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Article

Oblique Decision Tree Algorithm with Minority Condensation for Class Imbalanced Problem

Artit Sagoolmuang[a] **and Krung Sinapiromsaran**[b,*]

Applied Mathematics and Computational Science, Department of Mathematics and Computer Science, Faculty of Science, Chulalongkorn University, Bangkok, Thailand
E-mail: [a]a.sagoolmuang@gmail.com, [b]krung.s@chula.ac.th (Corresponding author)

Abstract. In recent years, a significant issue in classification is to handle a dataset containing imbalanced number of instances in each class. Classifier modification is one of the well-known techniques to deal with this particular issue. In this paper, the effective classification model based on an oblique decision tree is enhanced to work with an imbalanced dataset that is called oblique minority condensed decision tree (OMCT). Initially, it selects the best axis-parallel hyperplane based on the decision tree algorithm using the minority entropy of instances within the minority inner fence selection. Then it perturbs this hyperplane along each axis to improve its minority entropy. Finally, it stochastically perturbs this hyperplane to escape the local solution. From the experimental results, OMCT significantly outperforms six state-of-the-art decision tree algorithms that are CART, C4.5, OC1, AE, DCSM and ME on 18 real-world datasets from UCI in term of precision, recall and F1 score. Moreover, the size of a decision tree from OMCT is significantly smaller than others.

Keywords: Class imbalanced problem, minority entropy, oblique decision tree, minority condensation.

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1. Introduction

Creating an effective classification model plays the important role in data mining and machine learning in the past several years. However, there is a critical issue that significantly affects the classifier performance occurring when the number of instances in each class of a dataset is extremely different. Most well-known classifiers, such as the neural network and the decision tree which work well on a balanced dataset, deviate toward a class having a large number of instances called the majority class and neglect the one with a small number of instances called the minority class [1]. This problem is known as a class imbalanced problem which appears in many real world situations such as the fraud detection [2, 3] and the diseases diagnosis [4, 5]. For the fraud detection, the number of fraudulent transactions is very small compared with non-fraudulent cases. To minimize classification error, the built classifier mostly predicts unknown transactions to be non-fraudulent transactions. It means that some fraudulent cases are announced to be non-fraudulent ones. According to this behavior, this appears to have an undesirable outcome because the fraud is not detected. Similarly, the classifier for the diseases diagnosis may predict all patients normal to achieve the highest accuracy. This will misclassify the real patients causing them to loss an opportunity to receive the treatment. Furthermore, the class imbalanced problem is also appeared in network intrusion detection [6], sentiment analysis [7], protein/DNA identification [8] and e-mail spam filtering [9]. Consequently, It is absolutely necessary to handle this problem cautiously.

Several techniques for dealing with the class imbalance problem can be divided into four categories [10] which are (1) data-level approach, (2) algorithm-level approach, (3) cost-sensitive learning approach, and (4) ensemble-based approach. This research focuses on algorithm-level approach which can handle the class imbalanced problem without changing the original training dataset or assigning unrealistic cost for misclassifying minority instances. A remarkable classification model, oblique decision tree [11], is enhanced to make it suitable for any imbalanced dataset.

Intelligibility, infallibility and efficiency of the decision tree algorithm make the decision tree to be one of the successful classification models [12]. It recursively partitions the training dataset in each node using the axis-parallel hyperplane which gives the least impurity measure. However, almost all impurity measures are not designed for an imbalanced dataset because it treats importance of each class equally. So it will bias toward the class with a large number of instances. Establishing the decision tree modification for solving a class imbalanced problem has been a challenging concern with numerous publications offering various improved impurity measures. Nonetheless, changing the impurity measure is not sufficient due to insufficient information appearing along each parallel axis. Hence building a decision tree using an oblique hyperplane is more attractive, see [11]. Previous publications have been proposed for building the oblique decision tree such as using optimizations [13, 14, 11], using heuristics [15, 16], using feature extractions [17, 18] and applying genetic algorithm [19]. Unfortunately, most of these did not address the problem of imbalanced dataset. Until 2019, Chabbouh et al. proposed a Multi-Objective Evolutionary Algorithm (MOEA), called ODT-based-Θ-Nondominated Sorting Genetic Algorithm-III (ODT-Θ-NSGA-III) [20], to optimize simultaneously both precision and recall values providing an optimal Pareto concept in the process of building oblique decision trees. It is the first emerging algorithm to deal with the binary class imbalanced problem after the construction of the oblique decision tree.

Accordingly, this research proposes an oblique decision tree for classifying imbalanced dataset called oblique minority condensed decision tree (OMCT). It reduces the influence of majority instances by limiting the range of minority instances within the inner fence of Tukey boxplot, along the axis of the hyperplane.

The contribution of this paper are threefold:

1. First, the enhancement of an existing oblique decision tree algorithm is proposed to deal with the class imbalanced problem.
2. Second, it introduces the strategy for defining the boundary of minority instances within their cluster to avoid excess range of minority instances.
3. Third, the proposed approach provides an empirical and experimental results showing the improved performance comparing with existing methods.

The remaining of this paper is organized as follows. A brief review on the oblique decision tree and the decision tree for an imbalanced dataset are shown in Section 2. Next, Section 3 introduces the proposed algorithm. The results and discussions of the experiments are presented in Section 4. Finally, Section 5 offers the discussion and conclusion of this research.

2. Related Works

This section reviews related works that are the core of this research. It covers the solutions to solve a class imbalanced problem, the decision tree algorithm, and the oblique decision tree algorithm.

All algorithms in this section will deal with a binary imbalanced classification problem. Let $D = \{\vec{x}_1, \vec{x}_2, ..., \vec{x}_n\} \subseteq \mathbb{R}^d$ be a d-dimensional imbalanced training dataset containing n instances $\vec{x}_i = (x_1^i, x_2^i, ..., x_d^i)$ with respect to a set of class $C = \{c_1, c_2, ..., c_n\}$ where $c_i \in \{+1, -1\}$ for $i = 1, ..., n$. D is separated into two partitions which are a set of minority class or a positive class, $D^+ = \{\vec{x}_i \in D \mid c_i = +1\}$ having size n^+ and a set of majority class or a negative class, $D^- = \{\vec{x}_i \in D \mid c_i = -1\}$ having size n^-, where $n^+ + n^- = n$ and $n^+ \ll n^-$.

2.1. Class Imbalanced Techniques

As discussed in the previous section, the solution techniques used to handle the class imbalanced problem can be divided into four categories.

- Firstly, data-level approach modifies a training dataset before building a classifier. It attempts to rebalance the training dataset using under-sampling technique [21] and over-sampling technique [22] which eliminates the majority instances and synthesizes the minority instances, respectively.
- Secondly, algorithm-level approach [23] revises an existing model to deal with an imbalanced dataset. It also includes presenting a novel classifier that addresses this issue directly. Their mechanisms are biased toward identifying instances in the class containing the tiny number of instances.
- Thirdly, cost-sensitive learning approach [24] increases the importance of the minority class by assigning the large misclassified cost to inaccurate instances, while the lower cost is assigned to majority instances. Minimizing this total cost will make the model bias toward identifying the minority class.
- Fourthly, ensemble-based approach [25] combines one of the methods mentioned above with the ensemble learning algorithm such as Bagging and Boosting technique.

2.2. Decision Tree Algorithm

Decision tree algorithm is a recursive partitioning algorithm based on a tree structure consisting of a set of nodes connecting by branches. Each non-leaf node presents a splitting condition by a hyperplane $H : a_0 + \vec{a} \cdot \vec{x} = 0$, where $\vec{a} = (a_1, a_2, ..., a_d)$ is the normal vector and a_0 is the intercept. The leaf nodes represent the specific class of the instances. Traditionally, most decision tree algorithms apply to the axis-parallel hyperplane for splitting the set of instances. In order to select the axis-parallel hyperplane, a greedy approach is used via impurity measures of all axis-parallel hyperplanes (see Fig. 1). The hyperplane that provides the least impurity is chosen to be the splitting condition of the node. The algorithm stops when all instances are in the same class or user's specified criteria are met.

