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RADIATION TRANSFER IN ARC PLASMAS

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Abstract. In this paper, attention has been given to the absorption properties of the arc plasma at the different pressure conditions. Calculations of the absorption coefficients for a thermal plasma have been performed as a function of the temperature and the frequency. Methods for prediction of the average absorption coefficients were described and compared in detail.

Keywords: air plasma, radiation transfer, absorption coefficient.

1. Introduction

Currently arc discharges are used increasingly in various fields of modern science and technology. Certain plasma parameters make its employment very promising for creation of new technologies. Applications of the plasma are quite diverse: from equipment for the modification and processing of polymer materials and plasma cutting machines till using plasma in high voltage circuit breakers and as well as household applications, etc.

In most cases optimization of technological conditions is the result of experiment during development of advanced equipment employing plasma. However, the experimental conditions are rather extreme (high temperature, pressure and gas velocity). Also, carrying out experiments is associated with substantial material expenditures. Therefore, it is very important to develop mathematical approximations and process modeling of parameters in the arc plasma. Mathematical modeling is also quite complex and it requires certain computational resources, so it is carried out in several stages. Absorption coefficients are primary parameters in the simulation. Based on that it is possible to define other parameters, which are necessary for analyzing of energy transfer in arc plasma.

2. Energy balance of arc plasma

Electric discharge plasmas transmit relatively great deal of power. The ratios while transmission are described by the energy balance of the plasma. In the theory of the energy balance of the plasma only the element of the plasma volume is taken into consideration and heat convection is neglected.

The energy balance of the arc plasma in stationary state can be described by Ehlenbaas–Heller equation [1]

$$\sigma_E E^2 = -\mathrm{div}\,[\lambda \cdot \mathrm{grad}(T)] + \mathrm{div}(\mathbf{W}_\mathrm{R}), \qquad (1)$$

where $\sigma_E E^2$ denotes the input electric power (σ_E is the electric conductivity, E is the electric field), $\lambda \cdot \mathrm{grad}(T)$ denotes the energy flux due to heat conduction (λ is the heat conductivity, T is the plasma temperature, $\mathrm{div}(\mathbf{W}_\mathrm{R})$ denotes the losses of energy by radiation ($\mathbf{W}_\mathrm{R}$ is the radiation flux).

Therefore, for modeling of arc plasma including radiation it is necessary to determine the radiation flux and its divergence. The solution of the equation of radiation transfer makes possible to estimate these parameters.

The complete stationary equation of radiation transfer for an absorbing and emitting medium is

$$\mathbf{\Omega} \cdot \mathrm{grad}(I_\nu) = \kappa_\nu [B(\nu) - I_\nu], \qquad (2)$$

where $\mathbf{\Omega}$ is the unit direction vector, I_ν is the radiation intensity, κ_ν is the absorption coefficient, $B(\nu)$ is Planck's spectral radiation intensity for equilibrium radiation, ν is the frequency [2].

The equation of transfer is obviously very complicated. The spectral intensity, which is the dependent variable in this equation, depends in general upon independent variables $(\mathbf{r}, \nu, \mathbf{\Omega})$. One must approximate the equation of transfer, either analytically or numerically, in order to obtain a solution. Due to the non-linearity of equations describing the radiation field and strong dependence of input parameters on the radiation frequency, various approximate methods are used. One of them is the method of spherical harmonics – P_N-approximation [3–6].

The great advantage of the method of spherical harmonics is the conversion of the equation of transfer to relatively simple partial differential equations. However, the low-order approximations are usually only accurate in optically thick media and for higher-order approximations the mathematical complexity increases rapidly. It is known from neutron transport theory that approximations of odd orders are more accurate than even ones. Due to its simplicity, mainly the lowest order P_N solution corresponding to $N = 1$

(P_1-approximation) is usually used (so called diffusion approximation). The diffusion approximation describes in good accuracy the radiation field in many problems of radiation hydrodynamics.

