# The Focusing Effect in Negotiations* 

Andrea Canidio ${ }^{\dagger}$ and Heiko Karle ${ }^{\ddagger}$

This Version: September 2, 2020


#### Abstract

Two players with preferences distorted by the focusing effect (Kőszegi and Szeidl, 2013) negotiate an agreement. Our main result it that, as long as their preferences are differentially distorted, an issue will be inefficiently left out of the agreement or inefficiently included in the agreement whenever the importance of the other issues on the table is sufficiently large. In extreme cases, this could lead to an inefficient breakdown of the negotiation. Anticipating this possibility, the negotiating parties may negotiate in stages, by first signing an incomplete agreement and later finalizing the outcome of the negotiation. As in Raiffa (1982), these incomplete agreements may impose bounds on some dimensions of the bargaining solution in order to reduce their salience.


JEL classification: C78, D03, D86, F51.
Keywords: Salience, Focusing Effect, Bargaining, Negotiations, Incomplete Agreements.

[^0]
## 1 Introduction

The literature on decision theory and behavioral economics has long argued that preferences are context dependent, that is, a person's preference ranking may depend on the available consumption possibilities. A well understood implication is that a person's choices may be suboptimal when evaluated from some ex-ante view point, which implies that this person may prefer to eliminate certain options from her choice set before making her final choice. ${ }^{1}$ However, the implications of context dependence for how people negotiate joint decisions received considerable less attention and are therefore much less understood. This is despite the fact that, according to practitioners, behavioral and psychological elements play a key role in determining the outcome of negotiations. Motivated by this observation, in this paper we are interested in establishing whether two bargaining parties with context-dependent preferences may reach a bargaining outcome that is suboptimal from the ex-ante viewpoint, such as, for example, a breakdown of the negotiation. ${ }^{2}$ We are also interested in establishing whether the bargaining parties benefit from negotiating in stages, by first signing an agreement that eliminates certain options from their future bargaining set, and then finalizing the outcome of the negotiation.

We consider a specific source of context dependence, namely the focusing effect. The focusing effect (or focusing illusion) occurs whenever a person places too much importance on certain aspects of her choice set (i.e., when certain elements are more salient than others). Intuitively, an agent's attention is unconsciously and automatically drawn toward certain attributes, which are therefore overvalued when making a choice. Kőszegi and Szeidl (2013) formalize this concept by assuming that agents

[^1]maximize a focus-weighted utility
$$
\tilde{U}\left(x_{1}, x_{2}, . ., x_{n}\right)=\sum_{s=1}^{n} h_{s} u_{s}\left(x_{s}\right)
$$
where $x=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ is a given good with $n$ attributes. The focus weights $h_{s}$ are defined as:
$$
h_{s}=h\left(\max _{x \in C} u_{s}\left(x_{s}\right)-\min _{x \in C} u_{s}\left(x_{s}\right)\right)
$$
where $C$ is the choice set and $h()$ is the focusing function, assumed strictly increasing.
In this formalization, an agent overweighs the utility generated by the attributes in which her options differ more, where these differences are measured in utility terms. There is ample empirical evidence for this. For example, Schkade and Kahneman (1998) show that when asked about comparing life in California and in the Midwest, most people report California as the best place to live and cite the weather-i.e., the dimension in which the two choices differ the most-as the main reason. Despite this, actual measures of life satisfaction in the two regions are similar. Similarly, Kahneman, Krueger, Schkade, Schwarz and Stone (2006) show that people place too much weight on differences in monetary compensation when asked to compare job offers, which is the dimension in which these offers differ the most. ${ }^{3}$ The focusing effect plays an important role also in negotiations. For example, according to Raiffa (1982), the importance that the bargaining parties attach to an issue depends on the range of possible outcomes on that issue (see later for more discussion about Raiffa, 1982, and, in particular, his account of the Panama Canal negotiation).

In our model, two players negotiate over $n$ discrete issues and one continuous issue which we interpret as a transfer. In the next subsection (Subsection 1.1), we provide an illustrative example considering a peace negotiation between a government and a rebel group, but the model is sufficiently general to encompass buyer-seller negotiations as well as complex international negotiations. Both players' preferences are

[^2]distorted by the focusing effect, but not necessarily in the same way: we allow each player to have his own focus function, so that one player could be "more focused" than the other. The players' focus weights are determined by their focus function and the consideration set: the set of bargaining outcomes that are considered possible. We assume that the consideration set contains all bargaining outcomes that satisfy the participation constraint of at least one player. This way, extreme, unreasonable bargaining outcomes are not considered possible by the players, and hence do not affect their preferences. This assumption implies that the players' participation constraints determine the players' focus weights, while, at the same time, the focus weights determine the players' preferences and their participation constraints. Hence, solving for the focus weights is a fixed-point problem for which we show existence and uniqueness. Finally, we assume that the bargaining parties achieve a bargaining solution satisfying Nash (1953)'s axioms for given focus weights. As we will see, this implies that irrelevant alternatives affect the bargaining solution exclusively via the players' preferences, and not via the specific bargaining solution chosen. ${ }^{4}$

We derive conditions under which the outcome of the negotiation is materially inefficient relative to a rational benchmark in which all focus weights are equal to 1. A first necessary condition for an inefficient outcome to occur is that the players' preferences are differentially distorted by the focusing effect, i.e., the two players' focusing functions are not identical. A second necessary condition is that there is more than one issue on the negotiating table. When these two conditions are satisfied, inefficiencies arise if the issues under consideration are uneven in their importance. More precisely, a given issue will be inefficiently included into the agreement or inefficiently excluded from the agreement whenever the importance of this issue is sufficiently small relatively to the other issues on the table. This could even lead to a total breakdown of the negotiation.

Intuitively, the player whose preferences are more strongly distorted by the fo-

[^3]cusing effect is also more focused on the transfer dimension than the opponent. This player therefore dislikes making a transfer more than the opponent enjoys receiving that transfer. The difference in how the two players evaluate a transfer increases when the importance of an issue increases. When this difference is sufficiently large, then an issue may inefficiently be excluded from (resp. included into) the agreement if the more focused player should make (resp. should receive) the additional transfer required to include that issue. ${ }^{5}$ Hence, unlike most behavioral models, here the emergence of inefficiencies does not depend on the distortion of the players' preferences, but rather on their relative distortion.

We then allow the players to negotiate in stages. In the first stage, the players can sign an incomplete agreement that imposes a constraint on the future bargaining set. If they fail to find an agreement at this stage, they will move to a negotiation over the entire bargaining set. If, instead, they agree on an incomplete agreement, in the following period, they finalize the negotiation by choosing an outcome from the constrained bargaining set. At this stage of the negotiation, the players may wish to eliminate an issue that would otherwise be inefficiently included in the final agreement, or to include an issue that would otherwise be inefficiently excluded from the final agreement. In the first case, the players can easily achieve their goal by agreeing ex-ante that such issue will not be on the table in the future. In the second case, however, simply agreeing that a given issue will be included in the future agreement may not work. The reason is that in the last stage of the negotiation the players can disagree - and hence, quite trivially, not include any issue in the agreement. It follows that such commitment constraints the players future options without affecting their future preferences. Alternatively, the players may achieve the inclusion of an issue by reducing the salience of the transfer dimension. One way to do so is to eliminate from the bargaining set redundant issues, that is, issues over which the players do not expect to reach an agreement. To reduce the salience of the transfer dimension further, the players may also impose bounds on the transfer dimension. Imposing such bounds, however, generates a cost because they may

[^4]be binding in the future and hence prevent the players from implementing their preferred transfer. We provide a numerical example in which such bounds are used in equilibrium.

In our model, the one-step negotiation is the outside option of the negotiation over incomplete agreements. Hence, if the players agree to impose an incomplete agreement, then they both must be better off relative to the outcome of the one step negotiation. If their preferences are rational when negotiating the incomplete agreement, this immediately implies that if an incomplete agreements is used in equilibrium, then efficiency is strictly larger relative to the one-step negotiation. However, the negotiation over incomplete agreements is a Non Transferrable Utility (NTU) problem, because the players are not allowed to make transfers at this stage. Hence, there is no presumption that the players will always use an incomplete agreement to increase the efficiency of the negotiation when it is feasible to do so. Interestingly, we show that these results hold also when the players' preferences are distorted by the focusing effect while negotiating the incomplete agreement. The reason is that the set of bargaining outcomes achievable by imposing a specific incomplete agreement is, in general, strictly smaller than the full bargaining set. Hence, when negotiating the incomplete agreement the players' preferences are less distorted than in the one-step negotiation. Also in this case, the possibility of imposing an incomplete agreement is welfare improving.

In our model, therefore, the players may use incomplete agreements in order to reduce the salience of certain dimensions of the problem. This mechanism is well understood by practitioners, who believe that in negotiations "the importance of an issue might be lessened by the parties first narrowing the range of possible outcomes on that issue" (Raiffa, 1982, "The art and science of negotiation" p. 216). ${ }^{6}$ A textbook example for this negotiating procedure is the use of so-called "threshold agreements" during the Panama Canal negotiations between the Panamanian government and the US government. The threshold agreements guaranteed to the Panamanian government the achievement of minimum outcomes on three different issues, and were initiated by the American delegation to avoid the break-off of the

[^5]negotiation. ${ }^{7}$
Incomplete agreements are widely used, well beyond thresholds agreements. A case in point is the use of framework agreements in international negotiations and in procurement. In international negotiations, each round of negotiation is concluded by an agreement, which is not final but provides the framework for a later round of negotiations. ${ }^{8}$ In procurement, a framework agreement may define, for example, a set of prices and quality levels of a possible future transaction, with the understanding that the details of this transaction will be established in a future agreement. Our theory provides a possible explanation to why these incomplete agreements are used, especially when they impose bounds, reduce the number of issues on the table, or specify certain provisions of the final agreement. The main novelty of our theory relative to the existing literature on incomplete contracts (which we discuss in details at the end of this section) is that it applies to environments with no uncertainty. It, therefore, provides a rationale for scheduling different bargaining rounds independently from the arrival of new information.

We structure the paper in the following way. In the remainder of this section, we provide an illustrative example and then discuss the relevant literature. In Section 2 we introduce the model. In Section 3 we solve for the one-step negotiation. In Section 4 we introduce an additional bargaining round. In Section 5 we show that our results hold also when preferences are distorted by the focusing effect already when negotiating the incomplete agreement. In Section 6 we discuss a number of extensions, including the possibility that a previous incomplete agreement may be ignored, and other forms of context-dependent preferences different from the focusing

[^6]effect. The last section concludes. Unless otherwise noted all proofs are relegated to the appendix.

### 1.1 Illustrative example

For intuition, consider a peace negotiation between a government and a rebel group seeking independence for a region. Suppose that the parties reach an agreement in which the rebel group accepts to disband while the government makes a significant monetary transfer to that region, and that this agreement is materially efficient. Consider now the same negotiation with an additional issue on the table: whether the rebel group accepts stricter controls by the central government on the regional government (for example, by having the regional police force controlled by the central government). For both players, this second issue is very important - in the sense of generating a large benefit for the government and a large cost for the rebel group if included. Hence, it is possible to include this issue in the peace agreement only by significantly increasing the transfer made by the government. For the sake of the argument, assume that agreeing on this second issue is inefficient, because the cost for the rebel group is larger than the benefit for the government.