Various impurity measures have been proposed for evaluating the performance of the hyperplane such as Gini [26] using in the well-known decision tree algorithm, CART [14]. Another decision tree algorithm like C4.5 [27] applies the Shannon's entropy [28]. The Shannon's entropy used in OMCT algorithm is defined by (1). It equals to zero when all instances are in the same class and equals to one when the number of instances in all classes are identical.

$$Entropy(D) = -\frac{n^+}{n} log_2 \frac{n^+}{n} - \frac{n^-}{n} log_2 \frac{n^-}{n} \quad (1)$$

2.2.1. Decision Tree for Imbalanced Dataset

Traditional decision tree algorithms work well on the balanced datasets so they may not be appropriate for dealing with the imbalanced ones. Various impurity measures are proposed for improving the performance of the decision tree on the imbalanced datasets. In 2010, Chandra et al. presented the distinct class based splitting measure (DCSM) [29]. In 2006, the asymmetric entropy (AE) was proposed by Marcellin et al., [30]. The concept of the skew impurity measure is used instead of the symmetric one such as Shannon's entropy. The maximum value is shifted by the skewness parameter θ between 0 and 1. Another skew impurity measure for the imbalanced dataset is off-centered entropy (OCE) [31] that is suggested by Lenca et al., in 2008. In addition, the insensitive impurity measures are introduced for solving the class imbalanced problem such as DKM [32] and HDDT [33]. They are not affected by the ratio of the number of instances among all classes. Importantly, the technique that is an inspiration of this research is minority entropy (ME) [34]. It applies the concept of under-sampling in each node of the decision tree algorithm which is explained below.

Minority Entropy

Minority entropy (ME) is proposed in 2016 by Boonchuay et al. [34]. For each attribute j, the minority range $minrange_j$ is defined as the interval between the smallest and the largest values within attribute j of minority instances. Then, the splitting step will only be examined within this range $minrange_j$, i.e.

$$\begin{aligned} spr_j(D) = \{\vec{x} \in D \mid min_{\vec{z} \in D^+} proj_j(\vec{z}) \\ \leq proj_j(\vec{x}) \leq max_{\vec{z} \in D^+} proj_j(\vec{z})\} \end{aligned} \quad (2)$$

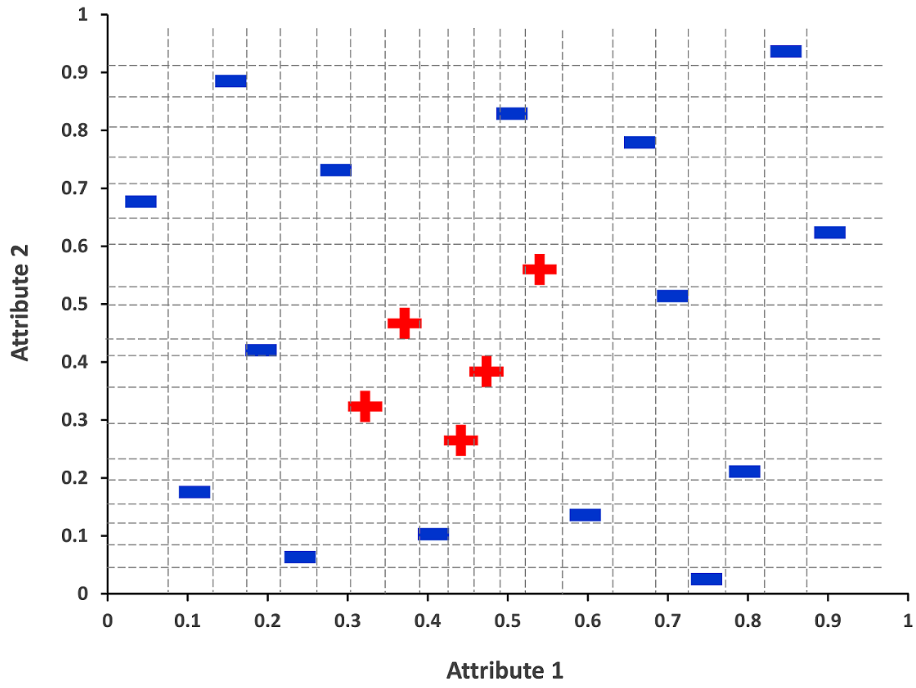


Fig. 1. All axis-parallel hyperplanes (dash line) considering in a greedy approach.

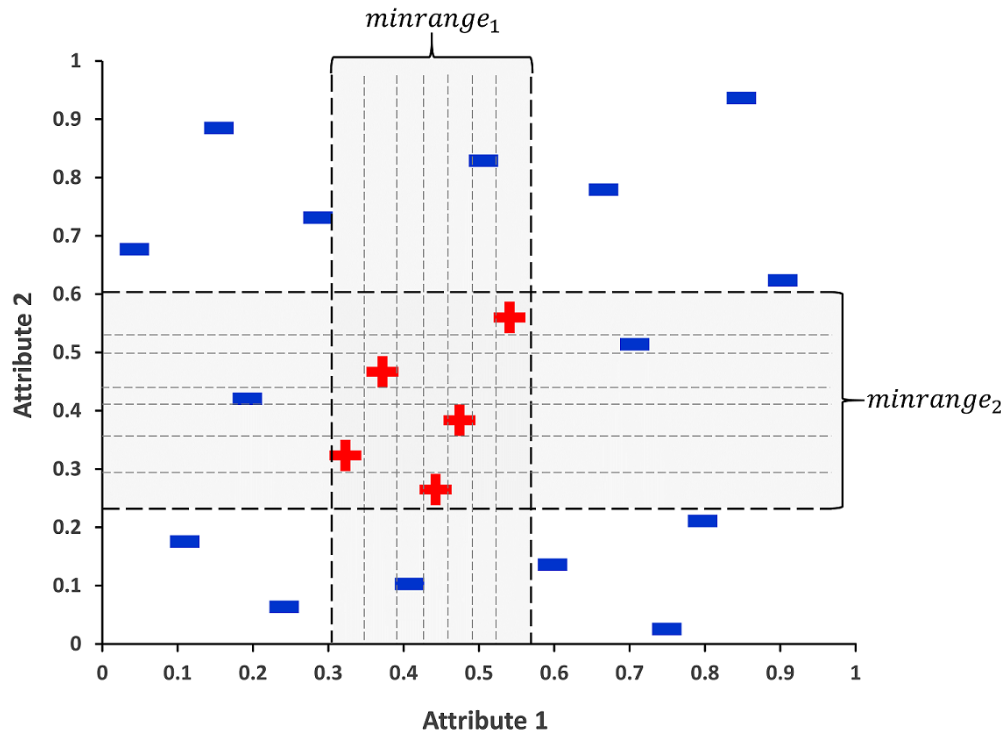


Fig. 2. All axis-parallel hyperplanes (dash line) considering in ME.

where $proj_j(\vec{a})$ is the j^{th} element of $\vec{a}$. It increases the ratio of the minority instances for ensuring that the selected hyperplane from ME will separate the datasets in the region of the minority distribution, as shown in Fig. 2.

2.3. Oblique Decision Tree

For a small number of the distinct axis-parallel hyperplanes, the greedy approach in a decision tree algorithm is practical. Nonetheless, the number of distinct oblique hyperplanes is up to $2^d \cdot \binom{n}{d}$ [13] having the exponential number of hyperplanes to explore. Precisely, this problem is proved to be NP-hard by Heath in 1993 [35]. Consequently, it is necessary to employ other techniques for finding the best oblique hyperplane. The linear combination version of CART or CART-LC [14] uses a deterministic hill-climbing algorithm for finding the best oblique hyperplane. In 1993, Simulated Annealing Decision Tree (SADT) is proposed by Heath et al. [11]. It applies the randomization of the simulated annealing algorithm to search for the best oblique hyperplane. These two methodologies are combined to obtain the well-known oblique decision tree algorithm as Oblique Classifier 1 or OC1 [13] in 1994 which is the core algorithm of this research. There are some other effective techniques to solve this problem. The algorithms based on heuristic arguments such as OC1-GA and OC1-ES [19] use genetic algorithm and evolutionary strategy to find the best oblique hyperplane, respectively. Using the feature extraction is another attractive concept. It transforms the original space before applying the greedy approach to find the best axis-parallel hyperplane such as the Fisher's decision tree [17] and HH-CART [18].

