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Time Series Forecasting and Analysis: A Study of American Clothing Retail Sales Data

Weijun Huang
University of Central Florida

TIME SERIES FORECASTING AND ANALYSIS: A STUDY OF AMERICAN CLOTHING RETAIL SALES DATA

by

WEIJUN HUANG

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ABSTRACT

This paper serves to address the effect of time on the sales of clothing retail, from 2010 to May 2019. The data was retrieved from the US Census, where N=113 observations were used, which were plotted to observe their trends. Once outliers and transformations were performed, the best model was fit, and diagnostic review occurred. Inspections for seasonality and forecasting was also conducted. The final model came out to be a ARIMA (2,0,1). Slight seasonality was present, but not enough to drastically influence the trends. Our results serve to highlight the economic growth of clothing retail sales for the past 8 years, cementing the significance of the production economy's stability. The quarterly GDP data was collected in order to find out the relationship with the differenced clothing data. Some observations of GDP data were affected by the clothing data before removing the seasonality. After removing the seasonality, the clothing expense is white noise and not predictable from the historical GDP.

DEDICATION

To my dad, mom: Thanks for all your help no matter mentally, physically, and financially.

To my soulmate: Thanks for supporting and always being by my side.

To myself: I kept telling myself I could do it, and I did it.

ACKNOWLEDGEMENTS

I would like to express my deepest appreciation to Dr. Mengyu Xu for being my thesis chair, leading me to achieve my research, and helping me with the programming code. I also extremely appreciate for Dr. Alexander Mantzaris for being my committee and giving me valuable comments to enhance my thesis research.

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CHAPTER ONE: INTRODUCTION

Clothing stores have been the link between designers, garment factory, and the people of a nation. The dependence of clothing stores has never wavered throughout the development of the United States. Some local stores have gone bankrupt, only to be replaced with more successful, well-known companies, such as Zara, H&M, Hollister, among others.

As a community evolves, they discard outdated practices for more modern ones. This can be seen across the entire economy; For example, goods can now be purchased online, and - depending on the service being provided - can be picked up in store, or at your door. Years ago, this service would have never worked, as stores did not have the infrastructure, or the wealth to do so. With the passage of time comes innovation and increases in quality of life.

The history of clothing store development is not as fleshed-out as other studies, however the important events can be broken up into two major events - *pre-modernization* and *modernization.* Before the clothing stores era was formed, people make their own clothes. The first documented clothing market was believed to be around the beginning of the 16^{th} century (Alison). Their selection of produce only included second-handed clothing. With the advancement of the times, there were a large number of clothes shops in the 19^{th} century and the clothing industry was steadily improving. Nowadays, the type of clothes shop can be divided into several types, which are designer stores, retail clothing stores (Urban Outfitter), and outlets.

The data for this paper was initially collected and compiled by the United States Census Bureau (https://www.census.gov/retail/marts/www/adv44800.txt). The data has been collected on a monthly basis from 1992 until May of 2019. The data is collected as part of the Monthly Retail

Trade Survey, in which the United States Census surveys various retailers specified under the Retail Trade and clothing and clothing accessories outlined by the North American Industry Classification System (census.gov). It is important to note that this data includes both retailers defined as store retailers, including classic establishments which operate at fixed locations, as well as non-store retailers, which would include retailers that sell merchandise via non-traditional means including but not limited to online sales, television sales, phone sales, et cetera (Tran, Adeline, et al.). The United States Census Bureau conducts sample revisions for the Monthly Retail Trade Survey almost every five to seven years in an effort to compensate for various changes in the market that may occur (Sartz, Rob, et. al.).

The question is, is there a trend in clothing sales? Could implementation of more modern advertising and selling tactics improved the revenue of clothing stores in the United States? Do the clothing sales follow a defined time series, and if so, can we forecast the change in sales over time? Is the GDP relative to the clothing retail sales data? If they are, are those relative every quarter, or only some of them are relative? These are the questions that will be answered in the following sections. These questions deserve thorough investigation relating to several parts, including literature review, methodology, and discussion.

CHAPTER TWO: LITERATURE REVIEW

The most basic model that can be used for statistical analysis is a general linear model. Although these models are helpful, they assume each value has significant weights for infinite lengths of time. For our purposes, this model alone would not suffice. Using model diagnostics, we explored several model types; specifically, the MA, AR, ARMA, and ARIMA. In later sections, we will also discuss model trends, as well as forecasting.

The Moving Average: MA

This model describes the weights of each lag K, and assigns them to each time t. The complexity of the model is determined by the maximum time lag that a meaningful conclusion can be made between two observations. For a MA model at q=2 (MA (2)), the largest meaningful lag between two times t and t' would be k=2. Any lags larger than two will not produce any correlation between values from times t and t'.

For example, if we were to fit a data set to a MA (2) model, then Y values for time t=10 and t=12 should be related by -θ2σ2e. This correlation should exist for any differences in time that are two units apart. If we were to compare the results for Y values at time t=10 and t=13, there would be 0 correlation. This idea of comparing values of Y_t from time values larger than the established lag K holds true for the rest of the models that will be discussed (Cryer, Chan 2011).

Auto-regression: AR

Generally, for an AR(p) model to be wide-sense stationary, the roots of the polynomial $\phi(z) = 1 - \sum_{i=1}^{p} \varphi_i z^{p-i}$ must lie inside the unit circle (Hamilton). When a model uses a previous value of Y_{t-k} to estimate the current Y_t, this is called an autoregressive model. These models depend on an "guessing term", in which this term will predict the value of the Y_t. This guessing term et, is a white noise process. Once again, observations at two different times t and t' can only be correlated if they are within the time lag=K values that the model specifies. If two Y values at times t = 5 and t = 6 were to be compared, they would be related by their autocorrelation value, which would be ϕ_k. This means that the lag directly affects the correlation between two values Y_t and Y_{t-k}. This is a classic example of an AR (1) process.

Specific to an AR (1) process, the model is considered stationary if the absolute value of the coefficient ϕ is less than 1. Should the model be an AR (2) process, then the requirements for a stationary time series can be expressed using the quadratic equation. The quadratic equation satisfies three criteria, which are the following:

1. The sum of coefficients ϕ_1 and ϕ_2 must be less than 1
2. The subtraction of ϕ_1 from ϕ_2 should be less than 1
3. The absolute value of the second coefficient ϕ_2 should be less than one.

If this criteria is not met, differencing must occur, which will be brought up later (Cryer, Chan 2011).

The Autoregressive-Moving Average: ARMA

The general ARMA model was described in the 1951 thesis of Peter Whittle, who used mathematical analysis (Laurent series and Fourier analysis) and statistical inference (Hannan). ARMA models were popularized by a 1970 book by George E. P. Box and Jenkins, who expounded an iterative (Box–Jenkins) method for choosing and estimating them. This method was useful for low-order polynomials (of degree three or less) (Hannan & Deistler). When a model cannot be explained without both autoregressive and moving average expressions, the two are combined to form an ARMA model. In these cases, both models will be incorporated into one expression, and the overall stationarity of the model depends whether the individual autoregressive and moving average models satisfy said conditions. Again, when dealing with non-stationary data sets, differencing (Cryer, Chan 2011).

The Integrated Autoregressive-Moving Average: ARIMA

When a model lacks the requirements to be considered stationary, modifications can be done to address this. Differencing occurs when one takes a Y_t and subtracts it with Y_{t-1}. This overall expression can be reduced to W_t, which is often times stationary. Usually, a model only requires differencing to the 1st or 2nd magnitude. Differencing any further would likely induce another problem - overdifferencing. The ARIMA model blends the previous ideas with differencing, and can be broken down to its ARI and IMA counter-parts.

Let's take the following autoregressive model as an example:

Y_t = 9Yt-1 + et

Immediately, it should be clear that the model cannot be sustained over a long period of time, as the values of Y_t will exponentially increase. However, when the model is differenced, it comes out to this:

$W_t = Y_t$ - Y_{t-1}

W_t = (1/9)($Y_{t+1} + Y_t + Y_{t-1}$)

The resulting expression can be maintained across a long period of time rather stably (Cryer Chan 2011). Seasonal ARIMA models are usually denoted ARIMA(p,d,q)(P,D,Q)m, where m refers to the number of periods in each season, and the uppercase P,D,Q refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model (Hyndman).

Trends

How the model changes in response to time can be addressed with a thorough understanding of trends. Models can have trends that suggest the average change across time, but these models vary. This concept can best be described using the following example:

Y_t = Ut + X_t ; (where U_t is the deterministic component, and X_t is the stochastic component.)

In this case, the model can have a trend that is linearly increasing or decreasing, due to U_t , but can have a certain randomness to it, due to X_t. The linear trend will be the simplest, however Ut can also be quadratic, square-root, exponential, logarithmic, among other possible trends.

Although not applicable to our data, it is important to note that intervention is not considered part of a trend. This is because intervention events are both rare and unexpected; these events are not part of the model, and are products of influences outside of our control. The following are examples of interventions:

1. Airplane ticket sales vs time, from 1990-2005*
2. Market value of homes vs time, from 2000-2010*
3. Number of marathon runners vs time, from 2010-2015*

*--In the U.S. alone.

Most cases of intervention occur by reducing the average values of the data set at a specific time t, but that is not always the case. A quick look at a dataset comparing amount of human blood donated in Florida vs. time, from 2010-2018, should show a large positive spike, at time t = 2017, for the months following the Pulse nightclub shooting. (Cryer, Chan 2011).

Forecasting

Observing the trend of a time series will often provoke the question, "what will happen next"? With forecasting, we use the previous data values from time t to predict further data values at times t + L. The usefulness becomes quickly apparent, as analysts for major companies can predict the growth of sales, or perhaps predict changes in the stock market (Cryer Chan 2011).

CHAPTER THREE: DATA, METHODOLOGY, AND RESULTS

The main objective of this project was to utilize statistical analysis software, R, to analyze a given dataset. As previously mentioned, the dataset used was collected by the United States Census Bureau, monthly from 1992 until the present day (Census.gov). At the time of analysis and writing of this paper, the last data point available for analysis is for May 2019. Due to restrictions in analysis, for the purpose of this project, the dataset was cut to only include data from January 2010 until May 2019. This was done in an effort to simplify the analysis process, as well as make the output easier to understand.