If the players' preferences are context dependent, then their preferences with respect to the first issue (that is, whether to disband) may change with the introduction of the second issue. Furthermore, if their preferences are context dependent because of the focusing effect, the possibility of including the second issue and making a very large transfer drives both players' focus towards this dimension of the negotiation. Suppose the rebel group is less focused than the government (which may be under pressure due to an upcoming election). In this case, the presence of the second issue makes the government more sensitive to the transfer dimension than the rebel group. It is therefore possible that there is a breakdown of the negotiation and no peace agreement is signed. If instead the rebel group is more focused than the government, then the rebel group overweighs receiving a transfer (relative to the government) and the players may sign a peace agreement including both issues.

Finally, government and the rebel group may benefit from signing an incomplete
agreement. If the rebel group is less focused than the government, than both players are better off by agreeing beforehand that they will only discuss the first issue - and therefore achieve an efficient peace agreement. In this case, incomplete agreements are used in equilibrium and they improve the efficiency of the negotiation. If instead the government is less focused than the rebel group, then the players may fail to sign an incomplete agreement. This happens whenever, by not imposing any restriction on the future bargaining set, the government obtains its most preferred outcome: an agreement on both issues while making a relatively low transfer. This is an example of a situation in which the players fail to improve the efficiency of a negotiation via an incomplete agreement.

### 1.2 Relevant Literature

A few studies have examined the impact of various behavioral biases in negotiations. For example, Bénabou and Tirole (2009) consider a bargaining model with selfserving beliefs and show that inefficient negotiation breakdown may occur. Inefficient breaksdowns and delays are also possible in models with heterogeneous beliefs (such as Yildiz, 2004 and Nageeb Ali, 2006). Inefficient delays also arise in Compte and Jehiel (2003), who introduce reference-dependent utility in a game of alternating offers. Because we consider a different behavioral mechanism, the conditions under which inefficiencies arise are different in our paper than in the papers mentioned above. Furthermore, novel with respect to the literature, we consider multiple issues and hence the possibility that issues are inefficiently included into or excluded from the final agreement. Also related is Shalev (2002), who considers loss aversion and shows that, similarly to risk aversion, loss aversion worsen the bargaining outcome of a player. Here instead we are interested in the welfare consequence of the focusing effect, and not so much in the way surplus is shared. However, we are close to Shalev (2002) methodologically because he solves the model cooperatively and then argues that the same solution can be implemented non-cooperatively.

The literature on incomplete contracts has long argued that behavioral biases and cognitive limitations may explain why agreements are often incomplete (see, for
example, Segal, 1999; Battigalli and Maggi, 2002; Bolton and Faure-Grimaud, 2010; Tirole, 2009; Hart and Moore, 2008; Herweg and Schmidt, 2015 and Herweg, Karle and Müller, 2018). However, to the best of our knowledge, all these explanations rely on the resolution of some uncertainty: after signing an incomplete contract, the arrival of new information makes the environment less complex, reduces the number of possible contingencies, and allows the contract to be either executed or, if renegotiation is allowed, completed in a final negotiating round. ${ }^{9}$ Instead, we consider a deterministic environment. This is justified by the observation that in many negotiations the start of a new bargaining round follows the end of the previous round and is not determined by the arrival of new information. With respect to this literature, our contribution is therefore to provide a foundation for the existence of incomplete agreements in contexts with no uncertainty. ${ }^{10}$

Despite this, in our model, when negotiating an incomplete agreement, the players face a tradeoff between the benefit of preference alignment and the cost of imposing restrictions to the bargaining set. This tradeoff is reminiscent of Hart and Moore (2008). In that paper, if a dimension of the future bargaining problem is left unspecified, each player will feel entitled to the best outcome within the set of possible outcomes in compliance with the initial contract. This feeling of entitlement will affect the final outcome because players may shade on performance. Similar to our model, Hart and Moore (2008) assume that this feeling of entitlement is not present when negotiating the initial contract. Hence, the players may use the initial contract to reduce the set of options available to their future selves, reduce the future sense of entitlement and align ex-post preferences with the ex-ante ones. These restrictions

[^7]have a cost because they may prevent the players from implementing the ex-post efficient trade.

Also, papers studying incomplete contracts assume that the players can make transfers both ex-ante (when signing the contract or allocating ownership) and expost (after the realization of the state of the world). In negotiations, instead, all utility-relevant events (including transfers) occur at the end, after the agreement is signed. In our model, therefore, we do not allow the players to make payments exante (that is, when discussing the incomplete agreement). We assume that only one agreement is possible, the final one, so that all transfers and exchanges will realize conditional on reaching an agreement during the last round of negotiation. This also distinguishes us from Esteban and Sákovics (2008), who allow the negotiating parties to sign separate agreements on different parts of the surplus. It also distinguishes us from the literature on agenda setting in negotiations, in which the players can reach several, issue-specific agreements. ${ }^{11}$

Finally, we employ here the model of focusing in economic choice proposed by Kőszegi and Szeidl (2013), in which the decision maker overweighs attributes in which her options vary the most. In their model of relative thinking, Bushong, Rabin and Schwartzstein (2017) make the opposite assumption: that a decision maker underweights attributes in which her options vary the most. Bordalo, Gennaioli and Shleifer (2013) also develop a model of salience. They assume that agents overvalue the attributes that differ the most with respect to a reference point. ${ }^{12}$ In Section 6.3 we discuss how our other results change when instead of using Kőszegi and Szeidl (2013), we use either Bushong, Rabin and Schwartzstein (2017) or Bordalo, Gennaioli and Shleifer (2013).

[^8]
## 2 The Model

Two players $a$ and $b$ are engaged in a negotiation over $i \in\{1, \ldots, n\}$ discrete issues and a continuous issue $t$. We interpret $t>0$ as a transfer $t$ from $b$ to $a$, while $t<0$ is a transfer $|t|$ from player $a$ to player $b$. If there is no agreement on issue $i$, the status quo on that issue is maintained, in which case the players' payoffs from issue $i$ are normalized to zero. If there is an agreement on issue $i$, player $a$ earns $\alpha_{i}$ while player $b$ earns $\beta_{i}$, where $\alpha_{i}$ and $\beta_{i}$ could be positive or negative real numbers. We call $q_{i} \in\{1,0\}$ the bargaining outcome with respect to issue $i$, and say that $q_{i}=1$ if there is an agreement on issue $i$, and $q_{i}=0$ if there is no agreement on issue $i$. The bargaining set is therefore $X \equiv\{1,0\} \times \ldots \times\{1,0\} \times \mathbb{R}$, with $x=\left\{q_{1}, q_{2}, \ldots, q_{n}, t\right\} \in X$ a possible bargaining outcome.

We make the following parametric assumptions:

1. there is at least an issue $i$ such that $\alpha_{i} \cdot \beta_{i}<0$.
2. there are no issues $i \leq n$ such that $\alpha_{i}<0$ and $\beta_{i}<0$.

As it will become clear later (see Equation 3.5), issues that generate costs to both players will never be included into an agreement, and issues that generate benefits to both players are always included into the final agreement. The first restriction therefore excludes from our analysis the uninteresting case in which, independently from the focusing effect, all issues are included into or excluded from the agreement. The second restriction is instead without loss of generality. The reason is that, as we will show later, issues that generate costs to both players are irrelevant in determining the shape of the consideration set (see Equations 3.1 and 3.2).

This environment nests several bargaining problems. For example, it corresponds to a standard buyer-seller negotiation whenever $\beta_{i}>0>\alpha_{i}$ for all $i \leq n$, so that all benefits accrue to player $b$ and all costs accrue to player $a$. Alternatively, we could have an international negotiation in which some benefits accrue to each player.

We allow the players to negotiate in stages. In the first stage, the players can sign an incomplete agreement that imposes a constraint on the future bargaining set, but they can also not sign any incomplete agreement. If they do not sign an
incomplete agreement, they move to a negotiation over the entire bargaining set. If they instead agree on an incomplete agreement, in the following period, they finalize the negotiation by choosing an outcome from the constrained bargaining set. Independently from what was agreed in period 1, during the last round of negotiation each player can unilaterally choose the no-agreement option. There is no time discounting. ${ }^{13}$

Definition 1 (Incomplete agreements). An incomplete agreement is a set $S \subset X$, assumed closed and bounded. A negotiation structure $\mathbb{S}$ is the collection of $S$ that can be chosen in period 1.

Note that, because there is no time discounting, a one-step negotiation is equivalent to a two-step negotiation in which the outcome of the period-1 negotiation is $S=X$. We assume that this option can be chosen unilaterally by either player, and hence constitutes the outside option of the first-stage negotiation.

Assumption 1 (Outside option). If in period 1 the parties disagree, then in period 2 they bargain over the unconstrained bargaining set $X$. If in period 2 the players disagree, then the negotiation ends with no agreement.

In the final negotiating round, players have context-dependent preferences à la Kôszegi and Szeidl (2013). Before writing these preferences, we first define the players' consideration set, that is, the set of outcomes that each player considers as possible. Kőszegi and Szeidl (2013) argue that the consideration set should be equal to the agent's choice set, with the possible exclusion of options that are dominated (that is, very bad in all attributes; see Kőszegi and Szeidl, 2013, Section 2.2, remark 2). Here, no issue is "dominated" in the sense that all issues have positive value for at least one player. Some transfers, however, may be so extreme that, if included in an agreement, would lead to a breakdown of the negotiation no matter how the other $n$ issues are resolved. The following assumption implies that those transfers are not part of the players' consideration set.

[^9]Assumption 2 (Consideration set). Consider a bargaining outcome $x=\left\{q_{1}, q_{2}, \ldots, q_{n}, t\right\} \in$ $X$ with $t>0(t<0)$. This bargaining outcome is in the consideration set if and only if it satisfies player b's (player a's) participation constraint and does not violate any prior incomplete agreement.

That is, a bargaining outcome is in the consideration set if it satisfies the participation constraint of the player making the transfer. ${ }^{14}$ Note also that the above assumption implies that prior incomplete agreements are binding. ${ }^{15}$

Having specified the players' consideration set, we can now write their utility functions. Call $h_{a}()$ player $a$ 's focus function and $h_{b}()$ player $b$ 's focus function, both assumed strictly positive and strictly increasing. The utility functions are

$$
\begin{aligned}
U^{a}(x) & =\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}(\bar{t}-\underline{t}) t \\
U^{b}(x) & =\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}(\bar{t}-\underline{t}) t
\end{aligned}
$$

where, for all $i \leq n, h_{a}\left(\left|\alpha_{i}\right|\right), h_{a}(\bar{t}-\underline{t}), h_{b}\left(\left|\beta_{i}\right|\right)$, and $h_{b}(\bar{t}-\underline{t})$ are the players' focus weights. The focus weight on a given issue depends on the utility generated by including that issue in the final agreement. The focus weight on the transfer dimension depends on $\bar{t} \geq 0$ and $\underline{t} \leq 0$, which are the largest and smallest transfer from player $b$ to player $a$ in the consideration set. The focusing effect causes each player to focus more on, and hence to overweigh, the dimension of the bargaining problem with the largest difference in terms of possible bargaining outcomes. Finally, the focus-

[^10]weighted utility is a decision utility, because it describes the decision maker's choice. We will contrast players' decision utility with their material utility corresponding to a rational benchmark in which all the focus weights are equal to one.

