

Some Inequalities under Random C - Condition

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Abstract

In this paper, the notion of random nonexpansive operators is generalized to operators satisfying random C – condition. Some new properties are obtained in uniformly convex Banach spaces, also well we get result of converge on random operator to random point in Banach space.

Keywords: Banach spaces, Random operators, random nonexpansive operators.

1. Introduction and Preliminaries

In 2007, Suzuk [1] introduced generalization of nonexpansive mappings by C – condition to prove some fixed point theorems and convergence theorems. Khan and Suzuki [2] proved a theorem about weak convergence via C – condition in Banach spaces whose dual has the *Kadec – Klee* property. In this manuscript, we present some of these results in the setting of random operators in uniformly convex separable Banach space. For more results in this area, see the works in [3],[4], [5] and [6]. Now a set Ω , a family Σ of subset of Ω is said to be σ –algebras if its closed complements and countable unions, i.e $\beta \in \Sigma$ implies $\beta^c \in \Sigma$ and $\beta_i \in \Sigma, i \in N$ implies $\bigcup_i \beta_i \in \Sigma$, the pair (Ω, Σ) is called a measurable sets [6].

Definition 1.1: [7] Let K be a separable Banach space and $\delta_n: \Omega \rightarrow K$ is measurable sequence.

Definition 1.2 : [7] A mapping $h : \Omega \rightarrow K$ is said to be measurable (Σ –measurable) if for any open subset V of K ,

$$h^{-1}(V) = \{\omega : h(\omega) \cap V \neq \emptyset\} \in \Sigma$$

Definition 1.3 : [7] A mapping $h : \Omega \times K \rightarrow K$ is random operator, if for each fixed $v \in K$ the mapping $h(\cdot, v): \Omega \rightarrow K$ is measurable .

Definition 1.4: [7] A random operator $h: \Omega \times K \rightarrow K$ is continuous if $h(v, \cdot): \Omega \rightarrow K$ is continuous, for each $v \in \Omega$.

Definition 1.5: [7] A measurable mapping $\delta: \Omega \rightarrow K$ is random fixed point of a random operator $h: \Omega \times K \rightarrow K$ if $h(\omega, \delta(\omega)) = \delta(\omega)$ For each $\omega \in \Omega$.

Definition 1.6 : [8] A Banach space K is said to be uniformly convex if there exist a strictly increasing function $\eta: [0,2] \rightarrow [0, 1]$ such that forevery $x, y, p \in K, R > 0$ and $r \in [0, 2R]$ The following implication holds:

$$\begin{cases} \|x - p\| \leq R \\ \|y - p\| \leq R \\ \|x - y\| \geq r \end{cases} \Rightarrow \left\| \frac{x + y}{2} - p \right\| \leq \left(1 - \eta\left(\frac{r}{R}\right) \right) \cdot R$$

Definition : 1.7 : [9] Let C be a subset of Banach space K , A mapping $h: C \rightarrow K$ is said to be demiclosed at $u \in K$ if for any sequence $\{v_n\}$ in C with v_n converges weakly to v and $hv_n \rightarrow u$, it follows that $v \in C$ and $hv = u$

Lemma 1.8 : [10] Let K be a uniformly convex Banach space , $0 < p \leq t_n \leq q < 1$. For all $n \in N$ suppose that $\{v_n\}$ and $\{u_n\}$ are two sequences of K such that $\limsup_{n \rightarrow \infty} \|v_n\| \leq r$, $\limsup_{n \rightarrow \infty} \|u_n\| \leq r$ and

$$\lim_{n \rightarrow \infty} \|t_n v_n + (1 - t_n)u_n\| = r \quad \text{for some } r \geq 0 \quad \text{Then } \lim_{n \rightarrow \infty} \|v_n - u_n\| = 0.$$