In P_1-approximation we suppose that the angular dependence of the specific intensity can be represented by the first two terms in a spherical harmonic expansion

$$I_\nu(\mathbf{r},\nu,\mathbf{\Omega}) = \varphi_1(\mathbf{r},\nu) + 3\boldsymbol{\varphi}_2(\mathbf{r},\nu)\cdot\mathbf{\Omega}, \tag{3}$$

where $\mathbf{r}$ is the position vector fixing the location of a point in space, φ_1 and $\boldsymbol{\varphi}_2$ correspond to the density of the radiation field multiplied by velocity of light c, and to the radiation flux [7].

The spectral density of the radiation field is

$$U_\nu(\mathbf{r},\nu) = \frac{1}{c}\int_0^{4\pi} I_\nu(\mathbf{r},\nu,\mathbf{\Omega})\mathrm{d}\Omega = \frac{4\pi}{c}\varphi_1(\mathbf{r},\nu). \tag{4}$$

Likewise, for radiation flux we obtain

$$\mathbf{W}_\nu = (\mathbf{r},\nu)\int_0^{4\pi} I_\nu(\mathbf{r},\nu,\mathbf{\Omega})\cdot\mathbf{\Omega}\mathrm{d}\Omega = 4\pi\boldsymbol{\varphi}_2(\mathbf{r},\nu). \tag{5}$$

By integration of the equation of radiation transfer Eq. (2) over all solid angles we obtain the diffusion equation for the spectral density of radiation field

$$\begin{aligned}-\mathrm{div}\left\{\frac{c}{3\kappa_\nu}\mathrm{grad}\left[U_\nu(\mathbf{r},\nu)\right]\right\} + \kappa_\nu c U_\nu(\mathbf{r},\nu)\\ = \kappa_\nu 4\pi B(\nu),\end{aligned} \tag{6}$$

and for the radiation flux

$$\mathbf{W}_\nu(\mathbf{r},\nu) = -\frac{c}{3\kappa_\nu}\mathrm{grad}\left[U_\nu(\mathbf{r},\nu)\right]. \tag{7}$$

The diffusion approximation is valid under assumption that the spectral radiation intensity is almost isotropic.

3. Multigroup method

One of the methods for handling the frequency variable in the equation of transfer is the multigroup method [8], which leads to its discretization. One assigns a given photon to one of G frequency groups, and all photons within a given group are treated the same from the point of view absorption properties of the medium, the absorption coefficient for given frequency group k is supposed to be constant with certain average value

$$\kappa_\nu(\mathbf{r},\nu,T) = \kappa_k(\mathbf{r},T), \nu_k \le \nu \le \nu_{k+1}, k = 1,\ldots,G. \tag{8}$$

The equation of transfer for the given frequency group can be treated as equation for grey medium:

$$\mathbf{\Omega}\cdot\mathrm{grad}\left[I_\nu(\mathbf{r},\mathbf{\Omega})\right] = \overline{\kappa_k}\left[\int_{\nu_k}^{\nu_{k+1}} B(\nu)\mathrm{d}\nu - I_k(\mathbf{r},\mathbf{\Omega})\right], \quad 1 \le k \le G, \tag{9}$$

where $\overline{\kappa}_k$ is the mean absorption coefficient defines as

$$\overline{\kappa_k} = \frac{\displaystyle\int_{\nu_{k-1}}^{\nu_k} \kappa_\nu(\nu)\left[B(\nu) - I_\nu(\mathbf{r},\nu,\mathbf{\Omega})\right]\mathrm{d}\nu}{\displaystyle\int_{\nu_{k-1}}^{\nu_k}\left[B(\nu) - I_\nu(\mathbf{r},\nu,\mathbf{\Omega})\right]\mathrm{d}\nu}. \tag{10}$$

For P_1-approximation the multigroup diffusion equation has the form

$$\begin{aligned}-\mathrm{div}\left\{\frac{c}{3\overline{\kappa_k}}\mathrm{grad}\left[U_k(\mathbf{r})\right]\right\} + \overline{\kappa_k} c U_k(\mathbf{r})\\ = \overline{\kappa_k} 4\pi \int_{\nu_{k-1}}^{\nu_k} B(\nu)\mathrm{d}\nu.\end{aligned} \tag{11}$$