For ease of notation, we define the focus wedge as

$$
\Delta(x) \equiv \frac{h_{a}(|x|)}{h_{b}(|x|)}
$$

which measures the distortion in player $a$ 's preferences relative to that of player $b$. In what follows, we will mostly be concerned with three cases:

- $\Delta(x)=1$ for all $x$, which implies that $h_{a}()=h_{b}()$. That is: the players' preferences are equally distorted by the focusing effect. We also say that the players are "equally focused."
- $\Delta(x) \geq 1$ and strictly increasing for all $x>0$. That is: player $a$ is always "more focused" than player $b$, the more so the larger is $x$.
- $\Delta(x) \leq 1$ and strictly decreasing for all $x>0$. That is: player $b$ is always "more focused" than player $a$, the more so the larger is $x$.

With respect to the players' preferences in period 1, for ease of exposition, we start by assuming that they are rational, that is, they do not depend on the set of possible options in period 1 (see Section 4). We then assume that the players' preferences are distorted by the focusing effect in period 1 as well (see Section 5).

We conclude the exposition of the model by introducing our last assumption:
Assumption 3 (Bargaining solution). Each bargaining round is solved by Nash bargaining.

That is, we assume that, within each bargaining round and for given preferences, the players will achieve an agreement that is invariant to affine transformations, Pareto optimal, symmetric and satisfies independence of irrelevant alternatives (see Nash, 1953). Of course, whereas in Nash (1953) the players' preferences are one of the primitives of the model, here instead they are endogenous and depend on
the consideration set. Once the endogeneity of preferences is taken into account, irrelevant alternatives might affect the bargaining solution.

This assumption allows us to focus on the sequence of agreements signed by the players, without explicitly modeling the sequence of offers and counteroffers within each negotiating round. Our results however easily extend to the case in which the negotiation is modeled as a non-cooperative game. This is immediate for the case of the variable threat game in Nash (1953). For the case of bargaining games of alternating offers à la Rubinstein (1982), note that the benefit and cost of agreeing on a given issue are the same in every period of the negotiation conditionally on reaching such period (that is because the disutility of waiting is, at that point, sunk). Therefore, if the consideration set is defined as in Assumption 2, the consideration set and the players' preferences do not change over time. This implies that the solution to the game of alternating offers is again the Nash bargaining solution. Finally, the period-1 bargaining game involves a positive outside option and non-transferable utility, and hence the non-cooperative implementation Nash bargaining solution is somewhat non standard. ${ }^{16}$ However, as we will see, our main results with respect to period 1 (namely Proposition 3 and 4) only rely on the fact that the agreement should be preferred by both players to their outside options, which is always true independently of the specific model of bargaining.

## 3 One-step negotiation

Suppose that no agreement was imposed by the players in period 1, that is, the players bargain in one step. By Assumption 2, in the one-step negotiation, the bounds on the transfer dimension are given by

$$
\begin{equation*}
\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}=-h_{a}(\bar{t}-\underline{t}) \underline{t} \tag{3.1}
\end{equation*}
$$

[^11]\[

$$
\begin{equation*}
\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}=h_{b}(\bar{t}-\underline{t}) \bar{t} . \tag{3.2}
\end{equation*}
$$

\]

The following lemma shows the existence of $\bar{t}$ and $\underline{t}$, and establishes an important preliminary result: absent a prior incomplete agreement, the salience of the transfer dimension is increasing in the benefit generated by the issues under consideration and in the number of issues under consideration.

Lemma 1. $\bar{t}$ and $\underline{t}$ exist and are unique. Furthermore:

- $\forall i \leq n, \bar{t}-\underline{t}$ is strictly increasing in $\alpha_{i}$ if $\alpha_{i}>0$, and constant in $\alpha_{i}$ otherwise,
- $\forall i \leq n, \bar{t}-\underline{t}$ is strictly increasing in $\beta_{i}$ if $\beta_{i}>0$ and constant in $\beta_{i}$ otherwise,
- $\bar{t}-\underline{t}$ is strictly increasing in $n$.

It follows that the Nash bargaining problem is: ${ }^{17}$

$$
\begin{equation*}
\max _{q_{1}, \ldots q_{n}, t}\left(\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}(\bar{t}-\underline{t}) t\right)\left(\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}(\bar{t}-\underline{t}) t\right) . \tag{3.3}
\end{equation*}
$$

Call the solution to the above problem $\left\{q_{1}^{*}, \ldots q_{n}^{*}, t^{*}\right\}$. Because of the focusing effect, we have that a given issue $i$ is included in the agreement (and hence $q_{i}^{*}=1$ ) if and only if the focus-weighted surplus generated by this issue is positive, that is

$$
\begin{equation*}
\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}+\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}>0 \tag{3.4}
\end{equation*}
$$

which can be rewritten as

$$
\begin{equation*}
\frac{\Delta\left(\alpha_{i}\right)}{\Delta(\bar{t}-\underline{t})} \alpha_{i}+\frac{h_{b}\left(\left|\beta_{i}\right|\right)}{h_{b}\left(\left|\alpha_{i}\right|\right)} \beta_{i}>0 \tag{3.5}
\end{equation*}
$$

[^12]Given this, we have:

$$
\begin{equation*}
t^{*}=\frac{1}{2} \sum_{i=1}^{n} q_{i}^{*} \cdot\left(\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}-\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}\right) . \tag{3.6}
\end{equation*}
$$

We first derive conditions under which the outcome of the negotiation is efficient.
Lemma 2. If $n=1$, then the outcome of the negotiation is efficient.
Hence, a necessary condition for the outcome to be inefficient is that there are at least 2 issues on the table. This is similar to what is found by Kőszegi and Szeidl (2013) (cf. their rationality in balanced trade-offs result in Proposition 3, applied to two-attribute choices).

Lemma 3. If the players are equally focused $(\Delta(x) \equiv 1)$, then the negotiating outcome is materially efficient. That is, an issue is included in the final agreement if and only if $\alpha_{i}+\beta_{i}>0$.

Quite interestingly, therefore, in this environment inefficiencies emerge not because players have preferences that are distorted by the focusing effect, but rather because players have preferences that are differentially distorted by the focusing effect. The next lemma shows that this is indeed the case.

Lemma 4. Assume $n \geq 2$, and consider an issue $i$ with $\alpha_{i} \cdot \beta_{i}<0$. Suppose the player who benefits from including issue $i$ in the agreement (i.e. who should make the additional transfer required to include that issue) is more focused than the other player:

- If $\alpha_{i}+\beta_{i}>0$ then issue $i$ is inefficiently excluded from the agreement for large $\bar{t}-\underline{t}$ but is efficiently included into the agreement for small $\bar{t}-\underline{t}$.
- If instead $\alpha_{i}+\beta_{i}<0$, then issue $i$ is always efficiently excluded from the agreement.

Suppose the player who bears the cost of including issue $i$ in the agreement (i.e. who would receive the additional transfer required to include that issue) is more focused than the other player:

- If $\alpha_{i}+\beta_{i}>0$, then issue $i$ is always efficiently included into the agreement.
- If instead $\alpha_{i}+\beta_{i}<0$, then issue $i$ is efficiently excluded from the agreement for small $\bar{t}-\underline{t}$ but is inefficiently included into the agreement for large $\bar{t}-\underline{t}$.

To say it more succinctly, if the player who should make the additional transfer required to include an issue in the agreement is more focused than the other, then it is possible that such issue is inefficiently excluded from the agreement. If instead the player who would receive the additional transfer required to include an issue in the agreement is more focused than the other, then it is possible that such issue is inefficiently included into the agreement. The inefficient outcome is more likely to occur whenever the salience of the transfer dimension is large. Intuitively, the willingness of the players to include an additional issue in the agreement depends on two elements: the perceived utility of receiving a transfer relative to the perceived disutility of making a transfer. Both of these elements increase when the salience of the transfer dimension increases. However, depending on who is more focused, one will grow faster than the other.

An important corollary of the above lemma is that when the salience of the transfer dimension is reduced, the players are more likely to behave as rational.

Corollary 1. Suppose that, following a reduction in $\bar{t}-\underline{t}$, the focus-weighted surplus on issue $i$ goes from positive to negative (from negative to positive). Then it must be that $\alpha_{i}+\beta_{i}<0\left(\alpha_{i}+\beta_{i}>0\right)$.

That is, if the salience of the transfer dimension decreases, the players become more likely to evaluate the surplus of including an issue in the agreement in the same way as rational players would.

Of course, the salience of the transfer dimension is endogenous. The next proposition provides sufficient conditions on the primitives of the problem for an inefficient outcome to emerge. ${ }^{18}$

[^13]Proposition 1. Suppose $n \geq 2$ and consider issue $i$ with $\alpha_{i} \cdot \beta_{i}<0$. Suppose that the player who should make (would receive) the additional transfer required to include that issue in the agreement is more focused than the other player and $\alpha_{i}+\beta_{i}>0$ $\left(\alpha_{i}+\beta_{i}<0\right)$. Then the agreement will be inefficient with respect to issue $i$ whenever

- some $\beta_{j}>0$ with $j \neq i$ is sufficiently large.
- some $\alpha_{j}>0$ with $j \neq i$ is sufficiently large.
- $n$ is sufficiently large.

Hence, the outcome on a given issue may be inefficient whenever the value of agreeing on some other issue is sufficiently large, or if there are sufficiently many issues on the table. The reason is that, as the value of an unrelated issue or the number of issues increases, then the salience of the transfer dimension increases (see Lemma 1). If inefficiencies are possible for some focus-weight on transfer (see Lemma 4), then the outcome of the negotiation will be inefficient.

An important corollary of the above proposition is that the focusing effect may cause an inefficient breakdown of the negotiation. For example, suppose that all benefits accrue to a player (say player b) who is also more focused than the other player, and that it is efficient to agree on issues $k<n$ only. By the above proposition, any issue $i \leq k$ will be inefficiently left out of the agreement whenever both benefit and cost of some other issue $j \in\{k, \ldots, n\}$ are sufficiency large. This implies that, if benefit and cost of issues $\{k, \ldots, n\}$ are sufficiency large, then no issue will be included in the agreement, leading to a breakdown of the negotiation.

Another interesting observation is that fixing the benefit and costs generated by each issue, the allocation of these costs and benefits matters for the material efficiency of the negotiation.

Proposition 2. Consider a negotiation with a vector of costs and benefits $\Gamma=$ $\left\{\alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}, \ldots, \alpha_{n}, \beta_{n}\right\}$. Construct a new negotiation $\Gamma^{\prime}=\left\{\alpha_{1}^{\prime}, \beta_{1}^{\prime}, \alpha_{2}^{\prime}, \beta_{2}^{\prime}, \ldots, \alpha_{n}^{\prime}, \beta_{n}^{\prime}\right\}$ with $\alpha_{i}^{\prime}=\beta_{i}>0$ and $\beta_{i}^{\prime}=\alpha_{i}<0$ for some issue $i$, and $\alpha_{j}^{\prime}=\alpha_{j}$ and $\beta_{j}^{\prime}=\beta_{j}$ for all other issues $j \neq i$. If player $a$ is more focused than player $b$ (vice versa), then the material efficiency of the negotiation is higher (lower) under $\Gamma^{\prime}$ than under $\Gamma$.

