depends on the value of the coefficient of variance , in which the appropriate service time distribution can be determined .

Now , we will describe the ATM application from previous studies, the service time is calculated for three months in different times of the ATM application, and proposed that the service times follows the deterministic distribution .

We reanalyze the application as the following steps:

- 1- The mean of the service time , the standard deviation, and the variance are computed .
- 2- For more accuracy results we used 95 % confidence intervals for service time .
- 3- The coefficient of the service time is calculated , which is equal 0.443277 as shown in table 4.4.

4- The values of the coefficient of the service time is used to determine the approximate service time distribution.

**Table 4.4 Table of our calculations for the coefficient of service time**

ATM application	service time (Si)	Si-AV	(Si-AV) <sup>2</sup>
	90	31	961
	60	1	1
	30	-29	841
	60	1	1
	15	-44	1936
	75	16	256
	30	-29	841
	50	-9	81
	90	31	961
	90	31	961
sum	590	total(F)	6840
AV	59		
N	10	F/N	684
		f/n-1	760
SD	26.15339366	SD /n-1	27.5680975
COV	0.443277859		0.46725589

Average Service Time (AV) =  $\sum_{i=1}^n Si/n$   
 Standard Deviation ( $\sigma$ ) =  $\sqrt{(\sum_{i=1}^n (Si - AV)^2) / (n-1)}$   
 coefficient =  $\sigma / AV$ .

Where:

- n is the number of observations for that service time,
- Si is the service time for observation i.
- CI: confidence Interval

SE	8.717798	8.270429	plus	75.21004	0.347738
Se*1.96	17.08688	16.21004	minus	42.78996	0.611204
pluse	76.08688				
minus	41.91312	co			

95% Confidence Intervals for Service Time:  
 Mean(service time) - 1.96 (SE(service time))  
 [Mean(service time) + 1.96 (SE(service time))]  
 SE = SD/sqrt(n) = 26.15 (standard error of the mean)  
 95% Confidence Intervals for Service Rate:  
 1/(Mean(service time) + 1.96 (SE(service time)))  
 99% = 2.326  
 90% = 1.645

The following table shows the values of the coefficient of confidence of mean 95% for different applications from previous studies, and the expected probability distribution the service time .

**Table 4.5 Table of coefficient of variance for service time for different applications**

Application name	Mean	Standard derivation	Coefficient	$\mu$	Coefficient of Confidence Of mean 95%	The expected distribution by our study	The distribution from previous studies
Bank (free days)	120	32.86335	0.273861	0.008333	[ 0.234121 - 0.32985 ]	Deterministic	Deterministic
Bank (busy days)	95	75	0.789474	0.010526	[0.741121- 0.844576]	Exponential	Exponential
Bank (free days)	108	33.40659	0.30932	0.009259	[ 0.259558- 0.382689]	Deterministic	deterministic
seaport	5.58	1.43	0.256272401	0.18	N/A	Erlang	Erlang

<b>Electronic data system</b>	8.857142857	6.127888742	0.611858406	0.112903226	[0.5811243-0.633453]	Erlang	Erlang
<b>Supermarket system</b>	0.01818	0.0146	0.80310	55	[0.7884 - 1.1388]	Exponential	Exponential
<b>Telephone Call Center</b>	201	248	1.233	0.004951	N/A	Exponential	Exponential

After we analyzed a large number of previous studies in queuing models and adopted the measures of variability in order to identify approximately the service time distribution, we have tables of numeric results for coefficient of variance for different distributions which means:

- 1- If the coefficient of the service time is close to zero , then the service time follows the deterministic distribution.
- 2- If the coefficient of the service time is close to one , the service time follows the exponential distribution. A Chi-squared test and the rule thumb are to define the range of close to one is [0.7-1.3].
- 3- If the coefficient of service time is between zero and one ,but not considerably close to any one of them ,then the service time follows the Erlang distribution.
- 4- If the coefficient of service time is greater than one, then the service time follows the Hyper exponential distribution .
- 5- If the service time is none of these values, then the service time follows the General distribution .

## Chapter Five

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### 5.1 Simulation and Validation

Simulation is a powerful tool to use for systems that change with time. Different simulation techniques are helpful in decision making, especially when the analytical methods are inapplicable or unavailable. Computer simulation is very useful to analyze the future systems and to predict the parameters that affect the behavior and system performance.