4. Mean absorption coefficients

For the multigroup method to be useful, one must be able to compute or estimate the mean values of absorption coefficients Eq. (10). An exact calculation of these group constants involves knowledge of the spectral intensity $I(\mathbf{r},\nu,\mathbf{\Omega})$ which is unknown. The underlying assumption in the multigroup method is that the group constants are relatively insensitive to the weighting functions $I(\mathbf{r},\nu,\mathbf{\Omega})$ used in computing the averages over frequency. The accuracy of this method depends also on the interval splitting. As the group width approaches zero the group constants become independent upon the estimate made for $I(\mathbf{r},\nu,\mathbf{\Omega})$.

The absorption coefficients are generally complex and widely varying functions of frequency (as can be seen at Fig. 1. The use of different weighting functions $I(\mathbf{r},\nu,\mathbf{\Omega})$ can lead to the quite different results. Generally, $\overline{\kappa}_k$ is taken as either Rosseland or Planck mean.

The Rosseland mean, also called mean free path of radiation is appropriate when the system approaches equilibrium (almost all radiation is reabsorbed). It has the form

$$\overline{\kappa_\mathrm{R}^{-1}} = \frac{\displaystyle\int_{\nu_{k-1}}^{\nu_k} \kappa_\nu^{-1}\frac{\mathrm{d}B(\nu,T)}{\mathrm{d}T}\mathrm{d}\nu}{\displaystyle\int_{\nu_{k-1}}^{\nu_k}\frac{\mathrm{d}B(\nu,T)}{\mathrm{d}T}\mathrm{d}\nu}. \tag{12}$$

The Planck mean is appropriate in the case of optically thin and emission dominated system. It can be expressed as

$$\overline{\kappa_\mathrm{P}} = \frac{\displaystyle\int_{\nu_{k-1}}^{\nu_k} \kappa_\nu B(\nu,T)\mathrm{d}\nu}{\displaystyle\int_{\nu_{k-1}}^{\nu_k} B(\nu,T)\mathrm{d}\nu}. \tag{13}$$

5. Absorption properties of plasma

Radiation in arc plasma depends, besides others physical quantities, on concentrations of all chemical species occurring in the plasma. Equilibrium compositions of