In other words, start with a given negotiation and, for a given issue, switch benefit and cost between players, so to give more benefits to player $a$. By (3.1) and (3.2), this has an effect on the focus weight on the transfer dimension. The proposition shows that if player $a$ is more focused than player $b$ the efficiency of the negotiation increases (while the opposite holds if $b$ is more focused than $a$ ). Intuitively, the salience of a given benefit $\beta_{i}$ is larger for the more focused player. At the same time, also the salience of the transfer dimension is larger for that player. When we reallocate benefits from the less focused player to the more focused player, therefore, there are two competing forces at play: although the player receiving the benefit values it more than the other, he is also less willing to make additional transfers. The proof of the proposition shows that the second effect always dominates. Hence, reallocating benefits to the more focused player leads to an overall decrease in $\bar{t}-\underline{t}$, and, by Corollary 1 , to an increase in the efficiency of the negotiation.

Finally, an interesting implication of the above proposition is that it may be possible to rank different negotiations in terms of efficiency as a function of which player earns the benefit/bears the cost of each issue. Suppose that player $a$ is more focused than player $b$. Consider a negotiation in which player $b$ earns all benefits, so that $\beta_{i}>0$ for all $i \leq n$. As discussed earlier, this situation corresponds to a negotiation between a buyer (player $b$ ) and a seller (player $a$ ), where the seller is more focused than the buyer. Starting from this negotiation, we can reallocate some benefits from player $b$ to player $a$, this way reaching a situation resembling international negotiations in which benefits and costs accrue to both players. At the other extreme, when all benefits are reallocated to player $a$, we achieve a buyer/seller negotiation in which the buyer is more focused than the seller. The above proposition shows that the buyer-seller negotiation in which the seller is more focused achieves the lowest material efficiency, the buyer-seller negotiation in which the buyer is more focused achieves the highest material efficiency, and international negotiations achieve intermediate levels of material efficiency.

## 4 Two steps negotiation

The starting point in solving for the two step negotiation is specifying a negotiation structure (see Assumption 1.2): what type of incomplete agreements the players can $\operatorname{sign}$ in period 1. Here we restrict our attention to incomplete agreements that: ${ }^{19}$

- commit the players to include an issue in the final agreement.
- commit the players to exclude an issue from the final agreement.
- impose an upper bound $\bar{T} \geq 0$ and a lower bound $\underline{T} \leq 0$ to the transfers that can be made by the players.

We say that $Q_{i}=\{1\}$ if the players commit to include issue $i$ in the final agreement, $Q_{i}=\{0\}$ if the players commit to exclude issue $i$ from the final agreement, and $Q_{i}=\{0,1\}$ otherwise.

An important observation is that, no matter what agreed in period 1 , in period 2 the players may disagree, so that the bargaining outcome $\overline{0}=\left\{q_{1}=0, \ldots, q_{n}=0, t=\right.$ $0\}$ is always in the consideration set. Hence, agreeing to include an issue in the future agreement imposes a constraint on the future bargaining set without affecting the players' preferences. On the other hand, both agreeing to exclude an issue and imposing bounds on the transfer dimension will affect the period-2 preferences.

### 4.1 Period 2

The presence of a prior incomplete agreement affects the focus weight on the transfer dimension as follows. The largest and lowest transfer in the consideration set are now

$$
\bar{t}^{o}=\min \{\bar{T}, \tilde{t}\} \quad \underline{t}^{o}=\max \{\underline{T}, \underline{t}\}
$$

[^14]with $\tilde{t}$ and $\underset{\sim}{t}$ implicitly defined as
\[

$$
\begin{align*}
& \sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\} \mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=-h_{a}(\tilde{t}-\underset{\sim}{t}) \underset{\sim}{t}  \tag{4.7}\\
& \sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\} \mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=h_{b}(\tilde{t}-\underset{\sim}{t}) \tilde{t} \tag{4.8}
\end{align*}
$$
\]

where $\mathbb{1}\{$.$\} is the indicator function, so that \mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=0$ whenever issue $i$ was excluded via an incomplete agreement, and $\mathbb{1}\left\{Q_{i} \neq\{0\}\right\}=1$ otherwise. Hence, whenever $\bar{T} \geq \tilde{t}$ the players are effectively negotiating without an upper bound on transfers (and similarly whenever $\underline{T} \leq \underset{\sim}{t}$ ). Nevertheless, if a prior incomplete agreement removes an issue from the table, this reduces the salience of the transfer dimension (i.e. changes players' preferences) via $\tilde{t}$ or $\underset{\sim}{t}$. If instead either $\bar{T} \leq \tilde{t}$ or $\underline{T} \geq \underset{\sim}{t}$, the bounds of the consideration set are determined explicitly by the incomplete agreement signed by the players in period 1 . Importantly, however, these bounds may be binding, and therefore have a direct effect on the outcome of the negotiation (in addition to their indirect effect via the players' preferences).

The set of feasible bargaining outcomes is now $x \in Q_{1} \times \ldots \times Q_{n} \times \mathbb{R} \cup \overline{0}$ where $x=\left\{q_{1}, q_{2}, \ldots, q_{n}, t\right\}$ and $\overline{0}$ is a vector of $n+1$ zeros representing the no agreement outcome. The solution to the period-2 negotiation therefore solves

$$
\begin{equation*}
\max _{x \in Q_{1} \times Q_{2} \times \ldots Q_{n} \times \mathbb{R} \cup \bar{o}}\left(\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}\left(\bar{t}^{o}-\underline{t}^{o}\right) t\right)\left(\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}\left(\bar{t}^{o}-\underline{t}^{o}\right) t\right), \tag{4.9}
\end{equation*}
$$

For a given $x^{*}=\left\{q_{1}^{*}, \ldots, q_{n}^{*}, t^{*}\right\} \neq \overline{0}$ solution to the above problem, it must be that

$$
\begin{equation*}
t^{*}=\max \left\{\min \left\{\frac{1}{2} \sum_{i=1}^{n} q_{i}^{*} \cdot\left(\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}\left(\overline{t^{o}}-\underline{t}^{o}\right)}-\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}\left(\overline{t^{o}}-\underline{t}^{o}\right)}\right), \bar{T}\right\}, \underline{T}\right\} \tag{4.10}
\end{equation*}
$$

An important observation is that, for any issue $i$ with $Q_{i}=\{0,1\}$, also here
this issue will be included only if it generates positive focus-weighted surplus (as in condition (3.5)). Here, however, this condition is not sufficient to determine the number of issues included in the agreement. The reason is that the constraints on the transfer dimension may be binding and therefore affecting the ability of a player to compensate the other for the inclusion of an additional issue in the agreement, even if this issue generates positive focus-weighted surplus. That is, we are dealing with a Non Transferable Utility (NTU) bargaining problem, in which the players are constrained in the way they can share surplus. Because the Nash bargaining program maximizes the product of the two utilities, in an NTU bargaining problem, an inefficient solution that shares the available surplus equally may be chosen over an efficient solution in which all surplus is captured by one of the two players.

### 4.2 Period 1

We now analyze the decision in period 1 to restrict the future bargaining set. Here we assume that players are rational in period 1 , while in the next section we assume that the players' preferences are distorted by the focusing effect in period 1 as well.

Call $\left(q_{1}^{O S}, \ldots, q_{n}^{O S}, t^{O S}\right)$ the bargaining outcome in case they bargain in one step, which is the outside option of the period-1 negotiation. The period-1 problem is:

$$
\begin{aligned}
& \max _{Q_{1}, \ldots, Q_{n}, \underline{T}, \bar{T}}\left(\sum_{i=1}^{n} \alpha_{i} \cdot\left(q_{i}^{*}-q_{i}^{O S}\right)+\left(t^{*}-t^{O S}\right)\right)\left(\sum_{i=1}^{n} \beta_{i} \cdot\left(q_{i}^{*}-q_{i}^{O S}\right)-\left(t^{*}-t^{O S}\right)\right) \\
& \quad \text { s.t. } \\
& \left\{\begin{array}{l}
\left\{q_{1}^{*}, \ldots, q_{n}^{*}, t^{*}\right\}= \\
\underset{x \in Q_{1} \times Q_{2} \times \ldots Q_{n} \times \mathbb{R} \cup \overline{0}}{\operatorname{argmax}}\left(\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}\left(\bar{t}^{o}-\underline{t}^{o}\right) t\right)\left(\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}\left(\bar{t}^{o}-\underline{t}^{o}\right) t\right), \\
\overline{t^{o}}=\min \{\bar{T}, \tilde{t}\} \\
\underline{t}^{o}=\max \{\underline{T}, \underline{t}\}
\end{array}\right.
\end{aligned}
$$

That is, the players will choose a bargaining outcome among those that can be reached via an incomplete agreement. Note that, in case a period-1 agreement is
reached, both players must be better off relative to the one-step negotiation. This immediately leads to the following proposition.

Proposition 3. If the players agree on an incomplete agreement in period 1, then the material efficiency of the negotiation is greater than in one-step negotiation.

Note however that the proposition does not say that the players will choose the materially efficient period-1 agreement, neither that they will always agree on an incomplete agreement if doing so increases the efficiency of the negotiation. The reason is that, again, the period-1 negotiation is a NTU bargaining problem: there is no presumption that the outcome will be materially efficient. However, because the outside option is the one-step negotiation, it must be that whatever is agreed in period-1 is better for both players than the one-step negotiation. Because the players have rational preferences, then whatever is agreed also increases the material efficiency of the negotiation (relative to the one-step negotiation).

Practically speaking: how can the players achieve such an increase in the efficiency of the negotiation? As discussed earlier, the possible inefficiencies arising in the one-step negotiation are twofold: an issue can be inefficiently included into the agreement or inefficiently excluded from the agreement. From period-1 point of view, the first type of inefficiency can be easily resolved by eliminating such issue from the future bargaining set. The second type of inefficiency is more complicated to address. The players may agree in period 1 to include such issue in the future agreement. As already discussed, however, such agreement does not affect the period-2 preferences and may still lead to an inefficient outcome - for example a breakdown of the negotiation. For this reason, the players may also decide to reduce the salience of the transfer dimension. This can be easily achieved by eliminating from the bargaining set all issues on which players expect no agreement to occur anyway. We call these issues redundant issues. These observations imply the following proposition.

Proposition 4. The players always eliminate redundant issues and inefficient issues from the bargaining set.

Proof. Directly from Corollary 1: players will always want to reduce the salience of the transfer dimension when doing so is beneficial (because it eliminates inefficient
issues) or has no other consequences (because it eliminates redundant issues that will not be included in the agreement anyway).

The salience of the transfer dimension can be further reduced by imposing bounds on future transfers. These bounds, however, may eliminate from the bargain set the period-1 preferred outcome. When this is the case, then period-1 players are facing a tradeoff: manipulating period-2 preferences to make them similar to the period1 preferences comes at the cost of excluding valuable bargaining outcomes. The solution depends on how "focused" are period-2 players and therefore how costly it is to make period-2 preferences similar to period-1 preferences.

We turn to a numerical example to demonstrate that players may use bounds on transfers. In our example $n=2, \beta_{1}=2, \beta_{2}=1$ and $\alpha_{1}=\alpha_{2}=-0.95$, so that the materially efficient outcome is to agree on both issues. Hence, rational players cannot do better than bargaining in one step and agreeing on both issues. Preferences, however, are distorted by the focusing effect, where the focus functions here take the form $h_{b}(x)=\kappa|x|+1$ and $h_{a}(x)=\kappa / 4 \cdot|x|+1$ with $\kappa>0$. This implies that player $b$ is more focused than player $a$, which means that, in the onestep negotiation, the second issue may be inefficiently excluded from the agreement. Also, the parameter $\kappa$ measures the strength of the focusing effect. Thus, in period 1 the players may want to impose bounds on the transfer dimension in order to induce their future selves to include also issue 2 in the agreement. Note also that, because all costs accrue to player $a$ and all benefits accrue to player $b, \underset{\sim}{t}=0$ and hence the only bound on transfer the players may impose in period 1 is $\bar{T}$.