Time Series Data Analysis

The raw dataset obtained from the Census Bureau was originally not in time series format. Therefore, the first step in analysis involved transferring the raw data into a time series. Following this, the corresponding time series was plot and interpreted. The corresponding graph can be seen in [Figure 1], where the time series dataset is plotted to represent monthly clothing revenue, in millions of dollars, as reported to the United States Census Bureau.

From this it can be seen that the time series follows a positive linear trend. It does not appear to level off in latest years. In this time series plot, one can see that separate plotting symbols are used to distinguish between the different months of the year. This is done in an effort to determine whether or not there is any evidence to indicate seasonality in our data. From the plot of the time series, there does not appear to be any strong indication of seasonality.

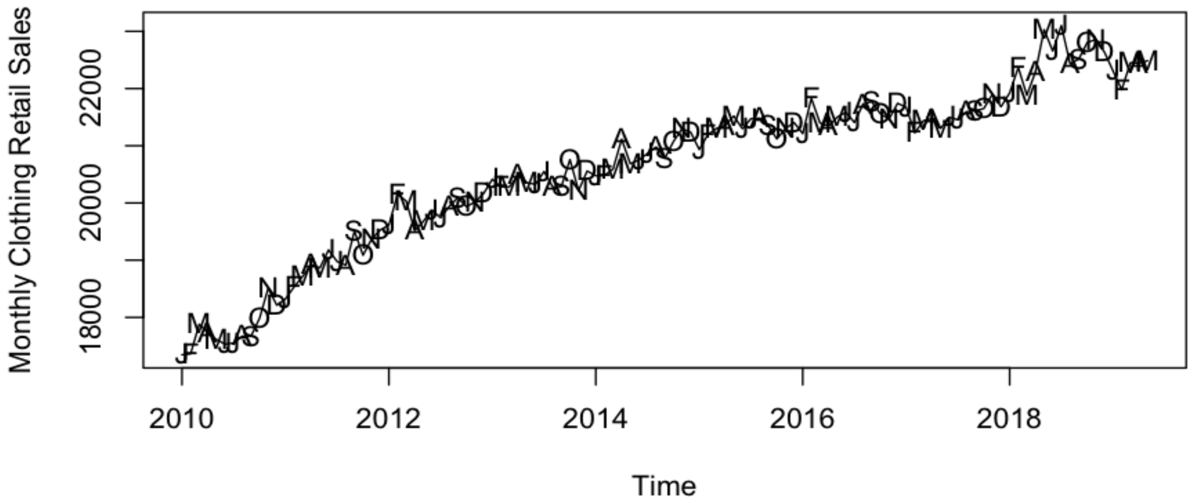

Figure 1: Time Series Plot for the Clothing Retail Sales Data.

Seasonal-means Trend

As previously stated, from [Figure 1], it was determined that the sales are not stationary in the time series plot. In fact, it slowly increases almost linearly with time. With observations from this time series plot, the method of least squares is used to establish two models as seasonal-means trend and seasonal-means trend plus quadratic time trend in order to accurately represent the dataset. The regression output, multiple R-squared, and standardized residuals of two models were interpreted and analyzed.

Initially the method of least squares was used to fit a seasonal-means trend to this time series, as shown in [Figure 2]. One can note that the multiple R-squared was 0.0052 percent, which is very low, and Adjusted R-squared has a negative number. The residuals interval for the first linear model is between -3177.8 and 2533.2. In addition to this, most of the beta coefficients are not statistically significant. In brief, this model is not a good trend fit for our dataset.

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	20422.00	476.37	42.870	<2e-16	***
month.February	117.80	673.69	0.175	0.862	
month.March	176.40	673.69	0.262	0.794	
month.April	246.40	673.69	0.366	0.715	
month.May	281.80	673.69	0.418	0.677	
month.June	99.89	692.15	0.144	0.886	
month.July	146.78	692.15	0.212	0.832	
month.August	136.33	692.15	0.197	0.844	
month.September	198.00	692.15	0.286	0.775	
month.October	241.44	692.15	0.349	0.728	
month.November	348.89	692.15	0.504	0.615	
month.December	374.22	692.15	0.541	0.590	

Figure 2: Model for a Linear Seasonal Means Trend

A plot of the standardized residuals of the aforementioned linear seasonal means trend model was created and can be seen in [Figure 3]. In this, one could clearly see the trend in residuals for this model. The standardized residuals of linear model indicated that the interval of standardized residuals was approximately between -2 to 1. Because of the lack of normality and the poor significance statistics, another model is suggested to be fit to determine the trend of the time series.

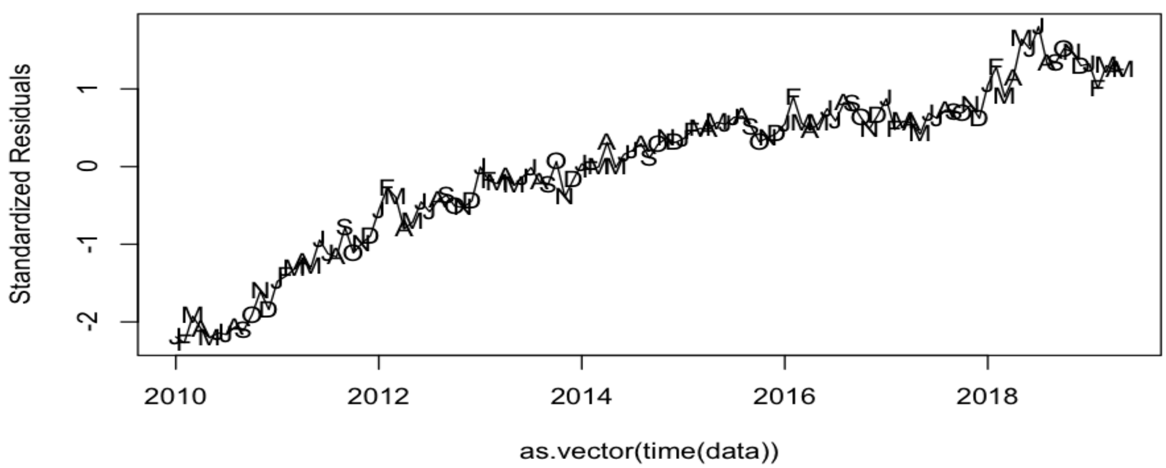

Figure 3: Standardized Residuals of Linear Seasonal Means Model

In short, the linear model to this time series analyzed above is not an ideal model of the trend of the time series because the multiple R-squared and the standardized residuals are not acceptable. Analysis of a second model is needed to compare against the first model to determine the proper equation of the trend of the time series.

Once again, the method of least squares is used to obtain a summary of a seasonal-means trend plus quadratic time trend equation, the output can be seen in [Figure 4]. The interval of residual is between -702.5 and 908.5. Comparing with the linear model above, the quadratic time trend interval had a smaller interval. Most of the terms for the second model are statistically significant. The multiple R-squared and Adjusted R-squared are 95.2 percent and 94.57 percent, which are much higher than that of the first model. Thus far, the quadratic time trend is much preferred over the linear trend, however, an analysis of the residuals of this linear seasonal means plus quadratic time trend model must be analyzed to determine whether this is the ideal for our time series.

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)     -1.900e+08  1.945e+07  -9.770 3.48e-16 ***
month.February   7.522e+01  1.495e+02   0.503    0.616    
month.March      9.188e+01  1.495e+02   0.615    0.540    
month.April      1.206e+02  1.495e+02   0.806    0.422    
month.May        1.154e+02  1.495e+02   0.771    0.442    
month.June       6.937e+01  1.538e+02   0.451    0.653    
month.July       7.303e+01  1.538e+02   0.475    0.636    
month.August     2.000e+01  1.538e+02   0.130    0.897    
month.September  3.973e+01  1.538e+02   0.258    0.797    
month.October    4.189e+01  1.538e+02   0.272    0.786    
month.November   1.087e+02  1.538e+02   0.707    0.482    
month.December   9.403e+01  1.539e+02   0.611    0.542    
time(data)       1.882e+05  1.931e+04   9.745 3.94e-16 ***
I(time(data)^2) -4.658e+01  4.792e+00  -9.719 4.48e-16 ***
---
```

Figure 4: Model for a Linear Seasonal Means Plus Quadratic Time Trend

Next, the standardized residuals of the of the linear seasonal means model plus the quadratic time trend model, and the resulting output can be seen in [Figure 5]. The interval of the model was approximately around -2 to 3. Compared to the residuals for the first model as shown in [Figure 3] the residuals of the second model are much more random.

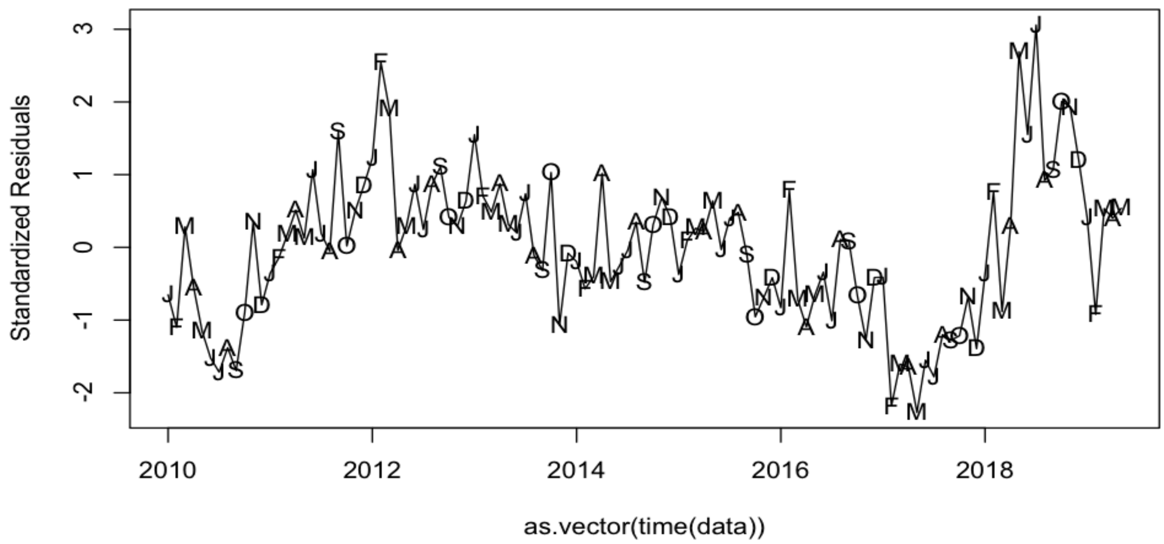

Figure 5: Standardized Residuals of Linear Seasonal Means Plus Quadratic Time Trend Model

The method of least squares is used to obtain a summary of a seasonal quadratic time trend, the output can be seen in [Figure 6]. The interval of residual is between -1020.99 and 856.59. Comparing with the linear model above, the quadratic time trend interval had a smaller interval. Most of the terms for the second model are statistically significant. The multiple R-squared and Adjusted R-squared are 90.59 percent and 89.46 percent, which are much higher than that of the first model. Thus far, the quadratic time trend is much preferred over the linear trend, but the linear seasonal means plus quadratic time trend model is the best. However, an analysis of the residuals of this linear seasonal means plus quadratic time trend model must be analyzed to determine whether this is the ideal for our time series.

