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The Consistency of Moving Twins Measurements in Spatially Closed Universe

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Abstract

The ordinary twin paradox is a problem of ageing of two twin observers A and B, B travels with a relativistic speed and returns back to A where A is at rest. According to each twin's calculation, if each considers itself at rest, the other would be younger at the end of the trip. This is paradoxical, in fact the paradox appears due to the accelerating of the travelling twin (B) and so that B will be younger at the return point. This paradox has been resolved by T. Dray by doing the calculation in a closed cylindrical universe, were the acceleration problem has been resolved for the travelling twin. This means that the travelling (faster) twin will be younger no matter which twin performed the calculation. In the present calculation we have shown if both twins are travelling with a relativistic speed in such universe and the faster twin (B) made the calculations. The result will always shows that the faster twin will have less proper time. This result is consistent with that of T. Dray, when one twin is at rest and other is moving.

Keywords: Special relativity; proper time; Lorentz transformations; compact spaces; twin paradox.
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1. Introduction

The problem of twin paradox arising from the special relativity is subject of study from the date of special relativity to nowadays. The problem studies the aging of two twins one twin (say A) stays at home while another (B) travels with constant velocity [1, 2]. The twin paradox arises from the measurement of time interval between two events in the theory of special relativity [3]. To give a better understanding to this paradox the typical resolution is made that the accelerated twin will mostly be younger throughout the journey [4-7]. To describe twin paradox, it is better to classify it into its types, with care taken to distinguish between the normal twin paradox and twin paradox in spatially periodic Universe.

1.1 Normal twin paradox:

In 1911, Paul Langevin was first to show the form of *normal twin paradox* differently from the others, in which the difference appears due to the emergence of acceleration of one twin. Hence, the absolute meaning of acceleration comes to Langevin's point of view. While, two reference frames solely, as presented by Max von Laue in 1913, are ample to interpret the difference, regardless of the acceleration involvement [8-11].

Consider a twin A and B, one of them A stays at earth while B travels at a relativistic speed into space and returns back to earth. Since twin B moves at a relativistic speed *v* away from A it is considered to be younger from viewpoint of A. While, in B's perspective it is A whose moving with the velocity -*v*, hence A should be younger as it is seen by B [12].

This result is paradoxical, the reality is that twin B is younger because it undergoes an acceleration and change of frame during the return point while twin A undergoes no such changes [12, 13].

1.2 Periodic Universe:

In a space such as spatially periodic universe (spatial dimension compactified), the situation is more complicated. The fact of this complexity is that, in this universe the frame of traveling twin remains unchanged, which means twin B will stay inertial throughout the whole journey. Now, both twins are considering the other to be experiencing time dilation. In other words, each sees the other to be younger; this seems to be more paradoxical [13-15]. The paradox is due to the space being topologically multiply connected [16, 17]. In such topology a preferred inertial reference exists, to which the twins' velocities are related, that is due to global break down of Lorentz invariance and cylindrical universe is topologically multiply connected [1, 18-20]. Despite this global breakdown, which does not affect the local structure of spacetime, the concept of spacetime interval ($\Delta X^2 - \Delta t^2$) is preserved under Lorentz transformations as a necessary step in special relativity [21]. In the present paper a complete calculation has been made in order to show that there is an agreement between both twins' calculations after their recombination in a spatially compact universe [1]. In the following section the twin paradox resolution is discussed for the compact cylindrical universe. This is by performing the calculation in the reference frame of the faster twin (related to the preferred reference), and showing that the faster twin B has less proper time than A.

2. Preliminary

In this section we briefly review some terms that have been used in the following calculations, such as frame of reference, Lorentz transformations, and proper time.

Frame of reference is a coordinate system used to formulate and express the physical measurements of an observer. A frame in which there is no change in acceleration for a moving observer is said to be inertial. Frames of different observers with different velocities, as in twin observers, can be related by Lorentz transformations [13, 14] an in Fig. (1).