Figure 4.1 provides a numerical example of how $\bar{T}$ affects the solution of the negotiation (for $\kappa=1 / 20$ ). For $\bar{T}=\tilde{t}=2.845$, the second issue is inefficiently excluded from the agreement. For $\bar{T}<\tilde{t}$ with $\bar{T}$ slightly below $\tilde{t}$, imposing such cap reduces the salience of the transfer dimension and increases $t^{*}$ relative to $t^{O S}=1.427$ without affecting the number of issues included into the agreement. Also, there is an intermediate range in which imposing a cap $\bar{T}$ below $\tilde{t}$ leads to the inclusion of the second issues into the agreement and a further increase in $t^{*}$ relative to $t^{O S}$. Finally, imposing an even lower cap $\tilde{T}$ leads to either only one issues being included into the


Figure 4.1: The period-2 transfer $t^{*}$ (solid black line) and the period-2 number of issues (dotted gray line; gray filling) as a function of the cap on transfer $\bar{T}$ set in period 1. Parameter values are $\beta_{1}=2, \beta_{2}=1, \alpha_{1}=\alpha_{2}=-0.95$ and $\kappa=1 / 20$, i.e. $h_{b}(x)=|x| / 20+1$ and $h_{a}(x)=|x| / 80+1$. $\tilde{t}=2.845$ and $t^{O S}=1.427$.
agreement or a breakdown of the negotiation.
The parameter $\kappa$ measures the intensity of the focusing effect. It is a convenient way to measure the disagreement between period-1 and period-2 players. As $\kappa$ increases, period-2 preferences move further away from rational. As a consequence generating a given degree of alignment between period-1 and period-2 preferences requires a tighter cap. Hence, $\kappa$ proxies how stringent is the tradeoff between the benefit of preferences alignment and the cost of imposing restrictions on the future bargaining set. In Table 4.1 we report the numerical solution of the period-1 problem for a number of different $\kappa$, all chosen so that in the one-step negotiation the second issue is inefficiently left out of the agreement. Indeed, for lower values of $\kappa$ the players may impose a cap on the transfer dimension so to achieve the materially efficient outcome. In this case, the players always impose the optimal cap, and are indifferent
between forcing their future self to include any subset of issues in the agreement or not (that is, they are indifferent between $Q_{i}=\{0,1\}$ or $Q_{i}=\{1\}$ for $i \in\{1,2\}$ ). For high enough $\kappa$, instead, the players cannot do better than to negotiate in one step.

Table 4.1: Solution to the negotiation

| $\kappa$ | one step |  |  | two steps |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 20$ | $t^{O S}=1.427$ | $q_{1}^{O S}=1$ | $q_{2}^{O S}=0$ | $\bar{T}=2.385$ | $t^{*}=2.385$ | $q_{1}^{*}=1$ | $q_{2}^{*}=1$ |
| $3 / 50$ | $t^{O S}=1.420$ | $q_{1}^{O S}=1$ | $q_{2}^{O S}=0$ | $\bar{T}=2.375$ | $t^{*}=2.375$ | $q_{1}^{*}=1$ | $q_{2}^{*}=1$ |
| $3 / 40$ | $t^{O S}=1.410$ | $q_{1}^{O S}=1$ | $q_{2}^{O S}=0$ | $\bar{T}=$ none | $t^{*}=1.410$ | $q_{1}^{*}=1$ | $q_{2}^{*}=0$ |
| $1 / 10$ | $t^{O S}=1.397$ | $q_{1}^{O S}=1$ | $q_{2}^{O S}=0$ | $\bar{T}=$ none | $t^{*}=1.397$ | $q_{1}^{*}=1$ | $q_{2}^{*}=0$ |

## 5 Context-dependent preferences in period 1

If the negotiation is heated, conducted under pressure, or conducted by groups of people who first have to agree among themselves and then with the other party, then the players' preferences may be far from rational already in period 1. In this subsection we therefore consider the possibility that the focusing effect distorts also the players' period- 1 preferences.

The point we want to make is that the period- 1 context is different from period-2 context and hence, even if the focusing effect is present in both periods, period-1 preferences will be different from period-2 preferences. Hence, also here, in period 1 the players may value restricting the future bargaining set so to make their future selves' preferences better aligned with those of their present selves. Furthermore, because the set of bargaining outcomes achievable from period- 1 viewpoint is smaller than the full bargaining set, the period-1 preferences will be "closer" to rational than the period-2 preferences. Hence, also here, if the players impose an incomplete agreement, this leads to an increase of the material efficiency of the negotiation.

In constructing the period-1 context, we maintain Assumption 2: a feasible bargaining outcome is in the consideration set if and only if it satisfies one of the player's participation constraint. The key, however, is to realize that the bargaining outcomes
that are feasible from period-1 view point are only those that can be achieved as an outcome of the negotiation for some incomplete agreement $S \in \mathbb{S} .{ }^{20}$ So, for example, whereas in the one-step negotiation all issues are in the consideration set, this does not have to be the case in period 1. It is possible that some issues are not valuable (because costs exceed benefits by a large margin) and hence are never included in any agreement no matter what $S$ the players agree upon. These issues are therefore not in the period- 1 consideration set. The same applies to transfers. From period-1 viewpoint, the smallest and largest transfer in the consideration set are given by the smallest and largest $t^{*}$ (as in Equation 4.10) that can be achieved for some incomplete agreement.

Without loss of generality, order the issues such that:

- issues $i \in\{1, \ldots, z\}$ are included in the final agreement of the one-step negotiation, (where $z \in\{0, \ldots, n\}$, with the convention that if $z=0$, then the one-step negotiation leads to no agreement),
- issues $i \in\{z+1, \ldots, w\}$ are excluded from the final agreement of the onestep negotiation, but can be included in the final agreement of the two-step negotiation provided that a given incomplete agreement is signed in period 1 (where $w \in\{z, \ldots, n\}$, with the convention that if $w=z$, then such issues do not exist).

Call the smallest and largest $t^{*}$ that can be achieved as a function of an incomplete agreement (as in Equation 4.10) $\underline{t}^{*}$ and $\bar{t}^{*}$, respectively. ${ }^{21}$ Note that, $t \in\left[\underline{t}^{*}, \bar{t}^{*}\right]$ is the set of feasible transfers from period-1 viewpoint.

[^15]Define $\bar{t}^{1}$ and $\underline{t}^{1}$ as the largest and smallest transfers in the period- 1 consideration set (which we derive below). A feasible bargaining outcome $x$ satisfies the player's participation constraints if and only if

$$
\begin{aligned}
& U^{a}(x)=\sum_{i=1}^{w} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \cdot q_{i}+h_{a}\left(\bar{t}^{1}-\underline{t}^{1}\right) t \geq \sum_{i=1}^{z} h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{a}\left(\bar{t}^{1}-\underline{t}^{1}\right) t^{O S} \\
& U^{b}(x)=\sum_{i=1}^{w} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} \cdot q_{i}-h_{b}\left(\bar{t}^{1}-\underline{t}^{1}\right) t \geq \sum_{i=1}^{z} h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}-h_{b}\left(\bar{t}^{1}-\underline{t}^{1}\right) t^{O S}
\end{aligned}
$$

Define $\tilde{t}^{1}$ and ${\underset{\sim}{t}}^{1}$ as the transfers that satisfy the players participation constraints with equality:

$$
\begin{align*}
& \sum_{i=z+1}^{w} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}=-h_{a}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)\left({\underset{\sim}{t}}^{1}-t^{O S}\right)  \tag{5.11}\\
& \sum_{i=z+1}^{w} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}=h_{b}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)\left(\tilde{t}^{1}-t^{O S}\right) \tag{5.12}
\end{align*}
$$

Following the same logic discussed in Section 4.1, the largest transfer in the consideration set $\bar{t}^{1}$ is the minimum between the largest feasible transfer and the largest transfer satisfying the player $a$ 's participation constraint, and similarly for $\underline{t}^{1}$ :

$$
\bar{t}^{1}=\min \left\{\tilde{t}^{*}, \tilde{t}^{1}\right\} \quad \underline{t}^{1}=\max \left\{\underline{t}^{*},{\underset{\sim}{1}}^{1}\right\}
$$

The next lemma proves the existence and uniqueness of $\bar{t}^{1}$ and $\underline{t}^{1}$, and also characterizes them with respect to $\bar{t}$ and $\underline{t}$ (the largest and smallest transfer in the consideration set of the one-step negotiation given by Equation 3.1 and 3.2).

Lemma 5. $\bar{t}^{1}$ and $\underline{t}^{1}$ exist and are unique. Furthermore, $\hat{t}^{1}-\underline{t}^{1} \leq \bar{t}-\underline{t}$. The inequality is strict whenever $z \geq 1$, or $w<n$, or both.

Remember that when $z \geq 1$, then the one-step negotiation leads to an agreement with at least one issue included. When $w<n$, then there is at least one issue that will not be included in the final agreement for any incomplete agreement that may
be signed in period 1. When either conditions holds, then the period- 1 negotiation is, effectively, about fewer issues than the one-step negotiation. If $w<n$ this is immediate: issues $w+1$ to $n$ are not feasible from period- 1 viewpoint, while they are feasible in the one-step negotiation. When $z \geq 1$, the argument is more subtle: it is because if $z=0$, then the outside option of the period 1 negotiation is equal to the outside option of the period 2 negotiation. If instead $z \geq 1$, then the two outside options are different: whatever issue is included in the one-step negotiation constitutes the outside option of the period-1 negotiation and hence does not affect the value of reaching an agreement in period 1.

Hence, when either $z \geq 1$ or $w<n$, then $\bar{t}^{1}-\underline{t}^{1}<\bar{t}-\underline{t}$. That is, period- 1 preferences, although not rational, are "closer" to rational than the preferences in the one-step negotiation. What we mean is the following: suppose that, in the onestep negotiation, the focus-weighted surplus generated by an issue $i$ is positive while the "rational" surplus is negative. Then, because in period 1 the players are less focused on the transfer dimension than in the one-step negotiation, by Corollary 1, they may agree with their rational selves and, as a consequence, exclude such issue from the future agreement. This implies the following proposition.

Proposition 5. Suppose preferences are distorted by the focusing effect also in period 1. If either $z \geq 1$ or $w<n$, then Proposition 3 holds.

Proof. Directly from Lemma 5 and Corollary 1.
Hence, the possibility of signing an incomplete agreement increases the material efficiency of the negotiation, despite the fact that the players' preferences are distorted also when negotiating the incomplete agreement.

## 6 Discussion

We now discuss a number of variations in our basic assumptions. First, we consider the possibility that the negotiation structure (i.e., the set of incomplete agreements that can be chosen in period 1) is different from the one considered in the main
text. In Section 6.2 we discuss what happens when incomplete agreements are non binding in the sense that they can be ignored in subsequent bargaining rounds when in the current bargaining round, no final agreement is reached. In Section 6.3 we discuss how our results will change if we consider other forms of context dependence different than the focusing effect.