```
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)       -4.824e+05  1.625e+04 -29.684   <2e-16 ***
month.February     7.620e+01  2.082e+02   0.366    0.715
month.March        9.319e+01  2.082e+02   0.448    0.655
month.April        1.216e+02  2.082e+02   0.584    0.561
month.May          1.154e+02  2.083e+02   0.554    0.581
month.June         1.417e+02  2.139e+02   0.662    0.509
month.July         1.470e+02  2.139e+02   0.687    0.494
month.August       9.493e+01  2.139e+02   0.444    0.658
month.September    1.150e+02  2.139e+02   0.537    0.592
month.October      1.168e+02  2.140e+02   0.546    0.586
month.November     1.827e+02  2.140e+02   0.854    0.395
month.December     1.664e+02  2.140e+02   0.777    0.439
```

Figure 6: Model for a Linear Seasonal Quadratic Time Trend Model

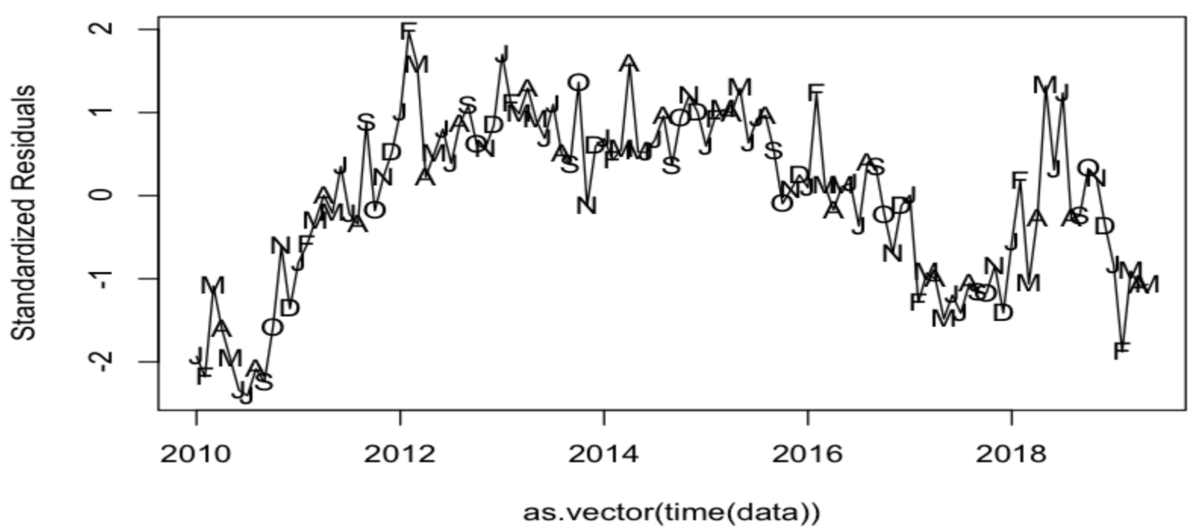

Figure 7: Standardized Residuals of Quadratic Time Trend Model

the standardized residuals of the of the linear seasonal quadratic time trend model, and the resulting output can be seen in [Figure 7]. The interval of the model was approximately around -2 to 2. Compared to the residuals for the first and second model as shown in [Figure 3] [Figure 5], the residuals of the second model are much more random.

The method of least squares was used to create two trend models in an effort to find the best trend fit for our time series. To compare the two, for the linear model almost all of the terms were not statistically significant, however its R-square and interval of residual were not acceptable when compared against the quadratic model. The plots of the residuals were not randomly placed on the linear model. Although not all of the terms in the second model containing the quadratic term were statistically significant, the second model had an almost perfect multiple R-squared and adjusted R-squared. In addition, the plots for the residuals of the second model were randomly placed. In summary, we will use the seasonal-means plus quadratic time trend as our model's trend.

Autocorrelation and Partial Autocorrelation Functions and Their Graphs

Following the comparison of least squares models and trends, the Autocorrelation Function of the original time series was plot in an effort to determine the best model fit for our time series. The autocorrelation function graph for our time series data is shown in [Figure 8]. One can easily see that the autocorrelation function for the given data does not die out at any given lag. This would be evidence of a lack of stationarity for the given time series.

One can see from [Figure 8] that all of the values of the autocorrelation function are notably and meaningfully far from zero. This only further points to the idea that the time series is not stationary. Because of this, differencing must occur for meaningful conclusions to be made. Following this, models such as the integrated moving average model (IMA) and the autoregressive integrated moving average model (ARIMA) should be considered.

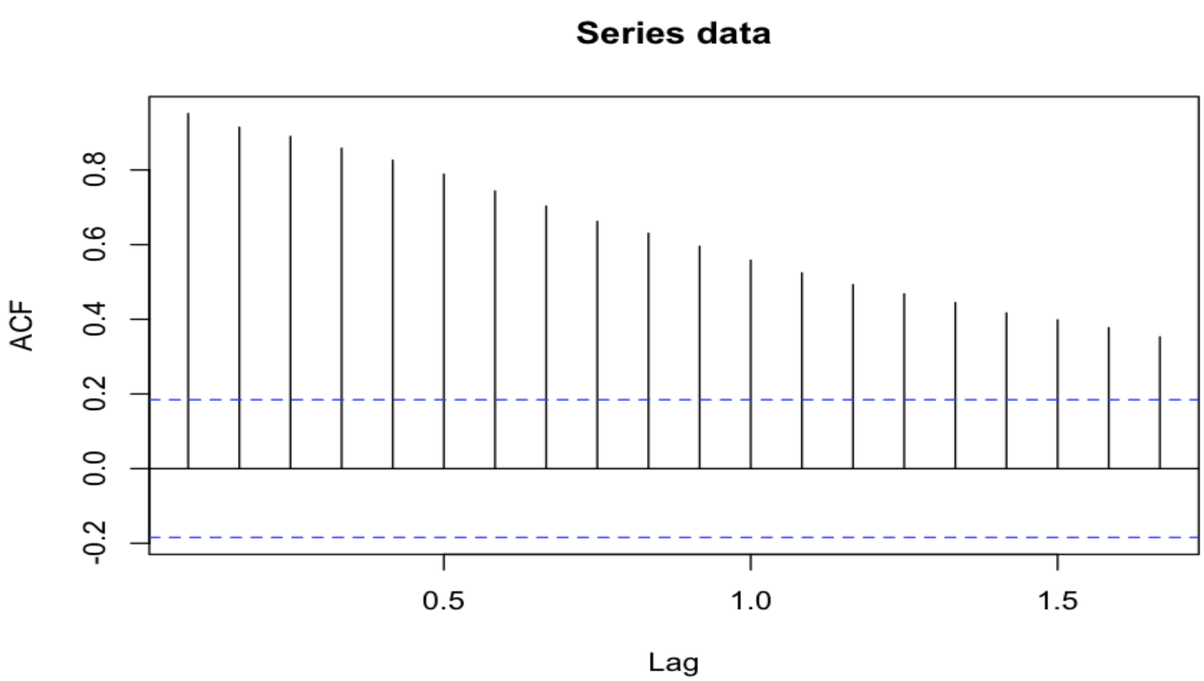

Figure 8: Autocorrelation Function Plot

Following this, the partial autocorrelation function for the time series is plot. The resulting plot is shown in [Figure 9]. From the plot, one can determine that there is a large cut off after the first lag. The remaining of the shown lags do not appear to hold much significance.

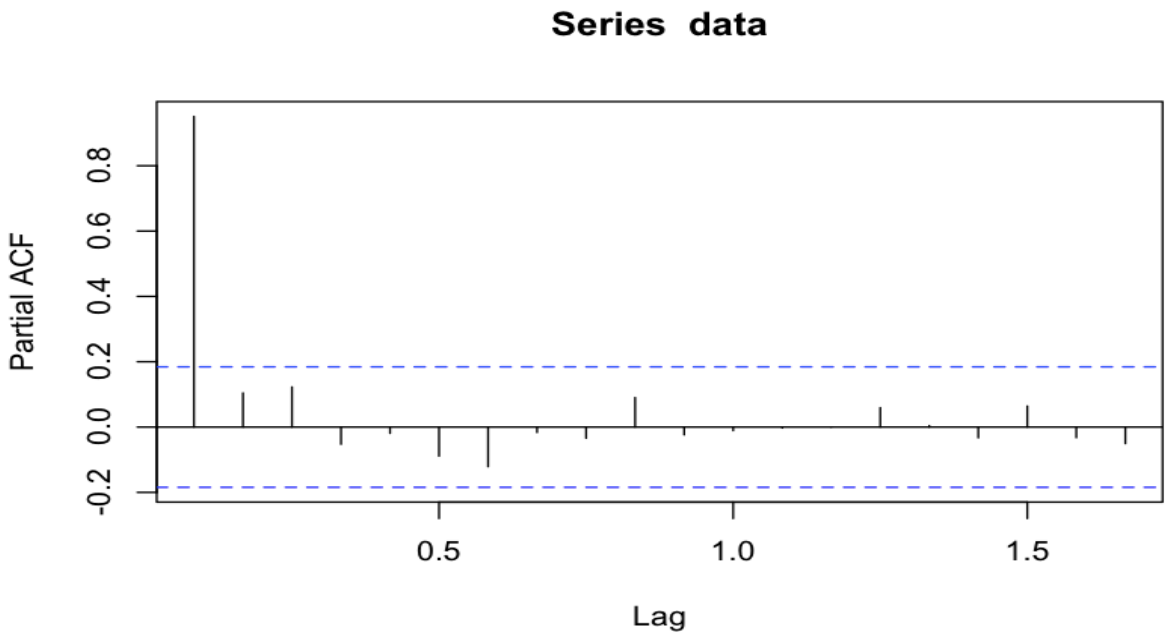

Figure 9: Partial Autocorrelation Function Plot

Following this analysis, the model was differenced. Differencing is a process used to make a nonstationary time series stationary. This process means that the average of the time series is constant over time. The autocorrelation function of the differenced time series is shown in [Figure 10].

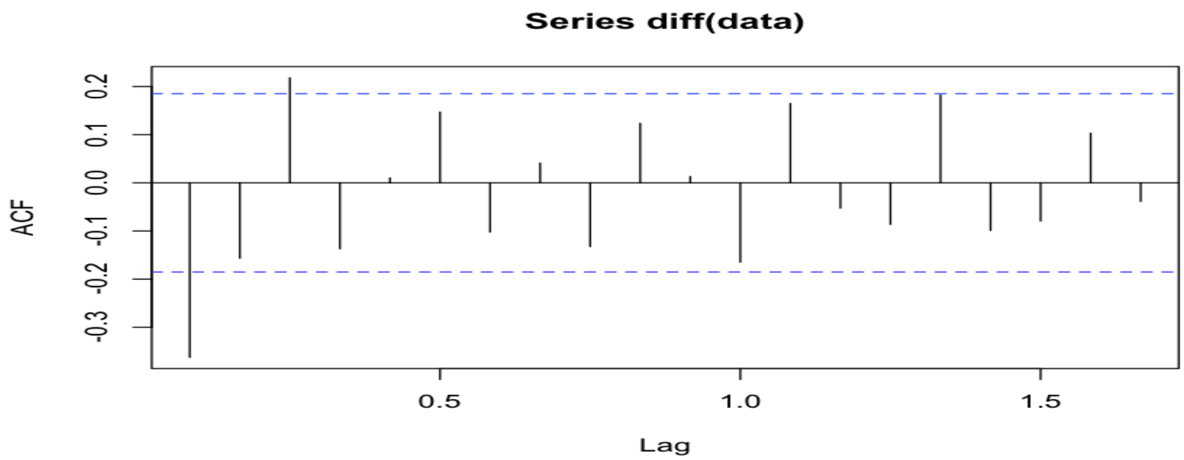

Figure 10: Autocorrelation Function of Differenced Time Series

Here, one can see that there is a significant drop off after the second lag, however there are multiple other significant lags throughout the series. Following this the partial autocorrelation function is plot and shown in [Figure 11].

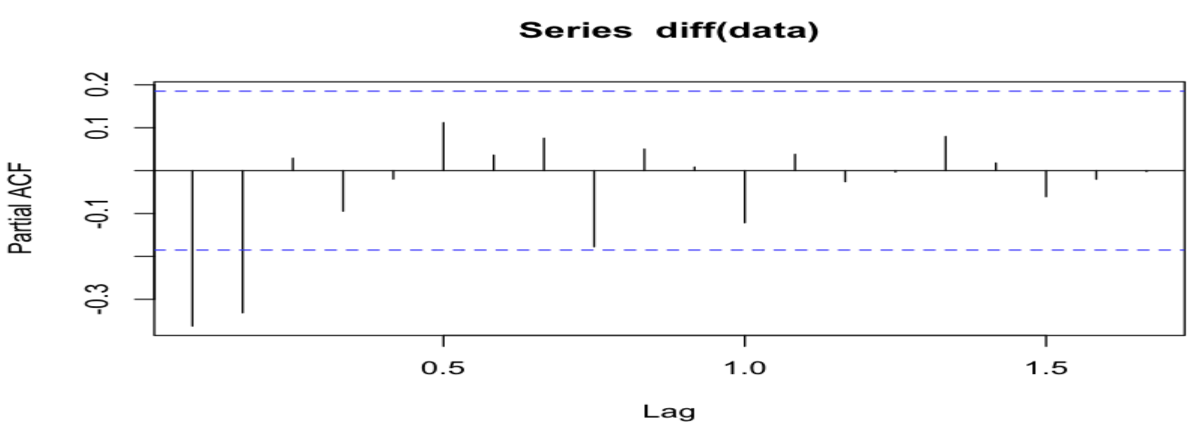

Figure 11: Partial Autocorrelation Function of Differenced Time Series

From the autocorrelation function and the partial autocorrelation function, an autoregressive integrated moving average model would be preferred for this given dataset. Given the lags shown in the autocorrelation function graph, one would be more inclined to fit an ARIMA (2,0,1) model to fit the given time series data. Further differencing will introduce over differencing, which is not ideal.

An autoregressive integrated moving average model is created using the r software. The resulting output is shown in [Figure 12]. As one can see, the value for $\theta 1$ is approximately equal to -0.5244, the value for $\theta 2$ is approximately equal to -0.3683, and the value for ψ is approximately equal to 0.1131.

