MODELLING OF RADIATIVE TRANSFER IN AIR ARC PLASMA

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Abstract. The objective of this work is to compare the accuracy of several approximate models of radiative properties for the prediction of radiative transfer in air arc plasma at the temperatures in the range of 300–25 000 K and the pressure of 0.1 MPa. Calculated absorption coefficients are used to generate the parameters of different models. The radiative transfer inside the cylindrically symmetrical air plasma with prescribed temperature profile was studied. The equation of radiative transfer was solved using the P_1 and SP_3 approximations, calculated radiative source term in the energy equation (net emission) was compared with results obtained by spectral integration.

Keywords: radiative transfer, P_1 approximation, SP_3 approximation, mean absorption coefficients.

1. Introduction

Opening of contacts in switching devices results in an electric arc, which has to be extinguished quickly. The switching arc is a very complex phenomenon, and its realistic simulation has to take into account many effects. A crucial role in the energy balance of the switching arc plasma plays the radiation transport of energy. However, the non-linearity of equations describing the radiation field and strong dependency of input parameters on the radiation frequency and properties of the medium make the mathematical plasma models very complicated.

Several approximate methods have been developed to simplify the solution of radiation transfer. The method of spherical harmonics (P_N approximation) provides a tool to obtain an approximate solution of arbitrary high order of accuracy by transforming the equation of radiation transfer into a set of simultaneous partial differential equations [1]. However, for higher order approximations the mathematical complexity increase extremely rapidly. Therefore, P_1 approximation is usually used in association with different spectral approximations. This method is rather popular since it reduces the equation of transfer from a very complicated integral equation to a relatively simple second-order partial differential equation [2, 3]. Furthermore, if overall energy conservation is computed (also a partial differential equation), compatibility of the solution methods is virtually assured. However, the P_1 approximation may be substantially in error in optically thin situations. Another drawback are the boundary conditions, especially when the surface emission dominates over medium emission.

The simplified SP_N method has been developed originally for neutron transport and was extended to radiation transfer [4]. It was shown that SP_3 approxi-

mation improves the P_1 approximation especially for high pressures [3]. Unlike P_1 method that requires for each frequency the solution of one elliptic differential equation, the simplified SP_3 approximation leads to the solution of two similar equations. It remains then very attractive in comparison with the full solution of the radiation transfer, as well as in comparison with the standard P_3 approximation.

Various spectral approximations have been studied in this work, including Planck and Rosseland mean absorption coefficients (MAC) for cutting the frequency interval in different number of bands, line limited Planck MAC, and Planck and Rosseland MAC in k-distribution type model [2]. Spectral approximations have been used in P_1 and SP_3 calculations.

2. Absorption coefficients and spectral approximations

Under the assumption of local thermodynamic equilibrium (LTE) and neglecting the scattering, the equation of radiation transfer can be written in the form

$$\mathbf{\Omega} \cdot \nabla I_\nu(r, \mathbf{\Omega}) = \kappa_\nu (B_\nu - I_\nu) \tag{1}$$

where I_ν is the spectral intensity of radiation, $\mathbf{\Omega}$ is a unit direction vector, B_ν is the Planck black-body intensity. The only system dependence quantity in eq. (1) is the absorption coefficient κ_ν.

At high arc temperatures, dissociation of molecules and ionization of atoms occurs in the gas. We assume that plasma is in a state of LTE, its equilibrium composition can be determined by, for example, minimizing Gibbs energy [5]. Neutral atoms and up to triple ions of N, O, Ar elements were considered. Molecules photo-absorption was taken into account for N_2, O_2, NO, NO_2 and N_2O molecular species and their relative ions.

Both continuum and line radiation have been considered in calculation of absorption coefficients. Continuum spectrum is formed by bound-free transitions (photoionization) and free-free transitions (inverse bremsstrahlung). Photoionization cross sections were taken from OPACITY project database [6] or calculated using Coulomb approximation for hydrogen-like species [7]. In the discrete radiation calculations (bound-bound transitions) Doppler, Stark, Van der Waals, natural and resonance broadening of spectral lines were taken into account, semi-empirical formulas [7] were used for calculation of various broadening mechanisms. In the calculation of cross sections for the diatomic molecular band systems we have used Franck-Condon principle [8]. Molecular photo-dissociation cross sections were taken from PHIDRATES database [9]. As can be seen in Fig. 1, the absorption spectrum is very complex and widely varying function of frequency.

