Added mass of whipping modes for ships at high Froude number by a free surface boundary element method coupled with strip theory

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Abstract

Accurate prediction of the whipping response of a ship's structure following a wave impact is fundamental to both the prediction of instantaneous local stresses and global fatigue life assessment. In particular the added mass effect of the surrounding water has a profound effect on the modal frequencies. "Strip theory", routinely used for analysis of rigid body motions of ships in waves, is extended in this paper to include ship flexure. Moreover, the theoretical foundation of the method is discussed and it is shown that, although the theory becomes invalid for rigid body motions of high-speed vessels,

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the ship flexure problem is an ideal application of the theory. The associated two-dimensional free surface gravity wave problem is solved using a boundary element method based on wave functions given by Wehausen and Laitone (1960), which is also described. Results are validated against a fully three-dimensional solution, and incorporation of the added mass into a finite element model is shown to give excellent agreement with full scale measurements.

Contents

1 Introduction

Substantial stresses in ship hulls arise from the flexural response to wave forces. This flexural response comprises a quasi-static component (that is, low frequency compared with the flexural natural frequencies) due to the encountered waves at the frequencies of significant wave excitation forces or vessel rigid body motions, and a relatively larger transient dynamic component due to impulsive excitations from “slams”, severe short duration impacts, typically near the bow, resulting from the coincidence of unusually large wave crests and plunging bow motions. The latter response is known as “whipping”, and while single slams may in rare cases cause structural failure [4, e.g.] it is generally the accumulated fatigue effect of the typically lightly damped whipping accompanying more frequent moderate slams that is most critical. Furthermore, the severity and frequency of slams (hence whipping) increase with vessel speed.

Flexural frequencies and mode shapes are strongly influenced by "added mass", the effective inertia of the fluid surrounding the hull that must be accelerated with the hull during vibrations, which is comparable in magnitude to the mass of fluid displaced by the stationary hull. The mass of air displaced by the superstructure is negligible, but global flexural modes associated with whipping, which involve the submerged part of the hull, are substantially modified by the added mass of the water.

Added mass computations for rigid bodies are typically based on a form of slender body theory known as "strip theory" [3], which derives significant corrections associated with the ship's forward motion. This paper extends strip theory to include longitudinal hull curvature so that it may be applied rigorously to the flexure problem. Vertical motions are considered, but lateral motions may be similarly analysed.

The current research arises from collaborative work with Incat, a manufacturer and exporter of state of the art large high speed catamaran passenger ferries. Recent ships built by Incat have approached 100 m in length, with service speeds of up to 45 kt representing high Froude numbers for ships of this size. The numerical results presented in this paper will be for these ships.

2 Sectional vertical forces at forward speed

The method of strip theory [3] is generalised below to express the vertical sectional added mass and damping with forward speed in terms of the local hull deflection, slope and curvature due to hull flexure.

Define x, y as horizontal and vertical in a ship hull cross section plane with the x axis coincident with the undisturbed free surface, and z as the longitudinal direction. Then the linearised flow around a ship hull moving with forward speed U is represented by the potential function $\phi\,(x, y, z, t)$, defined in the half space $y \leq 0$, satisfying

$$\begin{aligned} \nabla^2\phi &= 0 \quad \text{in the fluid domain,} & (1)\\ \left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial z}\right)^2\phi + g\frac{\partial\phi}{\partial y} &= 0 \quad \text{on } y = 0\,, & (2)\\ \nabla\phi\cdot\mathbf{n} &= \mathbf{V}\cdot\mathbf{n} \quad \text{on the ship hull.} & (3) \end{aligned}$$

From Bernoulli's equation, the dynamic pressure in a stationary reference frame is $p = -\rho(\partial\phi/\partial t + \frac{1}{2}\nabla\phi.\nabla\phi)$, thus in a reference frame moving with the ship at forward speed U the linearised vertical dynamic force is

$$F_y = -\rho\int_{\text{length}}\int_{\text{section}}\left(\frac{\partial\phi}{\partial t} - U\frac{\partial\phi}{\partial z}\right)n_y\,dl\,dz\,, \qquad (4)$$

where the path of l is around the submerged cross section girth, hence

$$\frac{\partial F_y}{\partial z} = -\rho\left(i\omega P - U\frac{\partial P}{\partial z}\right)\,, \qquad (5)$$

where $P = \int_{\text{section}}\phi n_y\,dl$. If $\eta\,(z, t) = \eta_0\,(z)\,e^{i\omega t}$ is the local vertical displacement of a cross section of the hull, then assuming a slender hull ($n_z \ll 1$) the hull boundary condition (3) becomes