```
Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = xmean, include.mean = FALSE, transform.pars = trans,
    fixed = fixed, optim.control = list(trace = trc, REPORT = 1, reltol = tol))

Coefficients:
          ar1      ar2     ma1    xmean
      -0.5244  -0.3683  0.1131  59.9089
s.e.   0.2369   0.1069  0.2524   4.8078

sigma^2 estimated as 7407:  log likelihood = -658.07,  aic = 1326.13

$degrees_of_freedom
[1] 108

$ttable
      Estimate     SE t.value p.value
ar1    -0.5244 0.2369 -2.2137  0.0290
ar2    -0.3683 0.1069 -3.4455  0.0008
ma1     0.1131 0.2524  0.4479  0.6551
xmean  59.9089 4.8078 12.4607  0.0000

$AIC
[1] 11.84049

$AICc
[1] 11.84383

$BIC
[1] 11.96185
```

Figure 12: Model for an ARIMA (2,0,1)

A summary of statistics for the ARIMA (2,0,1) Model is shown in [Figure 13]. As one can see, the standardized residuals and the Quantile-Quantile plot of the standardized residuals appear to be normal. Also, the p-values for the Ljung-Box statistic appear to be up to par with what would be ideal for the model of the time series.

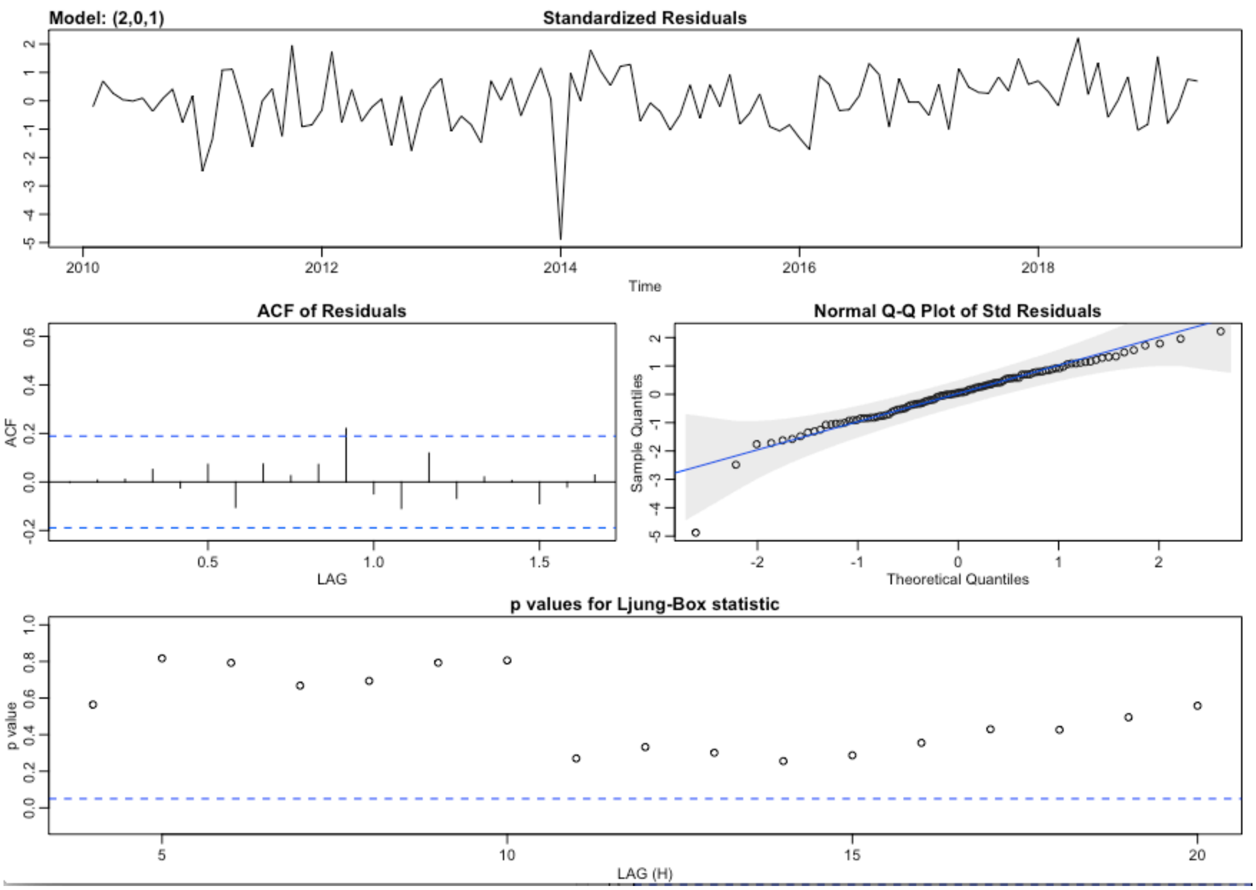

Figure 13: Summary of Statistics of the ARIMA (2,0,1) Model

Forecast for dataset

From the previously shown ARIMA (2,0,1) model the next step is to forecast the dataset future two years into the future to see the future trend of clothing stores. The forecast is shown in [Figure 14]. The last sample size of sales in the dataset is 22,487 millions of dollars. In [Figure 14] one can see the forecasting for the time series, which is denoted by the solid red points. One can see from the forecasted graph that the sales from June of 2019 to June of 2022 are around 22,500 to 23,500 millions of dollars. The forecast trend is a linear line without any factors affects and the increase rate for the future three years is based on the past ten years' sales rate. The gray area which is shown in the plot means if any factors affect the clothing retails market, the red point will flow up or down between the area.

If the trend continues, more and more clothing retail stores being established not only in the United States but also around the world because of its convenience to the consumers. Because of this the trend of the sales might go faster than predicted model. It is always possible to compare the actual data trend after two years and the forecasting trend below.

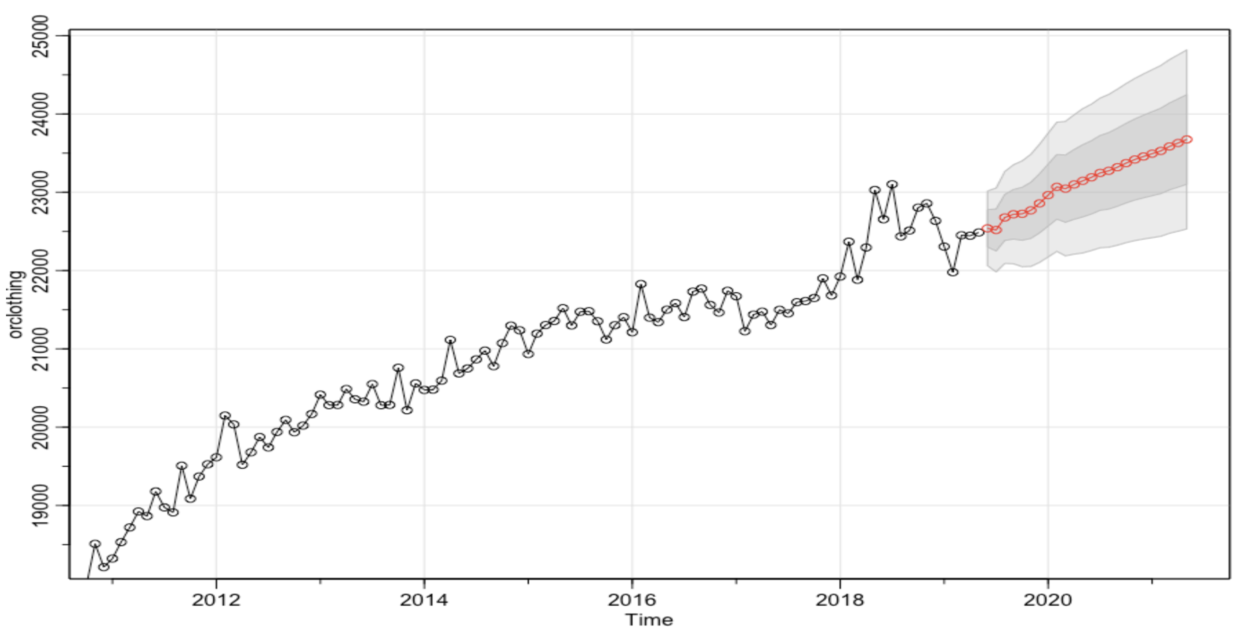

Figure 14: Forecast of the ARIMA (2,0,1) Model

CHAPTER FOUR: DISCUSSION

As the main objective for this project has been solved in the previous chapter. The researcher came out with another question upon his head. Is the GDP relative to the clothing retail sales data? If they are, are those relative every quarter, or only some of them are relative? The quarterly GDP dataset used was collected by the United States Bureau of Economic Analysis, quarterly from January 1993 until the present day (fred.stlouisfed.org). The last data point available for analysis is for July 2019. The research needed to fix his original clothing dataset from monthly to quarterly, since the frequency of the GDP dataset was quarterly.