One procedure for handling very complicated frequency variable in the equation of radiation transfer is to find an efficient way of averaging the absorption coefficients into frequency bands. Any averaging method depends strongly on the properties of the absorption spectrum, the physical properties of the system (such as typical temperature and size), and the accuracy required. The mean absorption coefficient (MAC) values are generally taken as either the Rosseland MAC $\overline{\kappa}_{\mathrm{R}}$ (based on the assumption that almost all radiation is reabsorbed) or the Planck MAC $\overline{\kappa}_{\mathrm{P}}$ (suitable in optically thin, emission dominated system):

$$\overline{\kappa}_{\mathrm{R}}^{-1} = \frac{\int_{\Delta\nu} \kappa_\nu^{-1} \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\,\mathrm{d}\nu}{\int_{\Delta\nu} \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\,\mathrm{d}\nu}, \qquad (2)$$

$$\overline{\kappa}_{\mathrm{P}} = \frac{\int_{\Delta\nu} \kappa_\nu B_\nu \,\mathrm{d}\nu}{\int_{\Delta\nu} B_\nu \,\mathrm{d}\nu}. \qquad (3)$$

There is no universal way of the definition of spectral bands $\Delta\nu$. Generally, the value of the absorption coefficient should not strongly vary over the band. However, increasing number of bands leads to increase of calculation cost. We have used 11 radiation bands with following cutting frequencies (in units of 10^{15} Hz) [3] which are shown in Fig. 1:

$$(0.001, 0.057, 0.296, 0.386, 0.746, 0.986,$$
$$1.71, 2.098, 2.64, 2.997, 4.49, 10) \qquad (4)$$

In the second approach, the band selection process was based upon numerical optimization with a mean value calculated as a line limited Planck MAC [10]. Using numerical optimization the three-band model was created with cutting frequencies (see Fig. 2)

$$(0.001, 2.938, 3.513, 10) \times 10^{15}\,\mathrm{Hz} \qquad (5)$$

Beside of standard Rosseland and Planck MAC (2), (3), the line limited Planck MAC was calculated with the absorption coefficient of spectral lines replaced by

$$\kappa_{\mathrm{lim}} = \frac{1}{L}(1 - e^{-\kappa_{\mathrm{line}}L}) \qquad (6)$$
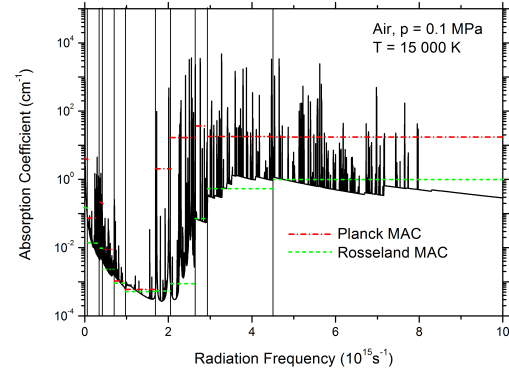
Figure 1. The high resolution spectrum of air plasma compared with standard Planck and Rosseland MAC, 11 frequency bands (eq. (4)).
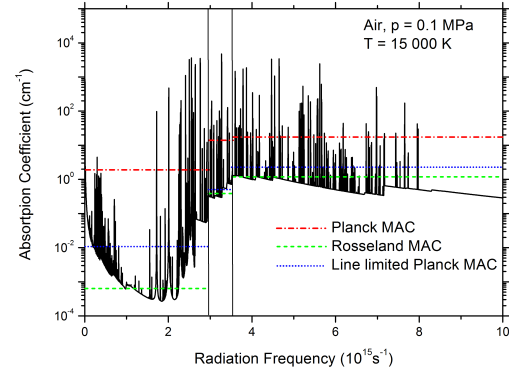

Figure 2. The high resolution spectrum of air plasma compared with standard Planck and Rosseland MAC, and line limited Planck MAC with L=10 mm; 3 frequency bands (eq. (5)).

where L is the characteristic absorption length.

It can be seen in Figs. 1, 2 that the standard Planck MAC strongly overestimate the influence of spectral lines.

Another way how to eliminate the overestimated influence of spectral lines on Planck MAC is the *k*-distribution type model [2]: in each band frequencies are further divided into four subgroups according to the value of κ_ν (in cm^{-1})

$$\kappa_\nu \in [0, 0.1], [0.1, 1], [1, 10], [10, \infty]. \qquad (7)$$

This approach was used for three frequency bands (5), and Rosseland and Planck MAC (2), (3) were calculated for each subgroup.