We present a simulation model that determine the appropriate queuing model for different applications. Approximately the proper distribution for the arrival rate is identifying by the values of mean and variance and the distribution for the service time is identifying by the value of the coefficient of variance. The simulation model will measure the effectiveness performance of A/B/1 models where A is interarrival distribution of type Poisson and B is distributions of the type Exponential, Erlang, Deterministic and Hyper-exponential. Computer simulation used to validate the results from the previous studies and proved the accuracy of the results obtained from our mathematical method .

## 5.2 Simulation Model Methodology

A novel technique is presented in defining the appropriate queuing model and evaluating the performance measures for different application. This simulation model will be a valuable tool in queuing theory for its ability to identify the proper probability distribution for arrival rate and service time without the need for the application analysis. In all the previous simulation models, the user has to identify the arrival rate, arrival rate distribution, service time and the service time distribution as inputs, then the simulation model evaluates the system performance.

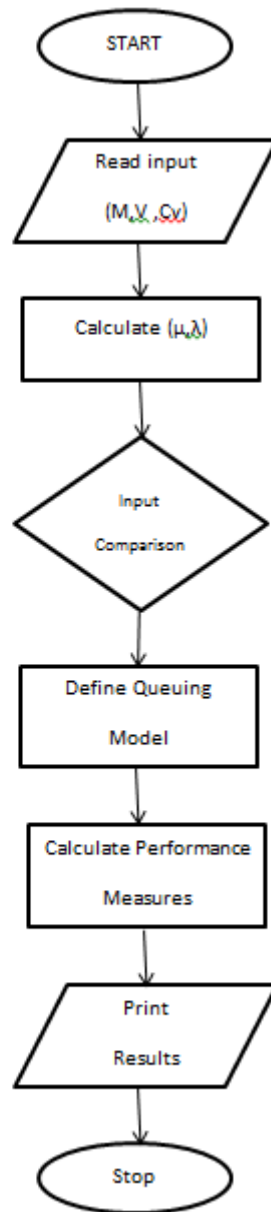
The simulation model will be used to simulate the interarrival time distribution of Poisson and the service time distributions of types ( Exponential, Erlang, deterministic and Hyper exponential distribution).

The methodology principle of simulation model as follows :

- 1- The probability distributions for both arrival and service processes are determined according to the measures of variability; The interarrival time distribution will be determined by comparing the mean and variance. The service time distribution will be determined according to the values of coefficient of variance.
- 2- The Inputs of the model are:
  - The mean and the variance for the arrival rate.
  - The mean and coefficient of variance for service time.
- 3- The outputs of the simulation models are:
  - The arrival rate ( $\lambda$ ).
  - The service rate ( $\mu$ ).
  - The arrival rate distribution .

- The service time distribution.
- 4- The queuing model will determined and the effectiveness of system performance will be measured :
- $L$  : The expected number of arrivals in the system.
  - $L_q$  : The expected number of arrivals in the queue.
  - $W$  : The expected time required a customer to spend in the system.
  - $W_q$  : The expected time required a customer to spend in Queue.
  - $U$  : the server utilization.

**Figure 5.1 Simulation Model Methodology**



### 5.3 Simulation Model

We used Anylogic software in our simulation, Anylogic is a multi-method simulation modeling tool. Our simulation model is divided in two major paradigms :

- 1- The first paradigm outputs are the arrival rate ( $\lambda$ ), service time ( $\mu$ ) and their approximate probability distributions, which identify the queuing model for the application .

- 2- The second paradigm outputs are the queue length ,the system length ,the waiting time in queue ,the waiting time in system and the server utilization

In this section, some simulation models for different applications and different queuing models will be presented. The following simulation model is M/M/1. As it is shown in this model, the mean is equal to the variance of the arrival rate, which means that the distribution of the arrival rate is approximate Poisson. The coefficient of the service time is equal 0.7, which means that the service time distribution is approximate Exponential.