Lemma 1.9 : [2] Let K be a uniformly convex Banach space. Let $\{u_n\}, \{v_n\}$ and $\{z_n\}$ be sequences in K let d and t be real numbers with $d \in (0, \infty)$ and $t \in (0,1)$ Assume that

$$\lim_{n \rightarrow \infty} \|u_n - v_n\| = d, \quad \limsup_{n \rightarrow \infty} \|u_n - z_n\| \leq (1 - t) d \quad \text{and}$$

$$\limsup_{n \rightarrow \infty} \|v_n + z_n\| < td, \quad \text{then :}$$

$$\lim_{n \rightarrow \infty} \|tu_n + (1 - t)v_n - z_n\| = 0.$$

2. Main Results

In this section we need the Banach space is separable and the following concept and properties for random condition (C)

Definition 2.1 (condition random C)

Let K be a subset of separable Banach space K . Let $G: \Omega \times K \rightarrow K$ be a random operator. Then G is said to be satisfy the random condition C (RC) if :

$$\frac{1}{2} \|v - G(\omega, v)\| \leq \|v - u\| \quad \text{implies} \quad \|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$$

For all $v, u \in K$ and for all $\omega \in \Omega$

Proposition 2.2 Let $G: \Omega \times C \rightarrow C$ be a measurable mapping where K a subset of a separable Banach space K . Assume that G satisfies condition (RC) , for all $v, u \in C$ and for all $\omega \in \Omega$, then te following hold:

- i) $\|G(\omega, v) - G^2(\omega, v)\| \leq \|v - G(\omega, v)\|$
- ii) Either $\frac{1}{2}\|v - G(\omega, v)\| \leq \|v - u\|$
or $\frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| \leq \|G(\omega, v) - u\|$ holds
- iii) Either $\|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$
or $\|G^2(\omega, v) - G(\omega, u)\| \leq \|G(\omega, v) - u\|$ holds

Proof:

- i) Follows from $\frac{1}{2}\|v - G(\omega, v)\| \leq \|v - G(\omega, v)\|$ and by the condition (RC) then we have :

$$\|G(\omega, v) - G^2(\omega, v)\| \leq \|v - G(\omega, v)\|$$

- ii) Arguing by contradiction we assume that $\frac{1}{2}\|v - G(\omega, v)\| > \|v - u\|$ and

$$\frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| > \|G(\omega, v) - u\|$$

Then we have by (i)

$$\begin{aligned} \|v - G(\omega, v)\| &\leq \|v - u\| + \|G(\omega, v) - u\| \\ &< \frac{1}{2}\|v - G(\omega, v)\| + \frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| \\ &\leq \frac{1}{2}\|v - G(\omega, v)\| + \frac{1}{2}\|v - G(\omega, v)\| \\ &= \|v - G(\omega, v)\| \end{aligned}$$

This is contradiction

- iii) Follows from (ii)
Either $\frac{1}{2}\|v - G(\omega, v)\| \leq \|v - u\|$ and by condition (RC) then implies.

$$\|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$$

$$\text{or } \frac{1}{2}\|G(\omega, v) - G^2(\omega, v)\| \leq \|G(\omega, v) - u\|$$

and by condition (RC) then implies :

$$\|G^2(\omega, v) - G(\omega, v)\| \leq \|G(\omega, v) - u\|.$$

Proposition 2.3 Let C be a subset of a separable Banach space K Let $G: \Omega \times C \rightarrow C$ be amasurable

mapping. Assume that G satisfies condition (RC) for all $v, u \in K$ and for all $\omega \in \Omega$ then :

- i) $\|v - G(\omega, u)\| \leq 3\|G(\omega, v) - v\| + \|v - u\|$

$$\text{ii) } \|u - G(\omega, u)\| \leq 3\|G(\omega, v) - v\| + 2\|v - u\|$$

By Proposition 2.2 (iii)

$$\text{Either } \|G(\omega, v) - G(\omega, u)\| \leq \|v - u\|$$

$$\text{or } \|G^2(\omega, v) - G(\omega, u)\| \leq \|G(\omega, v) - u\| \text{ holds}$$