Oblique Classifier 1

OC1 [13] is developed by Murthy et al. in 1994 for finding a multivariate split at each node of the decision tree algorithm. Two methodologies that are deterministic hill-climbing and randomization are used in selecting the split of the OC1 algorithm. It begins with the best axis-parallel hyperplane $H : a_0 + \vec{a} \cdot \vec{x} = 0$. Then, this hyperplane is perturbed along each axis using deterministic hill-climbing. The greedy approach is applied on a set U_j, see (3), for finding the best new value of coefficient a_j. Geometrically, when replacing a_j with u_j^i, the j^{th}-intercept is shifted that will pass through the instance $\vec{x}_i$. Fig. 3a shows all hyperplanes that are considered in deterministic hill-climbing with attribute 1.

$$U_j = \left\{ u_j^i = \frac{-\vec{a} \cdot \vec{x}_i + a_j \cdot x_j^i - a_0}{x_j^i} \mid i = 1, ..., n \right\} \tag{3}$$

However, the best hyperplane obtaining by this deterministic hill-climbing may get trap in the local solution. The randomization is offered to escape this local minimum. A random hyperplane $R : r_0 + \vec{r} \cdot \vec{x} = 0$ is generated by perturbing H with α, i.e. $H + \alpha R$. The optimal value of α is searched by the greedy approach on a set V defining by (4). Geometrically, each v^j gives a hyperplane that passes through instance $\vec{x}_i$ and the intersection of H and R. All hyperplanes that are considered in randomization are shown in Fig. 3b.

$$V = \left\{ v^i = \frac{-\vec{a} \cdot \vec{x}_i - a_0}{\vec{r} \cdot \vec{x}_i + r_0} \mid i = 1, ..., n \right\} \tag{4}$$

3. Oblique Minority Condensed Decision Tree

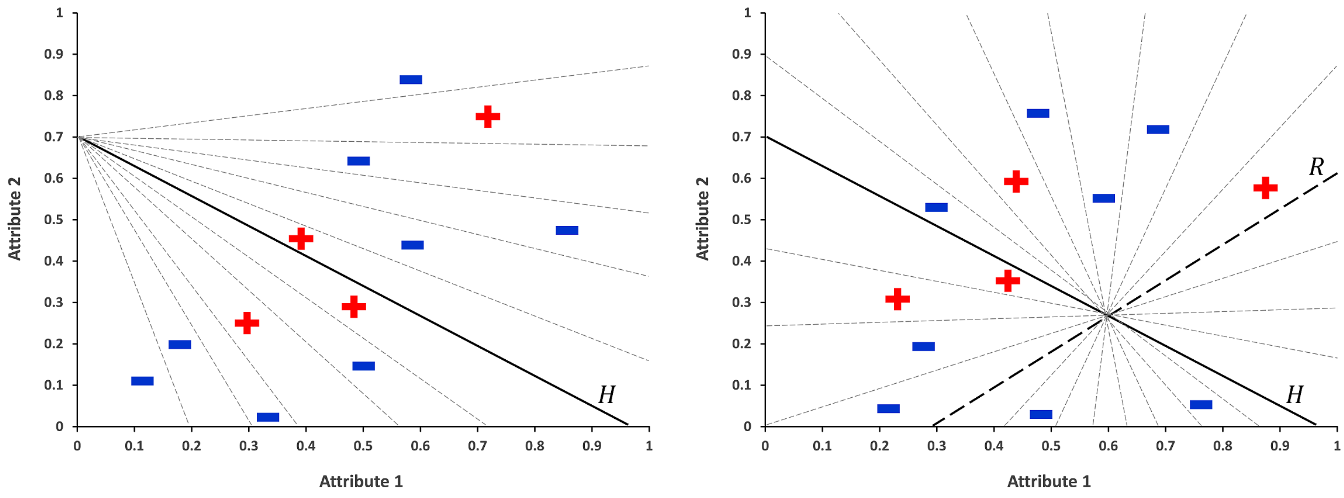
In this section, a novel oblique decision tree called minority condensed oblique decision tree or OMCT is proposed for handling the class imbalanced problem. It combines the oblique decision tree with the minority condensation.

3.1. Motivations

Although the axis-parallel decision tree algorithm based on minority entropy (ME) [34] shows the effectiveness of the decision tree algorithm for the class imbalanced problem. Nevertheless, class distribution of the real world datasets may not lie perfectly on any parallel axis so the best decision tree may not classify minority instances correctly. On the other hand, the oblique decision tree algorithm such as oblique classifier 1 (OC1) [13] uses a hyperplane that can deal with the oblique distribution. Oblique minority condensed decision tree (OMCT) integrates the advantages of ME and OC1 together. Furthermore, the disadvantage of each method is improved by the essence of one another. The minority condensation of minority instances is an essential ingredient of the OMCT algorithm. It is applied for condensing the minority instances before applying the greedy approach in each step of OMCT: determining the best axis-parallel hyperplane, improving the minority entropy using deterministic hill-climbing on each axis, and improving the minority entropy by randomization. Moreover, extra caution to treat some minority instances that extremely deviate from the others, called outliers [36], is applied. The interquartile range rule [37] is deployed to discard the minority outliers before applying the minority entropy. Values outside the minority inner fence computed by $[Q_1 - 1.5 * IQR, Q_3 + 1.5 * IQR]$ of minority instances are treated as the outliers and will be included after the partitioning step is done. The demonstration of the minority condensation is shown by Fig. 4 with the Tukey boxplot of the minority instances in dataset X. It defines left margin m_l and right margin m_r by the largest and smallest values of the minority instances within the minority inner fence of the current round. A set of instances within m_l and m_r is denoted by MC_X.

3.2. Minority Condensation

The formal definition of the minority condensation with some related formulae are defined in this section. Each instance both in minority and majority class is assigned a score. The minority condensation keeps instances inside the inner fence of the Tukey boxplot according this score, see Definition 1.



(a) (b)

Fig. 3. Mechanism of perturbing the initial hyperplane H (solid line) in each step of OC1 algorithm. (a) All hyperplanes (dash line) considering in deterministic hill-climbing with attribute 1. (b) All hyperplanes (dash line) considering in randomization with random hyperplane R (solid-dash line).