However, both of the quarterly GDP dataset and the quarterly clothing dataset are non-stationary. Following this analysis, two of the dataset were differenced. Differencing is a process used to make a non-stationary time series stationary. Also, one can find out the relationship between two data set each quarterly easier.

Time Series Data Analysis for Quarterly GDP

The raw dataset obtained from the United States Bureau of Economic Analysis was originally not in time series format. Therefore, the first step in analysis involved transferring the raw data into a time series. Following this, the corresponding time series was plot and interpreted. The corresponding graph can be seen in [Figure 15], where the time series dataset is plotted to represent quarterly GDP revenue, in billions of dollars, as reported to the United States Bureau of Economic Analysis.

From this it can be seen that the time series follows a positive linear trend. It does not appear to level off in latest years. In this time series plot, one can see that separate plotting symbols are used to distinguish between the different quarterly of the year. This is done in an effort to determine whether or not there is any evidence to indicate seasonality in our data.

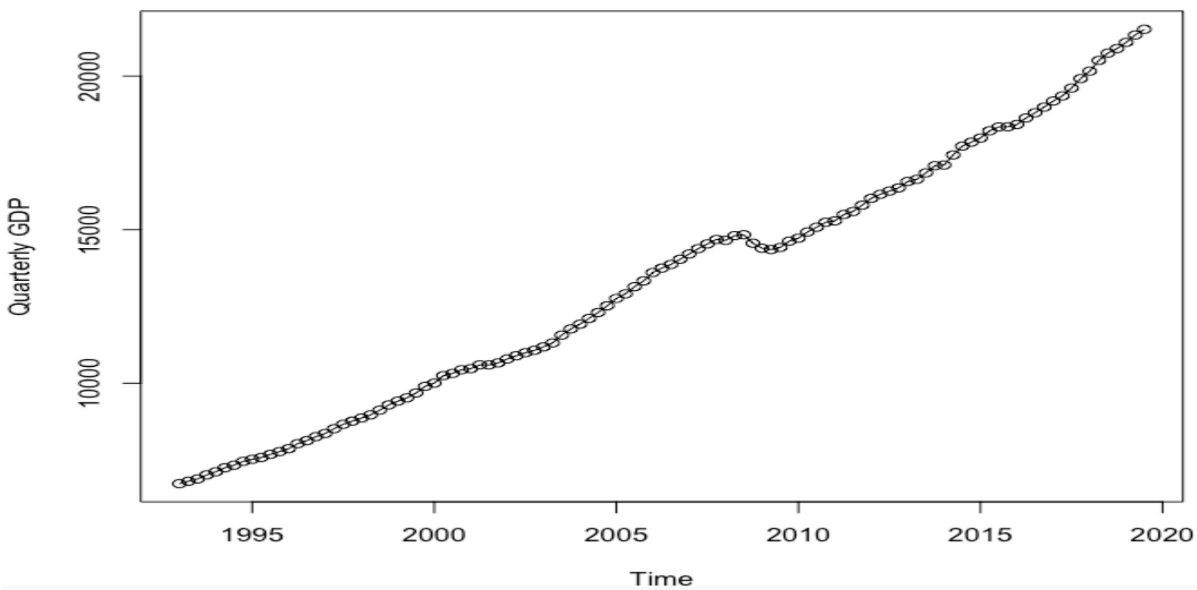

Figure 15: Time Series Plot for the Quarterly GDP Data

The differenced Autocorrelation Function of the Quarterly GDP time series was plot in an effort to determine the best model fit for our time series. The autocorrelation function graph for our Quarterly GDP time series data is shown in [Figure 16]. The partial autocorrelation function of the differenced time series is shown in [Figure 17].

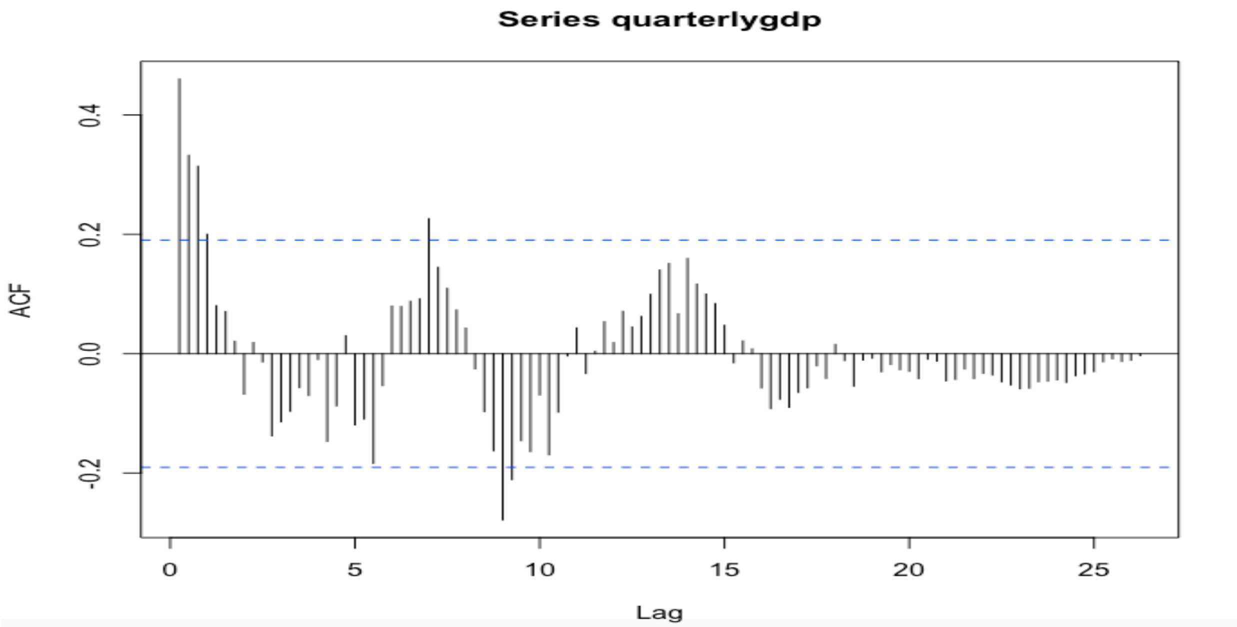

Figure 16: Autocorrelation Function of Differenced Time Series

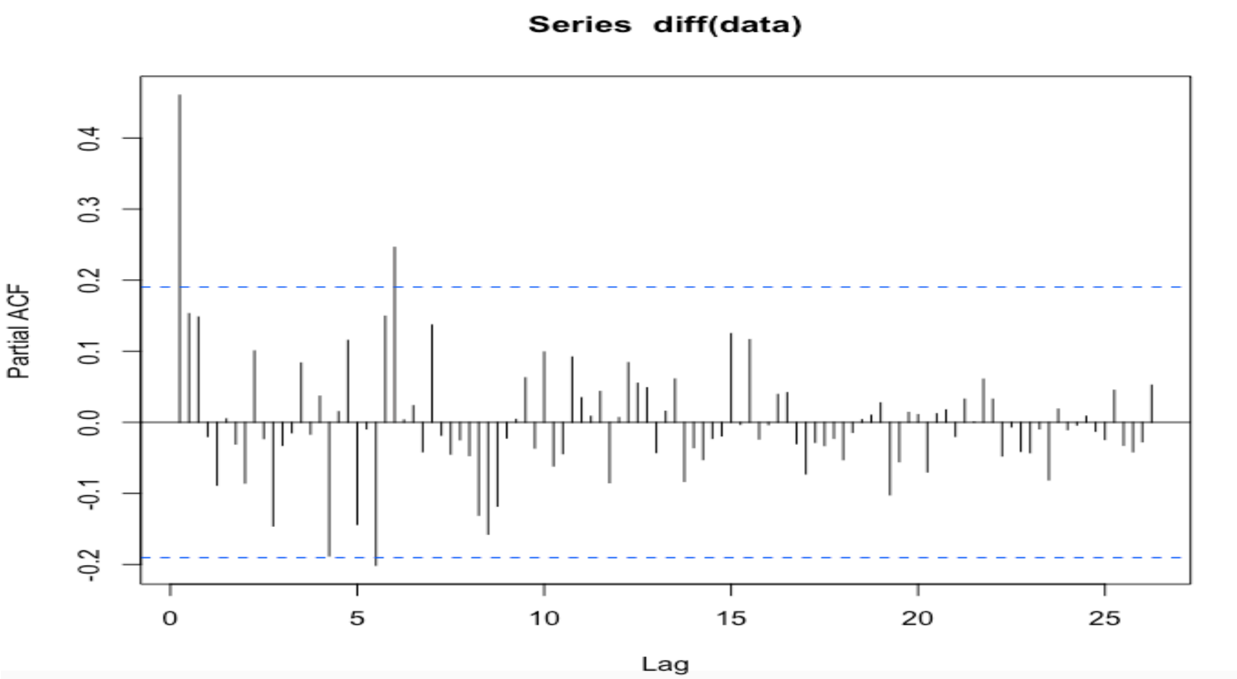

Figure 17: Partial Autocorrelation Function of Differenced Time Series

The Relationship Between Quarterly GDP and Clothing Retail Sales Using Multivariate ARMAX Models

Both frequency of the dataset was collected by quarter, and each quarter can be an independent variable. The researcher wants to know the relationship for the passing one, six and twelve month(s) between GDP and clothing retail sales. A multivariate ARMAX model was determined to solve the problem. For the two-dimensional series composed of quarterly GDP x_{t1}, and clothing retail sales x_{t2}. We can take x_t = (x_{t1}, x_{t2})' as a vector of dimension k = 2.

The main objective of the topic is how GDP might affect clothing. The first step in analysis involved transferring the passing one month of two data into a multivariate ARMAX model. Following this, the independent variable is clothing retail sales, and the dependent variable is GDP. The passing quarter of a year for two dataset summary is shown in [Figure 18]. One can see the P-value for the passing quarter of a year GDP was less than 0.05, so it rejected the null hypothesis. In other word, the passing quarter of a year GDP affected the clothing retail sales.