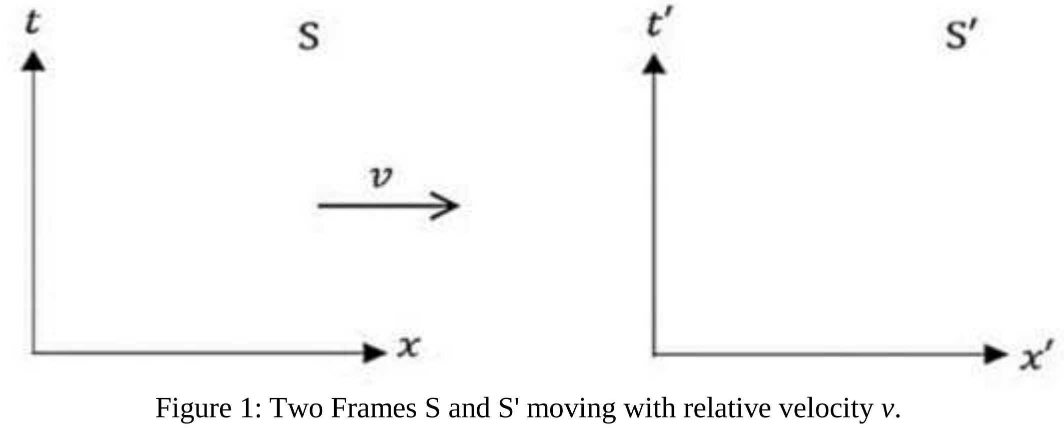

Figure 1: Two Frames S and S' moving with relative velocity *v*.

The Lorentz transformation for length of an observer in inertial frames *S* and *S'* is written as [22].

$$x' = \gamma(x - vt) \quad (1)$$

where γ is the Lorentz factor[1], x is the position coordinate and t is time coordinate. It is more significant when objects are moving with a relativistic speed at which the speed of light is universal [23-25].

Another term to be mentioned is 'proper time' τ the time measured by an observer's clock in its frame [26]. This time can be calculated from the distance between two events in space-time (frame) of an observer, which is called line element and it can be in four dimensional flat spacetime written as [27-30].

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \quad (2)$$

Dividing both sides of equation (2) by c^2, gives a proper time element from which proper time is calculated:

$$d\tau^2 = -dt^2 + (dx^2 + dy^2 + dz^2)/c^2 \quad (3)$$

3. Faster Twin Calculation

The results of measurements from both twins' frame, when they are both moving with different relativistic speed, should be consistent with the calculation of one twin's frame.

When frame of **B** is chosen, then the velocity of **A** as measured by **B** is not simple as it is in a non-relativistic situation, $v_A - v_B$.

In the present case, both velocities are relativistic, $v_B > v_A$. Thus, **B** will measure **A**'s velocity as follows:

$$v'_A = \frac{v_A - v_B}{1 - v_A / v_B} \quad (4)$$

v'_A is what **B** will measure from his rest frame.

Now, the rest frame is the straight line with negative slope for both twins (one can set a rest frame as earth's frame for simplicity). It can be seen from the diagram that **A** and earth's frame are no longer the same, which means we do not have a stay at home twin as it was in normal twin paradox as in Fig. (2).

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[1] $\gamma = \sqrt{\dfrac{1}{1 - v^2 / c^2}}$