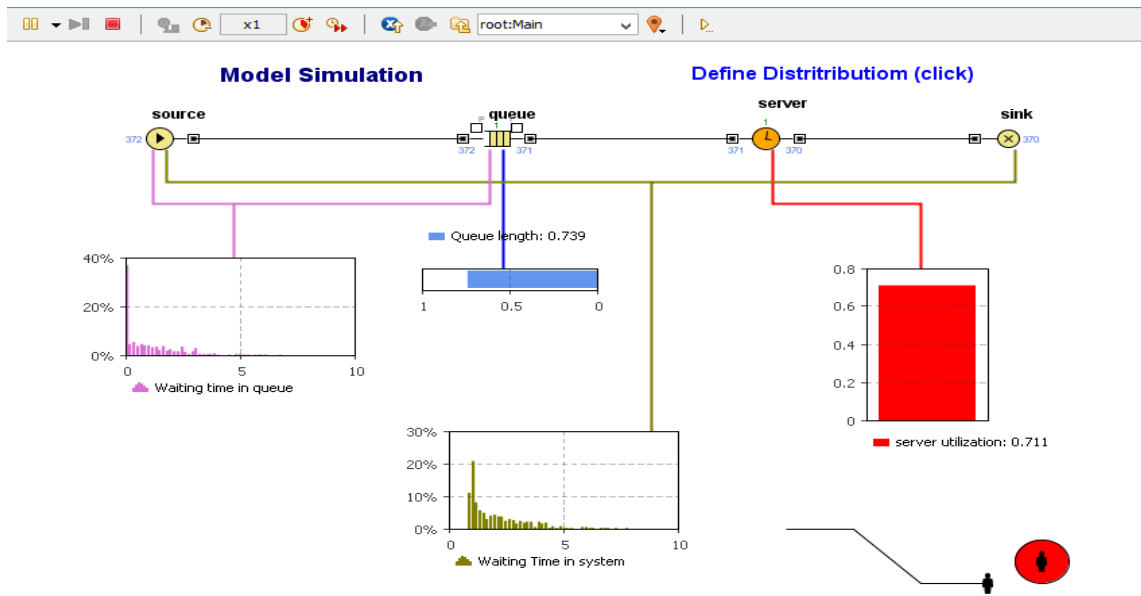
**Figure 5.2 Simulation model for M/M/1 model**

Define Distrtribution		Model Simulation (click)	
Enter The Arrival Mean	4.3	Enter the ServiceTime Mean	95
Enter The Arrival Variance	4.3	Enter the Service Time Coefficient	0.7
Arrival Mean (lamda)	0.232558139534	Service Rate ( $\mu$ )	0.010526315789
Arrival Distribution	Exponention	Service Distribution :	Exponention

The following simulation model illustrates the performance measures for M/M/1 queuing model ; the server utilization ,the queue length ,the system length ,the waiting time in queue and the waiting time in system.

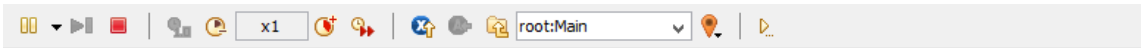


**Figure 5.3 The performance measure for M/M/1 model**



The following simulation model illustrates the queuing model M/E/1. The mean is equal to the variance for the arrival rate, which means that the distribution of the arrival rate is Poisson. The service time is equal 0.6 which means that the service time distribution is Erlang.

**Figure 5.4 Simulation model for M/E/1 model**



### Define Distribution

### Model Simulation (click)

Enter The Arrival Mean

Enter the ServiceTime Mean

Enter The Arrival Variance

Enter the Service Time Coefficient

Arrival Mean (lamda)

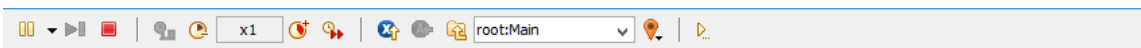
Service Rate ( $\mu$ )

Arrival Distribution

Service Distribution :