### 6.1 Other negotiation structures

In the main text, we consider a negotiation structure inspired by Raiffa (1982)'s threshold agreements: the players can include or exclude issues from the future agreement, or impose bounds on the transfer dimension. Yet, other negotiation structures are possible and may interact with context-dependent preferences. Quite clearly, Proposition 3 will continue to apply, i.e., whenever players agree on an incomplete agreement, the material efficiency of the negotiation is greater than in a one-step negotiation. Other features of the equilibrium may, however, change with the negotiation structure.

As a first example of another negotiation structure, consider the possibility that, in period 1, the players can directly agree on the negotiation outcome. That is, the negotiation structure $\mathbb{S}$ contains all singletons in $X$. It is reasonable to assume that, in this case, preferences should be distorted in period-1 as well, because it is effectively the moment in which the agreement is decided. However, if the players' preferences are distorted by the focusing effect in period 1 as well, then the outcome of the negotiation is the same as in the one-step negotiation. The reason is that the range of possible bargaining outcome in period 1 is the same as in the one-step negotiation, and hence the players' preferences are identical in the two cases. Hence, there is no difference between a one-step negotiation and a two-step negotiation in which negotiation structure contains all singletons in $X$.

Another possibility is to allow the players to "bundle" issues, that is, to decide in period 1 that if their future selves agree on issue $i$ they must also agree on issue $j$ (or disagree on both issues, or agree on one and disagree on the other). This is an interesting case and is reminiscent of the "nothing is agreed until everything is
agreed" formula used in negotiations. What it is however unclear in this case is how to write the period-2 focus weights. One could reasonably claim that if two issues become "bundled" then there should be a single focus weight on both issues (that is, they effectively become a single issue). We find this idea intriguing but prefer to leave this extension for future work.

### 6.2 Non-binding incomplete agreements

In the text, we assumed that the players must respect a prior incomplete agreement. This is, however, often not the case. In most jurisdictions, when facing a sequence of agreements between two parties, in case a later agreement contradicts an earlier agreement, courts typically enforce the most recent one. When courts are not available (for example, in the case of international negotiations), the same two parties are free to jointly ignore a previous agreement. Furthermore, in some type of incomplete agreements such as memorandum of understanding, it is explicitly stated that each party can unilaterally withdraw from the incomplete agreement.

In this subsection we explain that our framework can rationalize the existence of non-binding incomplete agreements. We introduce the following modification: in period 2, the players can either comply with a prior incomplete agreement, or trigger a third round of negotiation (at no cost). During the third round of negotiation, they will be bargaining over the entire bargaining set. Depending on the situation, triggering this additional negotiation round could be a joint decision, or even a unilateral decision by a player (as in memorandum of understanding). In either case, we say that incomplete agreements are non binding.

Non-binding incomplete agreements may nonetheless affect the outcome of the negotiation because they will be in place in period 2 and affect the players' preferences. The difference with the case considered in the main text is that, now, the bargaining solution to the third negotiating round must be an element of the period2 consideration set. The presence of this element imposes a constraint on the way in which period-1 players can manipulate period-2 preferences. For example, call the transfer solving the period- 3 bargaining problem $t_{3}^{*}$. If this transfer is positive,
then effectively period- 1 agents are restricted to choosing $\bar{T}>t_{3}^{*}$ because a lower cap will not have any effect on the player- 2 consideration set. To summarize, the fact that incomplete agreements are non binding imposes an additional constraint on the period-1 problem, without, however, changing its basic tradeoff.

### 6.3 Other models of context dependence

We develop our argument using Kőszegi and Szeidl (2013) because, as already discussed in the introduction, practitioners believe that reducing the range of possible outcomes on a dimension reduces the importance of this dimension within a negotiation (see Raiffa, 1982, p. 216). However, other models of context dependence have been proposed.

Bushong, Rabin and Schwartzstein (2017) propose a model in which the salience of a dimension decreases with the range of possible options in that dimension. Mathematically, the model is identical to that of Köszegi and Szeidl (2013), except that the focus functions $h_{a}()$ and $h_{b}()$ are decreasing. For our purposes, the fact that the focus functions are decreasing implies that a cap on transfers will increase the salience of the transfer dimension relative to no cap. It follows that the results derived in Proposition 4 are now reversed. The salience of the transfer dimension increases whenever $\beta_{i}>0$ or $\alpha_{j}>0$ decreases. Hence the inefficient outcomes on a given issue become more likely to occur when the value of reaching an agreement on other issues are small rather than large.

The goal of Bushong, Rabin and Schwartzstein (2017) is to model range-based relative thinking: the idea that given "the presence of greater ranges along a dimension, all changes along that dimension loom smaller." An implication of range-based relative thinking is the notion of diminishing sensitivity, where a fixed change is less salient the wider the range of utility differences. In their introduction, they propose the example of a 100 dollars optional "convenience" charge on a flight that costs 200 vs 500 . When the starting price is higher, the additional charge will be less salient and the consumer is more likely to purchase it.

An important element of this example is that the optional "convenience" charge
cannot be purchased independently from the flight. ${ }^{22}$ In our model, instead, the players can agree on any issue independently from all other issues. In other words, no issue is an "add on" to another issue. Given this interpretation, we do not think that diminishing sensitivity should be prominent in our analyses, and therefore prefer to use Kőszegi and Szeidl (2013). Of course, in some cases, some issues will be "add ons" to other issues, in which case the model proposed by Bushong, Rabin and Schwartzstein (2017) is probably better suited to study the bargaining problem.

Bordalo, Gennaioli and Shleifer (2013) propose a model of salience in which different options are evaluated relative to a reference point. This model could be applied to our framework by assuming that imposing bounds on transfers shifts the reference point of the transfer dimension. The effect of introducing these bounds will then depend on whether the transfer dimension becomes more or less salient. If its salience decreases, then we are back to a logic similar to Kőszegi and Szeidl (2013), leading to results qualitatively similar to the ones discussed in the body of the paper. If instead its salience increases, then we are back to a logic similar to Bushong, Rabin and Schwartzstein (2017), leading to results qualitatively similar to the ones discussed above. We will be in one or the other case depending on how exactly imposing a bound affects the reference point, which is an issue beyond the scope of this paper.

## 7 Conclusion

We have shown that, because of the focusing effect, too many or too few issues may be included in an agreement (relative to a rational benchmark). As a consequence, the players may negotiate in stages, by first restricting their future bargaining set via an incomplete agreement, and then finalizing the negotiating outcome. Incomplete agreements in the form of threshold or framework agreements are relevant and frequently used by practitioners in order to reduce the salience of certain issues (see

[^16]the discussion about Raiffa, 1982, in the Introduction). We show that when players agree on an incomplete agreement, they always increase the material efficiency of the negotiation (relative to a one-step negotiation). Our results therefore suggest that incomplete agreements help negotiating parties reduce the inefficiency caused by behavioral distortions, possibly even avoiding inefficient negotiation breakdowns. However, due to an inherent non-transferable utility problem, there is no presumption that the players will improve the outcome of the negotiation via an incomplete agreement, even if it is possible to do so. There is also no presumption that they will agree on the most efficient incomplete agreement.

The validity of our model is empirically testable, even without observing which player is more focused. For given parameters, consider the agreement of a one-step negotiation. Imagine now a counterfactual in which the importance of a given issue $i$ increases while the importance of all other issues is constant. By Proposition 1 , depending on which player is more focused, in this counterfactual some issues (other than $i$ ) that were part of the initial agreement may be dropped from the agreement, while some other issues (again, other than $i$ ) that were not part of the initial agreement may be included into the agreement. Importantly, once an issue $j \neq i$ is dropped from (resp. added to) the agreement, it won't be included in (resp. excluded from) the agreement again through further increases in the importance of issue $i$.

Suppose to repeat the above experiment sufficiently many times with different players randomly assigned to play as $a$ or $b$, so that in some experiments player $a$ will be more focused than player $b$, while the opposite is true in some other experiments. Under the null hypothesis that our model is valid we should observe that, unlike with rational preferences, the probability that an issue $j \neq i$ is included in the agreement changes with the value of issue $i$. Furthermore, such a change is monotonic. Hence, we should reject the validity of the model in two cases. First, if there is no issue $i$ such that changing its value affects the probability of reaching an agreement on the remaining issues. Second, if this effect is non monotonic, that is, as the value of issue $i$ increases, there is an issue $j \neq i$ that is first included, then excluded, then included again etc. (or first excluded, then included, then excluded again etc.).

We study restrictions on the bargaining set that are jointly agreed by the two negotiating parties. We do so because negotiations are often structured as a sequence of agreements. Nevertheless, one could also study restrictions to the bargaining set that are unilaterally imposed by one party. We speculate that this could give rise to a "theory of concessions" based on the focusing effect, where a party renounces to some beneficial bargaining outcomes so to manipulate the opponent's preferences before starting a negotiation. Of course, an important issue is to what extent such concessions are credible and hence influence the players' consideration set. Solving this issue, as well as determining the equilibrium of a game in which both players can make concessions, is left for future work. Similarly, the restrictions on the bargaining set could be imposed by a third party such as a mediator. An open question would then be why this third party can impose a constraint on the bargaining set (but not a bargaining solution directly) and what are his/her objectives. These considerations are also left for future research.

## A Appendix

Proof of Lemma 1. (3.1) and (3.2) together yield

$$
\begin{equation*}
\bar{t}-\underline{t}=\frac{\sum_{i=1}^{n} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}}{h_{b}(\bar{t}-\underline{t})}+\frac{\sum_{i=1}^{n} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}}{h_{a}(\bar{t}-\underline{t})} . \tag{A.1}
\end{equation*}
$$

The LHS is strictly increasing in $\bar{t}-\underline{t}$; the RHS is strictly decreasing in $\bar{t}-\underline{t}$, and they always cross only once. If $\bar{t}-\underline{t}$ exists and is unique, then also $\bar{t}$ and $\underline{t}$ exist and are unique. Also, the RHS of (A.1) is increasing in $\alpha_{i}$ if $\alpha_{i}>0$, in $\beta_{i}$ if $\beta_{i}>0$, and in $n$, which implies our statement.

Proof of Lemma 2. If $n=1$, then $\beta_{1} \cdot \alpha_{1}<0$. Suppose $\beta_{1}>0$ (the case $\alpha_{1}>0$ is analogous). By (3.1) and (3.2) we have $\bar{t}=\beta_{1}$ and $\underline{t}=0$. Hence, by (3.5), there is an agreement if and only if

$$
\frac{h_{a}\left(\left|\alpha_{1}\right|\right) \alpha_{1}}{h_{a}\left(\beta_{1}\right)}+\beta_{1}>0
$$

which is equivalent to $\beta_{1}>-\alpha_{1}$.
Proof of Lemma 3. If $\Delta(x)=1$, then by (3.5) there is an agreement if and only if

$$
\alpha_{i}+\frac{h_{b}\left(\left|\beta_{i}\right|\right)}{h_{b}\left(\left|\alpha_{i}\right|\right)} \beta_{i}>0
$$

which is equivalent to $\alpha_{i}+\beta_{i}>0$ since $h_{b}(|x|)$ is increasing in $|x|$.
Proof of Lemma 4. We limit our proof to the case that player $b$ is "more focused" than player $a$ (that is, $\Delta(x) \leq 1$ and strictly decreasing). The proof of the opposite case follows by interchanging cases A.) and B.), respectively.