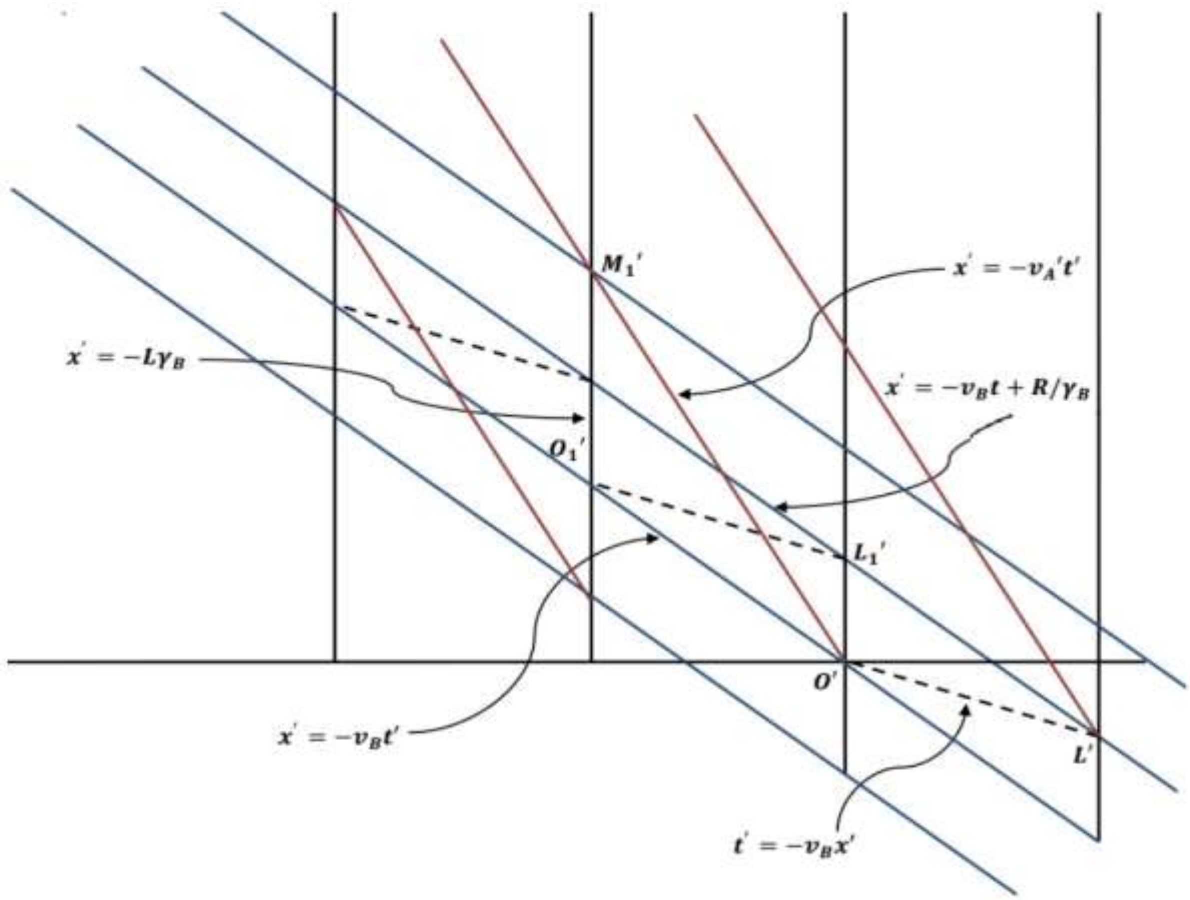

Figure 2: A's movement as seen by B in his reference frame. It is shown that how the universe would have been seen from the point of view of a faster moving twin B. The black fundamental lines are twin B's space time world lines while the red one is the twin A world lines, the universe have been identified through the dashed black lines. The Blue line are the earth coordinates as seen by both twins.

Now the line which joins $O'_1 L'_1$ can be calculated since it has the same slope as $t' = -v_B x'$ (*but with an interception k*)[2], hence:

$$t' = -v_B x' + k \tag{5}$$

We can use the fact that this line passes through $L'_1 (L / v_B \gamma_B, 0)$ to find the constant k (as $x' = 0$ if $t' = L / v_B \gamma_B$), therefore,

$$t' = -v_B x' + \frac{L}{v_B \gamma_B} \text{ and } x' = -v_B t + \frac{L}{\gamma_B} \tag{6}$$

From this point O'_1 coordinates can be found as it is the intersection of trajectories O'_1, L'_1 and $x' = -v_B t'$, hence

$$x' = -L\gamma_B \text{ and } t' = L\gamma_B v_B$$

One can conclude that the coordinate O'_1 is then equal to ($L\gamma_B / v_B, -L\gamma_B$). In this case, the second **B** line (or trajectory) around the universe is simply using O'_1, of $x' = 0 +$ constant,

[2] when passes through $p' \frac{L}{v_B \gamma_B} = 0 + k$.

hence, it is found that $x' = -L\gamma_B$. Now the position of the first meeting after the departure M'_1, which is intersection of the trajectories $x' = -L\gamma_B$ and $x' = v'_A t'$ is

$$M'_1 = (-L\gamma_B / v'_A, -L\gamma_B) \tag{7}$$

The proper time (using 2-dimensional line element) of **B** can be calculated as he travels from his first departure point to the first meeting at M'_1 from ($O' \to M'_1$). In his own frame, $\Delta x'_B$ must be zero as he would not see himself moving on his own frame. Thus, the coordinate time change of **B** will be the sum of $\Delta t_1(O' \to L'_1)$ and $\Delta t_2(O'_1 \to M'_1)$, as $\Delta t'_B = \Delta t_1 + \Delta t_2$, so:

$$\Delta t'_B = \left(\frac{L}{v_B \gamma_B} - 0\right) + \left(-\frac{L\gamma_B}{v'_A} - \frac{L\gamma_B}{v_B}\right) \tag{8}$$