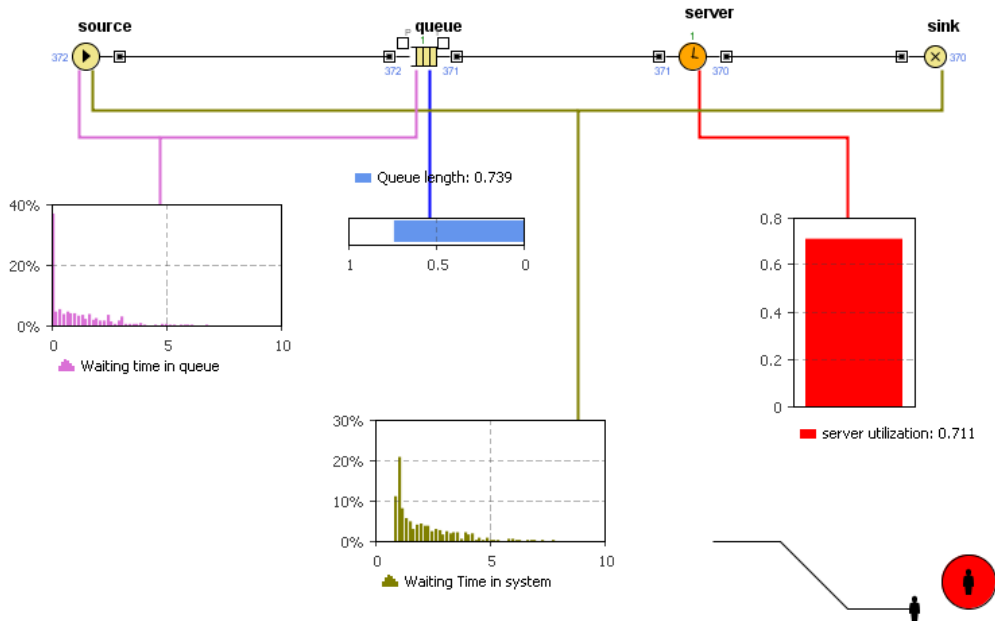
The following model shows the performance measures for M/E/1 model queuing model.

**Figure 5.5 The performance measures for M/E/1 model**



### Model Simulation

### Define Distribution (click)



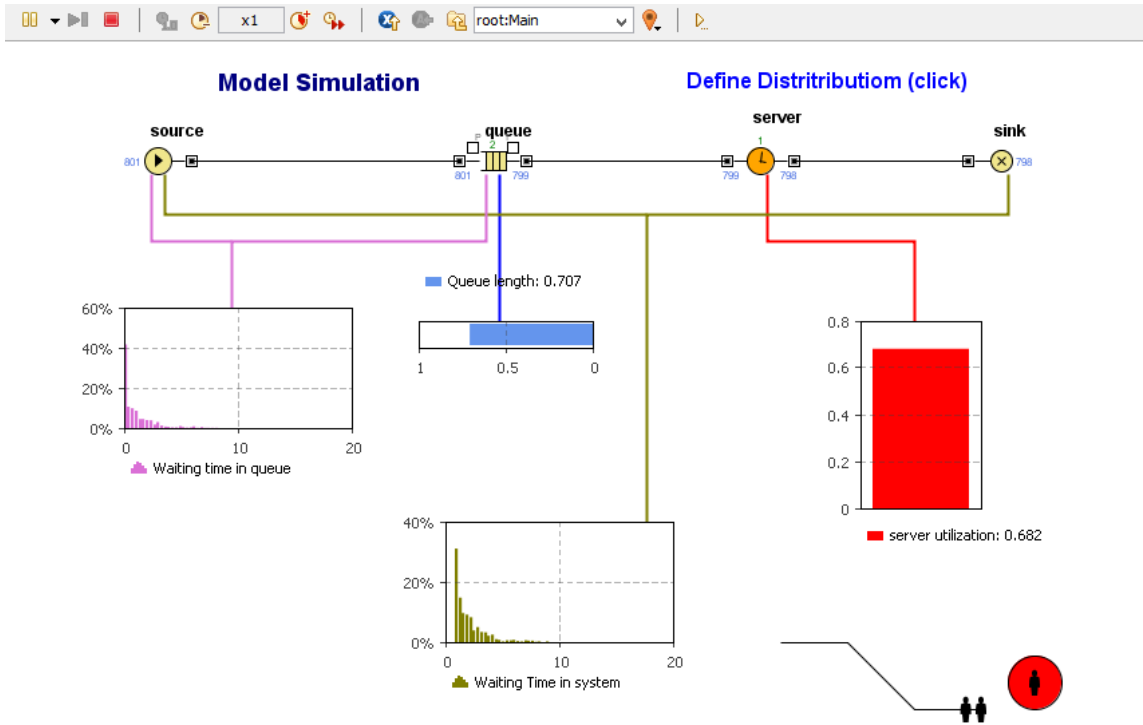
The following simulation model is the  $M/H_k/1$  model. The mean is equal to variance for the arrival rate, so the arrival rate distribution is Poisson. The service time is equal to 1.2, so the service time distribution is Hyper-exponential.