```
Estimation results for equation dqclothing:
===========================================
dqclothing = dqclothing.l1 + quarterlygdp.l1 + const + trend

                Estimate Std. Error t value Pr(>|t|)
dqclothing.l1    -0.2408     0.1130  -2.130   0.0356 *
quarterlygdp.l1   1.0531     0.4963   2.122   0.0363 *
const           -18.2052    88.5965  -0.205   0.8376
trend             0.2696     1.3057   0.206   0.8369
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 386.9 on 101 degrees of freedom
Multiple R-Squared: 0.06408,    Adjusted R-squared: 0.03628
F-statistic: 2.305 on 3 and 101 DF,  p-value: 0.08133
```

Figure 18: The passing quarter of a year for two dataset summary in ARMAX model

After repeating the same steps as previously, the passing six quarters for two dataset summary is shown in [Figure 19]. One can see the first quarter, the fourth quarter, and the fifth quarter were statistically significant.

```
Estimation results for equation dqclothing:
===========================================
dqclothing = dqclothing.l1 + quarterlygdp.l1 + dqclothing.l2 + quarterlygdp.l2 +
 dqclothing.l3 + quarterlygdp.l3 + dqclothing.l4 + quarterlygdp.l4 + dqclothing.
l5 + quarterlygdp.l5 + dqclothing.l6 + quarterlygdp.l6 + const + trend

                  Estimate Std. Error t value Pr(>|t|)
dqclothing.l1     -0.54467    0.13088  -4.162 7.47e-05 ***
quarterlygdp.l1    1.18380    0.53738   2.203  0.03028 *
dqclothing.l2     -0.24713    0.12837  -1.925  0.05751 .
quarterlygdp.l2    0.11639    0.53433   0.218  0.82807
dqclothing.l3     -0.16380    0.11739  -1.395  0.16648
quarterlygdp.l3    0.60901    0.52860   1.152  0.25247
dqclothing.l4     -0.07586    0.11100  -0.683  0.49614
quarterlygdp.l4    0.92812    0.53646   1.730  0.08720 .
dqclothing.l5      0.08265    0.10897   0.758  0.45027
quarterlygdp.l5    1.74195    0.54855   3.176  0.00208 **
dqclothing.l6      0.03065    0.10721   0.286  0.77565
quarterlygdp.l6    0.76669    0.60690   1.263  0.20990
const           -361.64211  114.84163  -3.149  0.00225 **
trend             -2.79358    1.45935  -1.914  0.05891 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 349.1 on 86 degrees of freedom
Multiple R-Squared: 0.3455,     Adjusted R-squared: 0.2466
F-statistic: 3.493 on 13 and 86 DF,  p-value: 0.000225
```

Figure 19: The passing six quarters for two dataset summary in ARMAX model

A summary of statistics for the passing twelve quarters Model is shown in [Figure 20]. As one can see, the sixth quarter and the eleventh quarter were statistically significant. The rest of them were non-statistically significant. It means only two quarters of the GDP affected the clothing retail sales.

```
Estimation results for equation dqclothing:
===========================================
dqclothing = dqclothing.l1 + quarterlygdp.l1 + dqclothing.l2 + quarterlygdp.l2 + dqclothi
ng.l3 + quarterlygdp.l3 + dqclothing.l4 + quarterlygdp.l4 + dqclothing.l5 + quarterlygdp.
l5 + dqclothing.l6 + quarterlygdp.l6 + dqclothing.l7 + quarterlygdp.l7 + dqclothing.l8 +
 quarterlygdp.l8 + dqclothing.l9 + quarterlygdp.l9 + dqclothing.l10 + quarterlygdp.l10 +
 dqclothing.l11 + quarterlygdp.l11 + dqclothing.l12 + quarterlygdp.l12 + const + trend

                  Estimate Std. Error t value Pr(>|t|)
dqclothing.l1     -0.47383    0.15028  -3.153  0.00241 **
quarterlygdp.l1    0.97358    0.60519   1.609  0.11231
dqclothing.l2     -0.23438    0.16284  -1.439  0.15464
quarterlygdp.l2    0.01638    0.62423   0.026  0.97915
dqclothing.l3     -0.04344    0.16828  -0.258  0.79707
quarterlygdp.l3    0.17343    0.62383   0.278  0.78184
dqclothing.l4     -0.01038    0.17545  -0.059  0.95299
quarterlygdp.l4    0.91931    0.62660   1.467  0.14695
dqclothing.l5      0.24584    0.17273   1.423  0.15923
quarterlygdp.l5    1.26234    0.61342   2.058  0.04344 *
dqclothing.l6      0.22055    0.17789   1.240  0.21931
quarterlygdp.l6    0.61580    0.66106   0.932  0.35488
dqclothing.l7      0.29513    0.17637   1.673  0.09885 .
quarterlygdp.l7   -1.05523    0.67413  -1.565  0.12215
dqclothing.l8      0.06042    0.14948   0.404  0.68731
quarterlygdp.l8    0.48279    0.68172   0.708  0.48125
dqclothing.l9      0.02688    0.12847   0.209  0.83491
quarterlygdp.l9   -0.32301    0.69909  -0.462  0.64553
dqclothing.l10    -0.15326    0.11928  -1.285  0.20317
quarterlygdp.l10  -0.10157    0.69776  -0.146  0.88470
dqclothing.l11     0.15208    0.11689   1.301  0.19763
quarterlygdp.l11  -1.50756    0.69109  -2.181  0.03261 *
dqclothing.l12    -0.13807    0.11702  -1.180  0.24215
quarterlygdp.l12  -0.13207    0.72826  -0.181  0.85663
const            -42.98095  240.30859  -0.179  0.85858
trend             -0.53865    2.26467  -0.238  0.81271
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 355.2 on 68 degrees of freedom
Multiple R-Squared: 0.4588,     Adjusted R-squared: 0.2599
F-statistic: 2.306 on 25 and 68 DF,  p-value: 0.003475
```

Figure 20: The passing twelve quarters for two dataset summary in ARMAX model

Up to this point, the researcher used a VAR(p) model to find the best model. The selection criteria used in the package are AIC, Hannan-Quinn (HQ), BIC (SC), and Final Prediction Error (FPE) (Shumway). The Variable selection result is shown below [Figure 21].

```
> VARselect(x, lag.max=12, type='both')
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     5      1      1      5

$criteria
                  1            2            3            4            5
AIC(n) 2.074371e+01 2.080530e+01 2.082713e+01 2.079491e+01 2.057300e+01
HQ(n)  2.083115e+01 2.093644e+01 2.100199e+01 2.101348e+01 2.083529e+01
SC(n)  2.096017e+01 2.112997e+01 2.126003e+01 2.133603e+01 2.122235e+01
FPE(n) 1.020765e+09 1.085868e+09 1.110367e+09 1.076006e+09 8.628906e+08
                  6            7            8            9           10
AIC(n) 2.060531e+01 2.065664e+01 2.062585e+01 2.069987e+01 2.073002e+01
HQ(n)  2.091131e+01 2.100636e+01 2.101928e+01 2.113702e+01 2.121089e+01
SC(n)  2.136288e+01 2.152245e+01 2.159987e+01 2.178212e+01 2.192050e+01
FPE(n) 8.927084e+08 9.418297e+08 9.159028e+08 9.898543e+08 1.024737e+09
                 11           12
AIC(n) 2.069939e+01 2.075348e+01
HQ(n)  2.122397e+01 2.132178e+01
SC(n)  2.199810e+01 2.216041e+01
FPE(n) 9.992287e+08 1.061623e+09
```

Figure 21: VAR selection for the best model

Fitting the model selected by BIC we obtain the first model which we analyzed was the most appropriate model. The prediction model for the relationship between GDP and clothing retail sales is estimated to be

$$\hat{C}_t = -18.21 + 0.27t - 0.24C_{t-1} + 1.05G_{t-1}$$

The Relationship Between Quarterly GDP and Clothing Retail Sales After Removing Seasonality

Following the prediction model, one might notice that the ACF of the original differenced GDP dataset had seasonality. It might impact the accuracy of the result. The researcher tried to remove the seasonality and extract the residuals. After removing seasonality, the multivariate ARMAX models and VAR(p) selection were would be used again in order to find out the best model equation for the relationship.

The seasonal ARIMA model for the clothing retail sales was ARIMA (1,1,1) x $(1,0,0)_{12}$. As one can see, the value for $\theta 1$ is approximately equal to -0.2363, and the value for ψ is approximately equal to -0.8415.