Then, by substituting the value of v'_A from equation (4) and $\gamma_B = 1/\sqrt{1-v_B^2}$, we obtain

$$\Delta t'_B = \frac{L}{\gamma_B (v_B - v_A)} \text{ and } \Delta x'_B = 0$$

The line element can be written in the following form:

$$\Delta \tau'^2_B = \Delta s'^2 / c^2 = \Delta t'_B{}^2 - \Delta x'^2_B \tag{9}$$

Substituting the above values of $x - t$ coordinates, proper time can be found to be:

$$\Delta \tau'_B = \frac{L}{\gamma_B (v_B - v_A)} \tag{10}$$

Now **A** moves in **B**'s frame from O' to M'_1, therefore

$$\Delta x'_A = -L\gamma_B \text{ and } \Delta t'_A = -L\gamma_B / v'_A$$

Using line element for **A** to calculate the proper time:

$$\Delta \tau'^2_A = (\Delta t_B)^2 - \Delta x'^2_B \tag{6}$$

Again, by substituting the values of $x - t$ coordinate for **A**, we obtain

$$\Delta \tau'_A = \frac{L}{\gamma_A (v_B - v_A)} \tag{7}$$

The results of equations (10) and (12) are consistent with the calculations made by an observer on the earth's reference frame which is easier to obtain, as it does not need to use the relativistic speed as it can be seen from Dray paper [1]. Hence, it can be seen that the faster twin, **B**, is younger than **A**.

4. Results and Discussion

It is clear that the results of the calculation reaches equations (10) and (12) that the proper time of twin **B** is smaller than proper time of twin **A**. Hence, twin **B** younger at the end of trip than twin **A**. The equations (10) and (12) contain the Lorentz factor, the faster velocity gives bigger Lorentz factor hence smaller proper time. These calculation is performed by the faster twin, and it would give identical results if it is performed by the slower twin. One point of discussion rises during this calculation, the existence of a preferred frame of reference, which is due to having a cylindrical universe that is topologically multiply connected. This existence of preferred frame does not affect the concept of spacetime interval which is conserved under Lorentz transformations.

5. Conclusions

A spatially closed universe has been used to resolve the problem of the acceleration of an observer in the twin paradox, which makes both twins to be inertial throughout a journey around the universe. The calculations in the closed universe model show that the observer with a faster relativistic speed will be younger. Similar result obtained by T. Dray, as if one observer stayed on earth whiles the other travels. While in the present calculation, both twins are moving. It has been shown that the faster twin is always younger no matter the calculation carried out by the faster or the slower observer in the closed universe.

Conflict of Interests.

There are non-conflicts of interest .

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الخلاصة

المفارقة التوأم العادية هي مشكلة شيخوخة المراقبين التوأمين A و B، B يسافر بسرعة النسبية ويعود مرة أخرى إلى A حيث A في حالة سكون. وفقاً لحساب كلا التوامين، إذا كان كل واحد يعتبر نفسه في حالة سكون، فسيكون الآخر أصغر سناً في نهاية الرحلة. وهذا الأمر المتناقض، في الواقع تظهر المفارقة بسبب تعجيل التوأم المتنقل (B) ولذلك يصبح B أصغر سناً في نقطة العودة. تم حل هذه المفارقة حسب .T Drayعن طريق إجراء الحساب في كون أسطواني مغلق، حيث مشكلة التعجيل يتم حلها للتوأم المسافر هذا يعني أن التوأم المسافر (أسرع) سيكون أصغر سناً بغض النظر عن التوأم الذي أجرى الحساب. حسب حساباتنا الحالية، أظهرنا ما إذا كان كلا التوأمين يسافران بسرعة نسبية في كون اسطواني مغلق و التوأم الأسرع (B) اجرى الحسابات. ستظهر النتيجة دائماً أن توأم الاسرع سيكون له وقت أقل. هذه النتيجة تتسق مع نتيجة .T Dray، عندما يكون احد التوأمين في حالة سكون والآخر يتحرك.

الكلمات الدالة: نظرية النسبية الخاصة، الزمن الحقيقي، تحويلات لورنتز، كون المدمج، المفارقة التؤام