3. P$_1$ and SP$_3$ approximations

In the P$_1$ approximation, the local intensity is approximated by a series of spherical harmonics, and the series is truncated after the first order terms. The directional equation (1) is transformed into one second-order elliptic partial differential equation for the density of

radiation U_ν (zero-order moment of the radiation intensity):

$$\nabla \cdot (-\frac{1}{3\kappa_\nu}\nabla U_\nu) + \kappa_\nu U_\nu = 4\pi\kappa_\nu B_\nu \qquad (8)$$

where $-\frac{1}{3\kappa_\nu}\nabla U_\nu = \mathbf{F}_\nu$ is the radiation flux. The associated Marshak boundary condition at a black-body diffuse wall take the form

$$\mathbf{n} \cdot \nabla U_\nu = \frac{3}{2}\kappa_\nu(4\pi B_\nu^{\text{wall}} - U_\nu) \qquad (9)$$

Using MAC in equations (8), (9) the net emission of radiation is

$$\nabla \cdot \mathbf{F}_R = \sum_j \nabla \cdot \mathbf{F}_j \qquad (10)$$

where j denotes the individual frequency bands.

Simplified P_3 equations (SP_3) are an asymptotic correction to standard diffusion theory and asymptotically relate to the slab geometry P_3 equations. In this approximation, from the equation (1) the system of two second-order partial differential equations for two variables $\psi_{1,\nu}$ and $\psi_{2,\nu}$ was derived in [3]

$$-\nabla \cdot \frac{\mu_1}{\kappa_\nu}\nabla\psi_{1,\nu} + \kappa_\nu\psi_{1,\nu} = 4\pi\kappa_\nu B_\nu$$
$$-\nabla \cdot \frac{\mu_2}{\kappa_\nu}\nabla\psi_{2,\nu} + \kappa_\nu\psi_{2,\nu} = 4\pi\kappa_\nu B_\nu \qquad (11)$$

where $\mu_i = (3 \mp 2\sqrt{1.2})/7$, $i = 1, 2$. These two equations are weakly coupled by the Marshak boundary conditions for a black-body diffuse wall

$$\alpha_1\psi_{1,\nu} + \frac{1}{\kappa_\nu}\mathbf{n}\cdot\nabla\psi_{1,\nu} = -\beta_2\psi_{2,\nu} + \eta_1 B_\nu^{\text{wall}}$$
$$\alpha_2\psi_{2,\nu} + \frac{1}{\kappa_\nu}\mathbf{n}\cdot\nabla\psi_{2,\nu} = -\beta_1\psi_{1,\nu} + \eta_2 B_\nu^{\text{wall}} \qquad (12)$$

where

$$\alpha_i = 5(34 \pm 11\sqrt{1.2})/96, \; i = 1, 2,$$
$$\beta_i = 5(2 \mp \sqrt{1.2})/96, \; i = 1, 2, \qquad (13)$$
$$\eta_i = 5\pi(3 \pm \sqrt{1.2})/2, \; i = 1, 2.$$

Using MAC in equations (11), (12) the net emission of radiation is

$$\nabla \cdot \mathbf{F}_R = \sum_j \nabla \cdot \frac{1}{\kappa_\nu}\nabla(a_1\psi_{1,j} + a_2\psi_{2,j}) \qquad (14)$$

with $a_i = (5 \mp 3\sqrt{1.2})/30$, $i = 1, 2$.

4. Results

The radiative transfer inside the cylindrical air plasma column at uniform pressure of 0.1 MPa with prescribed temperature profile was calculated using the P_1 and SP_3 approximations with various MAC. In Figures 3–6, the calculated net emission is compared with results of the full spectral integration [10].
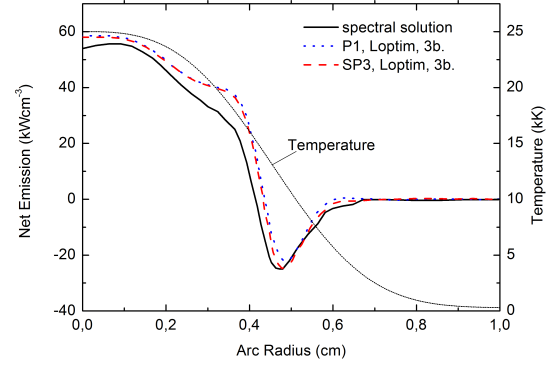

Figure 3. Net emission calculated using full spectral integration [10], and P_1 and SP_3 approximation with line limited Planck MAC, 3 frequency bands (eq. (5)).

Fig. 3 shows the results of P_1 and SP_3 approximations with line limited Planck MAC in three frequency bands (5). The characteristic absorption length is $L_{\text{optim}} = 3R_p = 1.38$ cm. R_p is the plasma radius which is defined as a distance from the cylinder axis, where the temperature reaches half point between the maximum and minimum value [10]. Good accuracy can be seen, SP_3 approximation leads to more accurate results in the strongly absorbing region at the plasma edge.