$$\nabla\phi\cdot\mathbf{n} = \left(\frac{\partial\eta}{\partial t} - U\frac{\partial\eta}{\partial z}\right)n_y\,, \qquad (6)$$

which, defining $\phi_0\,(x, y, z)$ such that $\nabla\phi_0\cdot\mathbf{n} = n_y$ (the vertical component of $\mathbf{n}$), has a solution of the form $\phi = (i\omega\eta_0 - U\eta_0')\,e^{i\omega t}\phi_0$ where $\eta' = \partial\eta/\partial z$. With ϕ_0 defined thus the zero speed added mass and damping, a and b (or the complex equivalent Z), are

$$a + \frac{b}{i\omega} = Z = \rho\int_{\text{section}}\phi_0 n_y\,dl\,, \qquad (7)$$

hence $P = (i\omega\eta_0 - U\eta_0')\, e^{i\omega t} Z/\rho$ and

$$\frac{\partial F_y}{\partial z} = e^{i\omega t}\left\{ Z\left(\omega^2\eta_0 + 2i\omega U\eta_0' - U^2\eta_0''\right) + \frac{\partial Z}{\partial z}\left(Ui\omega\eta_0 - U^2\eta_0'\right)\right\} . \qquad (8)$$

Finally, assuming that there is no phase difference between η' , η_0', and η_0'' (that is, the damping is small), and defining sectional added mass and damping coefficients with forward speed, a_u and b_u, as respectively the components of $\partial F_y/\partial z$ proportional to but opposing the local hull acceleration $-\omega^2\eta$ and velocity $i\omega\eta$, we obtain

$$a_u = a\left(1 - \frac{U^2}{\omega^2}\frac{\eta_0''}{\eta_0}\right) + \frac{U}{\omega^2}\left\{\frac{db}{dz} + \frac{\eta_0'}{\eta_0}\left(2b - U\frac{da}{dz}\right)\right\} , \qquad (9)$$

$$b_u = b\left(1 - \frac{U^2}{\omega^2}\frac{\eta_0''}{\eta_0}\right) - U\left\{\frac{da}{dz} + \frac{\eta_0'}{\eta_0}\left(2a + \frac{U}{\omega^2}\frac{db}{dz}\right)\right\} . \qquad (10)$$

Note that the vessel coefficients in the theory of [3] are recovered by observing that unit heave and pitch motions are $\eta_0 = 1$ and $\eta_0 = -x$, heave and pitch added mass are $\int_{\text{length}} a_u\eta\, dz$ and $-\int_{\text{length}} a_u\eta x\, dz$, and similarly for damping.

3 Zero speed force coefficients

We now seek $\phi_0\,(x, y, z)$ from which $a(z)$ and $b(z)$ are determined. The problem is greatly simplified if $\partial/\partial z$ terms in (1) and (2) are assumed negligible, from which it follows ϕ_0 represents a series of independent two-dimensional potentials satisfying

$$\frac{\partial^2\phi_0}{\partial x^2} + \frac{\partial^2\phi_0}{\partial y^2} = 0 \quad \text{in the fluid domain,} \qquad (11)$$

$$\phi_0 - \frac{g}{\omega^2}\frac{\partial\phi_0}{\partial y} = 0 \quad \text{on } y = 0\,, \qquad (12)$$

$$\frac{\partial \phi_0}{\partial x} n_x + \frac{\partial \phi_0}{\partial y} n_y = n_y \quad \text{on the hull.} \tag{13}$$

The omission of $\partial^2\phi/\partial z^2$ from (1) requires, in addition to slenderness, that the radiated wave pattern must be supercritical, $\omega U/g \gg \frac{1}{4}$, producing negligible longitudinal wave particle motion in the vicinity of the ship hull, and the omission of $U\partial/\partial z$ terms from (2) requires that $\omega \gg U/L$ (where L is the ship length). Thus

$$\omega^* \gg \frac{1}{4\mathrm{Fr}} \quad \text{and} \quad \omega^* \gg \mathrm{Fr}\,, \tag{14}$$

where $\omega^* = \omega\sqrt{L/g}$, $\mathrm{Fr} = U/\sqrt{gL}$, which is readily satisfied for ship flexural vibrations ($\omega^* > 50$ for typical Incat vessels). This is in contrast to the ship rigid body motions, in which the frequency range of interest ($1 < \omega^* < 10$ for typical Incat vessels) is only moderately high, restricting the theory of [3] to $\mathrm{Fr} = \mathcal{O}\left(\frac{1}{2}\right)$ (Incat vessels currently exceed $\mathrm{Fr} = 0.7$). The current application is therefore an ideal application of strip theory.