In the first case, we have :

$$\begin{aligned} \|v - G(\omega, u)\| &\leq \|v - G(\omega, v)\| + \|G(\omega, v) - G(\omega, u)\| \\ &\leq \|v - G(\omega, v)\| + \|v - u\| \\ \therefore \|v - G(\omega, u)\| &\leq \|v - G(\omega, v)\| + \|v - u\| \end{aligned}$$

In the second case, we have by **Proposition 2.2 (i)**

$$\begin{aligned} \|v - G(\omega, u)\| &\leq \|v - G(\omega, v)\| + \|G(\omega, v) - G^2(\omega, v)\| \\ &\quad + \|G^2(\omega, v) - G(\omega, u)\| \\ &\leq \|v - G(\omega, v)\| + \|v - G(\omega, v)\| + \\ \|G(\omega, v) - u\| &\leq 2\|v - G(\omega, v)\| + \|G(\omega, v) - u\| \\ &\leq 2\|v - G(\omega, v)\| + \|G(\omega, v) - v\| + \|v - u\| \\ &= 2\|v - G(\omega, v)\| + \|v - G(\omega, v)\| + \|v - u\| \\ &\leq 3\|v - G(\omega, v)\| + \|v - u\| \end{aligned}$$

Now:

$$\|u - G(\omega, u)\| \leq \|u - v\| + \|v - G(\omega, u)\|$$

By using (i)

$$\begin{aligned} &\leq \|u - v\| + 3\|v - G(\omega, v)\| + \|v - u\| \\ \therefore \|u - G(\omega, u)\| &\leq 3\|v - G(\omega, v)\| + 2\|v - u\| \end{aligned}$$

Proposition 2.4 Let C be a bounded and convex subset of uniformly convex separable Banach space K . let $G: \Omega \times C \rightarrow C$ be arandom operator. Assume that G satisfies condition (RC).Then for any $\epsilon > 0$ there exist $\eta(\epsilon) > 0$ such that for any $t \in [0,1]$ and for any $u, v \in C$ with :

$$\|G(\omega, v) - v\| < \eta(\epsilon) , \|G(\omega, u) - u\| < \eta(\epsilon) \text{ we have :}$$

$$\|G(\omega, tv + (1-t)u) - (tv + (1-t)u)\| < \epsilon$$

Proof:

Arguing by contradiction we assume that there exist $\epsilon > 0$, sequences $\{v_n\}, \{u_n\}$ in C , $t_n \in [0,1]$ and

$$\|G(\omega, v_n) - v_n\| < \frac{1}{n} , \|G(\omega, u_n) - u_n\| < \frac{1}{n}$$

And

$$\|G(\omega, t_n v_n + (1 - t_n)u_n) - (t_n v_n + (1 - t_n)u_n)\| \geq \epsilon$$

Denote $z_n = t_n v_n + (1 - t_n)u_n$ and $G(\omega, z_n)$ using

Proposition 2.3 (ii) we have that

$$\begin{aligned} 0 < \epsilon &\leq \liminf_{n \rightarrow \infty} \|z_n - G(\omega, z_n)\| \\ &\leq \liminf_{n \rightarrow \infty} (3\|u_n - G(\omega, u_n)\| + 2\|z_n - u_n\|) \\ &< \liminf_{n \rightarrow \infty} \left(3\left(\frac{1}{n}\right) + 2\|z_n - u_n\|\right) \\ &= 2 \liminf_{n \rightarrow \infty} \|z_n - u_n\| \end{aligned}$$

Similarly we can show that $0 < \liminf_{n \rightarrow \infty} \|z_n - v_n\|$

Thus $0 < \liminf_{n \rightarrow \infty} \|u_n - v_n\|$. Since C is bounded and

$$\begin{aligned} 0 < \liminf_{n \rightarrow \infty} \|z_n - v_n\| &= \liminf_{n \rightarrow \infty} t_n \|u_n - v_n\| \\ &\leq \liminf_{n \rightarrow \infty} t_n \cdot \sup \|u_n - v_n\| \end{aligned}$$

We have that $0 < \liminf_{n \rightarrow \infty} t_n$, similarly we can prove that $\limsup t_n < 1$. So without loss of generality. We may assume that $\{\|u_n - v_n\|\}$ and $\{t_n\}$ converge to some real number.