```
Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
    optim.control = list(trace = trc, REPORT = 1, reltol = tol))

Coefficients:
          ar1      ma1     sar1  constant
      -0.2363  -0.8415  -0.1586    3.1694
s.e.   0.1131   0.0686   0.0979    4.4983

sigma^2 estimated as 145884:  log likelihood = -774.24,  aic = 1558.48

$degrees_of_freedom
[1] 101

$ttable
         Estimate     SE  t.value p.value
ar1       -0.2363 0.1131  -2.0901  0.0391
ma1       -0.8415 0.0686 -12.2746  0.0000
sar1      -0.1586 0.0979  -1.6196  0.1084
constant   3.1694 4.4983   0.7046  0.4827

$AIC
[1] 14.8427

$AICc
[1] 14.84651

$BIC
[1] 14.96908
```

Figure 22: Model for an ARIMA (1,1,1) x $(1,0,0)_{12}$

A summary of statistics for the ARIMA (1,1,1) x $(1,0,0)_{12}$. Model is shown in [Figure 23]. As one can see, the standardized residuals and the Quantile-Quantile plot of the standardized residuals appear to be normal. Also, the p-values for the Ljung-Box statistic appear to be up to par with what would be ideal for the model of the time series.

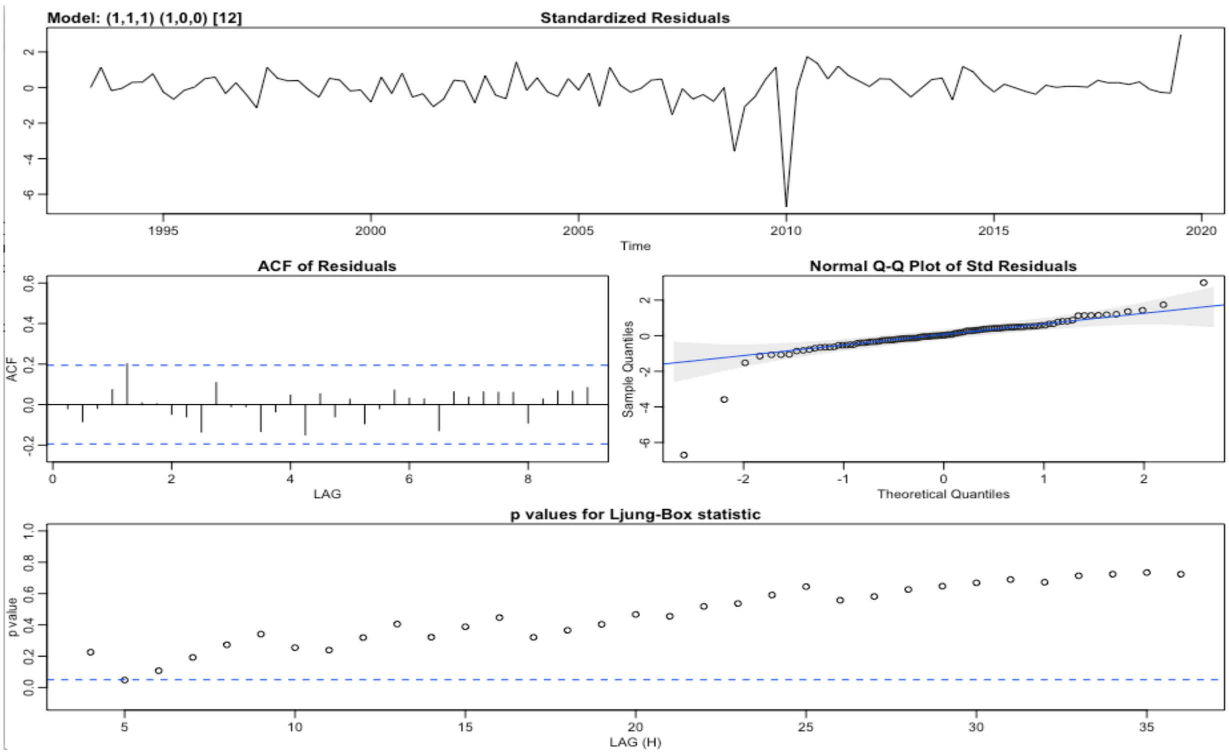

Figure 23: Summary of Statistics of the ARIMA (1,1,1) x $(1,0,0)_{12}$Model

The seasonal ARIMA model for the clothing retail sales was ARIMA (2,0,1) x $(1,0,1)_4$. As one can see, the value for $\theta 1$ is approximately equal to 0.7177, the value for $\theta 2$ is approximately equal to 0.0101, and the value for ψ is approximately equal to -0.3526.

```
Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = xmean, include.mean = FALSE, transform.pars = trans,
    fixed = fixed, optim.control = list(trace = trc, REPORT = 1, reltol = tol))

Coefficients:
         ar1     ar2      ma1      sar1    sma1     xmean
      0.7177  0.0101  -0.3526  -0.2941  0.3476  139.5467
s.e.  0.3509  0.1910   0.3385   0.5504  0.5319   18.1017

sigma^2 estimated as 5907:  log likelihood = -610.82,  aic = 1235.65

$degrees_of_freedom
[1] 100

$ttable
      Estimate      SE t.value p.value
ar1     0.7177  0.3509  2.0453  0.0435
ar2     0.0101  0.1910  0.0527  0.9581
ma1    -0.3526  0.3385 -1.0417  0.3001
sar1   -0.2941  0.5504 -0.5344  0.5942
sma1    0.3476  0.5319  0.6535  0.5149
xmean 139.5467 18.1017  7.7090  0.0000

$AIC
[1] 11.65705

$AICc
[1] 11.66505

$BIC
[1] 11.83293
```

Figure 24: Model for an ARIMA (2,0,1) x $(1,0,1)_4$

A summary of statistics for the ARIMA (2,0,1) x $(1,0,1)_4$. Model is shown in [Figure 25]. As one can see, the standardized residuals and the Quantile-Quantile plot of the standardized residuals appear to be normal. Also, the p-values for the Ljung-Box statistic appear to be up to par with what would be ideal for the model of the time series.

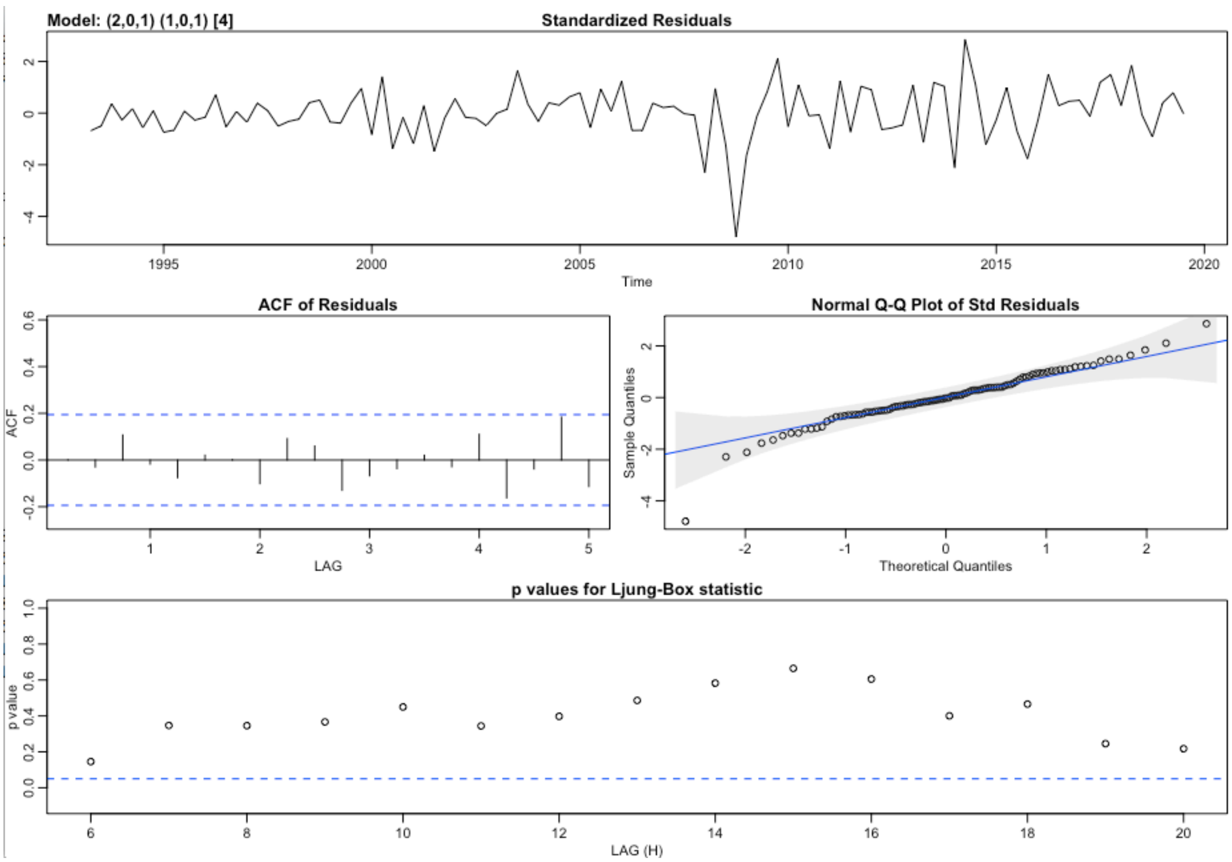

Figure 25: Summary of Statistics of the ARIMA (2,0,1) x $(1,0,1)_4$ Model

After the seasonality has removed, the researcher can extract the residuals from both seasonal ARIMA model. Plus, the multivariate ARMAX model would be used again to determine the best model for the data. A VAR(p) model is used to find the best model, and one can see the best model for the passing twelve is the first month.