A boundary element method based on that in [1] is used, using a source function given in [6] satisfying (11) and (12),

$$\begin{aligned} f(z) = {} & \frac{1}{2\pi}\left[\ln(z-c) - \ln(z-\bar{c}) + 2e^{-\mathrm{i}k(z-\bar{c})}\left(\mathrm{Ei}\left(\mathrm{i}k\left(z-\bar{c}\right)\right) + \delta\pi\mathrm{i}\right)\right] \\ & - \mathrm{j}\left[e^{-\mathrm{i}k(z-\bar{c})}\right], \end{aligned} \tag{15}$$

time-complex in j and space-complex in i, where space-complex z and c represent field and source locations, $k = \omega^2/g$ is the wavenumber, and $\delta = 1$ or -1 if $\Re_\mathrm{i}(z-c) > 0$ or < 0 respectively[1]. Each ship cross section is discretised into straight line elements over which sources of this form are distributed piecewise uniformly with time-complex intensities $\{Q\}$, determined by satisfying (13) at the mid-point of each element. This results in the system of linear time-complex equations

$$[A]\{Q\} = \{n_y\}\,, \tag{16}$$

[1] The subscripts i and j on $\Re$ and $\Im$ indicate space or time complex domains.

where

$$A_{ij} = -\Im_{\mathrm{i}} \left(e^{\mathrm{i}\alpha} \int_{c_1}^{c_2} \frac{\partial f}{\partial z}\, ds \right) , \tag{17}$$

in which c_1 and c_2 are endpoints of element j, z is the midpoint of element i, $ds = dc\, e^{-\mathrm{i}\beta}$ or $d\bar{c}\, e^{\mathrm{i}\beta}$ as appropriate, and α and β are slopes of elements i and j. The integral (without the $e^{\pm\mathrm{i}\beta}$) recovers the terms of $-f$, noting that z and c always appear as the combination $z-c$, and the first logarithm is interpreted as $-\mathrm{i}\pi$ when $i=j$.

Surface potentials are then obtained from

$$\{\phi\} = [B]\,\{Q\} , \tag{18}$$

where

$$\begin{aligned} B_{ij} &= \Re_{\mathrm{i}} \left(\int_{c_1}^{c_2} f\, ds \right) \\ &= \Re_{\mathrm{i}} \left(\left\{ \frac{-1}{2\pi} \left[e^{-\mathrm{i}\beta} (z-c) \ln(e^{-\mathrm{i}\gamma} (z-c)) - e^{\mathrm{i}\beta} (z-\bar{c}) \ln (z-\bar{c}) \right]_{c_1}^{c_2} \right.\right. \\ &\quad \left. - \frac{-\mathrm{i}e^{\mathrm{i}\beta}}{\pi k} \left(\left[e^{-\mathrm{i}k(z-\bar{c})} \left(\mathrm{Ei}\left(\mathrm{i}k\,(z-\bar{c})\right) + \delta\pi\mathrm{i} \right) \right]_{c_1}^{c_2} - \ln \left(\frac{z-\bar{c}_2}{z-\bar{c}_1} \right) \right) \right\} \\ &\quad \left. - \mathrm{j} \left\{ \frac{\mathrm{i}e^{\mathrm{i}\beta}}{k} \left[e^{-\mathrm{i}k(z-\bar{c})} \right]_{c_1}^{c_2} \right\} \right) \end{aligned} \tag{19}$$

in which γ is arbitrarily chosen so that the path of integration does not cross the negative real axis, for example, $\gamma = \arg(c_1 - c_2) + \frac{\pi}{2}$ $(i \neq j)$ or $\arg\left(z - \frac{c_1+c_2}{2}\right)$ $(i=j)$. Surface pressures, $-\rho \mathrm{j}\omega\phi$, are summed over elements to give forces, from which sectional added mass and damping coefficients are obtained, hence (7).

Extensive validation of this method is presented in [2].