The importance of spectral line limiting is shown in Fig. 4 where net emission calculated with standard Planck and Rosseland MAC in 3 frequency bands (5) is presented. Planck MAC overestimate very strongly the emission of radiation (so much that it is not shown in the figure due to the scale), on the other hand, Rosseland MAC underestimate it. Some improvement can be obtained by combination of Planck and Rosseland MAC - Planck MAC is used in frequency bands with low values of absorption coefficients, while Rosseland MAC in frequency bands with strong absorption. However, the set of three bands does not work well because even the combination of Planck and Rosseland MAC (Rosseland MAC in 1st and 3rd band, Planck MAC in 2nd band) increase the net emission ten times.

A small improvement in the results can be obtained by cutting the frequency interval in more bands, as can be seen in Fig. 5, where 11 bands (4) were used. In the combination of Planck and Rosseland MAC, Planck MAC were used only in 4th and 6th band. Planck MAC alone, or used in more bands lead to strong overestimation of emission.

Relatively good accuracy can be obtained by combination of frequency and κ-based cutting (k-distribution type model (7)), as it is shown in Fig. 6. In combination of Planck and Rosseland MAC, the Rosseland MAC were considered in subgroups with $\kappa > 1$ cm^{-1}.
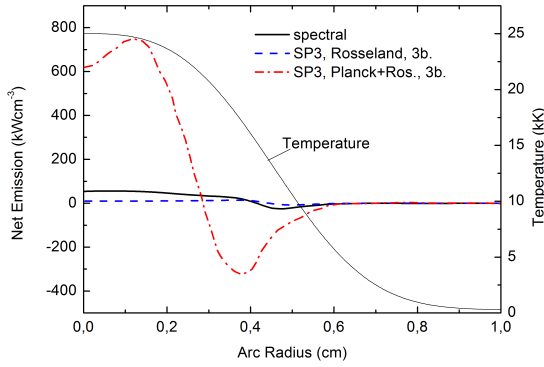
Figure 4. Net emission calculated using full spectral integration [10], and SP₃ approximation with Rosseland MAC and combination of Planck and Rosseland MAC for 3 frequency bands (eq. (5)).
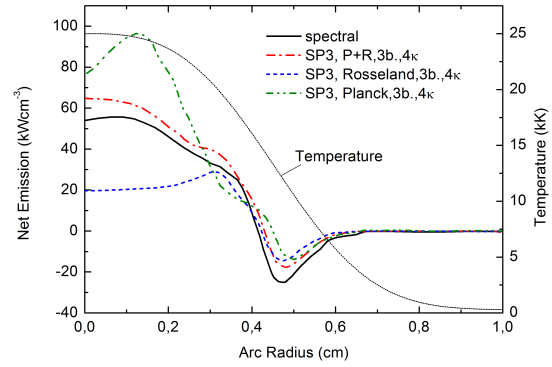

Figure 6. Net emission calculated using full spectral integration [10], and SP₃ approximation with Planck MAC, Rosseland MAC and combination of Planck and Rosseland MAC, k-type model, 3 frequency bands (eq. (5)).
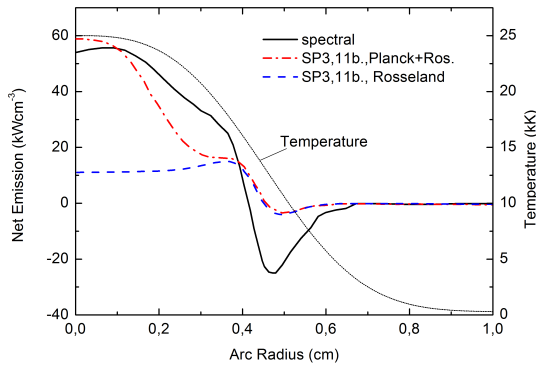

Figure 5. Net emission calculated using full spectral integration [10], and SP₃ approximation with Rosseland MAC and combination of Planck and Rosseland MAC for 11 frequency bands (eq. (4)).

5. Conclusions

The net emission has been calculated in air thermal plasma using P_1 and SP_3 method in association with different spectral approximations. SP_3 method is slightly more accurate than P_1 model. Rosseland MAC underestimate both emission and absorption of radiation, Planck MAC overestimate the influence of spectral lines. Good accuracy can be obtained using line limited Planck MAC and three-band model obtained by numerical optimization. Another suitable approach is the k-distribution type model with combination of Planck and Rosseland MAC. The advantage of this approach is that it does not depend on a certain thickness of the arc.

Acknowledgements

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