$d \in (0, \infty)$ and $t \in (0, 1)$, respectively.

Since $\lim_{n \rightarrow \infty} \|u_n - G(\omega, u_n)\| = 0$ and $0 < \liminf_{n \rightarrow \infty} \|u_n - z_n\|$ we have that $\frac{1}{2}\|u_n - G(\omega, u_n)\| \leq \|u_n - z_n\|$ for sufficiently Large $n \in N$. From

condition (RC) we get that :

$$\|G(\omega, u_n) - G(\omega, z_n)\| \leq \|u_n - z_n\|$$

Similarly we can show

$$\|G(\omega, u_n) - G(\omega, z_n)\| \leq \|v_n - z_n\|$$

Then now: Let $G(\omega, z_n) = x_n$

$$\limsup_{n \rightarrow \infty} \|v_n - x_n\| \leq$$

$$\begin{aligned} &\limsup_{n \rightarrow \infty} (\|v_n - G(\omega, v_n)\| + \|G(\omega, v_n) - G(\omega, x_n)\|) \\ &\leq \lim_{n \rightarrow \infty} (\|v_n - G(\omega, v_n)\| + \|v_n - z_n\|) \\ &= (1 - t)d \end{aligned}$$

From **Lemma (1.9)** we obtain :

$$0 < \epsilon < \lim_{n \rightarrow \infty} \|z_n - x_n\|$$

$$\leq \lim_{n \rightarrow \infty} (\|z_n - (tu_n + (1-t)v_n)\| + \|(tu_n + (1-t)v_n) - x_n\|)$$

$0 < 0$ which is a contradiction.

Proposition 2.5 Let C be abounded and convex subset of a uniformly convex separable Banach space K . Let $G: \Omega \times C \rightarrow C$ be a random operator. Assume G satisfies condition (RC). Then $I-G$ is demiclosed at zero. That is if $\{v_n\}$ in C converges weakly to $z \in C$ and $\lim_{n \rightarrow \infty} \|G(\omega, v_n) - v_n\| = 0$, then : $G(\omega, z) = z$

Proof:

Let η be a function from $(0, \infty)$ in to it self which satisfies the conclusion of **proposition (2.4)** we assume that $\{v_n\}$ converge weakly to z and $\lim_{n \rightarrow \infty} \|G(\omega, v_n) - v_n\| = 0$ let $\epsilon > 0$ be arbitrary chosen. Define a strictly decreasing sequence $\{\epsilon_n\}$ in $(0, \infty)$ by $\epsilon_1 = \epsilon$ and $\epsilon_{n+1} = \min\{\epsilon_n, \eta(\epsilon_n)\}/2$

It is obvious that $\epsilon_{n+1} < \eta(\epsilon_n)$. Choose a sub sequences $\{v_{ni}\}$ of $\{v_n\}$ such that ,

$\|v_{ni} - G(\omega, v_{ni})\| < \eta(\epsilon_n)$. since $\{v_{ni}\}$ converges weakly to z , v_n belongs to the closed convex hull of $\{v_{ni}: n \in N\}$ so there exist $u \in C$ and $p \in N$ such that $\|u - z\| < \epsilon$ and u belongs to the convex hull of $\{v_{ni}: n=1, 2, 3, \dots, p\}$

From **Proposition (2.4)** we get that

$\|G(\omega, u) - u\| < \epsilon$. So we have from **Proposition (2.3)** that :

$$\|G(\omega, z) - z\| \leq 3\|G(\omega, u) - u\| + 2\|u - z\| \leq 5\epsilon$$

Since $\epsilon > 0$ is arbitrary. We obtain $G(\omega, z) = z$.