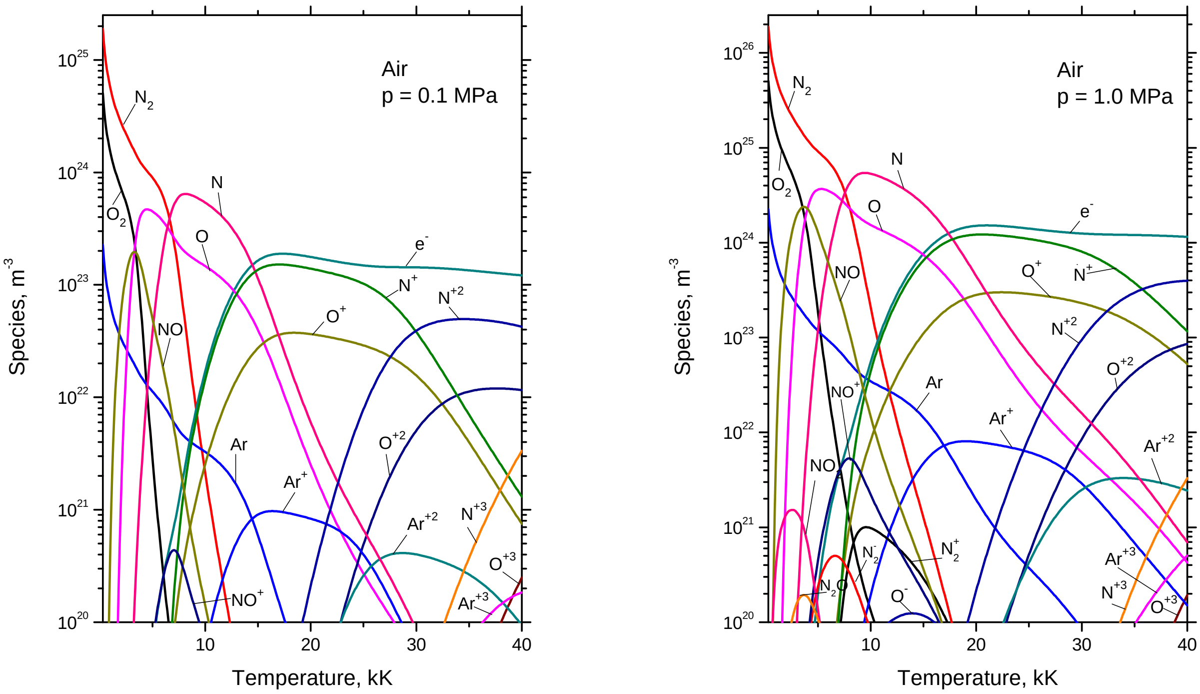

Figure 1. Concentration of different species in the thermal plasma as a function of temperature.

different types of plasma can be computed using some methods. At our Faculty, a computer code was developed for the calculation of absorption coefficients in thermal plasmas and a database of input parameters for many types of plasma was created. Here, these parameters were calculated with greater accuracy. One of the options considered is clean air without CO_2. In this case, we take into account availability in plasma some elements O_2, N_2, Ar, e^- and also ions of O, N, Ar. Figure 1 shows the concentrations of different species in the air plasma for two pressures 0.1 MPa and 1.0 MPa. It is obvious, that the concentration depends on the plasma pressure. Therefore values of these parameters influence the magnitude of the absorption coefficients.

The absorption coefficients of air plasma were calculated for eleven different pressures (0.01 MPa, 0.1 MPa, 0.2 MPa, 0.4 MPa, 0.6 MPa, 0.8 MPa, 1 MPa, 1.5 MPa, 2 MPa, 3 MPa and 4 MPa). The plasma temperature was considered in the range from 300 K to 35 000 K. The frequency interval for calculations was (0.01 to 10) $\cdot 10^{15}$ Hz with increment of 5×10^5 Hz. It allows the accuracy increasing of the calculated data.

An example of absorption coefficient for the pressure of 0.1 MPa and temperatures 5000 K and 15 000 K is shown in Fig. 2. This spectrum is a complex of discrete and continuous spectra. High complexity of radiation absorption coefficients (with respect to the frequency and temperature) can be seen.

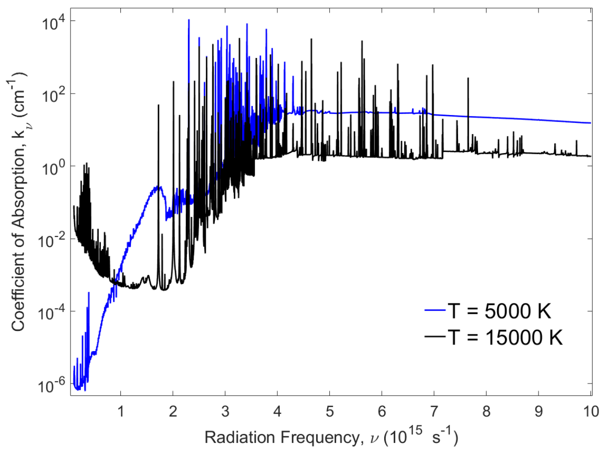

Figure 2. Absorption coefficient of air thermal plasma as a function of frequency.

After evaluation of obtained spectra for various pressures, expected regularities were revealed for values of absorption coefficient. The absorption coefficients at the pressure of 0.1 MPa and 1.0 MPa were taken for comparison. The corresponding graphs for temperatures of 5000 K and 15 000 K are shown in Fig. 3.