Case A.) $\alpha_{i}<0$ and $\beta_{i}>0$ : Suppose $\alpha_{i}+\beta_{i}<0$, that is, it is efficient not to include issue $i$ into the agreement. We consider two cases. The first one is $\bar{t}-\underline{t} \leq\left|\alpha_{i}\right|$. In this case, $\bar{t} \leq\left|\alpha_{i}\right|$ and the largest transfer player $b$ is willing to make does not cover the cost for player $a$ of including issue $i$. This issue will, therefore, not be included in the agreement. Second suppose that $\bar{t}-\underline{t}>\left|\alpha_{i}\right|$. Then, the largest transfer player $b$ is willing to make covers the cost for player $a$ of including issue $i$ and issue $i$ will be included into the agreement if the focus weighted surplus generated by issue $i$ is positive. By (3.4) the focus weighted surplus generated by issue $i$ is

$$
\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{h_{a}(\bar{t}-\underline{t})}+\frac{h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})}
$$

which is equivalent to

$$
\frac{h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}}{\Delta(\bar{t}-\underline{t})}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i} .
$$

Because $\Delta(x) \leq 1$ and $\alpha_{i}<0$ the above expression is always smaller than

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}
$$

which is negative whenever $\alpha_{i}+\beta_{i}<0$. Hence the players do not want to include issue $i$ in the agreement.

Suppose next that $\alpha_{i}+\beta_{i}>0$. Then,

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}>0
$$

because $h_{b}\left(\left|\beta_{i}\right|\right) \geq h_{a}\left(\left|\beta_{i}\right|\right)>h_{a}\left(\left|\alpha_{i}\right|\right)$ and if $\bar{t}-\underline{t}$ sufficiently large, the focus weighted surplus generated by including issue $i$ in the agreement is negative (despite being efficient to do so). On the other hand, if $\bar{t}-\underline{t}$ is sufficiently small the players will want to include issue $i$ into the agreement.

Case B.) $\alpha_{i}>0$ and $\beta_{i}<0$ : Suppose $\alpha_{i}+\beta_{i}>0$, that is, it is efficient to include issue $i$ into the agreement. This implies that $\left|\beta_{i}\right|<\alpha_{i}$. It further holds that $\alpha_{i}<h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}$ and by (3.1) $h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \leq-h_{a}(\bar{t}-\underline{t}) \underline{t}$. Therefore, $h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i} \leq$ $h_{a}(\bar{t}-\underline{t})(\bar{t}-\underline{t})$ which implies that $\alpha_{i} \leq(\bar{t}-\underline{t})$. By (3.5) the focus weighted surplus generated by issue $i$ is

$$
\frac{\Delta\left(\alpha_{i}\right)}{\Delta(\bar{t}-\underline{t})} \alpha_{i}+\frac{h_{b}\left(\left|\beta_{i}\right|\right)}{h_{b}\left(\left|\alpha_{i}\right|\right)} \beta_{i},
$$

where the first term must be (weakly) larger than $\alpha_{i}$ by $\alpha_{i} \leq(\bar{t}-\underline{t})$ and $\Delta(x)$ being decreasing in $x$ and the second term must be smaller than $\beta_{i}$ by $\left|\beta_{i}\right|<\alpha_{i}$. Thus, the focus weighted surplus generated by issue $i$ is positive whenever $\alpha_{i}+\beta_{i}>0$.

Suppose next that $\alpha_{i}+\beta_{i}<0$. Then,

$$
h_{a}\left(\left|\alpha_{i}\right|\right) \alpha_{i}+h_{b}\left(\left|\beta_{i}\right|\right) \beta_{i}<0
$$

because $h_{b}\left(\left|\beta_{i}\right|\right) \geq h_{a}\left(\left|\beta_{i}\right|\right)>h_{a}\left(\left|\alpha_{i}\right|\right)$ and if $\bar{t}-\underline{t}$ sufficiently small, the focus weighted surplus generated by including issue $i$ in the agreement is negative. On the other hand, if $\bar{t}-\underline{t}$ is sufficiently large the players will want to include issue $i$ into the agreement (despite being inefficient to do so).

Proof of Proposition 1. By Lemma 1, $\bar{t}-\underline{t}$ is

- increasing in $\beta_{j}$, strictly so for $\beta_{j}>0$, for all $j \leq n$.
- increasing in $\alpha_{j}$, strictly so for $\alpha_{j}>0$, for all $j \leq n$.
- increasing in $n$.

The statement then follows by Lemma 4.
Proof or Proposition 2. In the proof of Lemma 1, we show that $\bar{t}-\underline{t}$ is implicitly defined by (A.1). The RHS of (A.1) under $\Gamma^{\prime}$ is higher than the RHS of (A.1) under $\Gamma$ if and only if

$$
\frac{h_{a}\left(\beta_{i}\right) \beta_{i}}{h_{a}(\bar{t}-\underline{t})}>\frac{h_{b}\left(\beta_{i}\right) \beta_{i}}{h_{b}(\bar{t}-\underline{t})},
$$

or equivalently,

$$
\Delta\left(\beta_{i}\right)>\Delta(\bar{t}-\underline{t}) .
$$

Because $\bar{t}-\underline{t}>\beta_{i},{ }^{23}$ the above condition always holds when $b$ is more focused (and hence $\Delta$ is strictly decreasing), and is always violated when $a$ is more focused (and hence $\Delta$ is strictly increasing).

Hence, when $b$ is more focused, going from $\Gamma$ to $\Gamma^{\prime}$ leads to an increase of the RHS of (A.1) and therefore to an increase in the salience of the transfer dimension, and lower efficiency (by Corollary 1). Similarly, if $a$ is more focused, going from $\Gamma$ to $\Gamma^{\prime}$ leads to an decrease in the salience of the transfer dimension, and higher efficiency.

Proof of Lemma 5. (5.11) and (5.12) together yield

$$
\tilde{t}^{1}-{\underset{\sim}{t}}^{1}=\frac{\sum_{i=z+1}^{w} h_{b}\left(\left|\beta_{i}\right|\right) \max \left\{\beta_{i}, 0\right\}}{h_{b}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)}+\frac{\sum_{i=z+1}^{w} h_{a}\left(\left|\alpha_{i}\right|\right) \max \left\{\alpha_{i}, 0\right\}}{h_{a}\left(\tilde{t}^{1}-{\underset{\sim}{t}}^{1}\right)} .
$$

The argument is identical to the one presented in the proof of Lemma 1. The LHS is strictly increasing in $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$; the RHS is strictly decreasing in $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$, and they

[^17]always cross only once. Hence, $\tilde{t}^{1}-{\underset{\sim}{t}}^{1}$ exists and is unique. Together with (5.11) and (5.12), this implies that $\tilde{t}^{1}$ and ${\underset{\sim}{t}}^{1}$ exist and are unique, so that also $\hat{t}^{1}$ and $\underline{t}^{1}$ exist and are unique.

The second part of the proposition follows simply by comparing the above expression with (A.1) (see the proof of Lemma 1).

## References

Ali, S. M. Nageeb, "Waiting to settle: Multilateral bargaining with subjective biases," Journal of Economic Theory, 2006, 130 (1), 109-137.
Apffelstaedt, Arno and Lydia Mechtenberg, "Competition over ContextSensitive Consumers," Working Paper, 2017.
Auster, Sarah, "Asymmetric awareness and moral hazard," Games and Economic Behavior, 2013, 82, 503-521.
Bac, Mehmet and Horst Raff, "Issue-by-issue Negotiations: The Role Of Information And Time Preference," Games and Economic Behavior, 1996, 13 (1), 125-134.
Battaglini, Marco and Bård Harstad, "Participation and duration of environmental agreements," Journal of Political Economy, 2016, 124 (1), 160-204.
Battigalli, Pierpaolo and Giovanni Maggi, "Rigidity, Discretion, and the Costs of Writing Contracts," The American Economic Review, 2002, pp. 798-817.
Bénabou, Roland and Jean Tirole, "Over my dead body: Bargaining and the price of dignity," American Economic Review, 2009, 99 (2), 459-65.
Bernheim, B Douglas and Michael D Whinston, "Incomplete contracts and strategic ambiguity," The American Economic Review, 1998, pp. 902-932.
Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky, "The Nash bargaining solution in economic modelling," The RAND Journal of Economics, 1986, pp. 176188.

Bolton, Patrick and Antoine Faure-Grimaud, "Satisficing Contracts," The Review of Economic Studies, 2010, 77 (3), 937-971.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, "Salience and Consumer Choice," The Journal of Political Economy, 2013, 121 (5), 803-843.
Busch, Lutz-Alexander and Ignatius J Horstmann, "Endogenous Incomplete Contracts: A Bargaining Approach," Canadian Journal of Economics, 1999, pp. 956-975.
_ and _ , "Signaling via an Agenda in Multi-issue Bargaining with Incomplete Information," Economic Theory, 1999, 13 (3), 561-575.
Bushong, Benjamin, Matthew Rabin, and Joshua Schwartzstein, "A model of relative thinking," Working Paper, 2017.
Carroni, Elias, Andrea Mantovani, and Antonio Minniti, "Salience and Information Asymmetry," Working Paper, SSRN 3483514, 2019.
Chen, Ying and Hülya Eraslan, "Dynamic Agenda Setting," American Economic Journal: Microeconomics, May 2017, 9 (2), 1-32.
Compte, Olivier and Philippe Jehiel, "Bargaining with Reference Dependent Preferences," Working Paper, 2003.
Dekel, Eddie, Barton L Lipman, and Aldo Rustichini, "Representing preferences with a unique subjective state space," Econometrica, 2001, 69 (4), 891-934.
Dertwinkel-Kalt, Markus, Holger Gerhardt, Gerhard Riener, Frederik Schwerter, and Louis Strang, "Concentration bias in intertemporal choice," Working Paper, 2017.
_ , Mats Köster, and Florian Peiseler, "Attention-driven demand for bonus contracts," European Economic Review, 2019, 115 (C), 1-24.
Esteban, Joan and József Sákovics, "A Theory of Agreements in the Shadow of Conflict: The Genesis of Bargaining Power," Theory and Decision, 2008, 65 (3), 227-252.
Fearon, James D, "Rationalist explanations for war," International organization, 1995, 49 (3), 379-414.
_ , "Bargaining over objects that influence future bargaining power," Working paper, 1996.

Fisher, Roger, William L. Ury, and Bruce Patton, Getting to Yes: Negotiating Agreement Without Giving In, Penguin Group, 1991.

Flamini, Francesca, "First Things First? The Agenda Formation Problem for Multi-issue Committees," Journal of Economic Behavior \& Organization, 2007, 63 (1), 138-157.
Gul, Faruk and Wolfgang Pesendorfer, "Temptation and Self-Control," Econometrica, 2001, 69 (6), 1403-1435.
Harstad, Bård, "Harmonization and side payments in political cooperation," The American Economic Review, 2007, 97 (3), 871-889.
Hart, Oliver and John Moore, "Contracts as Reference Points," The Quarterly Journal of Economics, 2008, 123 (1), 1-48.
Herweg, Fabian and Klaus M. Schmidt, "Loss Aversion and Inefficient Renegotiation," The Review of Economic Studies, 2015, 82 (1), 297-332.
_ , Heiko Karle, and Daniel Müller, "Incomplete contracting, renegotiation, and expectation-based loss aversion," Journal of Economic Behavior $\mathcal{F}$ Organization, 2018, 145, 176-201.
Inderst, Roman, "Multi-issue Bargaining with Endogenous Agenda," Games and Economic Behavior, 2000, 30 (1), 64-82.