Theorem (2. 6)

Let C be a nonempty closed convex subset of a uniformly convex Banach space K . Let G and h be two random operators $G, h : \Omega \times C \rightarrow C$ satisfying condition (RC) such that :

$$\left\{ \begin{array}{l} \delta_1: \Omega \rightarrow C, \quad \bar{\delta}_1: \Omega \rightarrow C \\ \delta_{n+1} = (1 - a_n)G(\omega, \delta_n) + a_n h(\omega, \bar{\delta}_n) \\ \bar{\delta}_n = (1 - b_n)\delta_n + b_n G(\omega, \delta_n) \end{array} \right\} \quad (2.1)$$

Where $\{a_n\}$ and $\{b_n\}$ are sequences in $[\epsilon, 1 - \epsilon] n \in N$. And for some ϵ in $(0, 1)$ if $RF = RF(G) \cap RF(h) \neq \emptyset$ then $\lim_{n \rightarrow \infty} \|\delta_n(\omega) - \zeta(\omega)\|$

exists and $\lim_{n \rightarrow \infty} \|\delta_n(\omega) - G(\omega, \delta_n)\| = 0 = \lim_{n \rightarrow \infty} \|\delta_n - h(\omega, \delta_n)\|$

Proof :

Let $\zeta(\omega) \in RF$. By use of condition (RC) , we get : in $(0, 1)$

$$\frac{1}{2} \|\zeta(\omega) - G(\omega, \zeta(\omega))\| = 0 \leq \|\delta_n(\omega) - \zeta(\omega)\| \Rightarrow$$

$$\|G(\omega, \delta_n(\omega)) - G(\omega, \zeta(\omega))\| \leq \|\delta_n(\omega) - \zeta(\omega)\| \quad (2.2)$$

$$\frac{1}{2} \|\zeta(\omega) - G(\omega, \zeta(\omega))\| = 0 \leq \|\bar{\delta}_n(\omega) - \zeta(\omega)\| \Rightarrow$$

$$\|h(\omega, \bar{\delta}_n(\omega)) - h(\omega, \zeta(\omega))\| \leq \|\bar{\delta}_n(\omega) - \zeta(\omega)\| \dots \dots \dots (2.3)$$

Using inequalities (2.2) and (2.3) along with (2.1), we have :

$$\begin{aligned} \|\delta_{n+1}(\omega) - \zeta(\omega)\| &= \left\| (1 - a_n) \left(G(\omega, \delta_n(\omega)) - \zeta(\omega) \right) + a_n \left(h(\omega, \bar{\delta}_n(\omega)) - \zeta(\omega) \right) \right\| \\ &\leq (1 - a_n) \|G(\omega, \delta_n(\omega)) - \zeta(\omega)\| + a_n \|h(\omega, \bar{\delta}_n(\omega)) - \zeta(\omega)\| \\ &\leq (1 - a_n) \|\delta_n(\omega) - \zeta(\omega)\| + a_n \|\bar{\delta}_n(\omega) - \zeta(\omega)\| \\ &= (1 - a_n) \|\delta_n(\omega) - \zeta(\omega)\| + a_n \|(1 - b_n) \delta_n(\omega) + b_n G(\omega, \delta_n(\omega)) - \zeta(\omega)\| \\ &\leq (1 - a_n) \|\delta_n(\omega) - \zeta(\omega)\| + a_n (1 - b_n) \|\delta_n(\omega) - \zeta(\omega)\| + a_n b_n \|G(\omega, \delta_n(\omega)) - \zeta(\omega)\| \\ &\leq (1 - a_n) \|\delta_n(\omega) - \zeta(\omega)\| + a_n (1 - b_n) \|\delta_n(\omega) - \zeta(\omega)\| + a_n b_n \|\delta_n(\omega) - \zeta(\omega)\| \\ &= \|\delta_n(\omega) - \zeta(\omega)\| \end{aligned}$$