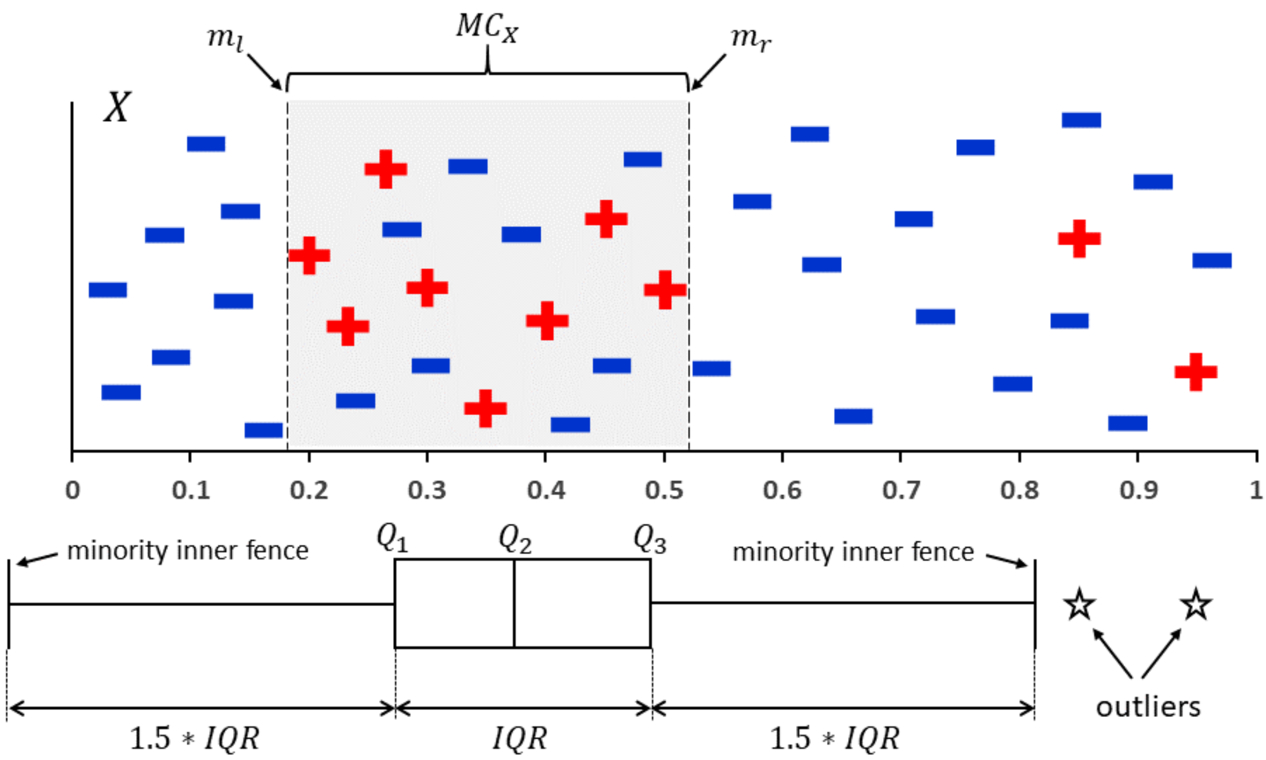


Fig. 4. The overview of the minority condensation.

Definition 1. Let $X^+ \subset \mathbb{R}$ be a set of scores of the minority class. IQR is the interquartile range computed from the third quartile Q_3 subtracting with the first quartile Q_1. Define $X^+_{wo} \subseteq X^+$ without outliers as follows:

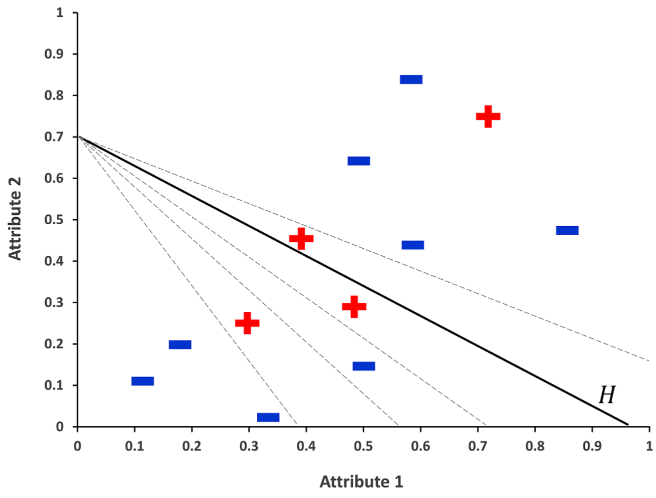
$$X^+_{wo} = \{x \in X^+ \mid Q_1 - 1.5 * IQR \leq x \leq Q_3 + 1.5 * IQR\}$$

The core definition of this research, a set of scores with the minority condensation, is presented in Definition 2. It combines scores in X^+_{wo} with the scores corresponding to the majority instances where their values lie within the range of X^+_{wo}.

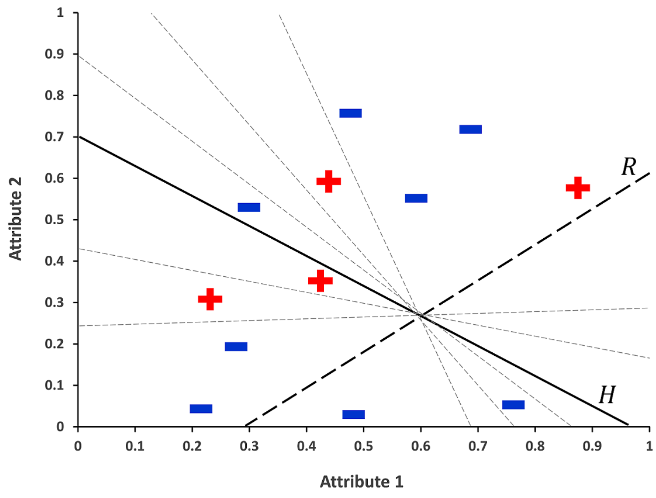
Definition 2. Let $X = X^+ \cup X^-$ be a set of scores of all instances. A set X with the minority condensation is defined as follows:

$$MC_X = \{x \in X \mid m_l \leq x \leq m_r\}$$

where the left margin m_l and the right margin m_r are



(a)



(b)

Fig. 5. Applying the minority condensation to each step of the oblique decision tree algorithm, i.e. deterministic hill-climbing (a) and randomization (b).

the minimum and maximum values of X^+_{wo}, respectively.

The set of scores from Definition 2 is constricted to intensify the importance of the instances in the minority class. It is used in three steps of the OMCT algorithm. In the process of determining the best axis-parallel hyperplane, the minority condensation is applied on each attribute of dataset D before the greedy approach. Minority instances that are discarded will be reconsidered in the next step. Due to the reduced range from the minority condensation, the number of discard majority instances is far more than ME. In the deterministic hill-climbing, for each j^{th} the minority condensation is applied on X where X is defined as U_j (3). It reduces the number of considered hyperplanes by a $\vec{a}$ without j^{th} component, see Fig. 5a. For improving the minority entropy by randomization, the minority condensation is applied on X where X is defined as V (4). It also reduces the number of considered hyperplanes at the intersection of given hyperplane H and the random hyperplane, see Fig. 5b.

3.3. OMCT Algorithm

The OMCT algorithm is constructed in a top-down fashion [38]. It separates the instances into two partitions using the oblique hyperplane as a criterion. Then, it recursively repeats on each partition until only one class instances are left in the partition or other specified stopping criteria are met.

The steps of the OMCT algorithm, which employs to search for an optimal oblique hyperplane, is illustrated by Fig. 6. It starts with calling the Axis Parallel Hyperplane step to generate the initial axis parallel hyperplane. The deterministic hill climbing step is applied to each attribute, which is restricted to the region of minority class by using the minority condensation. Then, it perturbs the hyperplane using the deterministic hill climbing step and the randomization step respectively. The use of the minority condensation is embedded in all steps to reduce the set of considered instances. They are repeated in a loop until the obtained oblique hyperplane cannot be improved.

Time Complexity of the Minority Condensation

The time complexity analysis of the minority condensation is shown in this section. There are three parts for applying the minority condensation to the set of scores X of size n. First, separating the class of instances in X to the minority or majority classes uses $O(n)$ time complexity. Second, determining the set X^+_{wo} by Definition 1 takes $O(n\ log\ n)$ running time using the merge sort for calculating the minority inner fence. Third, defining the set MC_X by Definition 2 spends $O(n)$ for considering that each instance is either inside or outside the interval between m_l and m_r. In summary, the overall time complexity is $O(n) + O(n\ log\ n) + O(n) = O(n\ log\ n)$ running time.

4. Experiments

This section purposes performance comparisons in classifying the imbalance datasets of OMCT with other effective decision tree algorithms in two aspects, the performance of classification and the size of a decision tree.