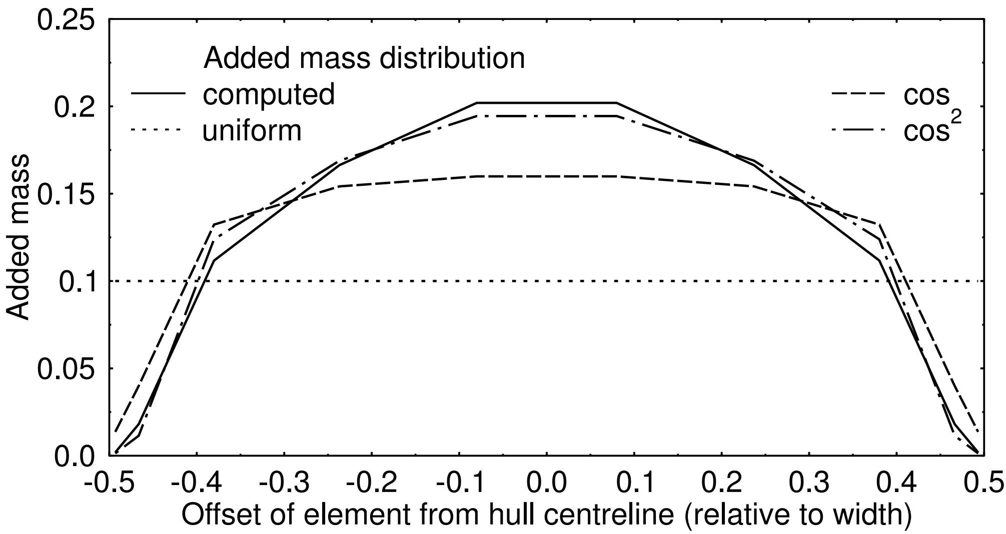

Figure 1: Proportion of added mass applied to 10 nodes around a typical section by different distribution models, as used in finite element modal analysis.

4 Results and conclusions

Results presented below are for three Incat high speed passenger catamarans of lengths 86–96 m and mass 880–1150 t. Full details are given in [5]. Modes studied were the first longitudinal bending mode, analogous to one dimensional beam bending, and the first torsional mode, in which the predominant hull deflection was one of rigid body rotation about a lateral axis, opposite hulls rotating in opposite directions.

In undertaking a modal analysis, note that the flexural problem (solved here using finite element software) and the hydrodynamic problem (solved using the above boundary element method) are coupled. However while added mass profoundly affects the natural frequencies, the effect of frequency

Table 1: Error in natural frequency for different added mass distributions

Mass distribution method	1st torsional mode	1st longitudinal mode
No added mass	66.1%	34.7%
Lumped at keel	−0.3%	−0.3%
Uniform	0.9%	0.7%
cos	0.3%	0.3%
$\cos^2$	0.1%	0.1%

on added mass, through (12), is small at the high frequencies associated with flexure. Similarly the effect of mode shape on added mass may be significant, especially near nodal points ($\eta = 0$), but the effect of added mass on mode shapes is small. The problem therefore is efficiently solved iteratively.

Distributing the added mass in the finite element model around each cross section exactly in proportion to the computed inertial pressure was very time consuming, so some simplified procedures were tested, including a single lumped mass distribution along the keel, and an mass areal density that was uniform around the submerged part of each cross section, or varied with the cos or $\cos^2$ of the angle made locally between the cross section curve and the horizontal, as illustrated in Figure 1. (Cosine was proposed to represent the vertical component of pressure, and $\cos^2$ additionally accounts for the right hand side of (13), which strongly influenced the local added mass areal density. As anticipated the $\cos^2$ distribution is seen to agree closely with the correct distribution). Table 1 shows for two modes studied that added mass has a profound effect on the natural frequency, but its girthwise distribution is not critical. Errors are consistent with the degree of approximation in the distribution, except the lumped mass model, which performs better than might be expected, probably due to the concentration of the correct distribution near the keel.

Figure 2 shows the effect of forward speed on added mass. While the relative total corrections appear small it is noted that the effect is more one