Therefore $\lim_{n \rightarrow \infty} \|\delta_n(\omega) - \zeta(\omega)\|$ exist for any $\zeta(\omega) \in RF$

Let $\lim_{n \rightarrow \infty} \|\delta_n(\omega) - \zeta(\omega)\| = a$. Consider

$$\begin{aligned} \|\bar{\delta}_n(\omega) - \zeta(\omega)\| &= \|b_n G(\omega, \delta_n(\omega)) + (1 - b_n) \delta_n(\omega) - \zeta(\omega)\| \\ &\leq b_n \|G(\omega, \delta_n(\omega)) - \zeta(\omega)\| + (1 - b_n) \|\delta_n(\omega) - \zeta(\omega)\| \\ &\leq b_n \|\delta_n(\omega) - \zeta(\omega)\| + (1 - b_n) \|\delta_n(\omega) - \zeta(\omega)\| \\ &= \|\delta_n(\omega) - \zeta(\omega)\|, \end{aligned}$$

which implies that

$$\limsup_{n \rightarrow \infty} \|\bar{\delta}_n(\omega) - \zeta(\omega)\| \leq a$$

Using (2.2) and (2.3) , we have

$$\limsup_{n \rightarrow \infty} \|G(\omega, \delta_n(\omega)) - \zeta(\omega)\| \leq a \text{ and}$$

$$\limsup_{n \rightarrow \infty} \|h(\omega, \bar{\delta}_n(\omega)) - \zeta(\omega)\| \leq a. \quad (2.4)$$

Moreover, we have

$$\begin{aligned}
 a &= \lim_{n \rightarrow \infty} \|\delta_{n+1}(\omega) - \zeta(\omega)\| \\
 &= \lim_{n \rightarrow \infty} \left\| (1 - a_n) \left(G(\omega, \delta_n(\omega)) - \zeta(\omega) \right) + a_n \left(h(\omega, \bar{\delta}_n(\omega)) - \zeta(\omega) \right) \right\| \quad (2.5)
 \end{aligned}$$

Therefore, by using (2.4) , (2.5) and **Lemma (1.8)** we have

$$\limsup_{n \rightarrow \infty} \left\| G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega)) \right\| = 0 \quad (2.6)$$

On the other hand, we have

$$\begin{aligned}
 \|\delta_{n+1}(\omega) - \zeta(\omega)\| &= \left\| (1 - a_n)G(\omega, \delta_n(\omega)) + a_n h(\omega, \delta_n(\omega)) - \zeta(\omega) \right\| \\
 &= \left\| \left(G(\omega, \delta_n(\omega)) - \zeta(\omega) \right) + a_n \left(h(\omega, \bar{\delta}_n(\omega)) - G(\omega, \delta_n(\omega)) \right) \right\| \\
 &\leq \left\| G(\omega, \delta_n(\omega)) - \zeta(\omega) \right\| + a_n \left\| G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega)) \right\|
 \end{aligned}$$

Taking *Lim inf* on both the sides, we get

$$a \leq \liminf_{n \rightarrow \infty} \left\| G(\omega, \delta_n(\omega)) - \zeta(\omega) \right\|$$

Which implies with (2.4) that $\left\| G(\omega, \delta_n(\omega)) - \zeta(\omega) \right\| = a.$ (2.7)

Using (2.7) , we have

$$\begin{aligned}
 \left\| G(\omega, \delta_n(\omega)) - \zeta(\omega) \right\| &\leq \left\| G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega)) \right\| + \left\| h(\omega, \bar{\delta}_n(\omega)) - \zeta(\omega) \right\| \\
 &\leq \left\| G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega)) \right\| + \left\| \bar{\delta}_n(\omega) - \zeta(\omega) \right\|
 \end{aligned}$$