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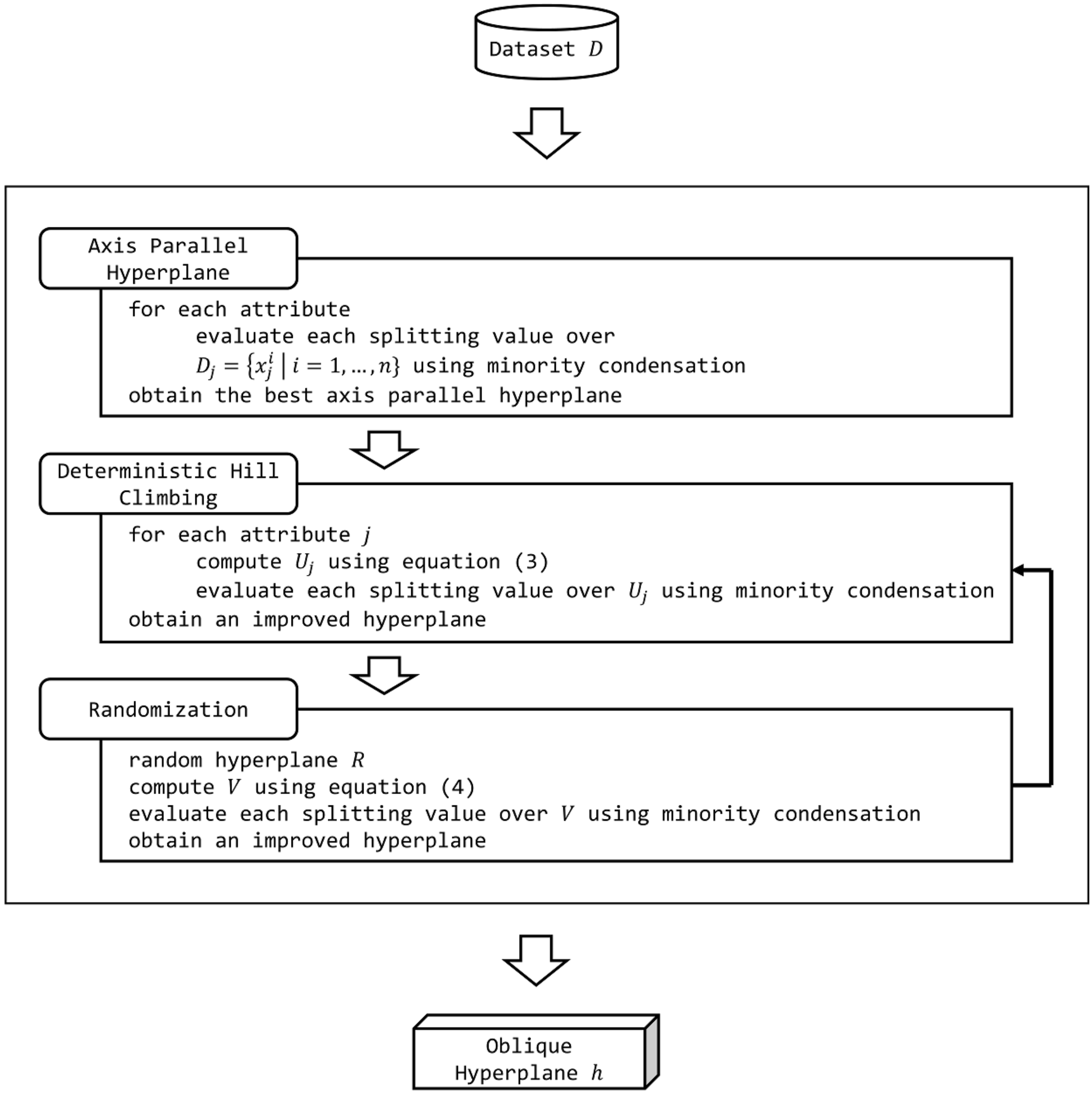

Fig. 6. Brief pseudo codes and basic iterations of the OMCT algorithm.

4.1. Datasets

In order to evaluate the performance of OMCT comparing with other decision tree algorithms, the real-world datasets from UCI repository [39] are used as experimental benchmark. Table 1 shows the summary of each dataset using in the experiments. The number and the name of each dataset appear in the first and the second columns, while the third and the fourth columns show the number of instances and the number of attributes. The fifth column presents the selected class for labeling as the minority class and the majority class. For the last three columns, they contain the percentage of the minority class, the majority class and the ratio of them called the imbalanced ratio, respectively. Each dataset applies the five-fold cross-validation and repeats it 20 times. Hence, one hundred experiments are performed on each dataset.

4.2. Comparative Methods and Evaluation

The performance of OMCT is evaluated comparing with six decision tree algorithms. Two out of six are well-known axis-parallel decision tree algorithms, CART [14] and C4.5 [27]. Others are the most famous oblique decision tree algorithm as OC1 [13], and three state-of-art decision tree algorithms for an imbalanced dataset, i.e. AE [30], DCSM [29] and ME [34].

There are two aspects to report in this paper which are (1) classification performances and (2) tree sizes. Due

Table 1. Summary of experimental datasets.

No.	Datasets	#Inst.	#Att.	Minor./Major. Class	%Minor.	%Major.	I.R.
1	Pima	768	8	1 / 0	34.90	65.10	1.87
2	Wine	178	13	3 / the rest	26.97	73.03	2.71
3	Haberman	306	3	2 / 1	26.47	73.53	2.78
4	Vehicle	846	18	bus / the rest	25.77	74.23	2.88
5	Glass	214	9	5,6,7 / the rest	23.83	76.17	3.20
6	Shuttle	58000	9	the rest / 1	21.40	78.60	3.67
7	Yeast	1484	8	VAC / the rest	20.49	79.51	3.88
8	Libras	360	90	1,2,3 / the rest	20.00	80.00	4.00
9	NewThyroid	215	5	2 / the rest	16.28	83.72	5.14
10	SatImage	2310	19	5 / the rest	14.29	85.71	6.00
11	Ecoli	336	7	imU / the rest	10.42	89.58	8.60
12	OpticDigits	5620	64	4 / the rest	10.11	89.89	8.89
13	Abalone	431	7	18 / 9	9.74	90.26	9.26
14	Landsat	6435	36	4 / the rest	9.73	90.27	9.28
15	PenDigits	10992	16	5 / the rest	9.60	90.40	9.42
16	Vowel	990	10	0 / the rest	9.09	90.91	10.00
17	PageBlocks	5473	10	2 / the rest	6.01	93.99	15.64
18	Letter	20000	16	A / the rest	3.95	96.06	24.35

to the extreme difference number of instances in each class, the traditional measure like accuracy is not suitable to evaluate the performance of classifying imbalanced datasets. It shows the detection rate of a whole dataset that has the minimum affect when the instances in the minority class are misclassified. In considering the binary class imbalanced problem, two performance aspects need to be evaluated, i.e., the percentage of instances correctly predicted to be the minority class and the percentage of the minority instances that are correctly classified. They are represented by two widely used measures which are precision and recall defined by equations (5) and (6), respectively [40]. Moreover, their harmonic mean, represented by F1 score in equation (7), summarizes the overall performance of each classifier. For the size of each decision tree, it is measured by the number of leaf nodes referring to the number of partitions. The smaller the number of nodes in a decision tree, the smaller the number of partitions.

$$Precision = \frac{TP}{TP + FP} \quad (5)$$

$$Recall = \frac{TP}{TP + FN} \quad (6)$$

$$F1 = 2 \cdot \frac{Precision \cdot Recall}{Precision + Recall} \quad (7)$$

where

- TP is the number of true predicted of minority instances.
- FP is the number of false predicted of minority instances.
- TN is the number of true predicted of majority instances.
- FN is the number of false predicted of majority instances.

In addition, the non-parametric statistical hypothesis test as Wilcoxon signed-rank test with 0.01 and 0.05 significance level (α) is applied for testing the performance difference of OMCT comparing with other algorithms. The null hypothesis (H_0) and the alternative hypothesis (H_1) of Wilcoxon signed-rank test are indicated as follows.

H_0 : The performance of OMCT and the comparative method are not different.

H_1 : The performance of OMCT and the comparative method are different.

4.3. Results and Discussions

The experimental results are demonstrated in this section. Each table shows the experimental results with different measurements. Each row of the table reports the performance of each decision tree algorithm working on the specific dataset.

4.3.1. Classification Performances

In the first experimental result, the precision of each decision tree algorithm is shown in Table 2. OMCT yields the least average ranking over other methods that means OMCT has the highest percentage of true predicted minority instances, while other trees predict majority instances as the minority instance more. Especially, the traditional decision tree algorithms such as CART and C4.5 show the poor precision performance. This happens because they give the importance to each class equally causing the boundary of partitioning setting in the region of the majority class.