**Figure 5.6 Simulation model for  $M/H_k/1$  model**

Define Distribution		Model Simulation (click)	
Enter The Arrival Mean	3.4	Enter the ServiceTime Mean	1.06
Enter The Arrival Variance	3.4	Enter the Service Time Coefficient	1.2
Arrival Mean (lamda)	3.4	Service Rate ( $\mu$ )	0.943396226415
Arrival Distribution	Exponention	Service Distribution :	Hyper-EXP

The following model illustrates the performance measures for  $M/H_k/1$

**Figure 5.7 The performance measure for  $M/H_k/1$**



## 5.4 Validation

This research is based on the previous studies which determined the queuing models and computed the performance measures. In our simulation models we determined the queuing models according to the measures of variability, and compute the effectiveness performance for each application from previous studies. The outcomes of the performance measures for applications that obtained from our simulation model were compared with theirs from previous studies, and the outcomes were approximately similar.

## Chapter five

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### Conclusion and Future work

In this research ,the appropriate queuing models is defined approximately for different applications. Computer simulation model is presented to determine the queuing model. The simulation model is based on measures of variability which using mean and variance to identify the arrival distribution; the coefficient of variance of service time to identify the service time distribution. We can also measure the effectiveness of system performance; the length of system, the length of queue , server utilization, waiting time in queue and waiting time in system.

This research will increase the understanding of the queuing models, improve the performance and productivity of them, and predict the future system behavior .

## **Future work**

In our study, we presented a simulation model for Markov queuing models with one server and infinite buffer. So we hope we develop our model to be used for Non-Markov queuing models with multiple servers and finite buffer.

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## المنهج الإحصائي لتحديد نموذج الطابور المناسب المستند على التطبيقات

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### الملخص

نظرية الانتظار هو دراسة رياضية طوابير أو خطوط الانتظار. يتم استخدامه لنمذجة العديد من الأنظمة في مختلف المجالات في حياتنا، سواء كانت أنظمة بسيطة أو معقدة. الفكرة الرئيسية في نظرية الطوابير من نموذج رياضي في تحسين الأداء والإنتاجية في التطبيقات. هي التي شيدت نماذج الطابور من أجل حساب مقاييس الأداء للتطبيقات والتنبؤ فترات الانتظار وطول قائمة انتظار. يعتمد هذا البحث على الأوراق السابقة لنظرية الانتظار في التطبيقات المختلفة الذي يقوم بتحليل سلوك هذه التطبيقات ويبين كيفية حساب إحصائية الطابور باستخدام قياسات التباين (يعني التباين ومعامل التباين) لمجموعة متنوعة من نظم الطوابير من أجل تحديد نموذج الطابور المناسب.

محاكاة الكمبيوتر هو أداة قوية وسهلة لتقدير ما يقرب من نموذج الطابور الصحيح وتقييم مقاييس الأداء للتطبيقات. يقدم هذا البحث نموذج محاكاة جديدة لتحديد النماذج المناسبة للتطبيقات وتحديد المتغيرات المعلمات التي تؤثر على مقاييس الأداء الخاصة بهم وذلك بالاعتماد على قيم المتوسط، التباين مع معامل التطبيقات الحقيقية ومقارنتها مع قيم خصائص نموذج الطابور، ثم وفقا للمقارنة يتم التعرف على نموذج الطابور المناسب تقريبا. إن نموذج المحاكاة سيقوم بقياس أداء فعالية الطابور:

(١) العدد المتوقع للوافدين في النظام.

(٢) العدد المتوقع للوافدين في قائمة الانتظار

(٣) الوقت المتوقع يتطلب العملاء لقضاء في النظام.

٤) الوقت المتوقع يتطلب العملاء لقضاء في الطابور.

٥) استخدام (فعالية) الخادم.