- and Martin Obradovits, "Loss Leading with Salient Thinkers," The RAND Journal of Economics, 2019, forthcoming.
Kahneman, Daniel, Alan B Krueger, David Schkade, Norbert Schwarz, and Arthur A Stone, "Would You Be Happier If You Were Richer? A Focusing Illusion," Science, 2006, 312 (5782), 1908-1910.
Kôszegi, Botond and Adam Szeidl, "A Model of Focusing in Economic Choice," The Quarterly Journal of Economics, 2013, 128 (1), 53-104.
Lang, Kevin and Robert W Rosenthal, "Bargaining Piecemeal or All At Once?," The Economic Journal, 2001, 111 (473), 526-540.
Nash, John, "Two-person cooperative games," Econometrica: Journal of the Econometric Society, 1953, pp. 128-140.
Noor, Jawwad, "Temptation and Revealed Preference," Econometrica, 2011, 79 (2), 601-644.

Powell, Robert, "Guns, butter, and anarchy.," American Political Science Review, 1993, 87 (01), 115-132.

Raiffa, Howard, The Art and Science of Negotiation, Harvard University Press, 1982.

Ray, DEBRAJ, "Costly Conflict under Complete Information," Manuscript, Dept. Econ., New York Univ, 2009.
Rubinstein, Ariel, "Perfect equilibrium in a bargaining model," Econometrica, 1982, pp. 97-109.

Sarver, Todd, "Anticipating regret: Why fewer options may be better," Econometrica, 2008, 76 (2), 263-305.
Schkade, David A and Daniel Kahneman, "Does Living in California Make People Happy? A Focusing Illusion in Judgments of Life Satisfaction," Psychological Science, 1998, 9 (5), 340-346.
Schumacher, Heiner and Heidi Thysen, "Equilibrium Contracts and Boundedly Rational Expectations," Working paper, 2017.

Segal, Ilya, "Complexity and Renegotiation: A Foundation for Incomplete Contracts," The Review of Economic Studies, 1999, 66 (1), 57-82.
Shalev, Jonathan, "Loss Aversion and Bargaining," Theory and Decision, 2002, 52, 201-232.
Slantchev, Branislav L, "Feigning weakness," International Organization, 2010, 64 (3), 357-388.
Strotz, R. H., "Myopia and Inconsistency in Dynamic Utility Maximization," The Review of Economic Studies, 1955, 23 (3), 165-180.
Thadden, Ernst-Ludwig Von and Xiaojian Zhao, "Incentives for unaware agents," The Review of Economic Studies, 2012, 79 (3), 1151-1174.
Tirole, Jean, "Cognition and Incomplete Contracts," The American Economic Review, 2009, 99 (1), 265-294.
Yildiz, Muhamet, "Waiting to persuade," The Quarterly Journal of Economics, 2004, 119 (1), 223-248.


[^0]:    *We are grateful to Chris Avery, Stefan Bechtold, Pedro Bordalo, Micael Castanheira, Markus Fels, Nicola Gennaioli, Dominik Grafenhofer, Bård Harstad, Oliver Hart, Paul Heidhues, Martin Hellwig, Fabian Herweg, Georg Kirchsteiger, Martin Kocher, Botond Köszegi, Patrick Legros, Guido Maretto, Marc Möller, Takeshi Murooka, Nick Netzer, Georg Nöldeke, Antonio Rosato, Armin Schmutzler, Martin Schonger, Heiner Schumacher, Adam Szeidl as well as seminar audiences at Central European University, Corvinus University, ETH Zurich, University of Zurich, LMU Munich, ECARES (ULB), Stony Brook International Conference on Game Theory, University of Oslo, Nordic Conference on Behavioral and Experimental Economics at Aarhus, Copenhagen Workshop on Limited Attention, UECE Lisbon Meetings on Game Theory, University of Bern, MPI for Research on Public Goods at Bonn, MaCCI Workshop on Behavioral IO, NHH Bergen, and ISNIE Conference at Harvard for their valuable comments and suggestions. This paper supersedes an earlier version titles "The Structure of Negotiations: Incomplete Agreements and the Focusing Effect."
    ${ }^{\dagger}$ IMT School for Advanced Studies, Lucca (Italy) and INSEAD, Fontainebleau (France); email: andrea.canidio@imtlucca.it.
    ${ }^{\ddagger}$ Department of Economics, Frankfurt School of Finance \& Management, 60322 Frankfurt am Main, Germany; email: h.karle@fs.de.

[^1]:    ${ }^{1}$ See, for example, Strotz (1955), Gul and Pesendorfer (2001), Dekel, Lipman and Rustichini (2001), Sarver (2008), Noor (2011), Kőszegi and Szeidl (2013), Bordalo, Gennaioli and Shleifer (2013), Bushong, Rabin and Schwartzstein (2017).
    ${ }^{2}$ A large literature has studied so called "rationalist explanations for war", that is, reasons why rational players may fail to find an efficient agreement. Those reasons are information asymmetries (see Slantchev, 2010), large indivisibilities (see Fearon, 1995), lack of commitment (see Fearon, 1996, Powell, 1993), and multilateral bargaining failures (see Ray, 2009). Here we abstract away from those mechanisms. By studying the implications of a well known behavioral bias for negotiations, we aim at providing a "non-rationalist" explanation to why players may fail to find an efficient agreement.

[^2]:    ${ }^{3}$ There is also a recent literature studying contract and menu design when consumers' preferences are distorted by the focusing effect; see, for example, Apffelstaedt and Mechtenberg (2017) and Dertwinkel-Kalt, Köster and Peiseler (2019).

[^3]:    ${ }^{4}$ Furthermore, as we argue in more detail in Section 2, also in our case the Nash bargaining solution can be implemented non-cooperatively via a game of alternating offers. As we discuss below, the focus of this paper is the sequence of agreements. For this reason, we use a cooperative bargaining solution allowing us to abstract away from the sequence of offers and counteroffers leading up to each agreement.

[^4]:    ${ }^{5}$ Interestingly, this result can be empirically tested even without knowing which player has the most distorted preferences. See the Conclusion.

[^5]:    ${ }^{6}$ See also Fisher, Ury and Patton (1991) "Getting to yes", p. 172.

[^6]:    ${ }^{7}$ The three threshold agreements were on the jurisdiction of the Panamanian canal, the Panamanian participation in its defense, and on the operation of the canal. The American delegations initiated the discussion of three threshold agreements to "avoid the break-off of the negotiations and to demonstrate the good will that would be necessary for later Panamanian concessions." See Raiffa (1982) p. 178.
    ${ }^{8}$ This principle is sometimes explicitly stated as "Nothing is agreed until everything is agreed", in the sense that any agreement that involves some specific issues cannot be considered final until all other issues are settled. Such an agreement should, therefore, be interpreted as a constraint on future bargaining rounds. See, for example, the rules governing the Doha round of trade negotiations. http://www.wto.org/english/tratop_e/dda_e/work_organi_e.htm (accessed on the 5th of October 2018).

[^7]:    ${ }^{9}$ An exception are models based on unawareness, such as, for example, Von Thadden and Zhao (2012), Auster (2013) and Schumacher and Thysen (2017), in which an informed principal offers a contract to an unaware agent. This contract determines what the agent will be aware of when choosing an action, which implies that the principal may want to offer an incomplete contract so to leave the agent unaware of certain elements. This framework is, however, not directly applicable to the study of negotiations in which both players could be identical.
    ${ }^{10}$ Some authors argue that, if information is perfect but players are prevented from writing a complete contract, then the players may decide to leave some potentially contractible aspects of an agreements unspecified (see Bernheim and Whinston, 1998, Battaglini and Harstad, 2016 and Harstad, 2007). In our paper, instead, the bargaining parties can reach an agreement without having first signed an incomplete agreement.

[^8]:    ${ }^{11}$ See Lang and Rosenthal (2001), Bac and Raff (1996), Inderst (2000), Busch and Horstmann (1999b), Busch and Horstmann (1999a), Flamini (2007), Chen and Eraslan (2017).
    ${ }^{12}$ Some recent papers study the implications of salience theory for contract theory; see, for example, Carroni, Mantovani and Minniti (2019) and Inderst and Obradovits (2019). See also Dertwinkel-Kalt, Gerhardt, Riener, Schwerter and Strang (2017) for an experimental comparison of these different models of focusing/salience.

[^9]:    ${ }^{13}$ Equivalently, the players earn their payoffs sufficiently far in the future relative to the moment in which the agreement is signed, so that whether a given agreement is achieved in one- or two-steps is irrelevant.

[^10]:    ${ }^{14}$ In an earlier version of the paper, instead of defining the consideration set as the set of bargaining outcomes that satisfies the participation constraint of either one or the other player, we assumed that a bargaining outcome is in the consideration set if it satisfies the participation constraint for both players. The results under this alternative definition are identical to those derived here, but there are a number of additional complications related to the existence and uniqueness of the consideration set. Since, a priori, neither the existing literature nor the specific problem at hands calls for one definition or the other, we choose to work with the one that allows for more straightforward derivations.
    ${ }^{15}$ We discuss in an extension (Section 6.2) the case of in which an incomplete agreement is binding only in the current round of negotiation but can be ignored in future rounds if no agreement is reached in the current round.

[^11]:    ${ }^{16}$ Although it can be done, for example, via a game of alternating offers with an exogenous probability of breakdown of the negotiation as in Binmore, Rubinstein and Wolinsky (1986).

[^12]:    ${ }^{17}$ Note that, in writing the Nash bargaining problem, we ignore the fact that $\underline{t} \leq t \leq \bar{t}$. The reason is that this constraint is never binding. If it was, one of the two players would earn zero surplus, which clearly does not maximize the objective function.

[^13]:    ${ }^{18}$ The conditions of the proposition are necessary and sufficient if we restrict to situations in which one player is more focused than the other (or they are equally focused), as discussed on page 14. Without such an assumption, however, they are only sufficient because some focus functions do not allow to rank players in terms of who is more focused.

[^14]:    ${ }^{19}$ In Section 6.1 we discuss what happens when other negotiating structures are considered.

[^15]:    ${ }^{20}$ This is equivalent to assuming that players are consequential. This assumption is already made in Koszegi and Szeidl (2013, p. 73) and it basically says that an agent's consideration set is determined by the set of future outcomes achievable as consequence of today's choices.
    ${ }^{21}$ The existence of $\underline{t}^{*}$ and $\bar{t}^{*}$ follows from the fact that, as already discussed in Section 4.1, when $\bar{T}$ is above a certain threshold, then the period- 2 focus weights (and the period- 2 solution as in Equation 4.10) do not depend on $\bar{T}$. The same happens when $\underline{T}$ is below a certain threshold. Without loss of generality, therefore, we can restrict $\bar{T}$ to belong to a closed interval (and similarly for $\underline{T}$ ). This, together with the fact that the objective function is continuous, guarantees that the maximum and minimum of (4.10) exist.

[^16]:    ${ }^{22}$ This is an important element of many examples of diminishing sensitivity. A famous one is the observation that people are willing to travel 10 minutes to another shop to save 30 dollars on a product whenever the initial cost of the product is low (say 100 dollars) but not when it is high (say 1000 dollars). Also here, it is not possible to get the discount independently from purchasing the product.

[^17]:    ${ }^{23}$ It is easy to check that $\bar{t}-\underline{t}=\beta_{i}$ whenever there is only issue $i$ on the table, but $\bar{t}-\underline{t}>\beta_{i}$ as long as there is at least one additional issue on the table.