In the second experimental result, the recall of each decision tree algorithm is shown in Table 3. It shows that OMCT yields the least average ranking over other trees. That is, OMCT has the highest percentage of minority instances predicting correctly, while there are more minority instances predicted as the majority instances by other algorithms. Especially, the decision tree algorithms designing for the imbalanced datasets such as AE and DCSM show the poor recall. This happens because that they focus on the minority instances excessively causing the boundary of partitioning to overfit the minority class.

The performance of each decision tree algorithm for handling the class imbalanced problem is represented by F1 score as in Table 4. Since OMCT has the lowest ranking in both precision and recall, it provides the best ranking in F1 score also. Statistically, the F1 score of OMCT is significantly better than CART, C4.5 and OC1 with a 99% confidence level. Note that those decision tree algorithms show poor performance since they are not designed for imbalanced datasets. For the decision tree algorithms inventing for the imbalanced dataset specifically such as AE, DCSM and ME, they have the lower performances than OMCT significantly as well at a 95% confidence level.

All experiments in this paper confirm that the proposed decision tree algorithm, OMCT, offers the improvement over other decision tree algorithms. Especially, it outperforms OC1 and ME which originate the concept of OMCT. Although OC1 shows the improvement over the traditional decision tree in both precision and recall, neglecting the importance of the minority class causes OC1 to misclassify a lot of minority instances, resulting in a low precision. ME shows the outstanding ability to discover the minority instances which is observed by the recall. Nevertheless, it gives low precision due to the unnecessarily broad range of minority instances. It results in the partitioning boundary of minority instances locates within the region of the majority class. OMCT integrates the advantages of both concepts, which consists of applying the oblique hyperplane to capture the distribution of the dataset, concentrating on the minority instances like ME to increase the chance of classifying minority instances, and avoiding outliers to reduce the affect of the broader range.

There are some datasets that OMCT exhibits lower rank such as Glass dataset, Letter dataset than six other decision tree classifiers. However, only 2 significant digits can be observed which may cause by sampling error.

4.3.2. Tree Size

The size of the decision tree from each method is represented by the number of leaf nodes indicating in Table 5. Ostensibly, OMCT provides the smallest size of the decision tree. The number of leaf nodes from OMCT is significantly less than other decision tree algorithms with a 99% confidence level. This causes by a smaller number of hyperplanes and a smaller number of partitions of OMCT than other methods. The reason is due to the ability in capturing the distribution of the minority class via the minority condensation and the flexibility of the oblique hyperplane. For the same reasons, applying the minority range of ME and using the oblique hyperplane of OC1 make tree size comparison to be within the second or the third ranks in this experiment. They have the similar number of leaf nodes, which are less than other methods, but they are still larger than OMCT significantly.

5. Conclusions

A novel oblique decision tree for handling the class imbalanced problem called oblique minority condensed decision tree or OMCT is introduced in this paper, which is an enhancement of the well-known oblique decision tree like OC1. The proposed minority condensation is embedded in the greedy approach for defining the boundary of minority instances within their cluster using the inner fence selection. OMCT is able to handle the class imbalanced problem by recursive partitioning the dataset using the combination of attributes, which concentrates on the region of the minority class. Although it requires additional process for determining the minority condensation. However, it takes $O(nlog(n))$ time complexity indistinguishable with the greedy approach without the minority condensation. Moreover, the number of splitting values that need to be considered is decreased when applying the minority condensation. At each node of the oblique decision tree algorithm, the greedy approach with the minority condensation is applied in three steps of building decision tree, i.e. to find the best axis-parallel hyperplane, to perturb the hyperplane along each axis with deterministic hill-climbing and to avoid the local solution with randomization.

The experimental results show that OMCT significantly outperforms CART, C4.5, OC1, AE, DCSM and ME in term of F1 score on eighteen real-world im-

Table 2. The performance comparison of precision on imbalanced datasets including with the rank which is shown in the parentheses.

No.	Datasets	CART	C4.5	OC1	AE	DCSM	ME	OMCT
1	Pima	0.5712 (3)	0.5708 (4)	0.5683 (6)	0.5739 (2)	0.5707 (5)	0.5681 (7)	0.5808 (1)
2	Wine	0.9293 (6)	0.9302 (5)	0.9387 (4)	0.9673 (1)	0.9291 (7)	0.9460 (3)	0.9485 (2)
3	Haberman	0.3343 (7)	0.3569 (3)	0.3446 (6)	0.3504 (5)	0.3678 (1)	0.3543 (4)	0.3666 (2)
4	Vehicle	0.9179 (5)	0.9041 (7)	0.9116 (6)	0.9211 (3)	0.9360 (1)	0.9232 (2)	0.9195 (4)
5	Glass	0.8556 (5)	0.8705 (2)	0.8662 (3)	0.8747 (1)	0.8652 (4)	0.8381 (7)	0.8493 (6)
6	Shuttle	0.9997 (6)	0.9996 (7)	0.9997 (5)	0.9998 (3)	0.9999 (1)	0.9998 (4)	0.9998 (2)
7	Yeast	0.6391 (6)	0.6182 (7)	0.6567 (3)	0.6756 (1)	0.6486 (5)	0.6553 (4)	0.6595 (2)
8	Libras	0.7553 (6)	0.6205 (7)	0.7765 (4)	0.8067 (1)	0.7892 (2)	0.7841 (3)	0.7554 (5)
9	NewThyroid	0.8948 (6)	0.8819 (7)	0.9107 (5)	0.9288 (2)	0.9500 (1)	0.9215 (4)	0.9275 (3)
10	SatImage	0.8929 (3)	0.8611 (7)	0.8805 (4)	0.8939 (1)	0.8780 (5)	0.8765 (6)	0.8930 (2)
11	Ecoli	0.5893 (4)	0.5570 (7)	0.6065 (3)	0.5666 (5)	0.5641 (6)	0.6164 (1)	0.6079 (2)
12	OpticDigits	0.9148 (6)	0.8655 (7)	0.9321 (2)	0.9240 (4)	0.9173 (5)	0.9293 (3)	0.9512 (1)
13	Abalone	0.3277 (4)	0.3064 (6)	0.3031 (7)	0.3169 (5)	0.3417 (3)	0.3434 (2)	0.3820 (1)
14	Landsat	0.5374 (6)	0.5073 (7)	0.5428 (5)	0.5661 (2)	0.5792 (1)	0.5556 (4)	0.5608 (3)
15	PenDigits	0.9446 (6)	0.9310 (7)	0.9480 (5)	0.9639 (2)	0.9587 (4)	0.9616 (3)	0.9691 (1)
16	Vowel	0.9135 (5)	0.9260 (3)	0.9063 (6)	0.8918 (7)	0.9400 (1)	0.9344 (2)	0.9247 (4)
17	PageBlocks	0.8634 (6)	0.8622 (7)	0.8758 (4)	0.8785 (2)	0.8654 (5)	0.8781 (3)	0.8830 (1)
18	Letter	0.9362 (5)	0.9320 (6)	0.9307 (7)	0.9508 (2)	0.9528 (1)	0.9467 (4)	0.9477 (3)
	Average rank	5.28	5.89	4.72	2.72	3.22	3.67	2.50

Table 3. The performance comparison of recall on imbalanced datasets including with the rank which is shown in the parentheses.