Taking *Lim inf* on be both sides and using (2.7), we find that

$$a \leq \liminf_{n \rightarrow \infty} \left\| \bar{\delta}_n(\omega) - \zeta(\omega) \right\| \quad (2.8)$$

Hence, by (2.3) and (2.8), we have

$$\lim_{n \rightarrow \infty} \left\| \bar{\delta}_n(\omega) - \zeta(\omega) \right\| = a \quad (2.9)$$

Since $a = \lim_{n \rightarrow \infty} \left\| \bar{\delta}_n(\omega) - \zeta(\omega) \right\|$

$$= \lim_{n \rightarrow \infty} \left\| (1 - b_n) \left(\delta_n(\omega) - \zeta(\omega) \right) + b_n \left(G(\omega, \delta_n(\omega)) \right) - \zeta(\omega) \right\|$$

We find from **Lemma (1.8)** that

$$\lim_{n \rightarrow \infty} \left\| G(\omega, \delta_n(\omega)) - \delta_n(\omega) \right\| = 0 \quad (2.10)$$

Since $\left\| \bar{\delta}_n(\omega) - \delta_n(\omega) \right\| = \left\| b_n G(\omega, \delta_n(\omega)) + (1 - b_n) \delta_n(\omega) - \delta_n(\omega) \right\|$

$$= \|G(\omega, \delta_n(\omega)) - \delta_n(\omega)\|$$

Making use of (2.10) , we get :

$$\lim_{n \rightarrow \infty} \|\bar{\delta}_n(\omega) - \delta_n(\omega)\| = 0 \quad (2.11)$$

Using (2.6), (2.10) , (2.11) and **Proposition (2.2) (ii)**, we have

$$\begin{aligned} \|\delta_n(\omega) - h(\omega, \delta_n(\omega))\| &\leq 3 \|\bar{\delta}_n(\omega) - h(\omega, \bar{\delta}_n(\omega))\| + 2\|\delta_n(\omega) - \bar{\delta}_n(\omega)\| \\ &\leq 3\|\bar{\delta}_n(\omega) - G(\omega, \delta_n(\omega))\| + 3 \|G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega))\| \\ &\quad + 2\|\delta_n(\omega) - \bar{\delta}_n(\omega)\| \\ &= 3\|(1 - b_n)(\delta_n(\omega)) + b_n G(\omega, \delta_n(\omega)) - G(\omega, \delta_n(\omega))\| \\ &\quad + 3 \|G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega))\| + 2\|\delta_n(\omega) - \bar{\delta}_n(\omega)\| \\ &= 3(1 - b_n)\|\delta_n(\omega) - G(\omega, \delta_n(\omega))\| + 3 \|G(\omega, \delta_n(\omega)) - h(\omega, \bar{\delta}_n(\omega))\| \\ &\quad + 2\|\delta_n(\omega) - \bar{\delta}_n(\omega)\| \end{aligned}$$

Yielding there by $\lim_{n \rightarrow \infty} \|h(\omega, \delta_n(\omega)) - \delta_n(\omega)\| = 0$

This concludes the proof.

Finally, we suggest using recent results to generalized the results in [11] or study of these properties for in other spaces, such as, in modular spaces [12].

Conflict of Interests.

There are non-conflicts of interest .

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الخلاصة

في هذا البحث تطرقنا الى تعميم للتطبيق المتمدد الذي يحقق الشرط العشوائي C . بعض الخواص الجديدة حصلنا عليها في فضاء بناخ الذي يحمل صفة التحذب العام كذلك نحن نحصل على نتيجة للتقارب حول التطبيق العشوائي الى نقطة عشوائية في فضاء بناخ .