Given by the appearance of these spectra, computer code was created to analyzing and evaluation of the absorption coefficients dependence on frequency. This computer code makes possible to calculate mean

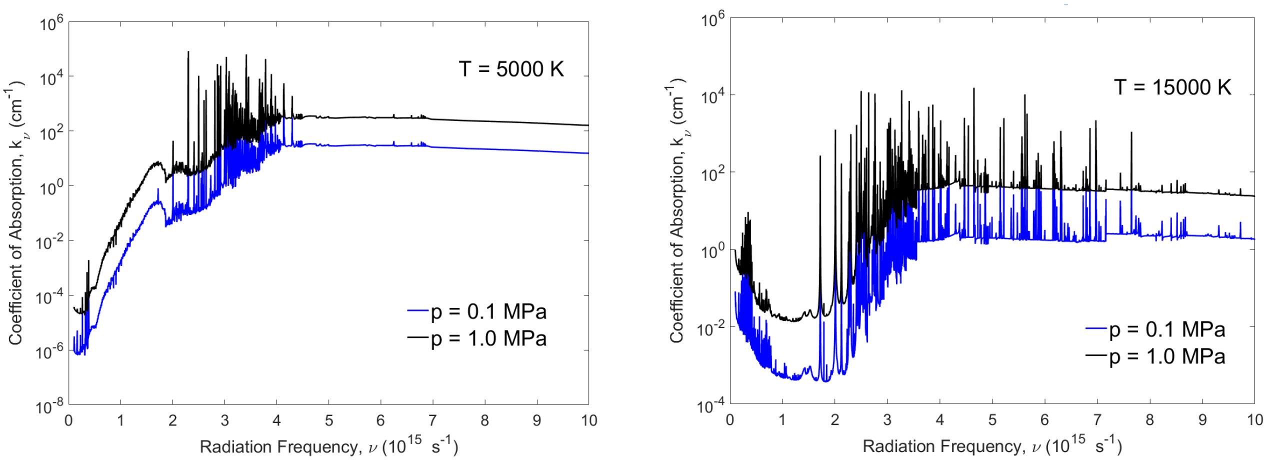

Figure 3. Comparison of absorption coefficients in air thermal plasma for two pressures.

absorption coefficients using methods described previously. Therefore, this software allows to divide the frequency interval into a certain number of groups.

As a result of these procedures, we obtain the mean absorption coefficients in relation to the temperature. Fig. 4 shows the example of results calculated for two frequency groups (the whole frequency interval was divided into 15 groups).

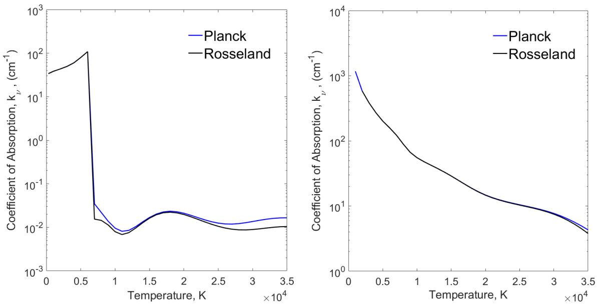

Figure 4. Mean absorption coefficients in air plasma for two frequency groups $(0.76 - 1.42) \cdot 10^{15}$ *Hz and* $(8.68 - 9.34) \cdot 10^{15}$ *Hz.*

6. Conclusions

Analyzing different ways of dividing into groups and different number of groups, we can conclude that these approximation methods give similar results in two thirds of our frequency interval. In estimating obtained data and graphs for an eleven different pressures we can conclude that the best results yields for Rosseland mean in frequency groups with small values of absorption coefficient and Planck's mean with high values of absorption coefficient. This occurs due to the properties of emission and re-absorption. From the comparison follows that Planck means generally overestimate the emission of radiation, and Rosseland means underestimate it.

In accordance with these conclusions we will use at each interval relevant method of obtainment the mean absorption coefficients for further calculation and determination of other parameters, which are necessary for resolve the equation of radiation transfer. Thus, using approximation we can significantly reduce computational time expenditures, while remaining a sufficiently high accuracy of estimations.

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