No.	Datasets	CART	C4.5	OC1	AE	DCSM	ME	OMCT
1	Pima	0.5738 (4)	0.5790 (2)	0.5765 (3)	0.5392 (7)	0.5560 (6)	0.5846 (1)	0.5653 (5)
2	Wine	0.9522 (4)	0.9447 (6)	0.9416 (7)	0.9584 (1)	0.9502 (5)	0.9558 (2)	0.9547 (3)
3	Haberman	0.3509 (5)	0.3530 (4)	0.3415 (6)	0.3163 (7)	0.3613 (3)	0.3630 (2)	0.3741 (1)
4	Vehicle	0.9175 (3)	0.8995 (7)	0.9055 (6)	0.9110 (5)	0.9220 (1)	0.9193 (2)	0.9166 (4)
5	Glass	0.8113 (7)	0.8609 (1)	0.8235 (5)	0.8162 (6)	0.8425 (2)	0.8395 (3)	0.8355 (4)
6	Shuttle	0.9997 (6)	0.9997 (4)	0.9997 (5)	0.9997 (1)	0.9992 (7)	0.9997 (3)	0.9997 (2)
7	Yeast	0.6530 (3)	0.6372 (7)	0.6517 (4)	0.6382 (6)	0.6435 (5)	0.6586 (1)	0.6537 (2)
8	Libras	0.6866 (3)	0.6247 (7)	0.6995 (2)	0.6377 (6)	0.6531 (5)	0.6760 (4)	0.7213 (1)
9	NewThyroid	0.8886 (2)	0.8543 (7)	0.8800 (4)	0.8857 (3)	0.9314 (1)	0.8800 (4)	0.8800 (4)
10	SatImage	0.8721 (5)	0.8561 (7)	0.8718 (6)	0.8836 (2)	0.8818 (3)	0.8779 (4)	0.8845 (1)
11	Ecoli	0.5943 (2)	0.5771 (5)	0.5829 (4)	0.5400 (6)	0.5114 (7)	0.5914 (3)	0.6086 (1)
12	OpticDigits	0.8993 (6)	0.8817 (7)	0.9389 (2)	0.9121 (4)	0.9011 (5)	0.9174 (3)	0.9484 (1)
13	Abalone	0.3583 (1)	0.3214 (5)	0.3042 (7)	0.3106 (6)	0.3286 (3)	0.3531 (2)	0.3261 (4)
14	Landsat	0.5596 (3)	0.5321 (7)	0.5508 (5)	0.5458 (6)	0.5592 (4)	0.5757 (1)	0.5636 (2)
15	PenDigits	0.9395 (5)	0.9264 (7)	0.9482 (2)	0.9428 (4)	0.9324 (6)	0.9464 (3)	0.9564 (1)
16	Vowel	0.9344 (4)	0.9611 (1)	0.9300 (5)	0.9133 (6)	0.8744 (7)	0.9400 (3)	0.9522 (2)
17	PageBlocks	0.8417 (5)	0.8496 (3)	0.8559 (2)	0.8332 (7)	0.8338 (6)	0.8432 (4)	0.8620 (1)
18	Letter	0.9441 (3)	0.9402 (4)	0.9283 (7)	0.9492 (2)	0.9380 (5)	0.9540 (1)	0.9346 (6)
	Average rank	3.94	5.06	4.56	4.72	4.50	2.56	2.50

balanced datasets from UCI. It shows the precision improvement in classifying the minority instances over the traditional decision tree algorithms. For the recall, it fixes the overfitting problem found in the decision tree algorithms for class imbalanced datasets. In addition, OMCT offers the smallest size of the decision tree in term of the number of leaf nodes which implies that the decision tree from OMCT is more general than other

Table 4. The performance comparison of F1 score on imbalanced datasets including with the rank which is shown in the parentheses.

No.	Datasets	CART	C4.5	OC1	AE	DCSM	ME	OMCT
1	Pima	0.5701 (5)	0.5732 (2)	0.5707 (4)	0.5545 (7)	0.5618 (6)	0.5745 (1)	0.5715 (3)
2	Wine	0.9371 (4)	0.9330 (7)	0.9364 (6)	0.9610 (1)	0.9371 (5)	0.9483 (3)	0.9490 (2)
3	Haberman	0.3390 (5)	0.3510 (4)	0.3375 (6)	0.3281 (7)	0.3600 (2)	0.3537 (3)	0.3662 (1)
4	Vehicle	0.9164 (4)	0.9004 (7)	0.9073 (6)	0.9149 (5)	0.9282 (1)	0.9201 (2)	0.9172 (3)
5	Glass	0.8247 (7)	0.8586 (1)	0.8377 (3)	0.8371 (4)	0.8480 (2)	0.8292 (6)	0.8346 (5)
6	Shuttle	0.9997 (5)	0.9996 (6)	0.9997 (4)	0.9997 (2)	0.9996 (7)	0.9997 (3)	0.9998 (1)
7	Yeast	0.6434 (6)	0.6257 (7)	0.6525 (4)	0.6543 (3)	0.6436 (5)	0.6553 (2)	0.6556 (1)
8	Libras	0.7127 (4)	0.6143 (7)	0.7281 (2)	0.7042 (6)	0.7057 (5)	0.7187 (3)	0.7329 (1)
9	NewThyroid	0.8842 (6)	0.8606 (7)	0.8889 (5)	0.8996 (2)	0.9363 (1)	0.8947 (4)	0.8989 (3)
10	SatImage	0.8816 (3)	0.8578 (7)	0.8754 (6)	0.8880 (2)	0.8790 (4)	0.8762 (5)	0.8881 (1)
11	Ecoli	0.5648 (4)	0.5477 (5)	0.5774 (3)	0.5360 (6)	0.5167 (7)	0.5837 (2)	0.5964 (1)
12	OpticDigits	0.9066 (6)	0.8731 (7)	0.9353 (2)	0.9177 (4)	0.9087 (5)	0.9230 (3)	0.9495 (1)
13	Abalone	0.3348 (3)	0.3031 (6)	0.2937 (7)	0.3054 (5)	0.3242 (4)	0.3391 (2)	0.3422 (1)
14	Landsat	0.5471 (5)	0.5182 (7)	0.5460 (6)	0.5547 (4)	0.5677 (1)	0.5642 (2)	0.5611 (3)
15	PenDigits	0.9419 (6)	0.9285 (7)	0.9480 (4)	0.9531 (3)	0.9453 (5)	0.9538 (2)	0.9626 (1)
16	Vowel	0.9215 (4)	0.9410 (1)	0.9157 (5)	0.8991 (7)	0.9022 (6)	0.9348 (3)	0.9363 (2)
17	PageBlocks	0.8515 (6)	0.8549 (4)	0.8651 (2)	0.8539 (5)	0.8485 (7)	0.8593 (3)	0.8715 (1)
18	Letter	0.9398 (5)	0.9358 (6)	0.9293 (7)	0.9497 (2)	0.9451 (3)	0.9502 (1)	0.9409 (4)
	Average rank	4.89	5.44	4.56	4.17	4.22	2.78	1.94

Table 5. The comparison of tree size on imbalanced datasets including with the rank which is shown in the parentheses.