```
> VARselect(x, lag.max=12, type='both')
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     1      1      1      1

$criteria
                  1            2            3            4            5            6
AIC(n) 2.076256e+01 2.083402e+01 2.089355e+01 2.094709e+01 2.079559e+01 2.081486e+01
HQ(n)  2.084999e+01 2.096516e+01 2.106841e+01 2.116567e+01 2.105788e+01 2.112087e+01
SC(n)  2.097901e+01 2.115869e+01 2.132645e+01 2.148822e+01 2.144494e+01 2.157244e+01
FPE(n) 1.040181e+09 1.117506e+09 1.186616e+09 1.252876e+09 1.078020e+09 1.100827e+09
                  7            8            9           10           11           12
AIC(n) 2.086354e+01 2.085789e+01 2.092615e+01 2.094085e+01 2.091807e+01 2.096223e+01
HQ(n)  2.121326e+01 2.125133e+01 2.136330e+01 2.142171e+01 2.144265e+01 2.153053e+01
SC(n)  2.172934e+01 2.183192e+01 2.200840e+01 2.213133e+01 2.221678e+01 2.236916e+01
FPE(n) 1.158317e+09 1.155112e+09 1.241210e+09 1.265237e+09 1.243473e+09 1.308070e+09
```

Figure 26: VAR selection for the best model After Removing Seasonality

Fitting the model selected by BIC we obtain the first model which we analyzed was the most appropriate model. The prediction model for the relationship between GDP and clothing retail sales is estimated to be

$$\hat{C}_t = -16.54 + 0.41t - 0.04C_{t-1} + 0.24G_{t-1}$$

The result is shown below [Figure 27]. One can tell the P-value for the GDP is not statistically significant. It did not reject the null hypothesis. It means the just passing quarter had no relationship with clothing. Plus, the P-value for the whole result is very large. However, the VAR selection chose the passing quarter was the best model, so the equation above would be the final result for the relationship between two data sets.

```
Estimation results for equation redqclothing:
==============================================
redqclothing = redqclothing.l1 + redquarterltgdp.l1 + const + trend

                    Estimate Std. Error t value Pr(>|t|)
redqclothing.l1     -0.03975    0.11200  -0.355    0.723
redquarterltgdp.l1   0.24084    0.53560   0.450    0.654
const              -16.53589   78.17479  -0.212    0.833
trend                0.40573    1.26765   0.320    0.750


Residual standard error: 388.6 on 101 degrees of freedom
Multiple R-Squared: 0.003962,   Adjusted R-squared: -0.02562
F-statistic: 0.1339 on 3 and 101 DF,  p-value: 0.9396
```

Figure 27: The passing quarter of a year for two dataset summary in ARMAX model

A summary of statistics for the passing six quarters Model is shown in [Figure 28]. As one can see, the fifth quarter was extremely statistically significant. The rest of them were not statistically significant. It means the passing year of the GDP was not affecting the clothing retail sale. However, the p-value of the fifth quarter was very small, and it affected the clothing retail sales.

```
Estimation results for equation redqclothing:
==============================================
redqclothing = redqclothing.l1 + redquarterltgdp.l1 + redqclothing.l2 + redquarterltgdp.l2 + redqclothin
g.l3 + redquarterltgdp.l3 + redqclothing.l4 + redquarterltgdp.l4 + redqclothing.l5 + redquarterltgdp.l5
 + redqclothing.l6 + redquarterltgdp.l6 + const + trend

                    Estimate Std. Error t value Pr(>|t|)
redqclothing.l1    -0.218078   0.123811  -1.761  0.08173 .
redquarterltgdp.l1  0.654189   0.548417   1.193  0.23620
redqclothing.l2    -0.161856   0.113105  -1.431  0.15605
redquarterltgdp.l2  0.048576   0.530848   0.092  0.92730
redqclothing.l3    -0.097096   0.111786  -0.869  0.38749
redquarterltgdp.l3  0.551839   0.525474   1.050  0.29658
redqclothing.l4    -0.017243   0.111237  -0.155  0.87717
redquarterltgdp.l4  0.804643   0.530112   1.518  0.13271
redqclothing.l5     0.080566   0.109997   0.732  0.46589
redquarterltgdp.l5  1.785433   0.539418   3.310  0.00136 **
redqclothing.l6    -0.001251   0.110980  -0.011  0.99104
redquarterltgdp.l6  0.975555   0.577729   1.689  0.09492 .
const              33.730220  86.429550   0.390  0.69731
trend              -0.722791   1.394044  -0.518  0.60545
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 373.2 on 86 degrees of freedom
Multiple R-Squared: 0.2069,     Adjusted R-squared: 0.08697
F-statistic: 1.725 on 13 and 86 DF,  p-value: 0.06975
```

Figure 28: The passing six quarters for two dataset summary in ARMAX model

The correlation plot between two data sets after removing seasonality is shown in [Figure 29]. The significant relationship is influenced by the extreme value. After removing seasonal effect, the clothing expense is a white noise and not predictable from the historical GDP. In short, the GDP did not have a correlative relationship with the clothing retail sales.

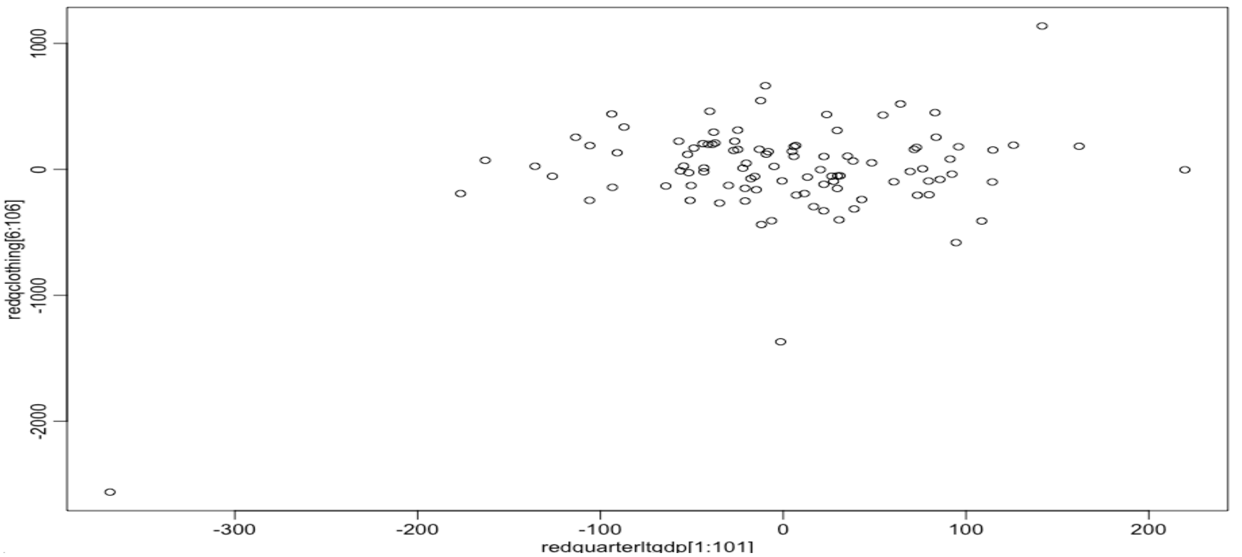

Figure 29: The correlation plot between two data sets after removing seasonality

CHAPTER FIVE: CONCLUSION

The first step of analyzing the dataset is transferring the dataset to time series format. Once the dataset is transferred to time series format, three steps were taken to analyze the time series data: determining the most appropriate trend and trend model for the dataset, analyzing a model fit for the time series, and forecasting the time series to analyze the future of the time series. The model for linear seasonal means plus quadratic time trend is better than the linear seasonal means time trend for the time series dataset because the linear seasonal means plus quadratic time trend had a better R-squared, and the points in the standardized residuals plot are placed randomly.

Following this, a time series model was fit to appropriately represent the model. Based on the partial autocorrelation and autocorrelation functions and their corresponding graphs, combined with the fact that our time series dataset was nonstationary, an ARIMA (2,1,1) model was chosen to be the best fit for our dataset. From there, a forecast was done to predict the future of the time series.

Furthermore, the quarterly GDP dataset used was collected in order to find out if there is a relationship between each other. Both frequency of the dataset was collected by quarter, and each quarter can be an independent variable. A multivariate ARMAX model was determined to solve the problem. For the two-dimensional series composed of quarterly GDP x_{t1}, and clothing retail sales x_{t2}. Before removing the seasonality for both dataset, one could see there is relationship between two data. The best model is

$$\hat{C}_t = -18.21 + 0.27t - 0.24C_{t-1} + 1.05G_{t-1}$$

After removing the seasonality, the null hypothesis is not rejected since the P-value is a large number. The best model for after removing the seasonality is

$$\hat{C}_t = -16.54 + 0.41t - 0.04C_{t-1} + 0.24G_{t-1}$$

The significant relationship is influenced by the extreme value. After removing seasonal effect, the clothing expense is a white noise and not predictable from the historical GDP.

LIST OF REFERENCES

[1] Alison, Toplis (2011). The Clothing Trade in Provincial England, 1800-1850. London: Pickering & Chatto Publishers.

[2] Cryer, Jonathan D., and Kung-sik Chan. Time Series Analysis with Applications in R. Springer, 2011.

[3] Hannan & Deistler (1988, p. 227): Hannan, E. J.; Deistler, Manfred (1988). Statistical theory of linear systems. Wiley series in probability and mathematical statistics. New York: John Wiley and Sons.

[4] Hannan, Edward James (1970). Multiple time series. Wiley series in probability and mathematical statistics. New York: John Wiley and Sons.

[5] Hamilton, James D. (James Douglas), 1954- (1994). Time series analysis. Princeton, N.J.: Princeton University Press. ISBN 0691042896. OCLC 28257560.

[6] Hyndman, Rob J; Athanasopoulos, George. 8.9 Seasonal ARIMA models. Forecasting: principles and practice. oTexts. Retrieved 19 May 2015.).

[7] Shumway, Robert H., and David S. Stoffer. *Time Series Analysis and Its Applications: with R Examples*. Springer, 2017.

[8] Swartz, Rob, et al. "US Census Bureau Retail Trade Definitions Page." *Definitions*, 16 Jan. 2009.

[9] Tran, Adeline, et al. "US Census Bureau Retail Trade Historical Data Page." *FAQs*, 16 Jan. 2009.

[10] U.S. Bureau of Economic Analysis, Gross Domestic Product [GDP], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GDP, November 20, 2019.