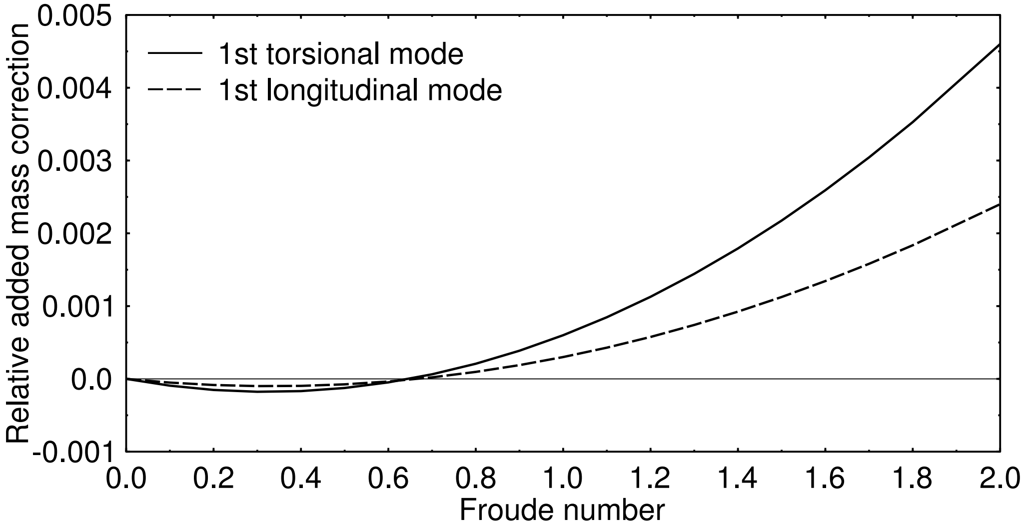

Figure 2: Effect of vessel forward speed ($\mathrm{Fr} = U/\sqrt{gL}$) on total vessel added mass.

of redistribution of mass as the correction terms change sign along the hull. The torsional mode experiences a larger correction as it is of lower frequency (around 1.6 Hz, depending on vessel loading) than the longitudinal mode (around 3.0 Hz).

Computed frequencies and mode shapes were compared with measurements on full scale ships obtained from impulse excitation tests, in which symmetrical modes for the moored vessel were initiated by dropping and braking the anchor, then measured at different locations with accelerometers, and from slam records extracted from strain measurements during operation. The first longitudinal bending mode dominated the anchor drop responses, while this and the first torsional mode were identified in the operational strain measurements by spectral analysis.

The natural frequency for the longitudinal mode from the anchor drop tests agreed with the computed value to within 0.3%, while the averaged operational strain measurement frequencies differed from computed values by 2.1%, 3.3% and 6.6% respectively for the longitudinal mode, and the torsional mode under two different loading conditions. The anchor drop result is well within the numerical error range, while the slightly poorer agreement of the operational measurements is attributed to less certainty about the loading condition (for example, the mass of fuel carried, whose variation during a voyage may be a significant proportion of the total ship mass) and noise associated with the generally much broader banded data spectra during the ship's operation due to continuous random excitation from the incident waves and ship rigid body motions. It is concluded therefore that natural frequencies can be accurately computed using the above method.

Mode shapes were measured only during the impulse excitation test, and these agreed reasonably well with numerical predictions (Figure 3). Differences are attributed to the fact that an impulse at the bow will inevitably excite numerous modes that are difficult to separate.

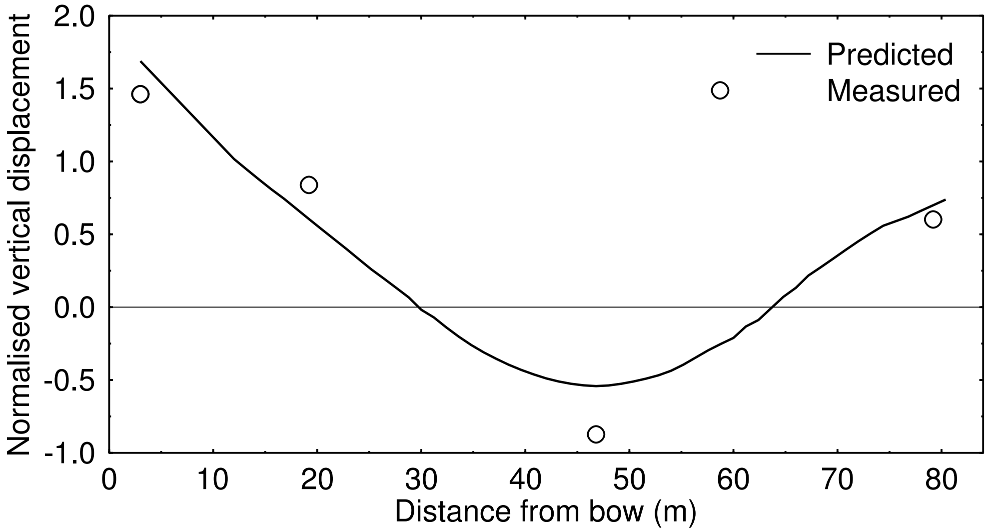

Figure 3: first longitudinal mode shape for Incat catamaran predicted using finite element method with added mass, and measured in anchor drop tests (normalised by RMS of values at four measurement locations).

Summary:

- the added mass effect on flexural natural frequencies is substantial;
- forward speed effects on flexural added mass are small, even at moderately high speed;
- strip theory, modified to include hull curvature, is an ideal method of calculating flexural added mass due to the high frequencies involved, both in terms of the validity of the assumptions made, and the quality of agreement of the computed and measured results;
- girthwise distribution of added mass is not critical for the calculation of global flexural modes.

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