No.	Datasets	CART	C4.5	OC1	AE	DCSM	ME	OMCT
1	Pima	110.64 (4)	146.62 (7)	83.98 (2)	125.12 (5)	145.10 (6)	104.48 (3)	75.02 (1)
2	Wine	4.56 (6)	4.56 (6)	4.32 (5)	3.70 (1)	4.10 (4)	3.96 (3)	3.78 (2)
3	Haberman	79.68 (3)	87.22 (7)	58.14 (2)	81.78 (5)	86.52 (6)	79.74 (4)	54.22 (1)
4	Vehicle	25.62 (5)	34.34 (7)	25.26 (4)	27.08 (6)	23.68 (3)	22.10 (2)	18.62 (1)
5	Glass	10.10 (5)	10.48 (6)	9.74 (3)	10.84 (7)	9.86 (4)	9.60 (2)	9.28 (1)
6	Shuttle	22.14 (5)	29.02 (7)	22.24 (6)	19.34 (4)	16.20 (2)	16.60 (3)	16.10 (1)
7	Yeast	132.56 (4)	155.34 (6)	109.02 (2)	142.96 (5)	167.24 (7)	124.44 (3)	99.94 (1)
8	Libras	21.48 (5)	38.44 (7)	20.90 (4)	21.70 (6)	18.84 (3)	17.58 (2)	16.14 (1)
9	NewThyroid	6.52 (5)	8.30 (7)	6.36 (4)	7.14 (6)	5.20 (1)	5.96 (3)	5.62 (2)
10	SatImage	54.46 (5)	70.26 (7)	51.26 (3)	55.82 (6)	53.18 (4)	48.54 (2)	40.82 (1)
11	Ecoli	20.12 (4)	22.42 (6)	17.36 (2)	21.12 (5)	23.42 (7)	18.18 (3)	16.02 (1)
12	OpticDigits	65.58 (5)	101.86 (7)	19.74 (2)	55.24 (4)	73.58 (6)	48.34 (3)	13.76 (1)
13	Abalone	36.14 (4)	43.16 (6)	33.48 (3)	36.76 (5)	44.90 (7)	33.08 (2)	29.86 (1)
14	Landsat	252.50 (4)	368.84 (7)	204.74 (2)	267.54 (5)	322.00 (6)	218.50 (3)	171.32 (1)
15	PenDigits	95.26 (6)	123.46 (7)	51.76 (2)	73.12 (4)	92.22 (5)	66.62 (3)	31.48 (1)
16	Vowel	13.40 (6)	13.26 (5)	12.02 (3)	16.18 (7)	13.16 (4)	9.58 (2)	9.32 (1)
17	PageBlocks	67.56 (4)	77.54 (6)	48.80 (2)	69.56 (5)	86.34 (7)	61.72 (3)	44.16 (1)
18	Letter	87.30 (5)	106.30 (6)	68.96 (2)	80.84 (4)	117.76 (7)	71.26 (3)	48.88 (1)
	Average rank	4.72	6.50	2.94	5.00	4.94	2.72	1.11

methods.

For some imbalance datasets such as Glass and Letter, OMCT has lower rank than other decision tree algorithms. It is conceivable that the local best split at the current step may not lead to the optimal decision tree [41]. Inducing the tree with an imperfect split at the beginning may result in the lower misclassified tree at the end. Furthermore, the use of oblique hyperplane may not handle the minority instances having spherical distribution perfectly since any oblique hyperplanes can be used to split this dataset.

Due to the complexity of finding the best oblique hyperplane, the algorithms for constructing OMCT takes long time to converge. Some improvements are needed for handling this situation. Moreover, the multiclass imbalanced problem with the nominal attributes should be investigated using OMCT technique.

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Appendix

Algorithm 1 OMCT (D,C,J)

Require: Dataset $D = \{\vec{x}_1, \vec{x}_2, ..., \vec{x}_n\}$ with respect to the set of classes $C = \{c_1, c_2, ..., c_n\}$ and the number of using randomization J.

1: Creating a node of the tree.
if All instances are in the same class. **then**
  return The node labeled as that class.
end if
2: $H : a_0 + \vec{a} \cdot \vec{x} = 0 =$ ObliqueHyperplane (D,C,J)
3: $D_l = \emptyset$
4: $C_l = \emptyset$
5: $D_r = \emptyset$
6: $C_r = \emptyset$
for $i = 1, ..., n$ **do**
  if $a_0 + \vec{a} \cdot \vec{x}_i \leq 0$ **then**
    7: $D_l = D_l \cup \{\vec{x}_i\}$
    8: $C_l = C_l \cup \{c_i\}$
  else
    9: $D_r = D_r \cup \{\vec{x}_i\}$
    10: $C_r = C_r \cup \{c_i\}$
  end if
end for
11: OMCT (D_l,C_l,J)
12: OMCT (D_r,C_r,J)

Algorithm 2 ObliqueHyperplane (D,C,J)

Require: The set of instances D with respect to the set of classes C and the number of using randomization J.

1: $H, I =$ AxisParallelHyperplane (D,C)
2: $H, I =$ DeterministicHillClimbing (D,C,H,I)
3: $count = 1$
while $count \leq 5$ **do**
  4: $H', I' =$ Randomization (D,C,H,I)
  if $I' < I$ **then**
    5: $I = I'$
    6: $H = H'$
    7: $H, I =$ DeterministicHillClimbing (D,C,H,I)
    8: $count = 0$
  end if
  9: $count += 1$
end while
return H

Algorithm 3 AxisParallelHyperplane (D,C)

Require: The set of instances D with respect to the set of classes C.

1: $SelectedAttr$ = None

2: $SelectedValue$ = None

3: $I = 1$

for $j = 1, ..., d$ **do**

  4: $D_j = \{x_j^i \mid i = 1, ..., n\}$

  5: MC_{D_j} = MinorityCondensation(D_j,C)

  6: Let $BestSplit \in MC_{D_j}$ be the best splitting value for attribute j obtaining from the greedy approach on MC_{D_j}.

  7: Let I' be the Shannon's entropy of partitioning D with $BestSplit$ for attribute j.

  if $I' < I$ **then**

    8: $I = I'$

    9: $SelectedAttr = j$

    10: $SelectedValue = BestSplit$

  end if

end for

11: Defining hyperplane $H : a_0 + \vec{a} \cdot \vec{x} = 0$, where all coefficients of $\vec{a}$ are 0, except for $SelectedAttr$th element is 1, and $a_0 = -SelectedValue$.

return H, I

Algorithm 4 DeterministicHillClimbing (D,C,H,I)

Require: The set of instances D with respect to the set of classes C, the hyperplane H and the impurity I.

for $j = 1, ..., d$ **do**

  1: Computing U_j using (3)

  2: MC_U = MinorityCondensation(U_j,C)

  3: Let $BestSplit \in MC_U$ be the best splitting value of MC_U obtaining from the greedy approach on MC_U.

  4: Let I' be the Shannon's entropy of partitioning MC_U with $BestSplit$.

  if $I' < I$ **then**

    5: $I = I'$

    6: $a_j = BestSplit$

  end if

end for

return H, I

Algorithm 5 Randomization (D,C,H)

Require: The set of instances D with respect to the set of classes C and the hyperplane H.

1: Randomly hyperplane $R : r_0 + \vec{a} \cdot \vec{r} = 0$

2: Computing V using (4)

3: MC_V = MinorityCondensation(V,C)

4: Let $\alpha \in MC_V$ be the best splitting value of MC_V obtaining from the greedy approach on MC_V.

5: Let I' be the Shannon's entropy of partitioning MC_V with α.

6: Defining hyperplane $H' : H + \alpha R$

return H', I'

Algorithm 6 MinorityCondensation (X,C)

Require: The set of score $X = \{x_1, x_2, ..., x_k\}$ with respect to the set of classes $C = \{c_1, c_2, ..., c_k\}$ where $c_i \in \{+1, -1\}$.

$X^+ = \{x_i \in D \mid c_i = +1\}$

Let $MinPos$ be the minimum value of X^+.

Let Q_1Pos be the first quartile of X^+.

Let Q_3Pos be the third quartile of X^+.

Let $MaxPos$ be the maximum value of X^+.

$IQR = Q3Pos - Q1Pos$

$X^+_{wo} = \{x_i \in X^+ \mid Q1Pos - 1.5 * IQR \leq x_i \leq Q3Pos + 1.5 * IQR\}$ (Definition 1)

Let m_l be the minimum value of X^+_{wo}.

Let m_r be the maximum value of X^+_{wo}.

$MC_X = \{x_i \in X \mid m_l \leq x_i \leq m_r\}$ (Definition 2)

return MC_X

Artit Sagoolmuang received the B.Sc. degrees (first-class honours) in Mathematics from Kasetsart University, Bangkok, Thailand, in 2014 and the M.S. degree in Applied Mathematics and Computational Science from Chulalongkorn University, Bangkok, Thailand, in 2016. Since 2017, he has been a Ph.D. candidate in Applied Mathematics and Computational Science program at Chulalongkorn University. His research interests include data mining and machine Learning algorithm especially in handling the class imbalanced problem.

Krung Sinapiromsaran received his B.S. in Mathematics from Chulalongkorn University, his M.S. and Ph.D. in Computer Science from the University of Wisconsin-Madison. He is currently an Assistant Professor in the Department of Mathematics, Chulalongkorn University. His ongoing research works are related to deep learning, machine learning, artificial intelligence, data mining, knowledge discovery, and optimization.